Superstrings + Topological Strings

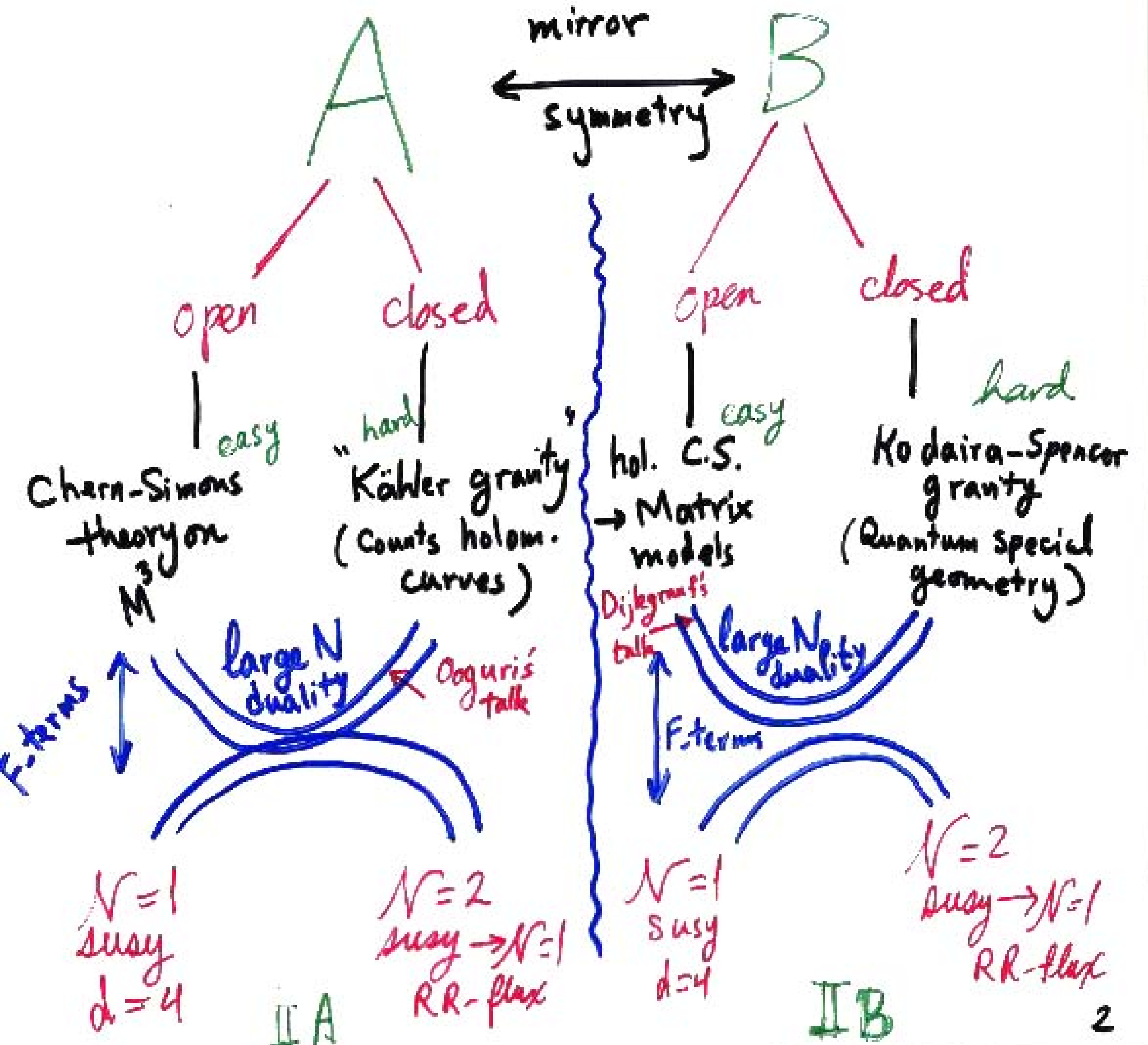
at Large N

based on { M. Aganagic + C.V. hep-th/0110171
M. Aganagic + M. Marino + C.V. hep-th/0206164

will hear later here { H. Ooguri + C.V. hep-th/0205297
R. Dijkgraaf + C.V. { hep-th/0206255
hep-th/0207106

closely related work { D.-E. Diaconescu, B. Florea + A. Grassi
{ hep-th/0205234
hep-th/0206163

Topological Strings on Calabi-Yau 3-folds



Direct Computation of genus g topological string amplitudes for closed strings has been very difficult →

mirror symmetry

for small g

localization techniques

⇓

F_g

<u>All genus ?</u>

Aim of this talk

A-model

Open $\longleftrightarrow$ closed
(large N)

use this to compute all genus F_g

Closed: string coupling, Kähler parameter, "counts" $(\Sigma^g \xrightarrow{\text{hol. maps}} CY)$

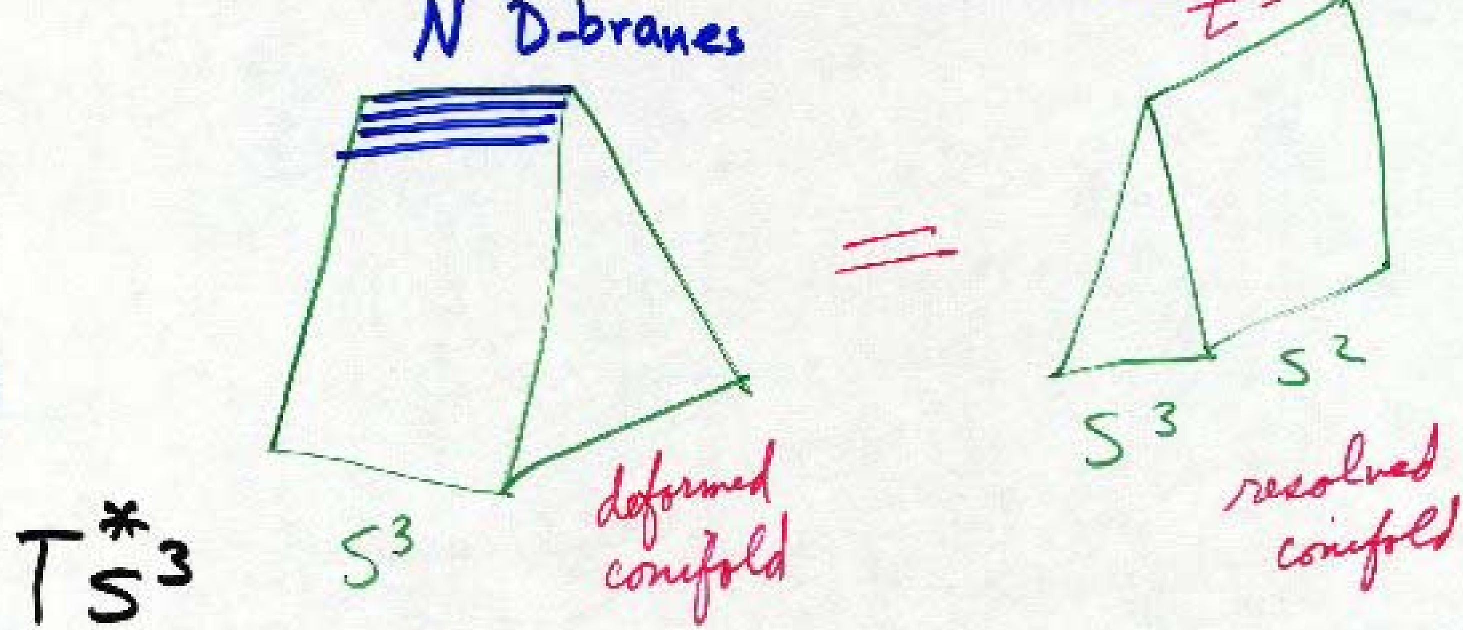
t: Kähler parameters

$$F(\lambda, t) = \sum \lambda^{2g-2} F_g(t)$$

$$= \sum_{m,d,s} n_{d,s} \frac{e^{-mdt}}{m\left[2\sin\frac{m\lambda}{2}\right]^{2-2s}}$$

counts BPS states of charge $d \in H_2(CY)$, "spin s"

