Analytic Progress in Open String Field Theory

– Strings, 2007 –

B. Zwiebach, MIT

In the years 1999-2003 evidence accumulated that classical open string field theory (OSFT) gives at least a partial description of the open string landscape:

Numerical solutions of the equations of motion

$$\boxed{Q\Psi + \Psi * \Psi = 0\,.}$$

show that OSFT encodes many backgrounds:

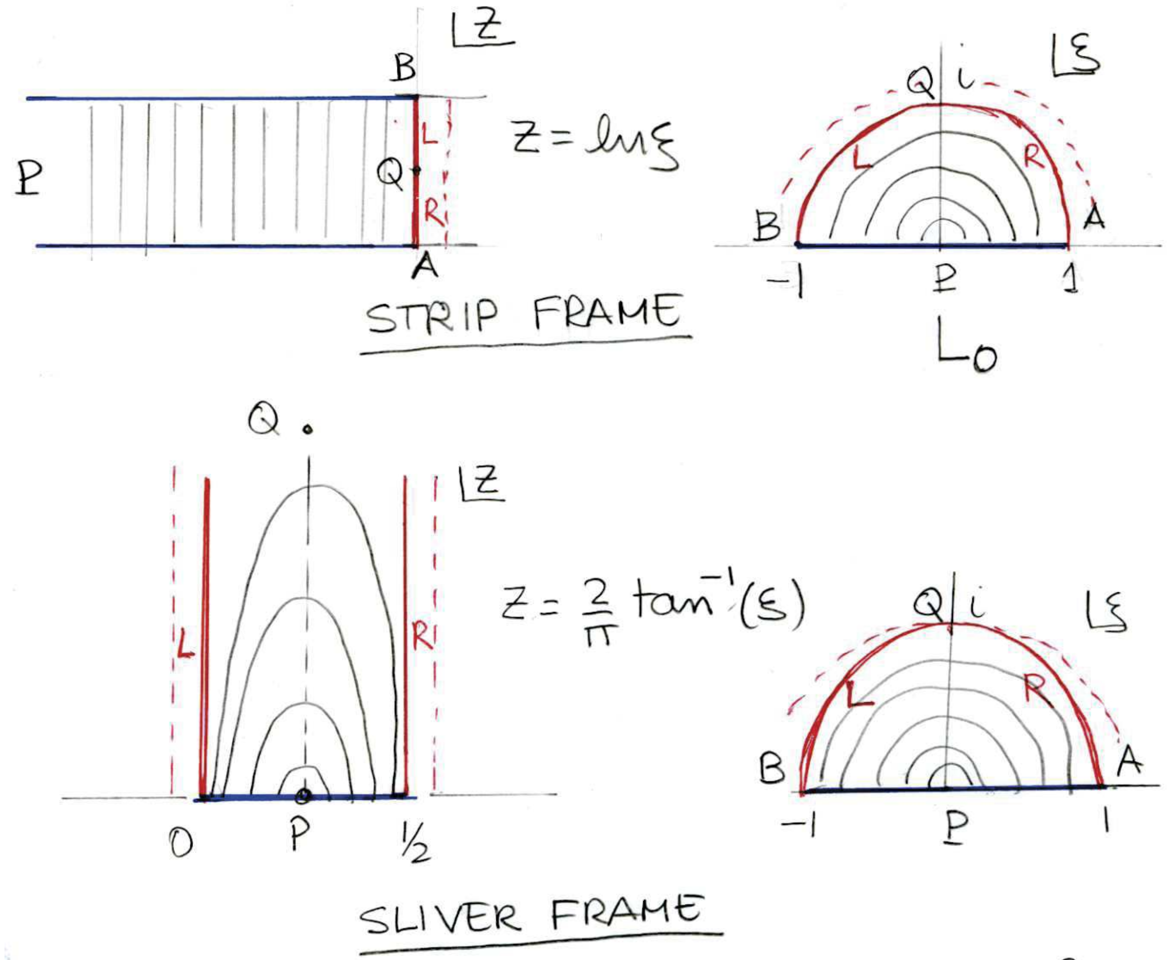
– The tachyon vacuum background

– Lower-codimension branes as solutions.

– Backgrounds related by marginal deformations.

To make progress towards a complete formulation of (open) string theory, analytic solutions were desired but not available.

In November 2005, Martin Schnabl (Cern) found an analytic tachyon vacuum solution [hep-th/0511286].



$$\mathcal{L}_0 \equiv L \longrightarrow L = L_0 + \frac{2}{3} L_2 - \frac{2}{15} L_4 + \cdots$$

L_n Virasoro operators refer to the ξ-frame.

L_0 has a simple interpretation in the strip frame.

$\mathcal{L}_0 \equiv L$ in the sliver frame is simple. It corresponds to a linear combination of ξ-frame Virasoros.

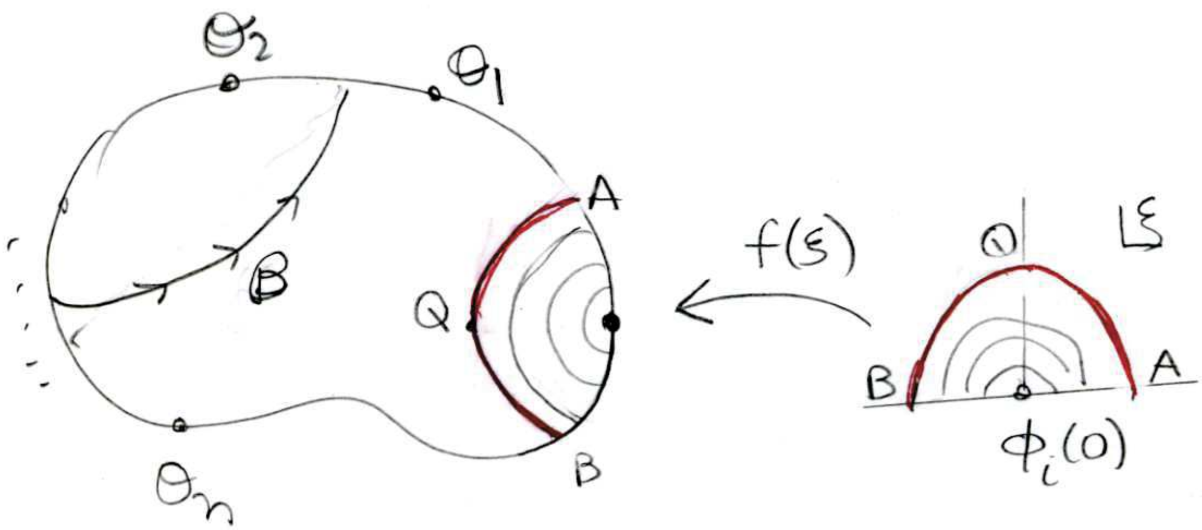
$\boxed{B\Psi = 0}$ Schnabl gauge

B is the antighost zero mode in the sliver frame:

$$B \equiv \oint \frac{dz}{2\pi i} z b(z) = \oint \frac{d\xi}{2\pi i} \frac{f(\xi)}{f'(\xi)} b(\xi)\,,$$

$$\rightarrow \quad B = b_0 + \frac{2}{3} b_2 - \frac{2}{15} b_4 + \frac{2}{35} b_6 - \ldots$$

