Semiclassical Quantization of superstring in $AdS_5 \times S^5$

Frolov, AT
hep-th/0204226
+ extensions

AdS/CFT : standard 't Hooft expansion

$$\lambda = g_{YM}^2 N = \frac{R^4}{\alpha'^2} = \text{fixed}, \quad g_s = \frac{\lambda}{N} \to 0$$

α' - expansion $\to$ $\frac{1}{\sqrt{\lambda}}$ - expansion $(\lambda \gg 1)$

Compute "interpolating functions":

$$S = f_1(\lambda) N^2 V_3 T^3 \qquad f_1 = c_1 + \underline{c_2} \lambda^{-3/2} + \dots$$

$$\langle W(C) \rangle \sim e^{-f_2(\lambda) \frac{T}{L}} \qquad f_2 = a_1 \sqrt{\lambda} + \underline{a_2} + \dots$$

$$\Delta = \Delta_0 + f_3(\lambda) \qquad f_3 = k_1 \sqrt[4]{\lambda} + \dots \text{ or } = n_1 \sqrt{\lambda} + \underline{n_2} + \dots$$

α'- expansion in (tree-level) GS string theory

Useful ! important information about AdS/CFT

Open string sector : studied earlier e.g. Förste, Ghoshal, Theisen; Drukker, Gross, AT

Closed string sector : recent progress BMN GKP

PLAN:

- GS action in $AdS_5 \times S^5$
- expansion near point-like string (null geodesic)
  - motion parallel to the boundary
  - rotation in S^5: $J \neq 0$

    pw limit = 1-loop approximation and beyond
- expansion near folded string rotating in AdS_5

  $S \neq 0$
- $S \neq 0$, $J \neq 0$ case: interpolation
- Non-conformal (near-AdS) generalization

related to talks by Maldacena, Klebanov, Polyakov, Schwarz, Minwalla

GS Action in R-R background

$$I = \frac{\sqrt{\lambda}}{2\pi}\int d\tau\, d\sigma\,\left(\mathcal{L}_B + \mathcal{L}_F\right)$$

$$\mathcal{L}_F = i\left(\sqrt{-g}\, g^{ab}\,\delta^{IJ} - \varepsilon^{ab} s^{IJ}\right)\underline{\bar{\theta}^I \rho_a \mathcal{D}_b \theta^J} + O\left(\theta^4\right)$$

$$I, J = 1, 2 \qquad\qquad s^{IJ} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\rho_a = \Gamma_A\, E^A_M(x)\, \partial_a X^M$$

$$\mathcal{D}_a = \partial_a X^M\, \mathcal{D}_M$$

$$\mathcal{D}^{IJ}_M = \delta^{IJ} D_M + \frac{1}{8\cdot 5!}\,\varepsilon^{IJ}\,\underline{F_{A_1\dots A_5}\Gamma^{A_1\dots A_5}\Gamma_M}$$

$$D_M = \partial_M + \tfrac{1}{4}\,\omega^{AB}_M\,\Gamma_{AB}\ , \qquad F_5 = \mathcal{E}_5 + *\,\mathcal{E}_5$$

$$\underline{\mathcal{D}_a \theta^I} = \left(\delta^{IJ} D_a - \tfrac{i}{2}\,\underbrace{\varepsilon^{IJ}\,\Gamma_* \rho_a}_{\text{"Mass term"}}\right)\theta^J$$

$$\Gamma_* \equiv i\,\Gamma_{01234}\ , \qquad \Gamma_*^2 = 1$$

GS string action in $AdS_5 \times S^5$

Covariant gauge

$$ds^2 = \frac{dx_m^2 + dz_p^2}{z^2}$$

Metsaev
AT
Kallosh

$$\mathcal{L} = \sqrt{\lambda}\left[\frac{1}{z^2}\left(\partial_a x^m - i\,\theta\gamma^m\partial_a\theta\right)^2 + \frac{\partial z_p\,\partial z_p}{z^2} + i\,\varepsilon^{ab}\,\frac{\partial_a z_p}{z^2}\,\theta\gamma_p\partial_b\theta\right]$$

$m = 0,1,2,3$
$p = 1,\dots,6$

$\partial x\,\theta\,\mathcal{D}\theta$: possible degeneracy for "small" strings

Light-cone gauge:

$x^m = (x^+, x^-, x_\perp)$

$\boxed{\Gamma^+\theta = 0}$ $\qquad \theta \to (\theta_i, \eta_i) \qquad i = 1,\dots,4$

$$\sqrt{-g}\,g^{ab} = \begin{pmatrix} -\frac{1}{z^2} & 0 \\ 0 & z^2 \end{pmatrix} \quad \text{and} \quad \boxed{x^+ = \tau}$$

Polyakov

or systematic phase-space approach

Metsaev
Thorn
AT

Non-degenerate action; well-defined QM

$$\mathcal{L} = \dot{x}_\perp^2 + \left(\dot{z}_p - \eta\gamma_{ps}\eta\,\frac{z_s}{z^2}\right)^2 - \underline{\lambda}\,\frac{x_\perp'^2 + z_p' z_p'}{z^4} + i\left(\theta\dot{\theta} + \eta\dot{\eta}\right) - \frac{1}{z^2}\eta^4 - \underline{\sqrt{\lambda}}\,\frac{1}{z^3}\,\eta\gamma^p z_p\left(\theta' - \frac{i}{z}x_\perp'\eta\right) + \text{h.c.}$$

$\frac{\sqrt{\lambda}}{2\pi}$ = string tension

String spectrum in $AdS_5 \times S^5$?