Recall C.S. / closed Top. String duality



$$\downarrow$$

U(N) Chern–Simons on S^3 $=$ A-Top. strings on $\mathcal{O}(-1)\oplus\mathcal{O}(-1)$

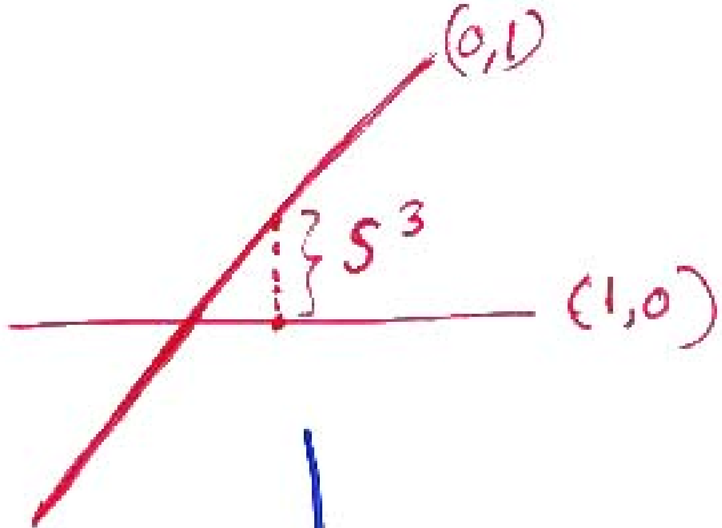
$$\downarrow$$

$$\mathbb{P}^1$$

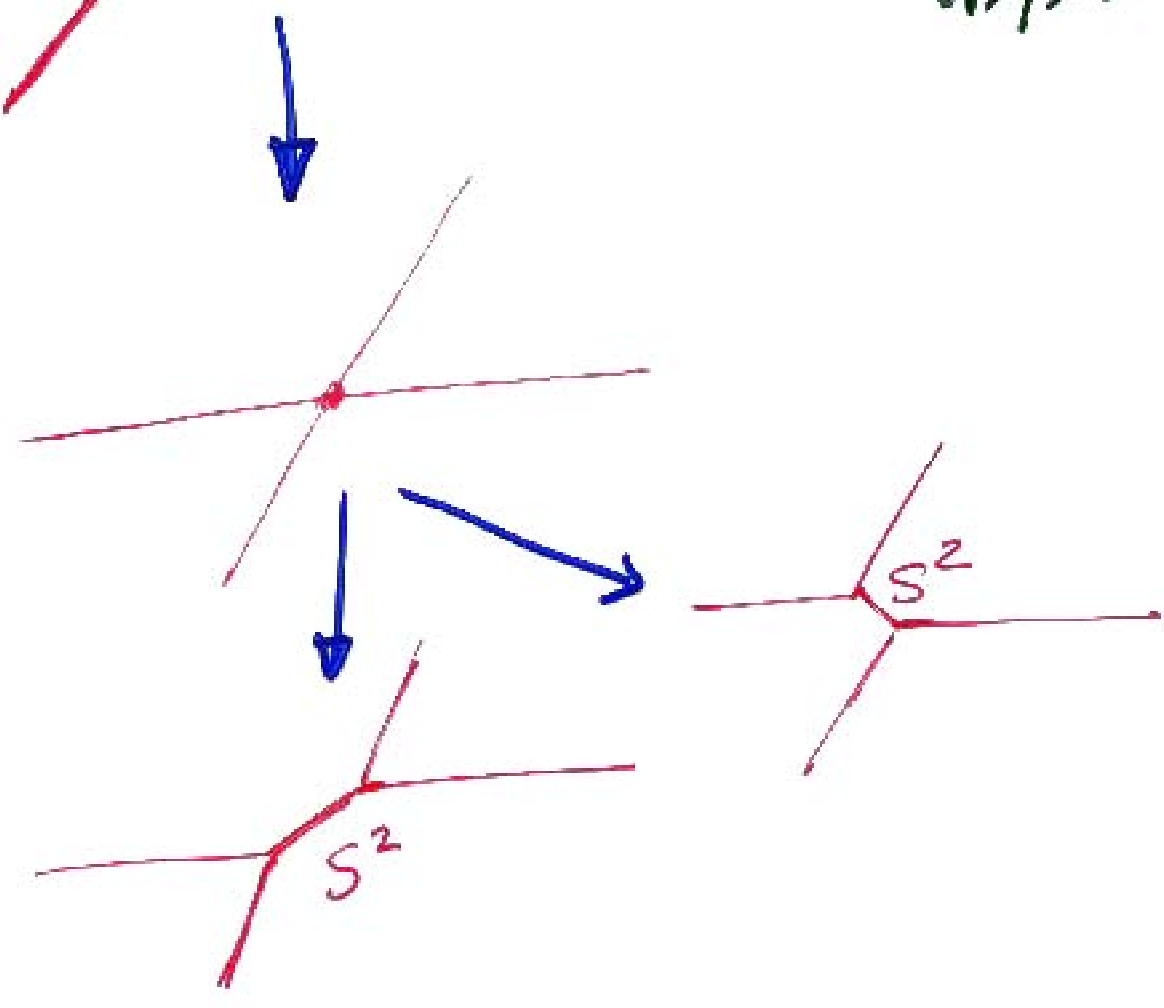
$$F(\lambda,t) = \sum_m \frac{e^{-mt}}{m\left[2\sin\frac{m\lambda}{2}\right]^2}$$

($n_{1,0}=1$, rest are $n_{d,s}=0$) simple.

Conifold transition has a IIB (p,q)-5-brane dual description $\longleftrightarrow$ geometrically $S^1 \times S^1$ action on CY



(p,q)-cycle $\in S^1 \times S^1$ shrinks at (p,q) 5-brane.



How to use this large N duality to compute F for more complicated CY

e.g. $F_{\mathbb{P}^2}(\lambda, t)$ or $F_{\mathbb{P}^1\times\mathbb{P}^1}(\lambda, t_1, t_2)$?