The relevant string fields are dressed surface states $|\Sigma\rangle$

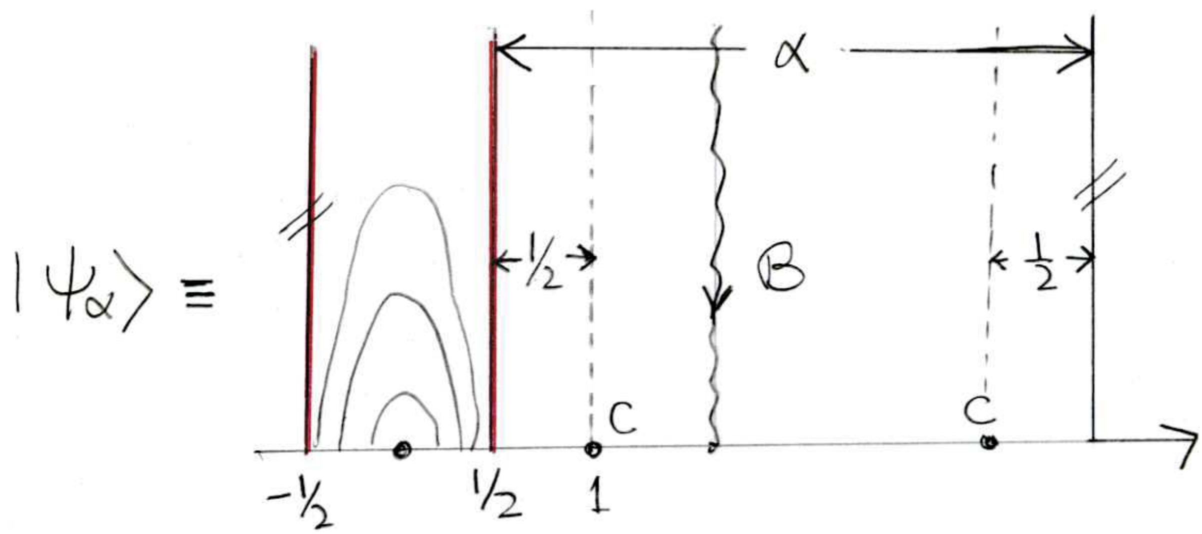


Using test (Fock) states ϕ_i, we define the surface states by the correlator

$$\langle \Sigma | \phi_i \rangle = \langle f \circ \phi_i(0) \; \mathcal{O}_1 \mathcal{O}_2 \ldots \mathcal{B} \ldots \mathcal{O}_n \rangle$$

The test states are inserted using $f(\xi)$ to map them from $\xi = 0$ to the surface.

The tachyon vacuum solution is written in terms of states ψ_α and their derivatives $\psi'_\alpha = \frac{d\psi_\alpha}{d\alpha}$:



$$\mathcal{B} = \int \frac{dz}{2\pi i} b(z)$$

$$\Psi = \lim_{N\to\infty}\Big[\underbrace{-\psi_N}_{\textit{phantom}} + \underbrace{\sum_{n=0}^{N} \psi'_n}_{\textit{ordinary}} \Big]$$

The "*ordinary*" piece suffices for a weak solution:

$$\langle f_i\,, Q\Psi + \Psi * \Psi\rangle = 0, \quad \text{for all Fock states } f_i$$

The "*phantom*" piece is needed for a strong solution (Okawa, Fuchs and Kroyter):

$$\langle \Psi\,, Q\Psi + \Psi * \Psi\rangle = 0\,.$$

In Strings 06 Schnabl presented this solution, and evidence that there are no physical open string states at the tachyon vacuum.

Progress this year:

1. Different projectors and different gauges.

   Okawa, Rastelli, and Zwiebach (hep-th/0611110)

2. New solution for (regular) marginal deformations.

   Schnabl (hep-th/0701248) and Kiermaier, Okawa, Rastelli, and Zwiebach (hep-th/0701249)

3. New solution for Superstring (regular) marginal deformations.

   Erler (arXiv:0704.0930) , Okawa (arXiv:0704.0936, arXiv:0704.3612)

4. Towards new solutions with general marginal deformations

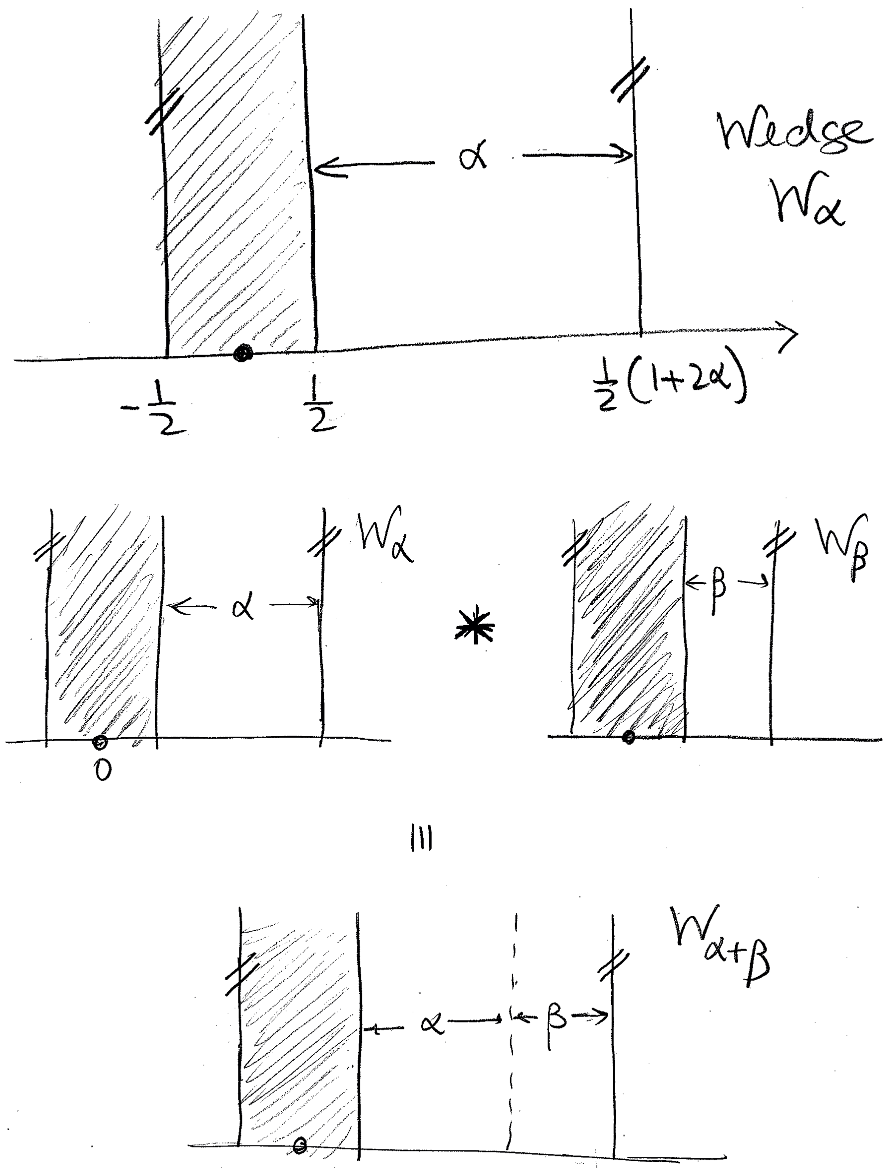
   Fuchs, Kroyter, and Potting (arXiv:0704.2222) , Fuchs and Kroyter (arXiv:0706.0717), Kiermaier and Okawa (to appear).