Special sectors of states with **large** quantum numbers → can be described by semiclassical quantization

Polyakov
BMN
GKP

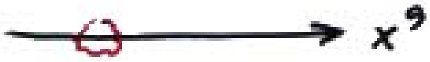
$= \alpha' \sim \frac{1}{\sqrt{\lambda}}$ -expansion near particular classical string solutions

$\lambda \gg 1$, parameters of solution fixed

Flat space examples :

- Expansion near point-like string :

$x^0 = x^9 = p\tau$

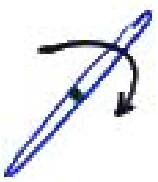


1-loop approximation is exact
equivalent to l.c. gauge quantization : $x^+ \sim \tau$

$$P^+ = E + P_9 = \frac{2p}{\alpha'}$$

$$P^- = E - P_9 = \frac{1}{P^+}\left(P_\perp^2 + \sum_{n=-\infty}^{\infty} |n| N_n\right)$$

- Expansion near classical rotating string:



$x_1 = r_0 \sin w\sigma \cos w\tau$ $\qquad x_0 = \kappa\tau$

$x_2 = r_0 \sin w\sigma \sin w\tau$ $\qquad r_0 = \frac{\kappa}{w}$

$w = n = 1, 2, \ldots$

$$E = \sqrt{\frac{2}{\alpha'} S}$$

cf. Leading Regge trajectory
$(a_1^+)^S (a_1^+)^S |0\rangle$

Classical solution – coherent state in osc. vac:

$|\psi\rangle_s = \exp(\sqrt{S} a_1^+) \exp(\sqrt{S} a_1^+) \; |0\rangle$

Expansions near different classical solutions – connected

Stable classical string configurations in $AdS_5 \times S^5$? ("non-topological solitons")

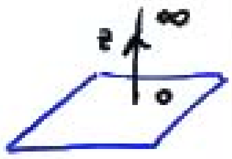
Understand spectrum (at large $\sqrt{\lambda}$) by expanding near different classical soln's and "interpolating"

Poincare and Global coordinates

$AdS_5 \times S^5$

$$ds^2 = \frac{1}{z^2}\left(dx_m dx_m + dz_p dz_p \right) \qquad \begin{array}{l} m=0,1,2,3 \\ p=1,\dots,6 \end{array}$$

$$= \underbrace{\frac{1}{z^2}\left(dx_m dx_m + dz^2 \right)} + d\Omega_5$$



$$\underline{ds^2_{AdS_5}} = -\cosh^2\rho \, dt^2 + d\rho^2 + \sinh^2\rho \, d\Omega_3$$

$$\frac{x_0}{z} = \cosh\rho \sin t$$

$$\frac{x_i}{z} = n_i \sinh\rho \qquad n_i^2 + n_4^2 = 1 \qquad i=1,2,3$$

$$\frac{1}{2z}\left(-1 + z^2 + x_m x_m\right) = n_4 \sinh\rho \qquad d\Omega_3 = dn_k dn_k \quad k=1,2,3,4$$

String energy :

$$E = \int d\sigma \, \mathcal{E} = \frac{1}{2}\left(P_0 + K_0 \right)$$ = energy in global coordinates

$$\mathcal{E} = \cosh^2\rho \, \dot{t} = \frac{1}{2}\left(1 + z^2 + x_m x_m\right)\mathcal{P}_0 - 2 x_0 \mathcal{D}$$

$$\int d\sigma \, \mathcal{P}_0 = \int d\sigma \, \frac{\dot{x}_0}{z^2}$$ = energy in Poincare coordinates

$$\mathcal{D} = \frac{1}{z^2}\left(z\dot{z} + x_m \dot{x}_m \right)$$

For all (simple) rotating solutions : $z^2 + x_m x_m = 1$

$$E = P_0$$

Point-like string : null geodesic

$$ds^2 = \frac{1}{z^2}\left(dx_m dx_m + dz_p dz_p\right)$$

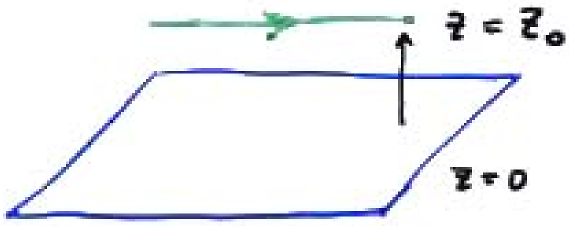
$$x_m = a_m + v_m f(\tau) \qquad a, b, u, v = \text{const}$$

$$z_p = b_p + u_p f(\tau) \qquad v^2 + u^2 = 0$$

$$f(\tau) = -\frac{b \cdot u}{u^2} + \frac{\nu}{u^2} \tan \nu\tau \qquad \nu^2 = b^2 u^2 - (bu)^2$$