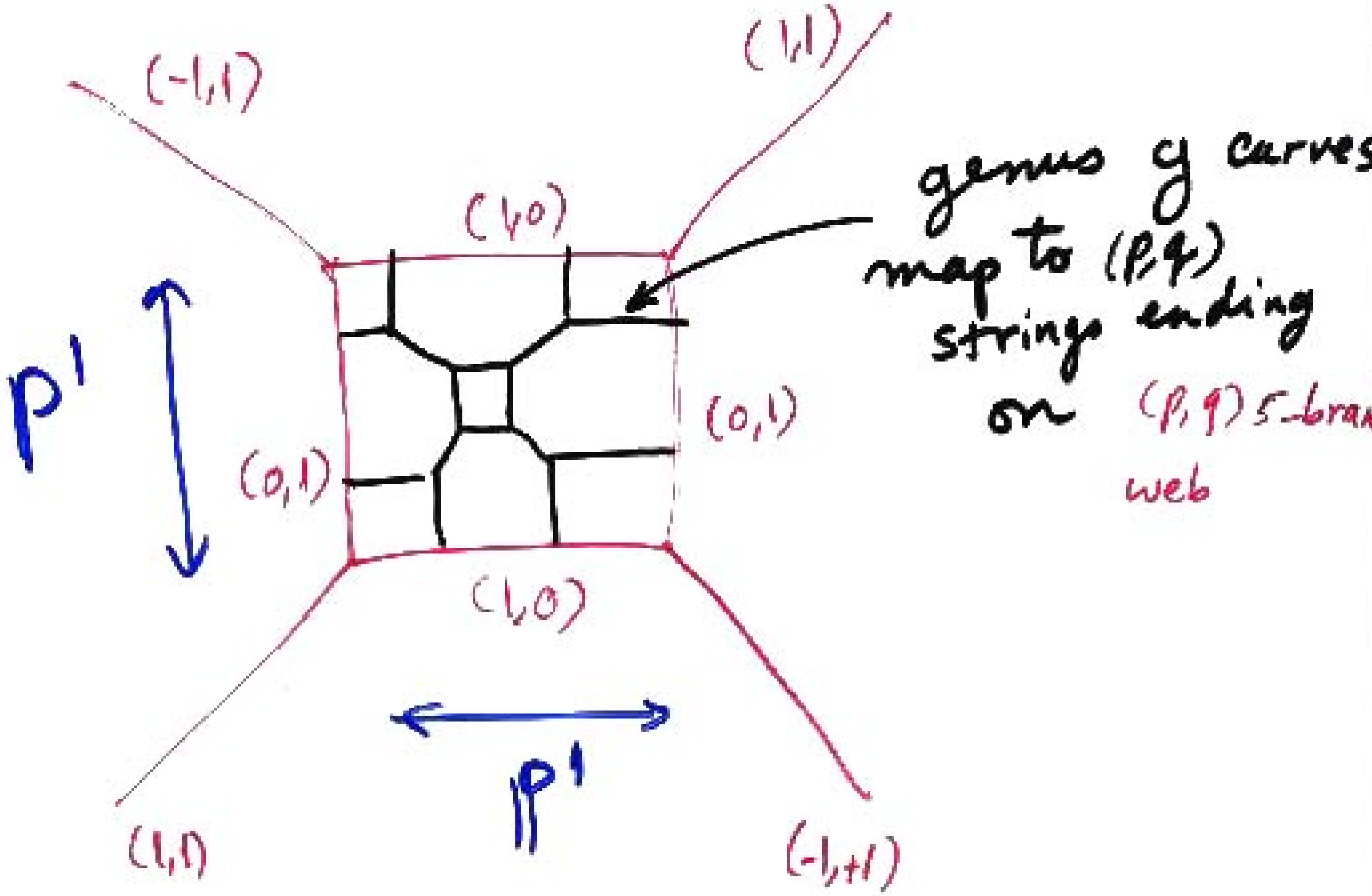
Expected to be very complicated.

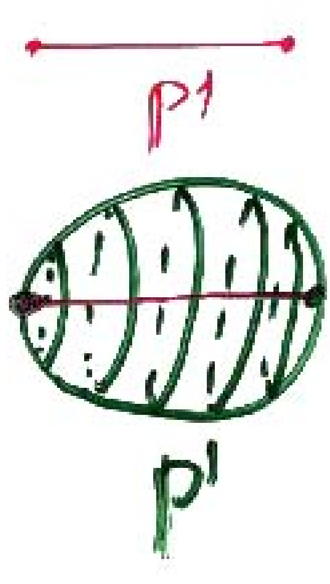
In particular $n_{d,g} \neq 0$

for infinitely many d, g.

[unlike $\mathbb{P}^1$ case where only $n_{1,0}=1$, $n_{d,g}=0$ otherwise]

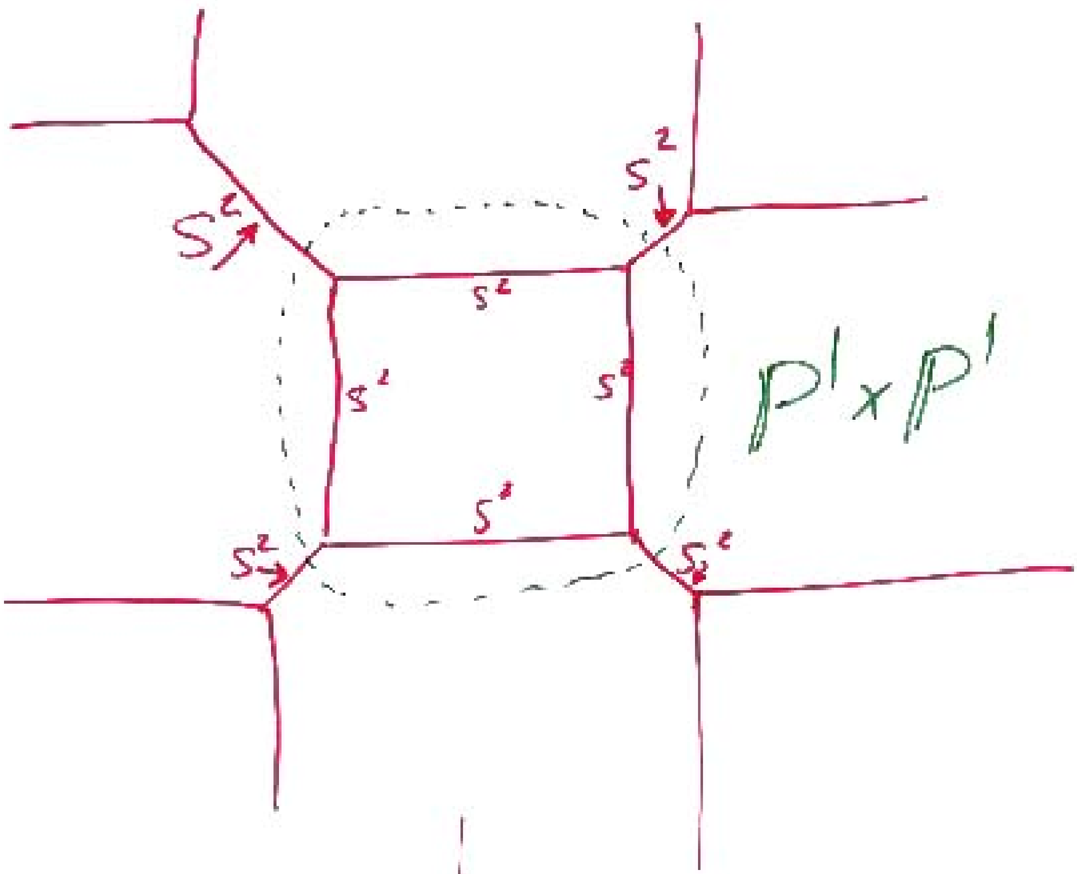
$P^1 \times P^1$ in a CY:

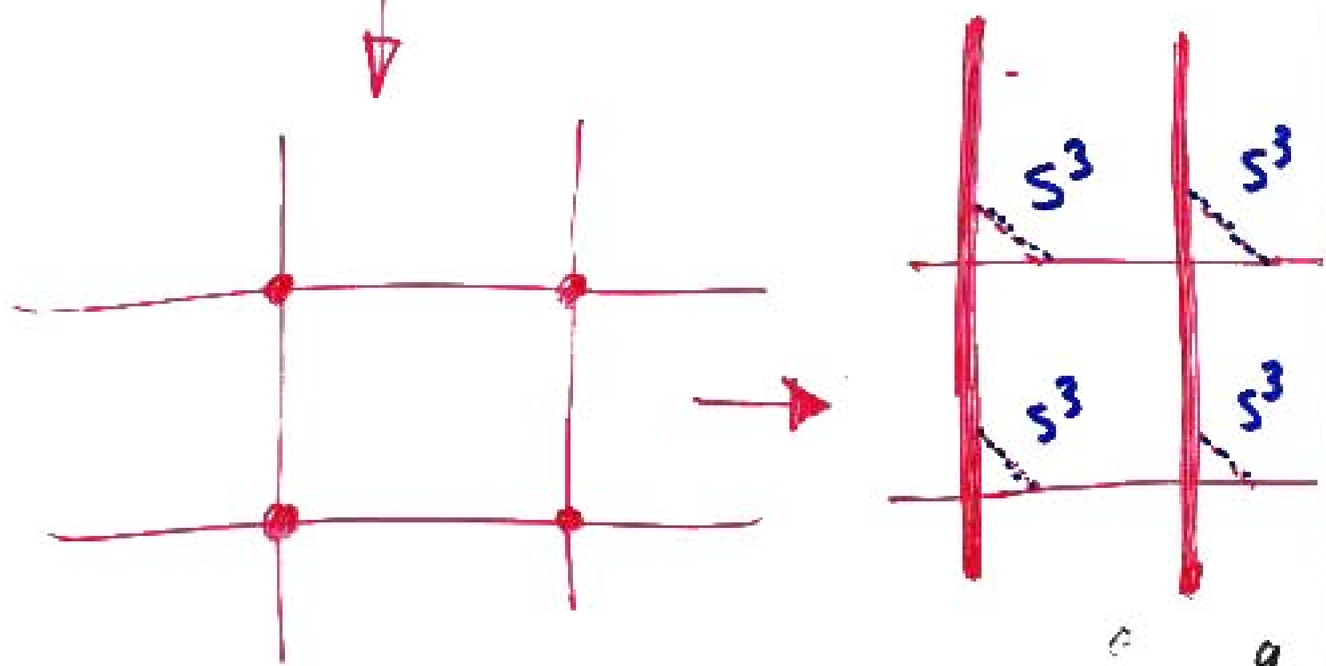


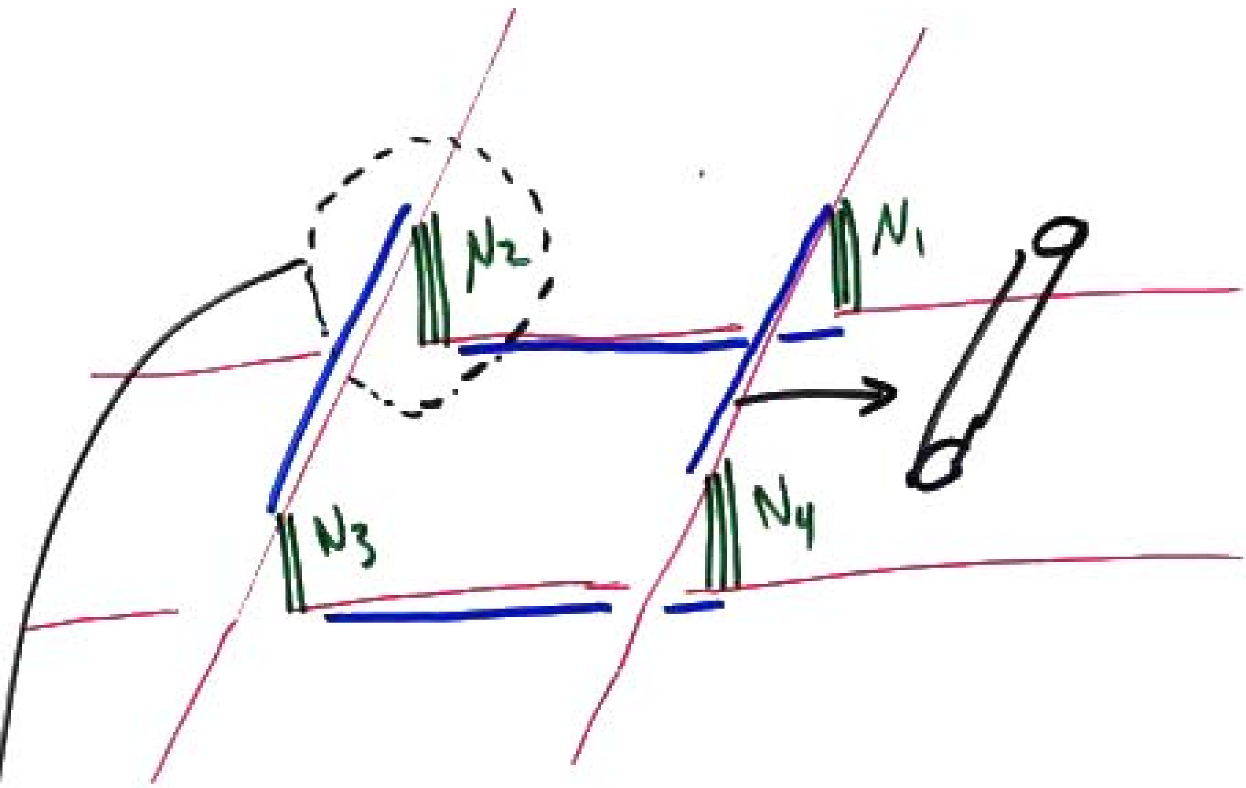
[recall : 

]

Consider instead





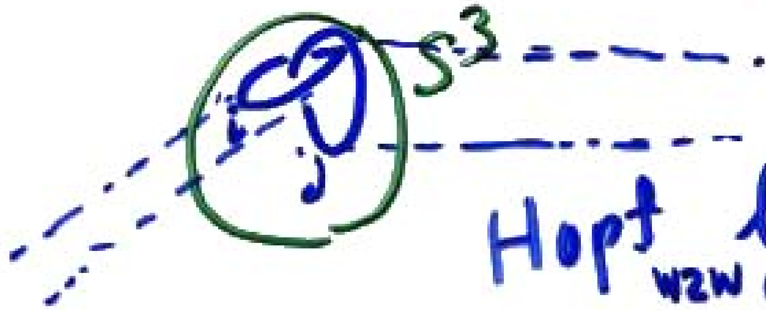


$$\prod_{i=1}^{4} U(N_i) \quad \text{Chern-Simons on } S^3_i .$$

$$+ \text{"}\underbrace{\text{bi-fundamental}}_{\text{integrate out}} \text{ matter"}$$