5. Off-shell amplitudes and string diagrams.

   Fuji, Nakayama, Suzuki (hep-th/0609047) Rastelli and B. Z., to appear.

Key structures.

The structure needed to build the solution is an abelian family of wedge states $\mathcal{W}_\alpha$ associated the sliver frame:



Just like for B:

$$L \equiv \oint \frac{d\xi}{2\pi i}\,\frac{f(\xi)}{f'(\xi)}\,T(\xi) = \oint \frac{d\xi}{2\pi i}\,(1+\xi^2)\,\tan^{-1}\xi\,T(\xi)$$

$$L = \; L_0 + \frac{2}{3}L_2 \; - \frac{2}{15}L_4 \; + \frac{2}{35}L_6 - \ldots$$

$$L^\star = L_0 + \frac{2}{3}L_{-2} - \frac{2}{15}L_{-4} + \frac{2}{35}L_{-6} - \ldots$$

$$\boxed{[L\,, L^\star\,] = L + L^\star \equiv L^+}$$

$\star$ is BPZ conjugation.

The operator L^+ has a decomposition into commuting left and right parts $L^+ = L^+_L + L^+_R$.

$$|W_\alpha\rangle = e^{-\alpha L_L^+}\,|\mathcal{I}\rangle\,.$$

One can show that

$$-L_L^+ = \int \frac{dz}{2\pi i} T(z) \equiv \mathcal{L}$$

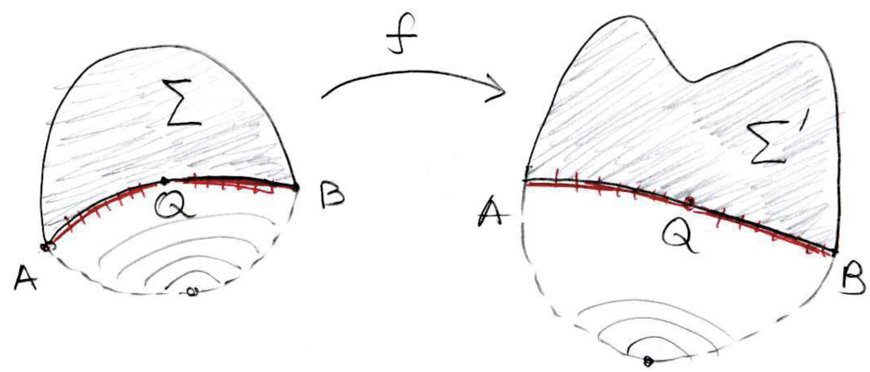
$$-B_L^+ = \int \frac{dz}{2\pi i} b(z) \equiv \mathcal{B}$$

$\boxed{\mathcal{L} \text{ changes width, } \mathcal{B} \text{ is the associated antighost .}}$

Different projectors and different gauges.

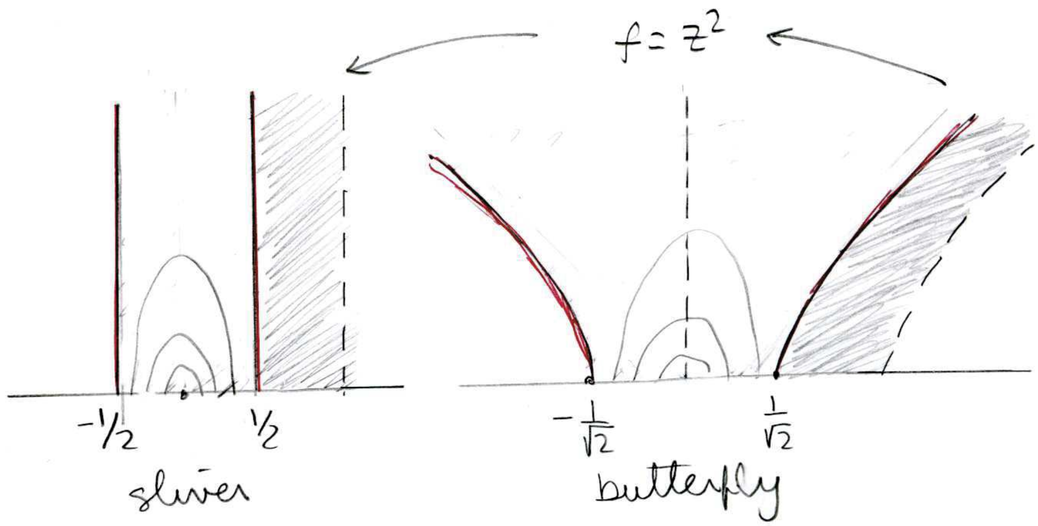
Okawa, Rastelli, and Zwiebach (hep-th/0611110)

All surface states are related by midpoint preserving reparameterizations (symmetries of the star product)

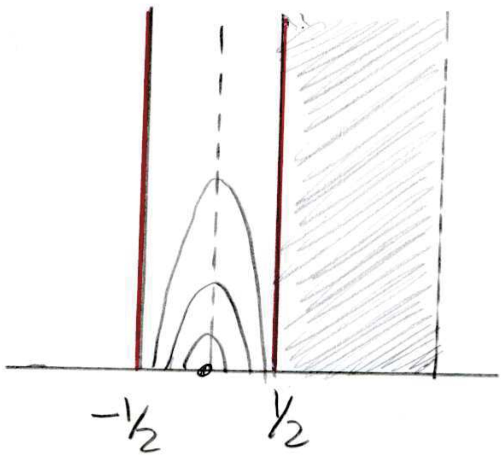


The conformal map $f : \Sigma \to \Sigma'$, uniquely determined by preserving A, Q, and B induces the reparameterization of the open string.

This applies to projectors: the sliver and the butterfly are related by reparameterizations ($f = z^2$). Obtain wedge-like states for the butterfly:



Self-reparameterizations of the sliver actually act on the wedge states.



$$z' - \frac{1}{2} = e^{-2\beta}\left(z - \frac{1}{2}\right)$$

Obtain a β-deformation of Schnabl's sliver solution.

For any projector, the gauge condition is

$$B\Psi = 0$$

where B is the zero mode of $b(z)$ in the conformal frame of the projector.

Upon β deformation the gauge condition is

$$\Big(B + \frac{1}{2}(e^{2\beta} - 1)(B + B^\star)\Big)\Psi = 0$$

Level expansion can be carried out explicitly for special projectors, those for which

$$\big[\mathcal{L}_0\,,\, \mathcal{L}_0^\star\big] = s(\mathcal{L}_0 + \mathcal{L}_0^\star)$$

with $s \geq 1$ a parameter (butterfly is $s = 2$).