Two basic cases:

① Straight line parallel to boundary



$$x_0 = p\tau \qquad x^+ = 2p\tau$$

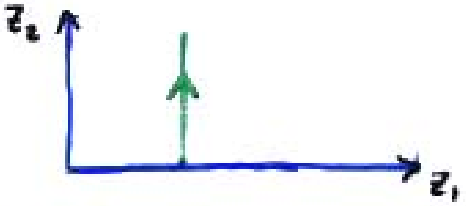
$$x_1 = p\tau \qquad x_\perp = 0$$

$$z_1 = z_0, \quad z_{2,\dots,6} = 0$$

② Rotation in S^5 (in (z_1, z_2) plane)

$$z_1 = z \cos\varphi \qquad z_2 = z \sin\varphi$$

$$z \equiv |z| = \frac{1}{\cos \nu\tau}, \quad x_0 = \tan \nu\tau, \quad \underline{\varphi = \nu\tau}$$

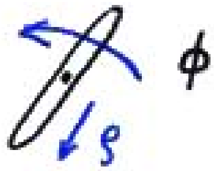


$$\underline{z_1 = 1, \quad z_2 = \tan\nu\tau = x_0}$$

Global coordinates: $t = \nu\tau, \quad \varphi = \nu\tau, \quad \rho = 0$

Folded string rotating in AdS_5

Global coordinates:



$$t = \kappa\tau, \quad \phi = w\tau, \quad \rho = \rho(\sigma)$$

$$\rho'^2 = \kappa^2 \cosh^2\rho - w^2 \sinh^2\rho$$

$0 \le \rho \le \rho_{max}$

$w = w(\kappa)$

Poincare coordinates:

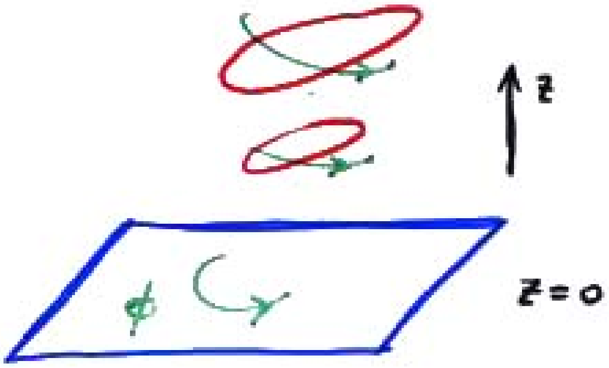
$$ds^2 = \frac{dx_m dx_m + dz^2}{z^2}$$

$$x_0 = \tan \kappa\tau$$

$$x_1 = r \cos w\tau, \qquad x_2 = r \sin w\tau$$

$$r(\tau,\sigma) = \frac{\tanh \rho(\sigma)}{\cos \kappa\tau}$$

$$z = \frac{1}{\cosh\rho(\sigma) \cos\kappa\tau} \qquad z^2 + x_m x_m = 1$$



string moves towards horizon
rotating and stretching

$$E_{glob.} = E_{Poinc.} = P_0, \qquad S = \sqrt{\lambda}\, s$$

$$E = E(S) = \begin{cases} \sqrt{\sqrt{\lambda}\, S} + \dots, & s \ll 1 \\ S + \frac{\sqrt{\lambda}}{\pi} \log S + \dots, & s \gg 1 \end{cases}$$

Point-like strings in $AdS_5 \times S^5$

- Geodesic parallel to the boundary:

$$X_0 = X_1 = p\tau \quad , \quad z = 1$$

$$E_0 = P_0 = P_1 = \sqrt{\lambda}\, p \quad \gg 1$$

1-loop correction : $E_1 = \frac{1}{2p} \sum_n |n| N_n$

same as l.c. gauge flat string spectrum

But 2-loop correction is non-trivial

$$E_2 = \frac{1}{\sqrt{\lambda}} f(p, n) \qquad = ?$$

- Geodesic rotating in S^5:

$$t = \nu\tau \quad , \quad \varphi = \nu\tau \quad ,$$

$\xi_{1,2,3,4} = 0$ other AdS_5

$\psi_{1,2,3,4} = 0$ other S_5 coordinates

$$E_0 = \sqrt{\lambda}\,\nu \quad \gg 1$$

$$J = \sqrt{\lambda}\,\nu \quad \gg 1$$

$\lambda \gg 1$

$\nu = \text{fixed} = \frac{J}{\sqrt{\lambda}}$

Expand GS action near solution →

Compute σ-model loop $\left(\frac{1}{\sqrt{\lambda}}\right)$ corrections to E

in sector of states $\{ |\Phi\rangle \}$ with

$$\langle \Phi | \hat{J} | \Phi \rangle = J = \sqrt{\lambda}\,\nu = \text{quantized}$$

