Non-Supersymmetric Attractors

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Outline

1. Motivation & Introduction

2. General Ideas

3. An Example: Type II on CY3

4. Rotating Black Holes: Entropy Function

5. Connections to Microscopic counting

6. Conclusions.

References:

hep-th/0507093, hep-th/0511117

hep-th/0512138, 0606244, 0611143

hep-th/0705.4554

Collaborators:

K. Goldstein, N. Iizuka, R. Jena, D. Astefanesci, S. Nampuri, A. Dabholkar, G. Mandal, A. Sen.

Some Related References:

1) Ferrara, Gibbons, Kallosh, hep-th/9702103

2) Denef, hep-th/0005049, …

3) A. Sen, hep-th/0506177 …

4) Kallosh et.al., …

5) Ferrara et. al., …

6) Kraus and Larsen, 0506173, 0508218.

Some Related References Cont'd

7) Ooguri, Vafa, Verlinde, hepth/0502211

8) Gukov, Saraikin, Vafa, hepth/0509109, hep-th/0505204

9) Saraikin, Vafa, hep-th/0703214

Non-Supersymmetric Attractors

Motivations:

1. Black Holes:

Non-supersymmetric extremal black holes are a promising extension.

2. Flux Compactifications:

Interesting Parallels.

II. What is an Attractor?

•4 Dim. Gravity, Gauge fields F^a,

moduli, ϕ_i

$$
\begin{aligned}
S \;=\; \int d^4x\sqrt{-G}\big(R - 2g_{ij}\partial\phi^i\partial\phi^j \\
-f_{ab}(\phi^i)F^a_{\mu\nu}F^{b\ \mu\nu} \\
-\frac{1}{2}\tilde{f}_{ab}(\phi^i)F^a_{\mu\nu}F^b_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}\big)
\end{aligned}
$$

(Two derivative action)

Attractors: General Ideas

Attractor Mechanism: Extremal Black Holes have a universal near-horizon region determined only by the charges.

(Extremal Black Holes carry minimum mass for given charge).

Attractor:

- Scalars take fixed values at horizon. Independent of Asymp. Values (but dependent on charges).
- Resulting near-horizon geometry of form, $AdS_2 \times S^2$ also independent of asymptotic values of moduli.
- The near-horizon region has enhanced symmetry $SO(2,1) \times SO(3)$