Solution for (regular) marginal deformations

Schnabl (hep-th/0701248) and Kiermaier, Okawa, Rastelli, and Zwiebach (hep-th/0701249)

For any dimension one matter primary V

$$\Psi^{(1)} = cV(0)|0\rangle$$

is BRST closed:

$$Q\Psi^{(1)} = 0\,.$$

If V is exactly marginal, expect a solution

$$\Psi_\lambda = \sum_{n=1}^{\infty} \lambda^n \, \Psi^{(n)}\,, \qquad \lambda \in \mathbb{R}$$

The EOM $Q\Psi_\lambda + \Psi_\lambda * \Psi_\lambda = 0$ gives

$$Q\Psi^{(n)} = \Phi^{(n)}\,, \qquad \Phi^{(n)} = -\sum_{k=1}^{n-1} \Psi^{(n-k)} * \Psi^{(k)}\,.$$

Needed: $\Phi^{(n)}$ must be BRST *exact* for all $n > 1$.

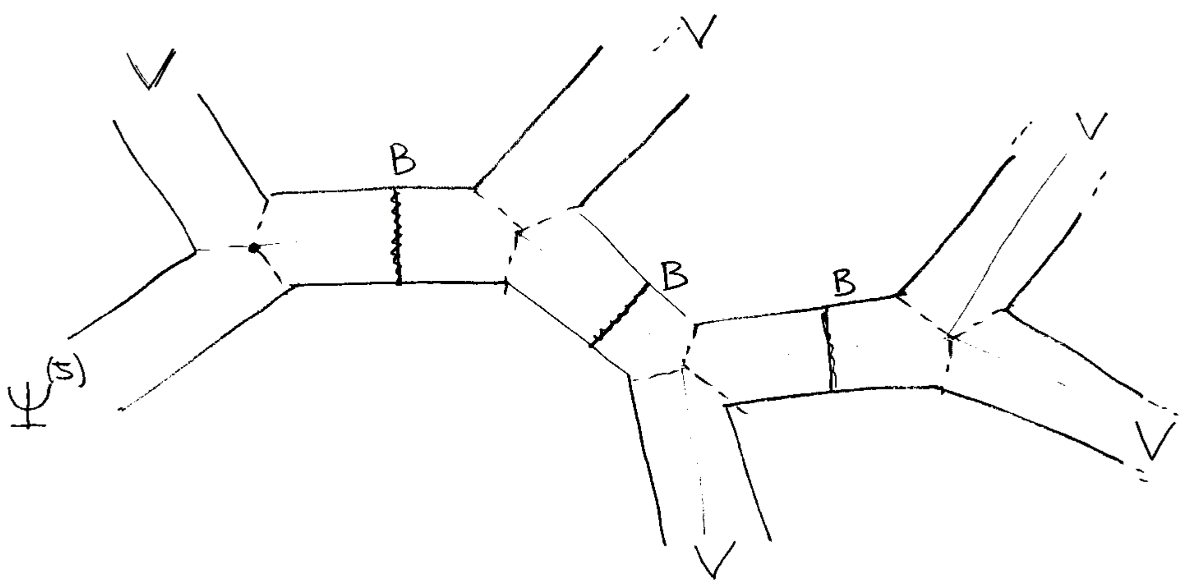
It is guaranteed to be closed, but it should be exact if V is exactly marginal.

In Siegel gauge solve

$$Q\Psi^{(n)} = \Phi^{(n)} \quad \to \quad \Psi^{(n)} = \frac{b_0}{L_0}\Phi^{(n)}.$$

$$\Psi^{(n)} = b_0 \int_0^\infty dt e^{-tL_0}\Phi^{(n)}.$$

In practice get a nested set of star products and operators b_0/L_0, whose Schwinger representation generates extremely complicated Riemann surfaces



Cannot do the conformal map to the UHP.

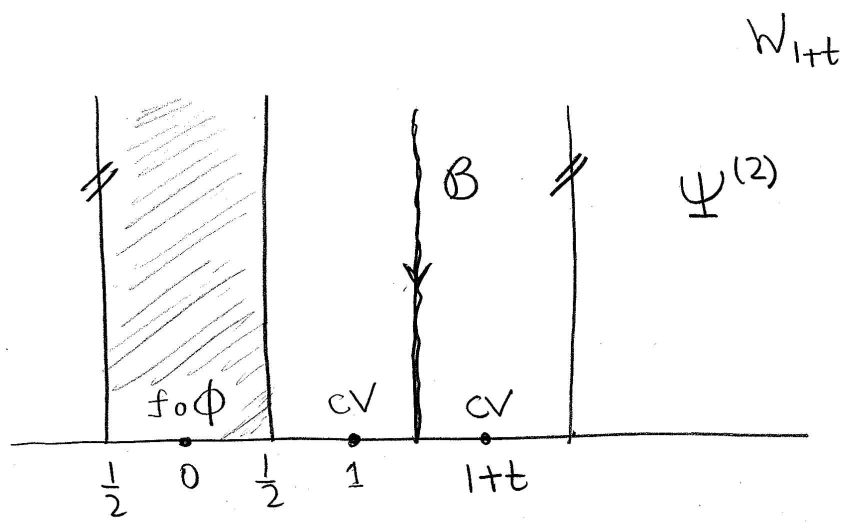
Cannot evaluate integrand in closed form.

Solve with wedges in $B\Psi = 0$ gauge

$$Q\Psi^{(1)} = 0\,, \qquad B\Psi^{(1)} = 0\,, \qquad L\Psi^{(1)} = 0\,.$$

$$Q\Psi^{(2)} = -\Psi^{(1)} * \Psi^{(1)} \quad \rightarrow \quad \Psi^{(2)} = -\frac{B}{L}(\Psi^{(1)} * \Psi^{(1)})$$

$$\boxed{\langle\, \phi, \Psi^{(2)}\,\rangle = \int_0^1 dt\, \langle\, f\circ\phi(0)\, cV(1)\, \mathcal{B}\, cV(1+t)\,\rangle_{\mathcal{W}_{1+t}}\,.}$$



As $t \to 0$ collision. At $t = 1$ maximum separation.

$$\langle\, \phi, Q_B\Psi^{(2)}\,\rangle = -\int_0^1 dt\, \langle\, f\circ\phi(0)\, cV(1)\, \mathcal{L}\, cV(1+t)\,\rangle_{\mathcal{W}_{1+t}}\,,$$

$$= -\int_0^1 dt\, \frac{\partial}{\partial t}\langle\, f\circ\phi(0)\, cV(1)\, cV(1+t)\,\rangle = -\Psi^{(1)} * \Psi^{(1)}\,,$$