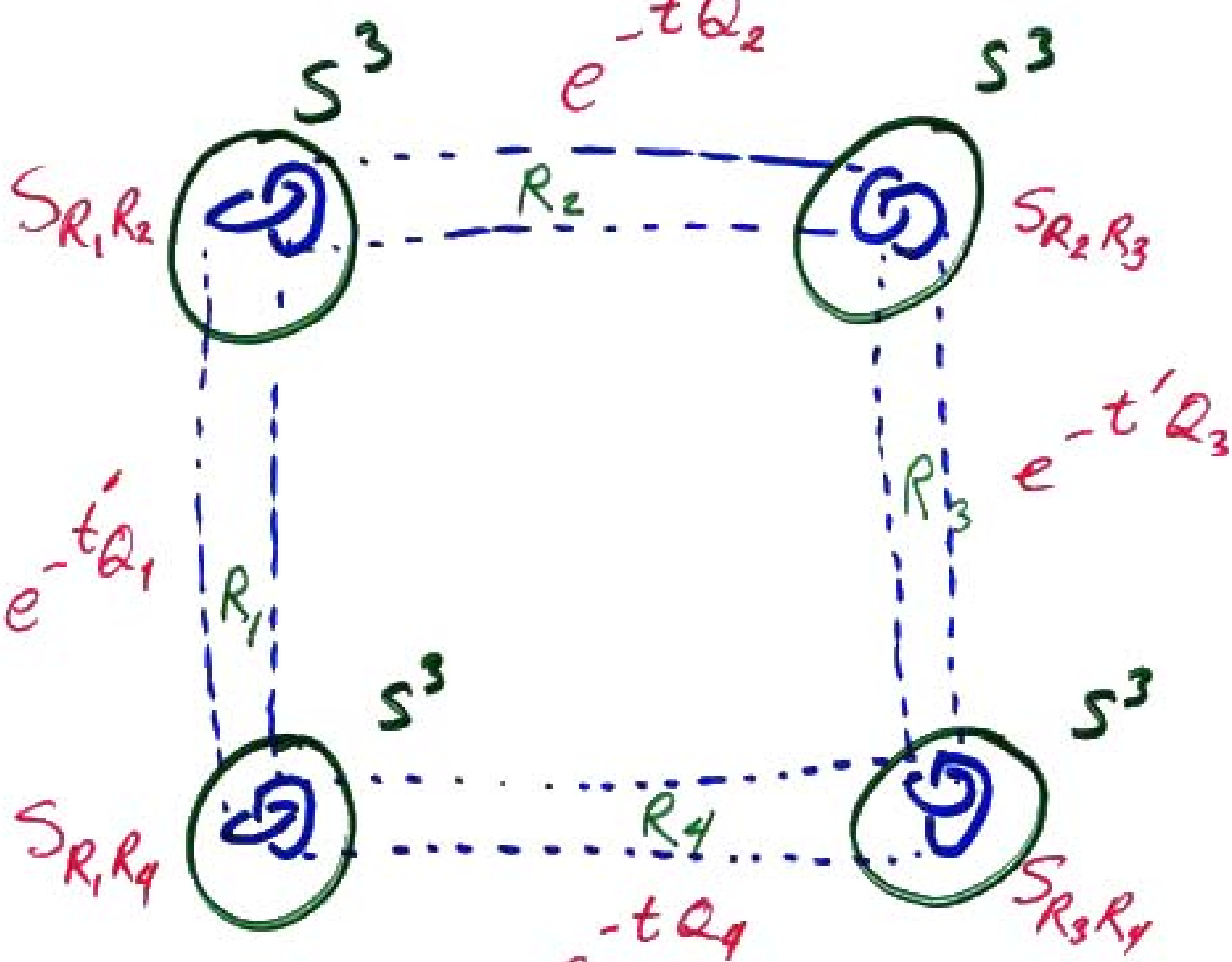
→ 1-loop exact

→ annulus



Hopf link $= S_{ij}$

WZW char. $\chi_i(-1/\tau) = S_{ij}\,\chi_j(\tau)$



$$= S_{R_1R_2}e^{-tQ_2}S_{R_2R_3}e^{-t'Q_3}S_{R_3R_4}e^{-tQ_4}S_{R_4R_1}e^{-t'Q_1}$$

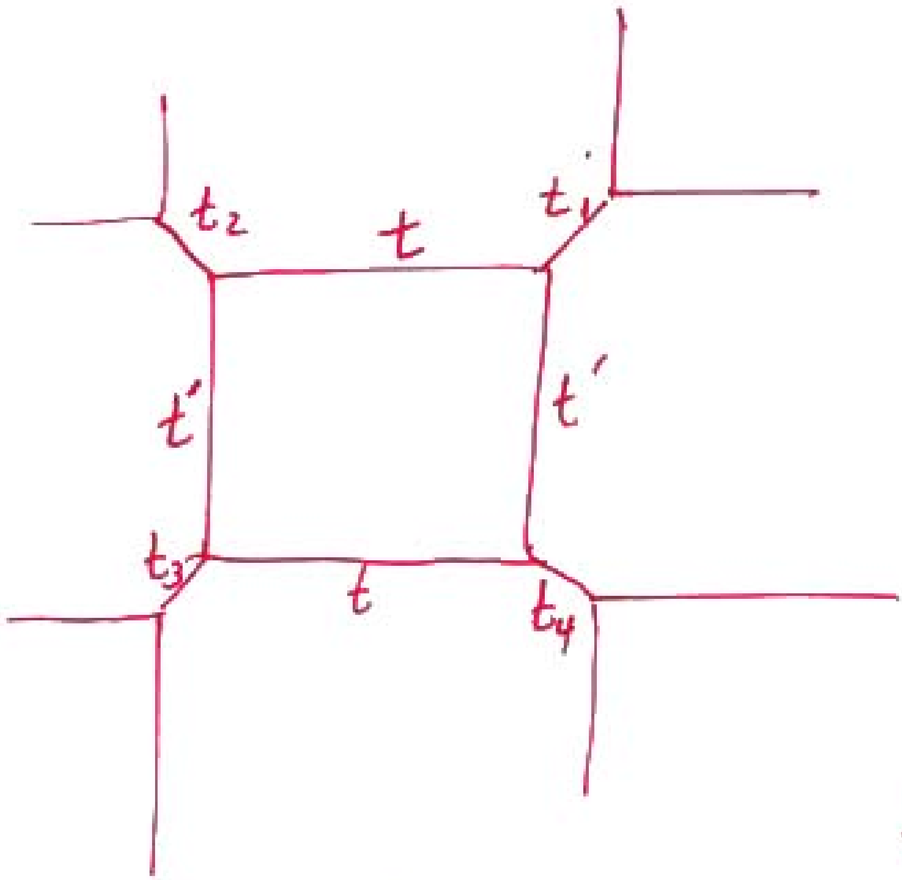
$S_{R_iR_j}$ modular transformation matrix $\tau \to -1/\tau$