1-loop approximation:

curvature

$$\mathcal{L}_B^{(2)} = -(\partial\tilde{t})^2 + (\partial\tilde{\varphi})^2 + (\partial\xi_i)^2 + (\partial\psi_\kappa)^2 + \underline{\nu}^2(\xi_i^2 + \psi_\kappa^2)$$

$$\mathcal{L}_F^{(2)} = -i\left(\eta^{ab}\delta^{IJ} - \varepsilon^{ab}s^{IJ}\right)\bar{\theta}^I \rho_a \mathcal{D}_b \theta^J$$

$$\rho_0 = \Gamma_M \partial_0 X^M = \nu(\Gamma_0 + \Gamma_\varphi)$$

$$\mathcal{D}_a^{IJ} = \delta^{IJ}\partial_a - \tfrac{i}{2}\varepsilon^{IJ}\Gamma_* \rho_a$$

$$\Gamma_* = i\,\Gamma_{01234}$$

Natural $\varkappa$-symmetry gauge ("imposed" by background)

$$\Gamma^+\theta^I = 0\,, \qquad \Gamma^\pm \equiv \mp\Gamma_0 + \Gamma_\varphi$$

$$\mathcal{L}_F^{(2)} = -i\nu\left(\bar{\theta}^1\Gamma^-\partial_+\theta^1 + \bar{\theta}^2\Gamma^-\partial_-\theta^2 - 2\underline{\nu}\,\bar{\theta}^1\Gamma^-\Pi\theta^2\right)$$

F_5-term

$$\Pi = i\Gamma_*\Gamma_0 = \Gamma_{1234}\,, \qquad \Pi^2 = \mathbf{1}$$

String in pw background and corresponding quadratic l.c. gauge GS action "rediscovered"

Metsaev

Penrose limit $(R\to\infty)$ equivalent to 1-loop approximation near null geodesic $(\sqrt{\lambda}\to\infty)$

BMN
GKP

General quantization procedure:

use constraints to eliminate $\tilde{x}^-$ or impose $\tilde{x}^+ = 0$

$t \sim \tau$

$E_{10d} \sim E_{2d}$

$$E - J = \left\langle \frac{1}{\nu} \int_0^{2\pi} \frac{d\sigma}{2\pi} \mathcal{H}_\perp (\tilde{\xi}_k, \tilde{\Psi}_s, \theta^2) \right\rangle$$

"transverse" 2-d Hamiltonian: 4+4 massive bosons
4+4 massive fermions
+ interactions

$$E - J = \frac{1}{\nu} \sum_{n=-\infty}^{\infty} \sqrt{n^2 + \nu^2}\, N_n + O\left(\frac{1}{\sqrt{\lambda}}\right)$$

$$E = \sqrt{\lambda}\,\nu + \sum_{n=-\infty}^{\infty} \sqrt{1 + \frac{n^2}{\nu^2}}\, N_n + O\left(\frac{1}{\sqrt{\lambda}}\right)$$

$$E = J + \sum_{n=-\infty}^{\infty} \sqrt{1 + \frac{\lambda}{J^2} n^2}\, N_n + O\left(\frac{1}{\sqrt{\lambda}}\right)$$

"Miracle" of $E(\underline{J}, \underline{\lambda})$ being analytic in λ

→ possibility of comparison to gauge theory BMN

True at higher-loop orders? Yes!

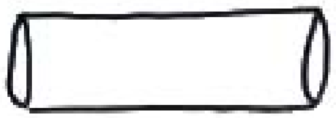
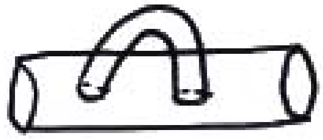
UV finite massive 2-d theory:

$m_B = m_F = \nu$: $\frac{1}{\nu}$ - expansion regular

$$\mathcal{L}_{GS} \sim \sqrt{\lambda} \left((\partial \xi)^2 + \nu^2 \xi^2 + \sum_{k=1}^{\infty} \left[c_k\, \xi^{2k} (\partial \xi)^2 + b_k\, \nu^2 \xi^{2k+2} \right] \right)$$

+ fermions

Parameters:

 $+ \; g_s^2$  $+ \; \ldots$

σ, τ

standard expansion in
String theory: $\quad \dfrac{\alpha'}{R^2} = \dfrac{1}{\sqrt{\lambda}} \ll 1, \quad \nu = \dfrac{J}{\sqrt{\lambda}}$ fixed, $\quad g_s \to 0$

$$\frac{1}{\nu^2} = \frac{\lambda}{J^2} \equiv \lambda'$$

$$\frac{1}{\sqrt{\lambda}} = \frac{\nu}{J} = \frac{1}{J}\,\frac{1}{\sqrt{\lambda'}}$$

$$g_s = g_{YM}^2 = \frac{\lambda}{N} = \lambda' \cdot \frac{J^2}{N} \equiv \lambda' \cdot g_2$$

Fixed λ', $\quad J \to \infty$, $\quad g_2 \equiv \dfrac{J^2}{N} \to 0 \; = \; \nu = \text{fixed}, \; \lambda \to \infty, \; g_s \to 0$

= 2-d 1-loop approximation in classical string theory

$$E\left(\frac{1}{\nu^2}, \frac{1}{\sqrt{\lambda}}, g_s\right) = E\left(\lambda', \frac{1}{J\sqrt{\lambda'}}, \lambda' \cdot g_2\right)$$

ℓ- loop correction: $\langle n | E | n \rangle$

$$E_\ell = \frac{1}{(\sqrt{\lambda})^{\ell-1}} F_\ell(\nu, n)$$

$\ell = 1, 2, \ldots$

$$F_\ell(n=0) = 0$$

2-d susy:
$\langle 0 | E | 0 \rangle = 0$

$$F_\ell(\nu \to \infty) = \frac{1}{\nu^{\ell-1}}\left(c_\ell(n) + \frac{1}{\nu^2} b_\ell(n) + \ldots\right)$$

large mass exp'n

$$E_\ell = \frac{1}{J^{\ell-1}} f_\ell\left(\frac{\lambda}{J^2}, n\right) \to \text{series in } \lambda' = \frac{\lambda}{J^2} = \frac{1}{\nu^2}$$

$$E_\ell(J \to \infty, \lambda' = \text{fixed}) \xrightarrow{(\ell > 1)} 0$$

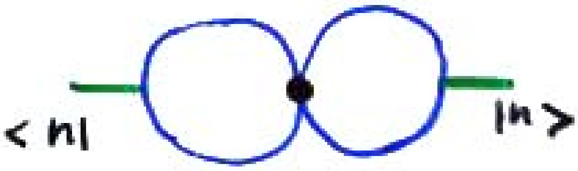
"non-renormalization" for near-BPS states

2- loop correction:

$$\mathcal{L}^{(4)} = -(\partial \tilde{t})^2 + (\partial \tilde{\varphi})^2 + (\partial \xi)^2 + (\partial \psi)^2 + \nu^2(\xi^2 + \psi^2)$$

$$+ \frac{1}{\sqrt{\lambda}}\Big[-\xi^2 (\partial \tilde{t})^2 + \psi^2 (\partial \tilde{\varphi})^2 + \frac{1}{2} \xi^2 (\partial \xi)^2 - \sum_k^{i-1} \psi_k^2 (\partial \psi_i)^2 + \nu^2 \Big(\frac{1}{2}\xi^4 + \frac{1}{3}\psi^4 + \sum_{s<s'} \psi_s^2 \psi_{s'}^2\Big)\Big]$$

$$+ \bar{\theta}(\partial + \nu)\theta + \theta^4\text{- terms}$$



$$E_2 = \frac{1}{\sqrt{\lambda}\,\nu} f_2(\nu, n) \rightarrow$$

$$\boxed{E_2 = \frac{1}{J} f_2\left(\frac{\lambda}{J^2}, n\right)} = c\,\frac{\lambda}{J^3} + \ldots$$

compare to 1- loop in gauge theory?

Rotating string in AdS_5:

2-d loop $\left(\frac{1}{\sqrt{\lambda}}\right)$ corrections to $E(S)$

Classical solution: $s =$ fixed, $\lambda \gg 1$

$$\underline{E = \sqrt{\lambda}\ \mathcal{E}(s)}\ , \quad \underline{S = \sqrt{\lambda}\ s}$$

$$\mathcal{E}(s) \simeq s + \frac{1}{\pi} \ln s + \ldots, \qquad s \gg 1$$

de Vega, Egusquiza   GKP

Non-BPS : expect corrections to ground-state energy

$$\boxed{E = \sqrt{\lambda}\ s + f_1(\lambda) \ln s + f_2(\lambda) \ln^2 s + \ldots}$$

$$f_1(\lambda) = \frac{\sqrt{\lambda}}{\pi} + a_1 + \frac{a_2}{\sqrt{\lambda}} + \ldots$$

$$f_2(\lambda) = 0 + k_1 + \frac{k_2}{\sqrt{\lambda}} + \ldots \qquad = 0$$

Gauge-theory side:

no $\ln^n s$ in Δ of $\mathrm{Tr}(\Phi^* \mathcal{D}_{m_1} \ldots \mathcal{D}_{m_S} \Phi)$