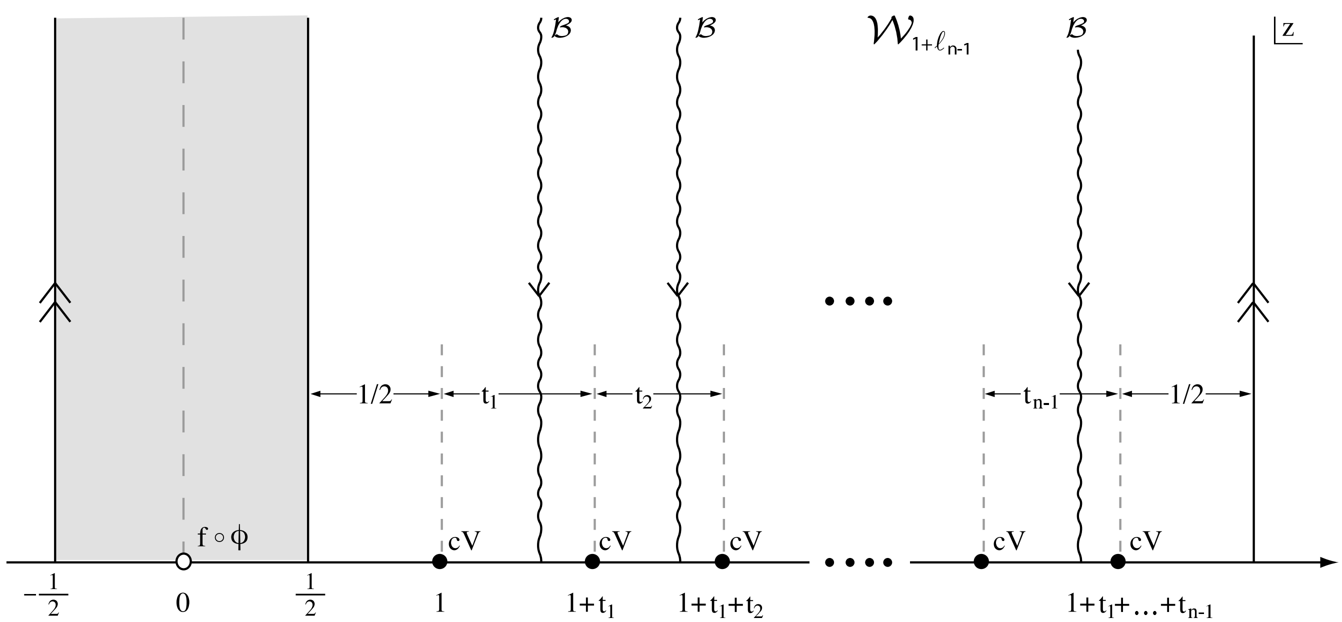
IFF

$$\lim_{t\to 0}\, cV(0)\, cV(t) = 0.$$

To n-th order one gets a REMARKABLY simple formula

$$\langle\, \phi, \Psi^{(n)}\,\rangle = \int_0^1 dt_1 \int_0^1 dt_2 \ldots \int_0^1 dt_{n-1}$$
$$\Big\langle\, f\circ\phi(0)\, cV(1) \prod_{i=1}^{n-1} \Big[\, \mathcal{B}\, cV(1+\ell_i)\Big]\,\Big\rangle_{\mathcal{W}_{1+\ell_{n-1}}} .$$

$$\ell_i \equiv \sum_{k=1}^{i} t_k \,,$$



Rolling Tachyon Solution

Perturb the unstable D-brane BCFT with

$$V(y) = e^{\frac{1}{\sqrt{\alpha'}} X^0(y)}$$

V is exactly marginal and has regular OPE

$$V(y)V(0) \sim |y|^2 V(0)^2 \,,$$

Deformation by cV represents a tachyon rolling from the perturbative vacuum at $x^0 = -\infty$

$$\Psi^{(1)} = e^{\frac{1}{\sqrt{\alpha'}} X^0(0)} \, c_1 |0\rangle \,,$$

Shifting time, the profile is

$$T(x^0) = \mp \, e^{\frac{1}{\sqrt{\alpha'}} x^0} + \sum_{n=2}^{\infty} (\mp 1)^n \beta_n \, e^{\frac{1}{\sqrt{\alpha'}} n x^0} \,.$$

A calculation gives:

$$\boxed{\begin{aligned}\beta_{n+1} = 2 \int d^n t \, (2+\ell_n)^{-n(n+3)} \left(\frac{g_0'}{2+\ell_n} - g_0 \right) \\ \cdot \frac{g_0^2}{{g_0'}^2} \left[\prod_{i=0}^{n} \frac{g_i'}{g_i^{2(n+1)}} \right] \prod_{0 \le i < j \le n} \big(g_i - g_j \big)^2 .\end{aligned}}$$

$$g(z) = \frac{1}{2} \tan(\pi z) \,, \quad g_i \equiv g\left(\frac{1+\ell_i}{2+\ell_n} \right) , \;\; \ell_0 \equiv 0 \,.$$

All β_n coefficients are positive.

$$
\begin{aligned}
T(x^0) = \mp\, e^{\frac{1}{\sqrt{\alpha'}}x^0} + \ 0.15206\ e^{\frac{1}{\sqrt{\alpha'}}2x^0} \quad &\mp 2.148 \cdot 10^{-3}\, e^{\frac{1}{\sqrt{\alpha'}}3x^0} \\
+ 2.619 \cdot 10^{-6}\, e^{\frac{1}{\sqrt{\alpha'}}4x^0} &\mp 2.791 \cdot 10^{-10}\, e^{\frac{1}{\sqrt{\alpha'}}5x^0} \\
+ 2.801 \cdot 10^{-15}\, e^{\frac{1}{\sqrt{\alpha'}}6x^0} &\mp 2.729 \cdot 10^{-21}\, e^{\frac{1}{\sqrt{\alpha'}}7x^0} + \ldots
\end{aligned}
$$

Top sign: the tachyon rolls towards the tachyon vacuum, it overshoots it, and then begins to perform larger and larger oscillations.

There is good numerical evidence that the series is

ABSOLUTELY CONVERGENT

The oscillations are real !

Consistent with earlier work of Moeller and BZ (2003), as well as Coletti *et.al* (2005).

Possible interpretation discussed by Ellwood (arXiv:0705.0013) and Jokela, Jarvinen, Keski-Vakkuri, Majumder (arXiv:0705.1916)

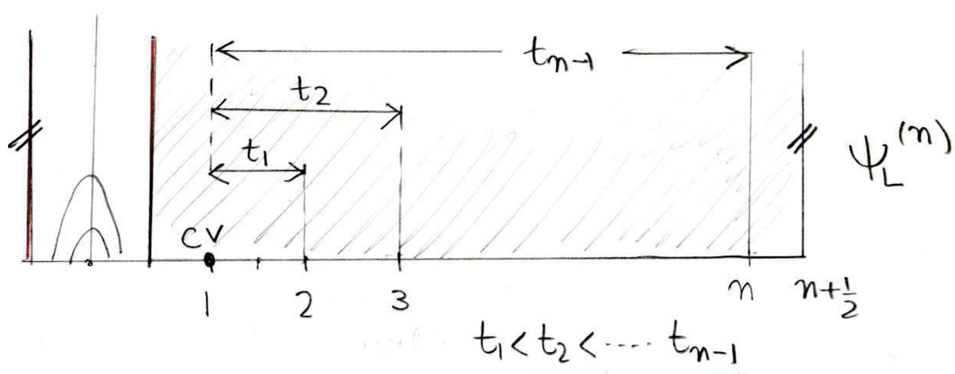
Marginals with singular OPE's

Use a point-splitting regulator and counterterms to construct solutions to quadratic and cubic order. Closed-form solutions were lacking.