Q_i = # boxes of R_i

$t, t' \longleftrightarrow$ Kähler classes of $P^1 \times P^1$

(up to some renormalization)

$$= \mathrm{Tr}[S U_t S U_{t'} S U_t S U_{t'}]$$



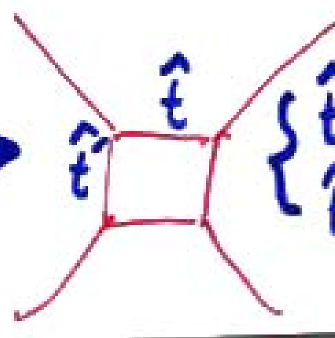
$t_i = N_i \lambda$

$\|$ Large N duality

$$\mathrm{Tr}\left[S_1 U_t S_2 U_{t'} S_3 U_t S_4 U_{t'}\right]$$

How to get $\mathbb{P}^1 \times \mathbb{P}^1$?

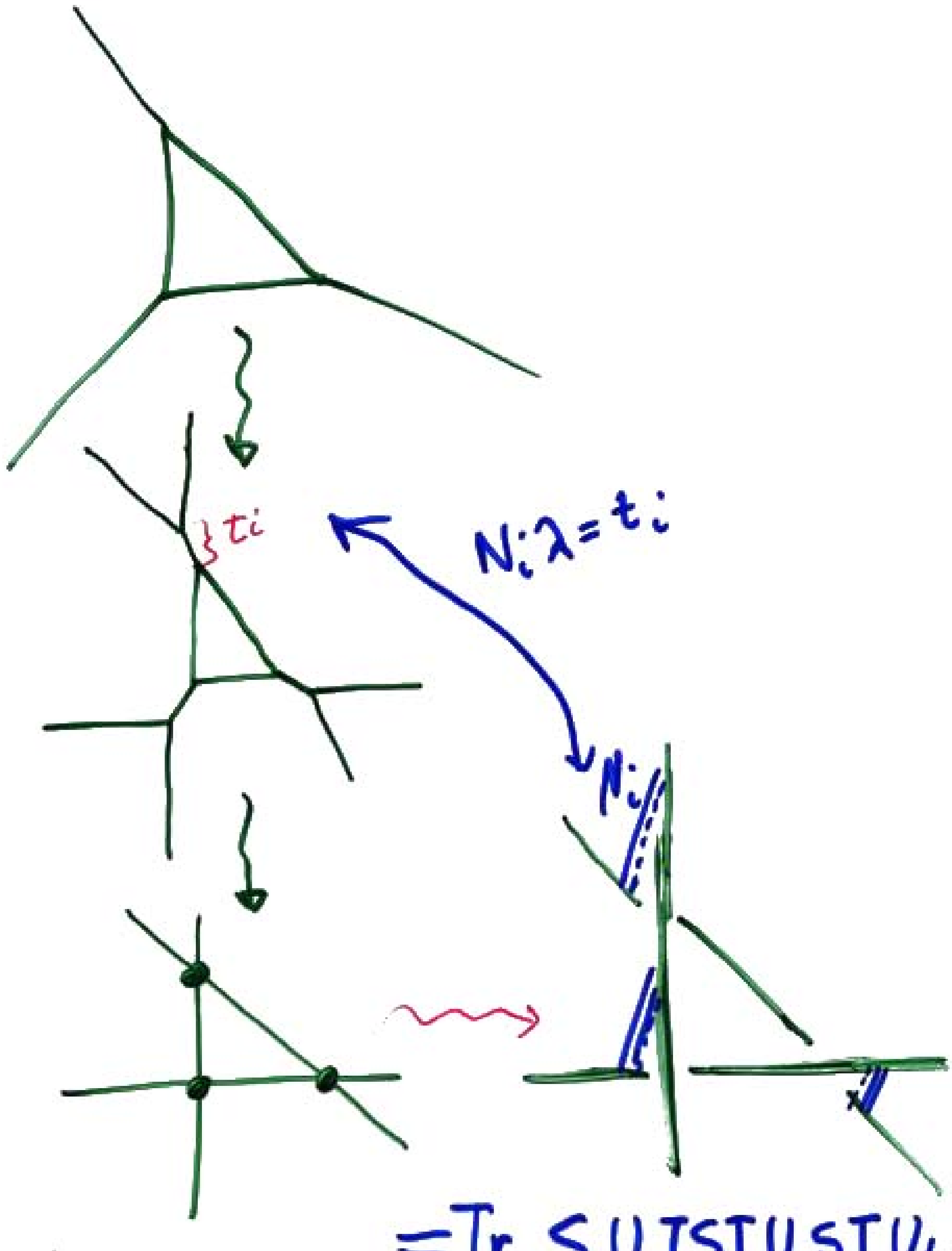
take $\begin{cases} t_i \to \infty & (N_i = \infty) \\ \text{fix } \hat{t}, \hat{t}' \end{cases}$ $\longrightarrow$



$$\begin{cases} \hat{t} = t - \frac{t_1 + t_2}{2} \\ \hat{t}' = t' - \frac{t_2 + t_3}{2} \end{cases}$$

"double scaling limit"

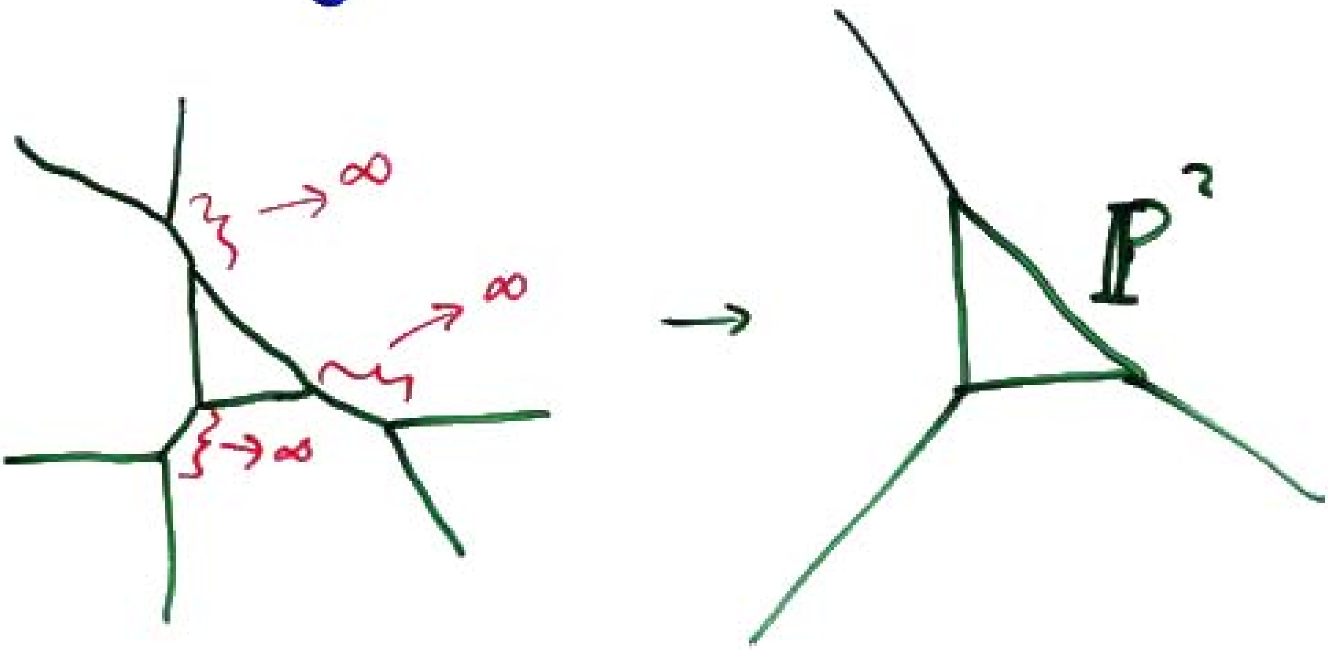
Similarly for $\mathbb{P}^2$:

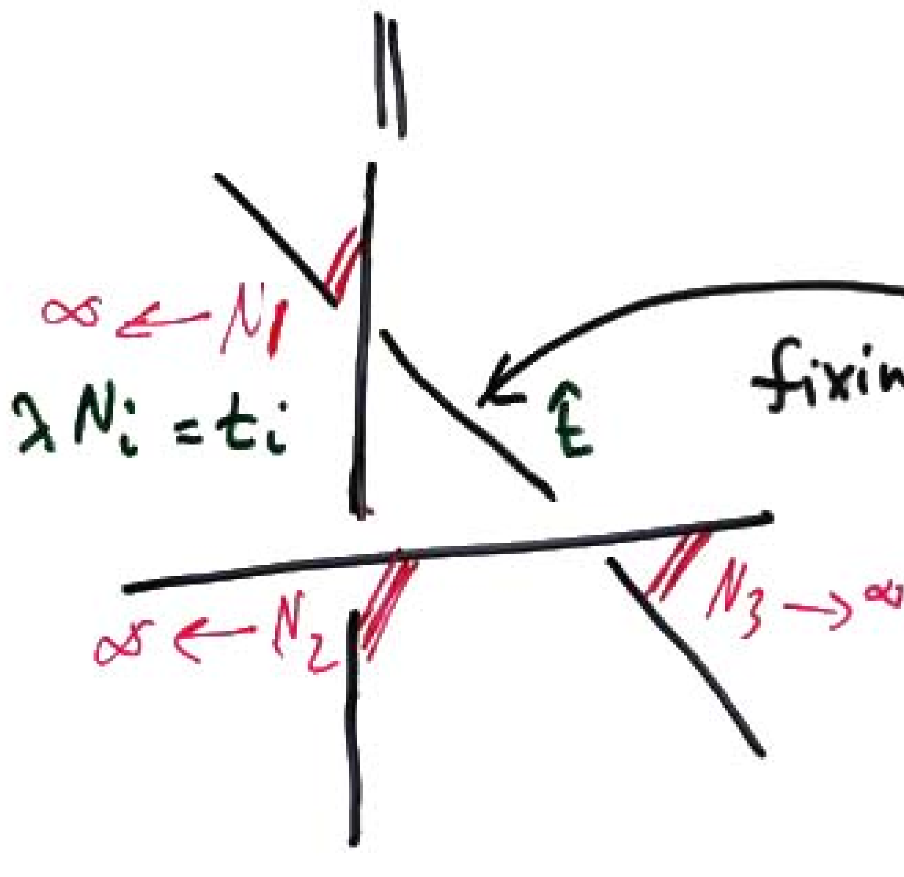


$$= \mathrm{Tr}\; S_1 U_t T S_2 T U_t S_3 T U_t$$

$T: (\tau \to \tau + 1)$

Again a suitable double scaling limit gives $\mathbb{P}^2$:





fixing $\hat{t} = t - \frac{1}{2}(t_1 + t_3)$

$t_i \to \infty$

g	$d=1$	2	3	4	5	6	7	8
0	3	−6	27	−192	1695	−17064	188454	−2228160
1	0	0	−10	231	−4452	80948	−1438086	25301295
2	0	0	0	−102	5430	−194022	5784837	−155322234
3	0	0	0	15	−3672	290853	−15363990	649358826
4	0	0	0	0	1386	−290400	29056614	−2003386626
5	0	0	0	0	−270	196857	−40492272	4741754985
6	0	0	0	0	21	−90390	42297741	−8802201084
7	0	0	0	0	0	27538	−33388020	12991744968
8	0	0	0	0	0	−5310	19956294	−15382690248
9	0	0	0	0	0	585	−9001908	14696175789
10	0	0	0	0	0	−28	3035271	−11368277886
11	0	0	0	0	0	0	−751218	7130565654
12	0	0	0	0	0	0	132201	−3624105918
13	0	0	0	0	0	0	−15636	1487970738
14	0	0	0	0	0	0	1113	−490564242
15	0	0	0	0	0	0	−36	128595720
16	0	0	0	0	0	0	0	−26398788
17	0	0	0	0	0	0	0	4146627
18	0	0	0	0	0	0	0	−480636
19	0	0	0	0	0	0	0	38703
20	0	0	0	0	0	0	0	−1932
21	0	0	0	0	0	0	0	45

Table 1: The integral invariants n_d^g for local $\mathbb{P}^2$, and degrees $d = 1, \cdots, 8$.

$\|$

$\#$ BPS states
charge d
Spin g

$\overset{=}{\text{roughly}}$ "$\#$" genus g curves
of degree d
embedded in
$\mathbb{P}^2$

g	$d=11$	12
0	4827935937	−66537713520
1	−135854179422	2380305803719
2	2145146041119	−48109281322212
3	−24130293606924	698473748830878
4	210503102300868	−7935125096754762
5	−1485630816648252	73613315148586317
6	8698748079113310	−572001241783007370
7	−42968546119317066	3786284014554551293
8	181202644392392127	−21609631514881755756
9	−658244675887405242	107311593188998164015
10	2074294284130247058	−466990545532708577390
11	−5702866358492557440	1791208287019324701495
12	13744538465609779287	−6085017394087513680618
13	−29157942375100015002	18384612378910358924791
14	54641056077839878893	−49578782776769125835658
15	−90735478019244786786	119723947998685791289164
16	133885726253316075984	−259634731498425150837576
17	−175976406401479949154	506961721474582218552270
18	206477591201198965488	−893407075206205808615238
19	−216671841840838260606	1424048002136300951108030
20	203674311322868998065	−2057099617415644933602618
21	−171730940091766865658	2697839037217627321703085
22	130015073789764141299	−3217397468483821476968358
23	−88451172530198637924	3494176460021369389735746
24	54098277648908454123	−3460084190968494003073062
25	−29751302949160261398	3127576636374963802648718
$\vdots$	$\vdots$	$\vdots$
55	0	91

Table 2: The integral invariants n_d^g for local $\mathbb{P}^2$, and degrees $d = 11, 12$.