Must have $f_2 \equiv 0$ on string side

for existence of strong/weak coupling interpolation

Indeed, $f_2, f_3, \ldots = 0$ follows from scaling argument for $s \gg 1$

Semiclassical quantization:

$$x = x_{class} + \xi$$

$$\langle \psi | \hat{E} | \psi \rangle = E(S)$$

$$\langle \psi | \hat{S} | \psi \rangle = S = \sqrt{\lambda}\, s$$

Finite 2-d theory

but no longer 2-d susy : $\langle 0 | \hat{E} | 0 \rangle \neq 0$

Quadratic fluctuation action:

$$\mathcal{L}_B^{(2)} = -(\partial \tilde{t})^2 - \mu_t^2 \tilde{t}^2 + (\partial \tilde{\phi})^2 + \mu_\phi^2 \phi^2$$
$$+ 4 \tilde{\rho} \left(\kappa \sinh\rho\, \partial_0 \tilde{t} - w \cosh\rho\, \partial_0 \tilde{\phi} \right)$$
$$+ (\partial \tilde{\rho})^2 + \mu_\rho^2 \tilde{\rho}^2 + (\partial \beta_i)^2 + \mu_\beta^2 \beta_i^2 + (\partial \psi_i)^2$$

+ constraints

$$\mu_t^2 = \underline{m^2(\sigma)} - \kappa^2$$
$$\mu_\phi^2 = \underline{m^2(\sigma)} - w^2$$
$$\mu_\rho^2 = \underline{m^2(\sigma)} - \kappa^2 - w^2$$
$$\mu_\beta^2 = \underline{m^2(\sigma)} \equiv 2 \rho'^2(\sigma)$$

κ, w functions of s

similar to open string case in Drukker, Gross, AT

$$\mathcal{L}_F^{(2)} = \left(\eta^{ab} \delta^{IJ} - \epsilon^{ab} s^{IJ} \right) \bar{\theta}^I \rho_a D_b \theta^J$$

apply σ-dependent rotation $\psi^I = U(\sigma)\, \theta^I$

$\varkappa$-symmetry gauge $\psi^1 = \psi^2 \longrightarrow$

<u>massive 2-d fermions</u>

$$\mathcal{L}_F^{(2)} = i\,\bar{\Psi}\,\tau^a \partial_a \Psi + \bar{\Psi} M \Psi$$

$$M = i\rho' \cdot \Gamma_{234}$$

4+4 massive 2-d fermions $\boxed{m_F = \pm \rho'(\sigma)}$

2-d susy spont. broken classical solution

Sum rule: $\sum m_B^2 - \sum m_F^2 = 0$

explicit check of UV finiteness

1-loop correction to ground-state energy:

$$E = \sqrt{\lambda}\,\mathcal{E}(s) + E_1(s) + O\left(\frac{1}{\sqrt{\lambda}}\right)$$

$$E_1 = \frac{1}{\kappa} \langle 0|\, \hat{H}_{2\text{-}d}\, |0\rangle$$

$t = \kappa\tau$

$\kappa = \kappa(s)$

"long-string" limit: $s \gg 1$

$\rho' \simeq \text{const} = \kappa$: Masses $\simeq$ const

$$\kappa(s \gg 1) \simeq \frac{1}{\pi} \ln s \gg 1, \qquad s = \frac{S}{\sqrt{\lambda}}$$

$$E_1 \overset{s\gg 1}{\simeq} \frac{1}{2\kappa} \sum_{n=1}^{\infty} \Big[\sqrt{n^2+4\kappa^2} + 2\sqrt{n^2+2\kappa^2} + 5\sqrt{n^2} - 8\sqrt{n^2+\kappa^2} \Big]$$

$$\overset{s\gg 1}{\simeq} -\underbrace{\frac{3\ln 2}{4\pi}}_{a_1} \cdot \ln s$$

No $\ln^2 s$, etc.

Higher orders: $\kappa \sim \ln s \gg 1$

$$E_\ell = \frac{1}{\kappa}\left(E_{2\text{-}d}\right)_\ell \simeq \frac{1}{\kappa}\left(c_\ell \kappa^2 + O(\kappa)\right)$$

$$\simeq c_\ell \kappa + \ldots \sim c_\ell \ln s + \ldots$$

Why:

$$\mathcal{L} = \sqrt{\lambda}\Big[\left(1 + a_1 \xi^2 + a_2 \xi^4 + \ldots\right)(\partial \xi)^2 + m^2\left(\xi^2 + b_1 \xi^4 + \ldots\right) + \text{fermions}\Big]$$

UV finite theory

on cylinder $0 < \tau < T$, $T \to \infty$

$0 < \sigma < 2\pi L$, $L = 1$

$$\Gamma_\ell = T\left(E_{2\text{-}d}\right)_\ell = \frac{1}{(\sqrt{\lambda})^{\ell-1}} \cdot V_{2d} \cdot m^2 \cdot f_\ell(mL)$$