Interesting solution by Fuchs, Kroyter and Potting for the case of $V = i\partial X$

Kiermaier and Okawa (to appear) have a general proposal:

1. Use fixed wedges and moving vertex operators to write solutions $\Psi_L^{(n)}$ and $\Psi_R^{(n)}$ that are complex conjugates of each other:



2. Construct a power-series for a (large) gauge parameter U that relates $\Psi_L^{(n)}$ and $\Psi_R^{(n)}$.

3. The real solution is obtained by transforming Ψ_L by a gauge transformation with $\sqrt{U}$.

4. The renormalization can be incorporated in U and Ψ_L if certain (expected) formal properties hold.

Off-shell amplitudes and string diagrams.

Fuji, Nakayama, Suzuki (hep-th/0609047) Rastelli, B.Z., to appear.

The propagator in $B\Psi = 0$ gauge is **not** B/L. It is (Schnabl)

$$\boxed{\mathcal{P} \equiv \frac{B}{L}\, Q\, \frac{B^\star}{L^\star}\,.}$$

With a C such that $C^2 = 0$ and $\{B, C\} = 1$, $P = BC$, is a projector to the gauge slice. The kinetic term K on the slice is

$$\frac{1}{2}\langle P\Psi, QP\Psi\rangle = \frac{1}{2}\langle \Psi, \underbrace{P^\star Q P}_{K}\,\Psi\rangle\,,\;\; K = C^\star B^\star Q B C\,.$$

The above $\mathcal{P}$ satisfies $\mathcal{P}K = P$.

Aim to find the general Feynman rules and their geometrical pictures.

Puzzling aspect of $\mathcal{P}$: Each line seems to carry two modular parameters. Physically we know only one is relevant.

Use

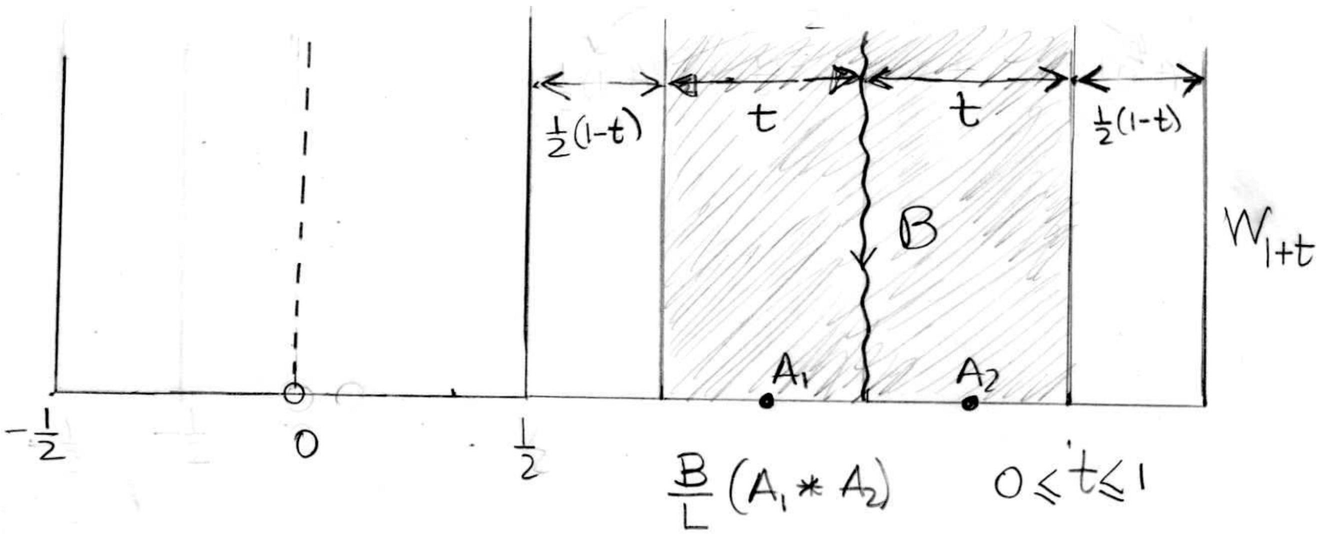
$$\mathcal{P} \equiv \frac{B}{L}\, Q\, \frac{B^\star}{L^\star} = \frac{B}{L} - \frac{B}{L}\frac{B^\star}{L^\star} Q$$

For a four string amplitude

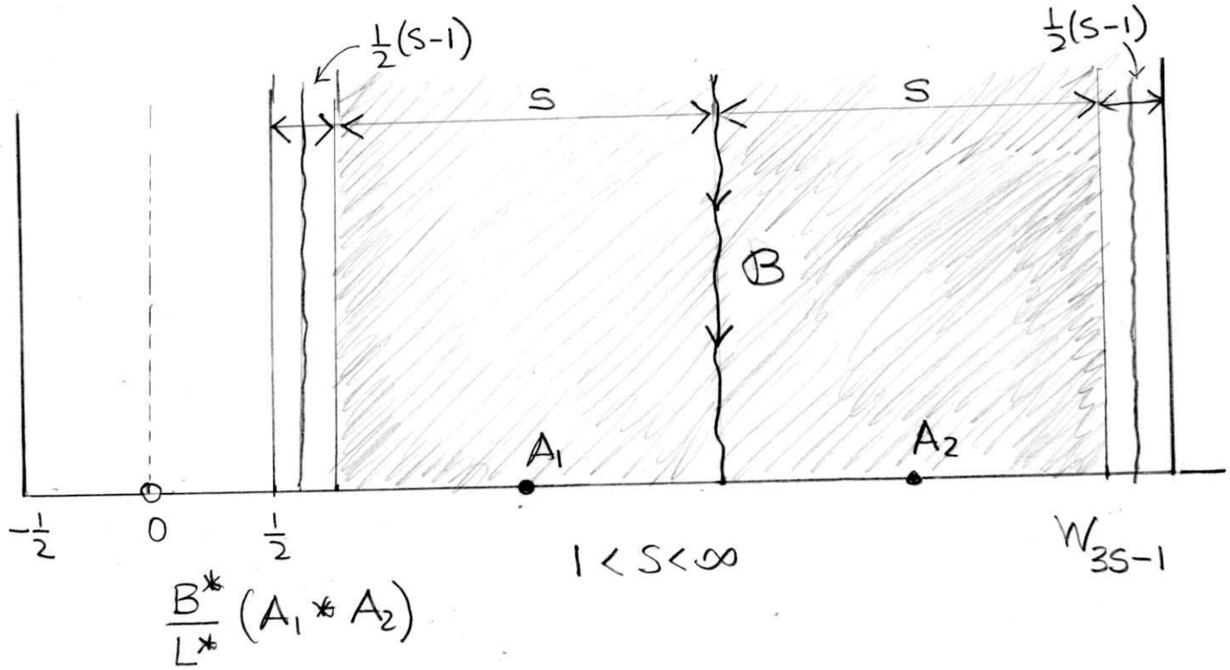
$$\begin{aligned}\mathcal{F}_4 &= \Big\langle \Psi_1 * \Psi_2, \mathcal{P}\Big(\Psi_3 * \Psi_4\Big)\Big\rangle \\ &= \Big\langle \Psi_1 * \Psi_2, \frac{B}{L}\Big(\Psi_3 * \Psi_4\Big)\Big\rangle \\ &\quad - \Big\langle \frac{B^\star}{L^\star}\Big(\Psi_1 * \Psi_2\Big), \frac{B^\star}{L^\star}\Big(Q\Psi_3 * \Psi_4 - \Psi_3 * Q\Psi_4\Big)\Big\rangle\end{aligned}$$