d_2	$d_1=0$	1	2	3	4	5	6
0		−2	0	0	0	0	0
1	−2	−4	−6	−8	−10	−12	−14
2	0	−6	−32	−110	−288	−644	−1280
3	0	−8	−110	−756	−3556	−13072	−40338
4	0	−10	−288	−3556	−27264	−153324	−690400
5	0	−12	−644	−13072	−153324	−1252040	
6	0	−14	−1280	−40338	−690400		

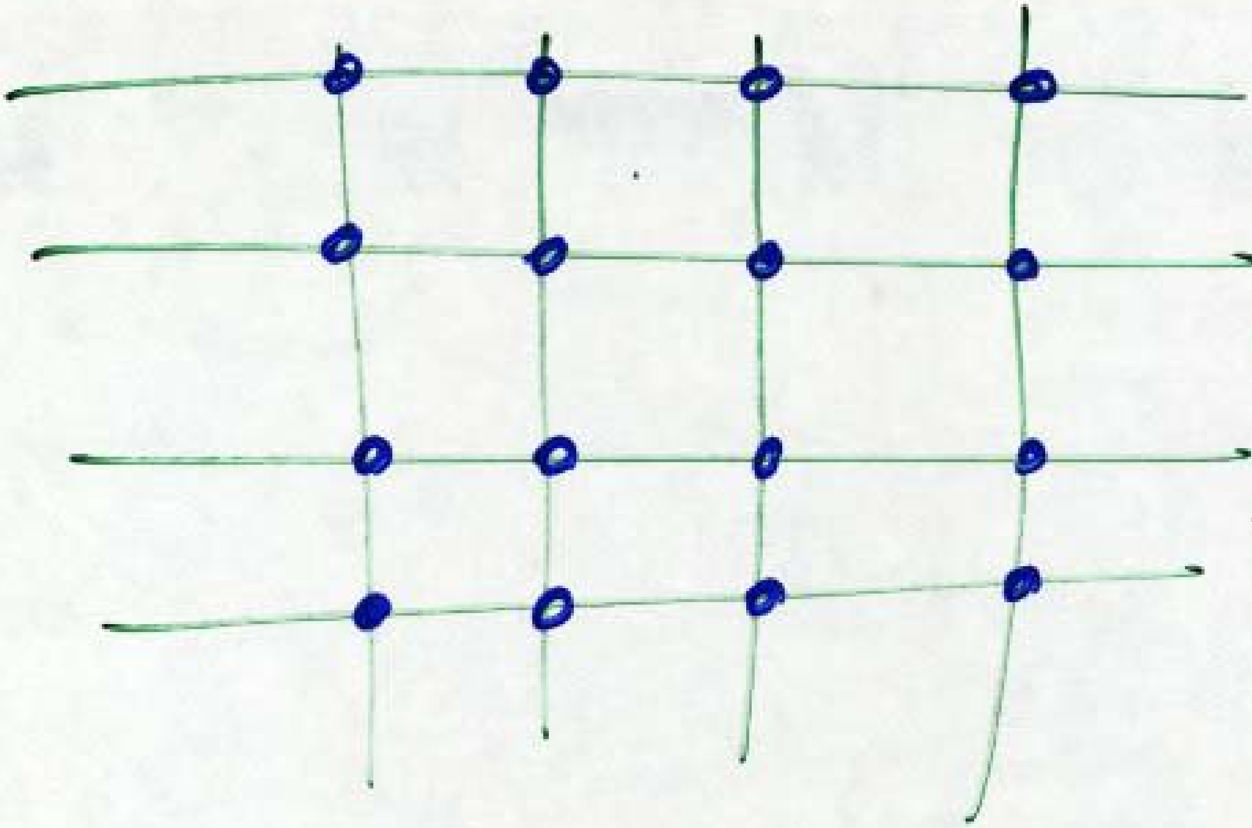
Table 3: The integral invariants n_d^0 for local $\mathbb{P}^1 \times \mathbb{P}^1$.

d_2	$d_1=2$	3	4	5	6
2	9	68	300	988	2698
3	68	1016	7792	41376	172124
4	300	7792	95313	760764	4552692
5	988	41736	760764	8695048	
6	2698	172124	4552692		

Table 4: The integral invariants n_d^1 for local $\mathbb{P}^1 \times \mathbb{P}^1$.

d_2	$d_1=2$	3	4	5	6
2	0	−12	−116	−628	−2488
3	−12	−580	−8042	−64624	−371980
4	−116	−8042	−167936	−1964440	−15913228
5	−628	−64624	−1964440	−32242268	
6	−2488	−371980	−15913228		

Table 5: The integral invariants n_d^2 for local $\mathbb{P}^1 \times \mathbb{P}^1$.



gives a "Lattice" model

Using local large N-dualities + considering certain double scaling limits

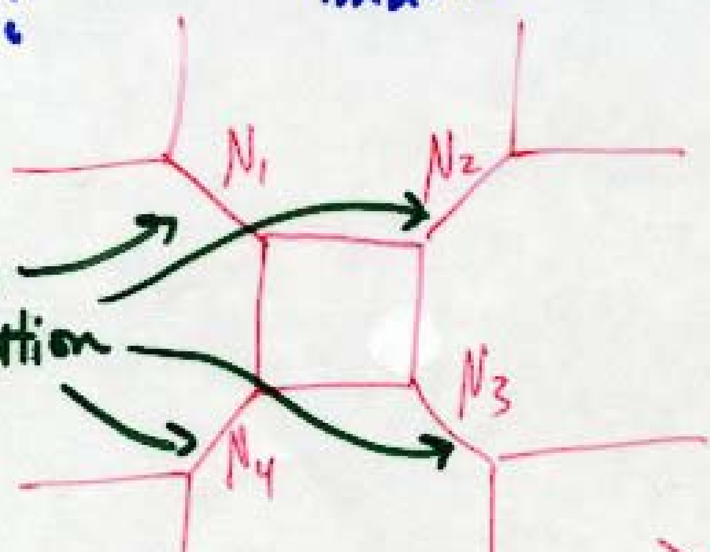
$\downarrow$

All A-model topological strings can be computed for local (toric) CY 3-folds

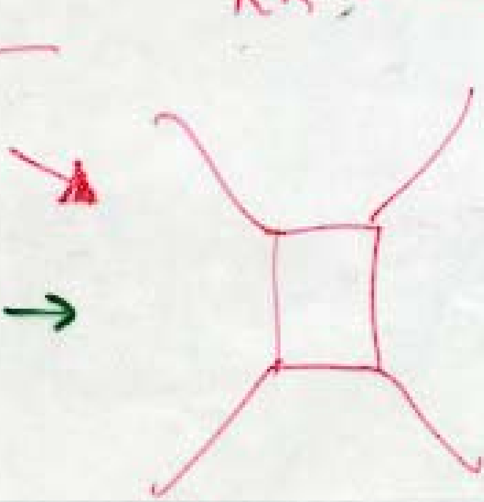
Embedding in IIA superstrings

$\mathcal{N}=1\ \prod_i U(N_i)$ + bi-fundamental matter   D6: $S^3_i \times R^4$

Gaugino condensation



N_i units of RR flux



→ double scaling limit →

RR flux/vol → 0

Significance of Double Scaling limits

$$\Big\downarrow$$

Superstring / Gauge theory duality

where RR-fluxes $\to 0$