$V_{2d} = T \cdot L$

$$f_\ell(mL)\Big|_{mL \to \infty} \to b_\ell = \text{finite}$$

$$\left(E_{2\text{-}d}\right)_\ell \to \frac{b_\ell}{(\sqrt{\lambda})^{\ell-1}}\, m^2 \to c_\ell \kappa^2$$

if $m \sim \kappa \sim \ln s \gg 1$

No $\ln^n s$ $n > 1$ terms!

as on gauge-theory side

Non-trivial check of AdS/CFT

String rotating in AdS_5 and boosted in S^5: $E = E(J, S)$

Flat space : $E = \sqrt{J^2 + \frac{2}{\alpha'} S}$

momentum spin

$AdS_5 \times S^5$: $E(J,S) \rightarrow J + \frac{\lambda S}{J} + \ldots$, $\; S + J^2 + \ldots$, $\; S + \ln S + \ldots$

$J \gg S$: classical spin can be built out of string oscillators near $E=J$ state

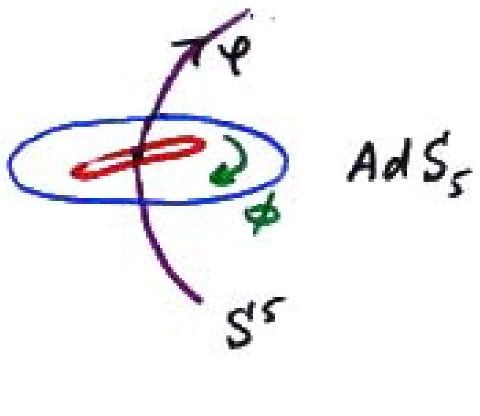
classical / quantum "overlap"

Solution:

$t = \kappa\tau, \quad \phi = \omega\tau, \quad \varphi = \nu\tau$

$\rho = \rho(\sigma)$: $\rho'^2 = \kappa^2 \cosh^2\rho - w^2 \sinh^2\rho - \nu^2$

$\omega = \omega(\kappa, \nu), \quad \kappa = \kappa(s)$



$S = \sqrt{\lambda}\, s$

$J = \sqrt{\lambda}\, \nu$

$E = \sqrt{\lambda}\, \mathcal{E}(s, \nu)$

Short string limit:

$E \simeq \sqrt{J^2 + 2\sqrt{\lambda}\, S}$ as in flat space

$\nu \gg S$:

$$\boxed{E \simeq J + S + \frac{\lambda}{2J^2} S + \dots}$$

compare to quantum correction to $E = J$ case:

$$E = J + \sum_n \sqrt{1 + \frac{\lambda n^2}{J^2}}\, N_n + O\left(\frac{1}{\sqrt{\lambda}}\right)$$

Leading Regge trajectory :

$|\psi\rangle = (a_1^+ a_{-1}^+)^S |0\rangle$: $n = 1$, $N_1 = S$

$$\boxed{E \simeq J + \left(1 + \frac{\lambda}{2J^2} + \dots\right) S + \dots}$$ same !

cf. $P^-_{\text{flat}} = -\frac{1}{\alpha' p^+} \sum_n |n| N_n \sim S$

Linear term in S: comes from curvature of AdS (mass of pw oscillators)

Checks consistency of semiclassical approach to spectrum

Gauge-theory operators: BMN Russo

$$\sum_{\ell_1, \dots, \ell_S = 0}^{J} \mathrm{Tr}\left(\dots Z D_{m_1} Z \dots Z D_{m_S} Z \dots\right) e^{\frac{2\pi i}{J}(\ell_1 + \dots + \ell_S)}$$

ℓ_i - position of D_{m_i}

Gauge-theory test of string prediction?

$$\Delta = J + S + \frac{\lambda}{2J^2} S + \dots$$

$J \gg S$ 1-loop in gauge theory

Long string limit: $S \gg 1$

$\nu \ll \ln S$: $$E \simeq S + \frac{\sqrt{\lambda}}{\pi} \ln \frac{S}{\sqrt{\lambda}} + \frac{\pi J^2}{2\sqrt{\lambda} \ln \frac{S}{\sqrt{\lambda}}} + \dots$$

$\nu \gg \ln S$: $$E \simeq S + J + \frac{\lambda}{2\pi^2 J} \ln^2 \frac{S}{J} + \dots$$

No $\ln S$ - term

Oscillation modes:

tower of string oscillators on top of $E(S,J)$ vac

same S and J but different oscillator #'s

generalization of $$E = J + \sum_n \sqrt{1 + \frac{\lambda n^2}{J^2}}\, N_n$$

— " —

Other cases: rotation in AdS_5 and S^5 Russo

— " — BH in AdS_5 Armoni, Barbon, Petkou

$AdS_5 \times S^5/Z_M$ Mandal, Suryanarayana, Wadia

Non-conformal (near-AdS) generalization

How $E = J$ and $E = S + \ln S$

modified in non-conformal cases ?

expect time-dependence – running couplings

cf. Pando-Zayas et al

$\dot{E} \neq 0$: corrections to anom. dimensions ?