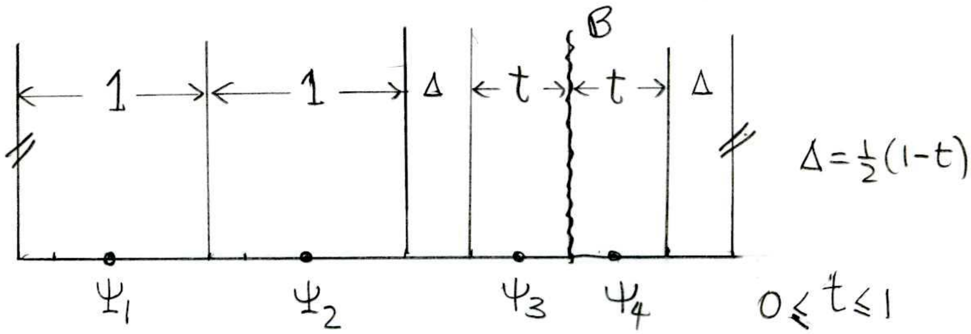
To evaluate these and other general amplitudes need to learn how to act with B/L and $B^\star/L^\star$ on star-products of general states:

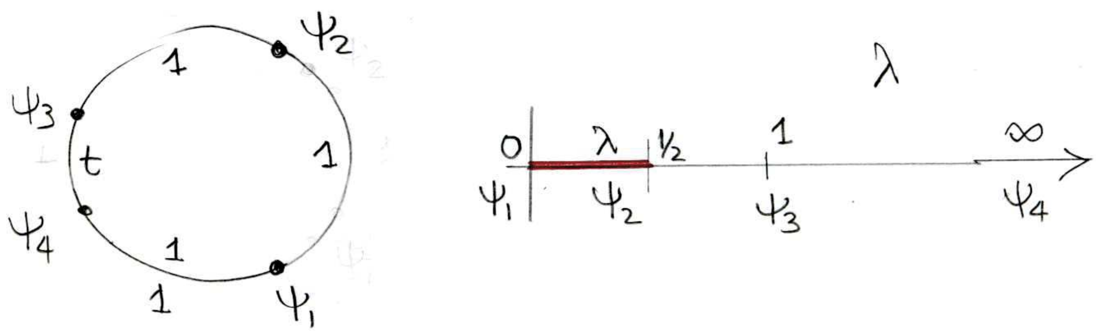
B/L action on $A_1 * A_2$



$B^\star/L^\star$ action on $A_1 * A_2$







$$\mathcal{F}_4^{(1)} = \ (2\pi)^D \delta\Big(\sum p_i\Big) \int_0^{1/2} d\lambda\, \lambda^{-\alpha' s - 2}(1-\lambda)^{-\alpha' t - 2}\, \mathcal{M}(\lambda, p_i)$$

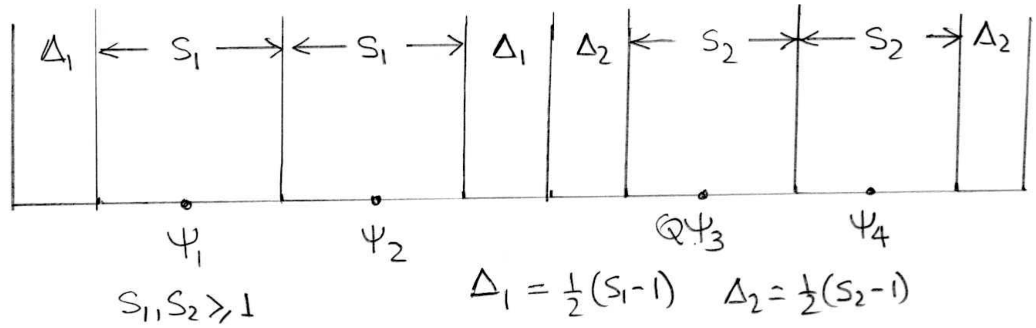
$$\mathcal{M}(\lambda, p_i) = \left[\frac{\pi(1-\lambda)^{-1}\left(\frac{3}{4}-\lambda\right)^{\frac{1}{2}}}{\sin^{-1}[(1-\lambda)^{-1}\left(\frac{3}{4}-\lambda\right)^{1/2}]}\right]^{4-\alpha'\sum p_i^2}$$

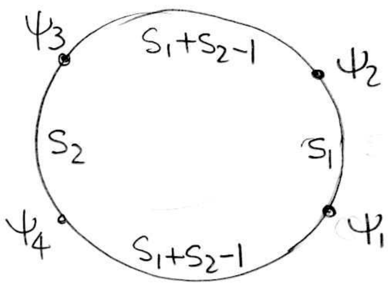
$$\cdot \left(\frac{\lambda}{t}\right)^{2-\alpha'(p_4^2+p_3^2)} \cdot (1-\lambda)^{2-\alpha'(p_1^2+p_2^2)}$$

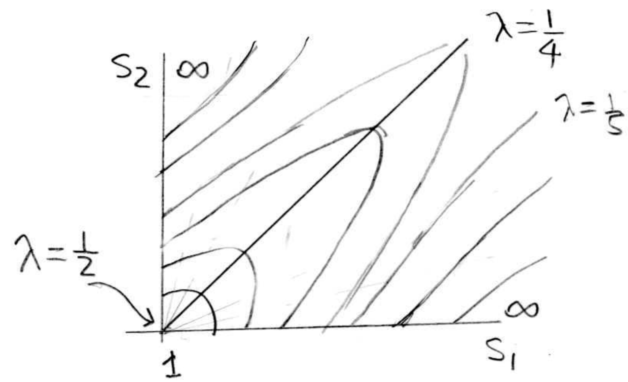
$$\text{Where: } \ 4\sin^2\left[\frac{\pi}{3+t}\right] = \frac{3-4\lambda}{1-\lambda}\,.$$

On-shell, $\mathcal{M} = 1$.

The full amplitude requires the addition of the t-channel. One cannot integrate over $\lambda \in [0,1]$ since $\mathcal{M}$ develops a singularity at $\lambda = 3/4$ (because even for $t \to \infty$ one only reaches there).