First-order perturbation from conformal point

Example: fractional D3 on conifold

in near-$AdS_5 \times T^{1,1}$ region

Klebanov AT

$$ds^2 = h^{-1/2}(r)\, dx_m dx_m + h^{1/2}(r)\left(dr^2 + r^2 dT_{1,1}\right)$$

$$h = \frac{1}{r^4}\left(1 + \varepsilon \ln r\right) \qquad \varepsilon = \frac{3 g_s}{2\pi}\frac{M^2}{N}$$

$$ds^2 = \frac{dx_m dx_m + dz^2}{z^2} + dT_{1,1} + \varepsilon \ln r\,[\ \dots\ ] + O(\varepsilon^2)$$

$z = \frac{1}{r}$

$O(\varepsilon)$ correction to point-like string

rotating in $T_{1,1}$:

$$x_0 = \tan \nu\tau + \varepsilon\, T(\tau) \ , \qquad \varphi = \nu\tau + \varepsilon\, \Phi(\tau)$$

$$z = \frac{1}{\cos \nu\tau}\left[1 - \varepsilon\, Z(\tau)\right]$$

$$\cos^2\tau\,\dot{T} = -2Z + \tfrac{1}{2}f$$

$$\dot{\Phi} = -\tfrac{1}{2}f \qquad\qquad f = -\ln z(\tau)$$

$$\tan\tau\,\dot{Z} = \frac{1}{\cos^2\tau}Z - \tfrac{1}{2}f$$

Compute energy (in global coordinates):

$$E = J + \varepsilon\, h_1(\tau) + O(\varepsilon^2)$$

$$h_1 = 0 \quad \text{for any } f(\tau)$$

Vanishing of leading $\frac{M^2}{N}$ correction to dimension of $Tr\,(A_2 B_2)^J$ operator in $SU(N)\times SU(N+M)$ theory

cf. Δ for $Tr\,(A_i B_j) = -\tfrac{1}{2} + \frac{\zeta(3)}{\sqrt{\lambda}}\left(\frac{M}{N}\right)^4 + \ldots$

Frolov
Klebanov
AT

Rotation in the boundary plane:

Similar leading-order analysis

$$E = S + \left(\frac{\sqrt{\lambda}}{\pi} + \varepsilon\,\gamma(\tau) + O(\varepsilon^2)\right)\ln S + \ldots$$

$\gamma \neq 0$ reflects running of gauge couplings

Results :

- Point-like string rotating in S^5 : $J = \sqrt{\lambda}\,\nu$

$$E = \sqrt{\lambda}\,\nu + \sqrt{1+\frac{n^2}{\nu^2}}\,N_n + \frac{1}{\sqrt{\lambda}\,\nu}F\left(\frac{1}{\nu^2},n\right) + O\left(\frac{1}{\lambda}\right)$$

$\hookrightarrow 0$ for $J\to\infty$, $\lambda' = \frac{\lambda}{J^2}$ = fix

Leading 2-loop string correction $\sim \frac{1}{J}\cdot\frac{\lambda}{J^2}$

Compare to 1-loop in gauge theory?

- Folded string rotating in AdS_5 : $S = \sqrt{\lambda}\,s$

$$\underset{s\gg 1}{E} = \sqrt{\lambda}\,s + f_1(\lambda)\ln s + f_2(\lambda)\ln^2 s + \ldots$$

$$f_1(\lambda) = \frac{\sqrt{\lambda}}{\pi} + a_1 + \frac{a_2}{\sqrt{\lambda}} + \ldots, \qquad a_1 \simeq -\frac{3\ln 2}{\pi}$$

$f_2(\lambda) = 0$ in agreement with gauge theory expectations

- Non-conformal generalization :

fractional D3 on conifold geometry in near $AdS_5 \times T^{1,1}$ region:

- $E = J + \varepsilon\, h(\tau) + O(\varepsilon^2)$

$\varepsilon \sim \frac{g_s M^2}{N}$, $h = 0$: $SU(N)\times SU(N+M)$ gauge theory explanation?

- $E = S + \left(\frac{\sqrt{\lambda}}{\pi} + \varepsilon\,\gamma(\tau) + \ldots\right)\ln S + \ldots$ running couplings

Conclusions

- GS action – practical tool for exploring string / gauge theory duality
  especially in semiclassical expansion
- Progress in clarification of part of string spectrum: (J, S) - sector
  in and beyond leading order in $\frac{1}{\sqrt{\lambda}}$
- Extension to non-conformal (and non-supersymmetric) cases

e.g. expected universality of $E = S + \ln S$