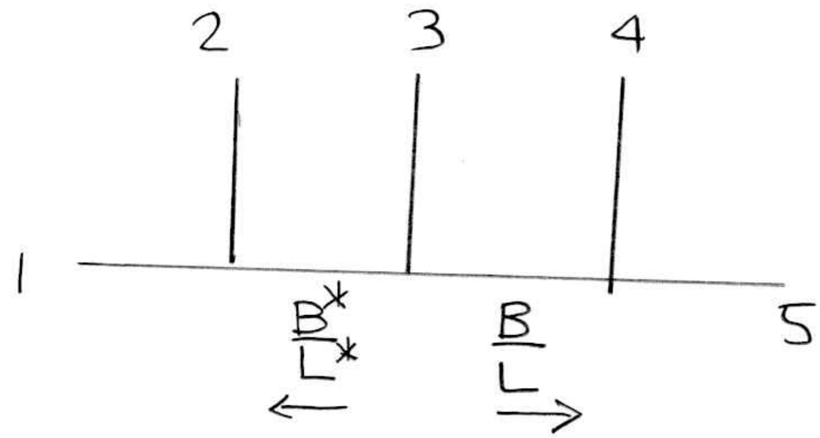


$$
\begin{aligned}
\mathcal{F}_4^{(2)} = \; & -(2\pi)^{26}\delta\Big(\sum p_i\Big)\frac{1}{\pi}\left(\frac{2}{\pi}\right)^{\alpha'\sum p_i^2-4}(\alpha'(p_3^2+p_4^2)-2) \\
& \cdot \int_{\widehat{\mathcal{M}}_2} dt_1 dt_2\, \gamma^2\,(\gamma t_2)^{\alpha'(p_1^2+p_2^2)-2}\,(\gamma t_1)^{\alpha'(p_3^2+p_4^2)-2} \\
& \qquad \cdot (\cos(\gamma t_2)+\cos(\gamma t_1))\left(\gamma\cos(\gamma t_1)-\frac{\sin(\gamma t_1)}{t_1}\right) \\
& \qquad \cdot \Big[\sin(\gamma t_1)\Big]^{-\alpha'(s+p_3^2+p_4^2)}\cdot\Big[\sin\gamma\Big]^{-\alpha'(2t+\sum p_i^2)} \\
& \qquad \cdot \Big[\sin(\gamma(t_1+1))\Big]^{\alpha'(2s+2t+\sum p_i^2)}\cdot\Big[\sin(\gamma t_2)\Big]^{-\alpha'(s+p_1^2+p_2^2)}\,.
\end{aligned}
$$

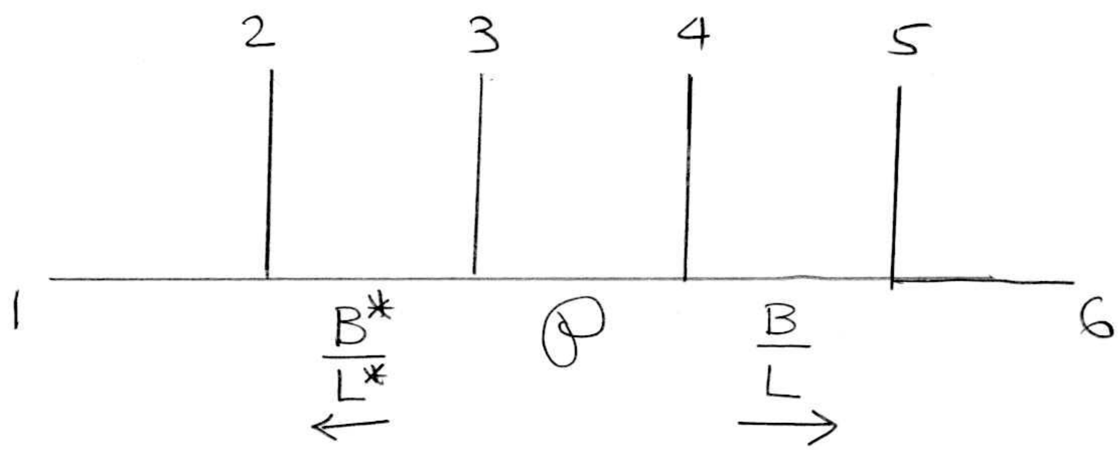
$$
\gamma = \frac{\pi}{2+t_1+t_2}, \quad \widehat{\mathcal{M}}_2 \equiv \{t_1, t_2\,|0 \le t_1 \le 1, 0 \le t_2 \le 1\,, t_1+t_2 \ge 1\}\,.
$$

Off-shell amplitudes are **not** simply integrals of suitable forms over **moduli space**.

On-shell 5 point functions do not require double moduli parameters on any lines



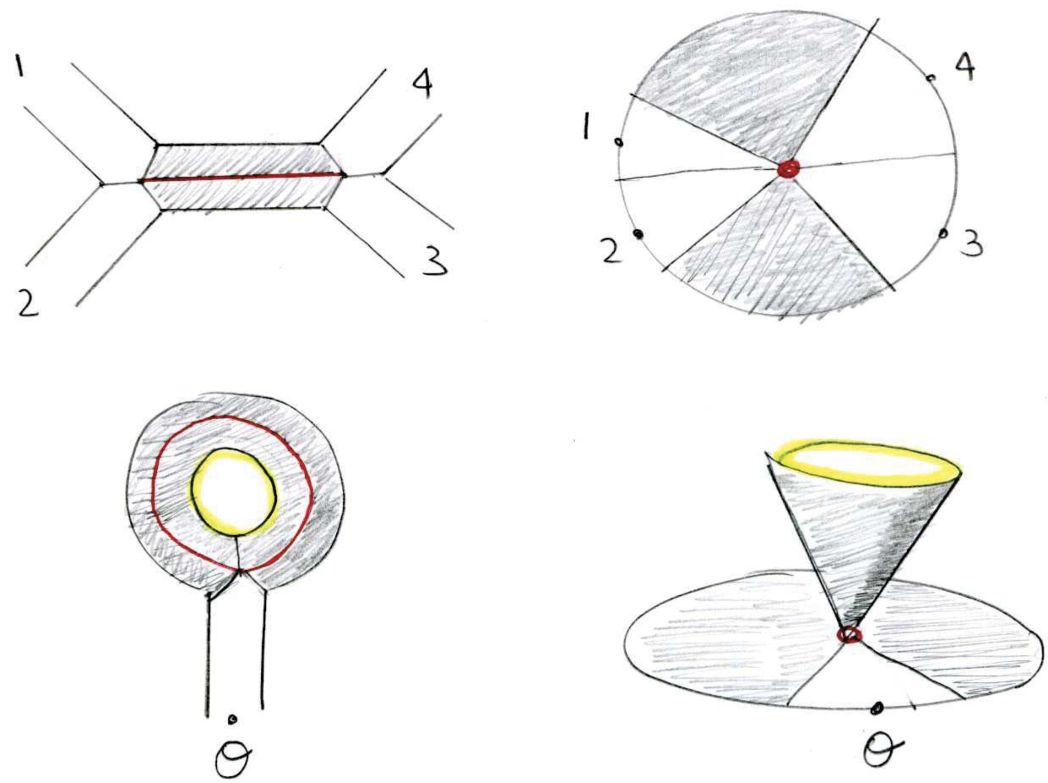
For on-shell six-point amplitudes at least one line must have the full $\mathcal{P}$



There may not be a theorem that each surface is produced only once – but the physical requirement of decoupling of trivial states will hold.

Loop amplitudes?

Not clear how the Feynman rules can create regular surfaces.



Difficulties with the fact that for an internal propagator there is no midpoint line

Conclusions

- Schnabl gauge is a projector gauge.
- Deformations by marginal operators with regular operator products are truly simple. Describe D-brane decay. Interpretation remains puzzling.
- Progress for regular superstring marginals, and for marginal operators with singular operator products.
- Some understanding the novel properties of off-shell amplitudes in this gauge.

Despite this, the lessons from the development have **not yet been fully absorbed**.

- Missing superstring tachyon vacuum.
- Missing lump solutions (D-branes as solitons)
- Some truly new solutions.
- Implications for closed strings