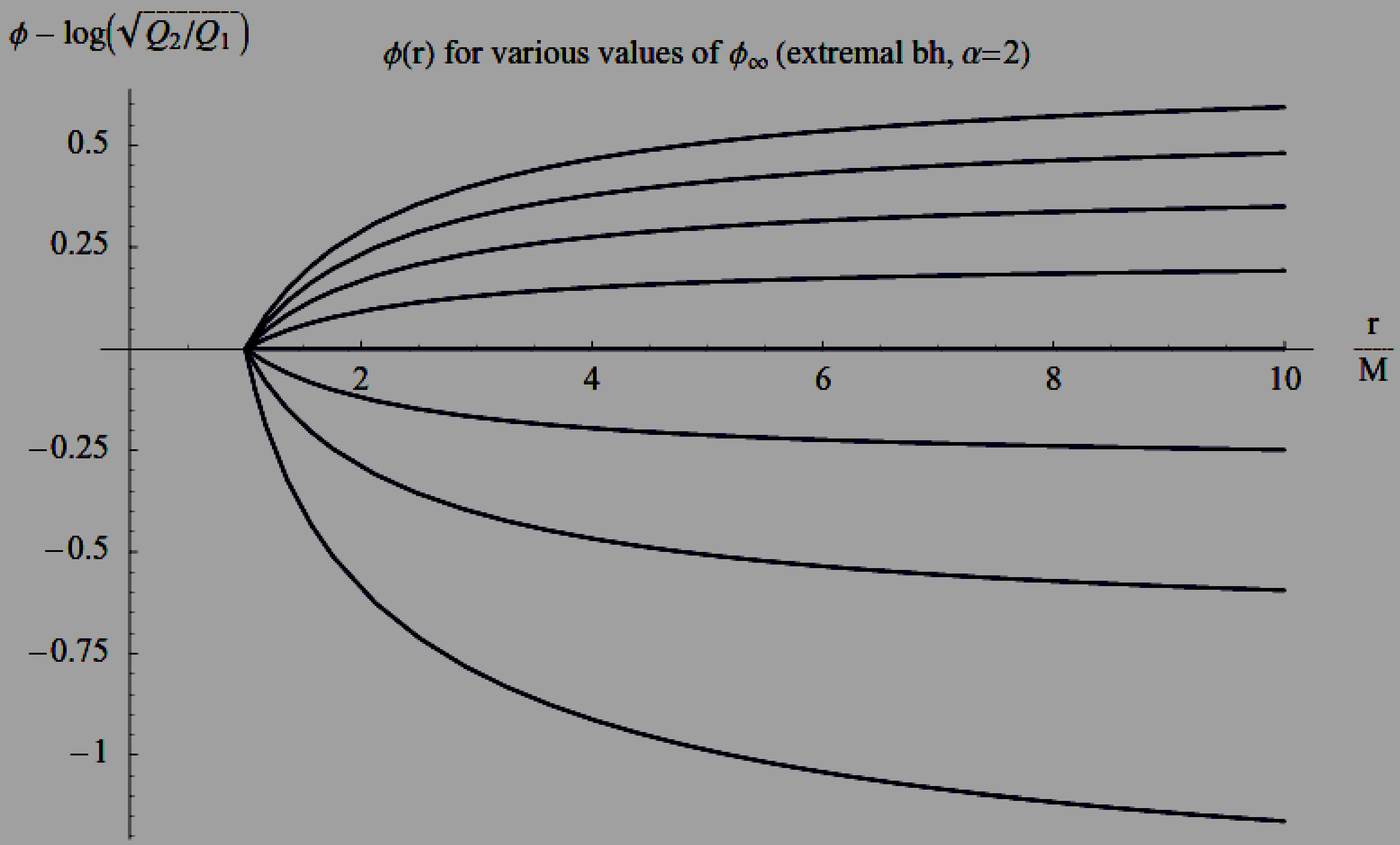


Figure 1: Attractor behaviour for the case $\gamma = 1$; $\alpha, -\tilde{\alpha} = 2$

Attractors:

- So far Mainly explored in Supersymmetric Cases.
- In this talk we are interesting in asking whether non-supersymmetric extremal black holes exhibit attractor behaviour.

Main Result:

- Extremal Black Holes generically exhibit attractor behaviour.

4 dimensions or higher, Spherically symmetric or rotating, Spherical or non-spherical horizon topology etc.

- Some conditions must be met, for attractor to exist and for it to be stable.

Spherically Symmetric Extremal Black Holes in 4 Dim:

Simplification: Reduces to a one dimensional problem.

$$\begin{aligned} S &= \int d^4x\sqrt{-G}(R - 2g_{ij}\partial\phi^i\partial\phi^j \\ &\quad -f_{ab}(\phi^i)F^a_{\mu\nu}F^{b\ \mu\nu} \\ &\quad -\frac{1}{2}\tilde{f}_{ab}(\phi^i)F^a_{\mu\nu}F^b_{\rho\sigma}\epsilon^{\mu\nu\rho\sigma}) \end{aligned}$$

Spherical Symmetric Case Cont'd:

$$ds^2 = -a(r)^2 dt^2 + a(r)^{(-2)} dr^2 + b(r)^2 d\Omega^2$$

$$\begin{aligned} F^a &= f^{ab}(Q_{ea} - \tilde{f}_{bc} Q^c_m) \frac{1}{b^2} dt \wedge dr \\ &\quad + Q^a_m \sin\theta d\theta \wedge d\phi \end{aligned}$$

Non-Supersymmetric Attractors

$$V_{eff}(\phi^i) \equiv f^{ab}(Q_{ea} - \tilde{f}_{ac}Q^c_m)(Q_{eb} - \tilde{f}_{bd}Q^d_m) + f_{ab}Q^a_m Q^b_m$$

$$S = \int dr((a^2 b)' b' - a^2 b^2 g_{ij}\phi^{i'}\phi^{j'} - \frac{V_{eff}}{b^2})$$

V_{eff} : Effective Potential

Ferrara, Kallosh, Gibbons, '96-'97; Goldstein, Iizuka, Jena, Trivedi, '05.

Conditions for an Attractor Cont'd

- There is an attractor phenomenon if two conditions are met by V_{eff}

1) It has a critical point:

$$\partial_i V_{eff}(\phi_{i0}) = 0$$

2) Critical point is a minimum:

$$M_{ij} \equiv \partial_i \partial_j V_{eff}(\phi_{i0}) > 0$$

(Stability)

Conditions For An Attractor Cont'd

- The attractor values moduli are ϕ_{i0}
- Attractor geometry: $AdS_2 \times S^2$

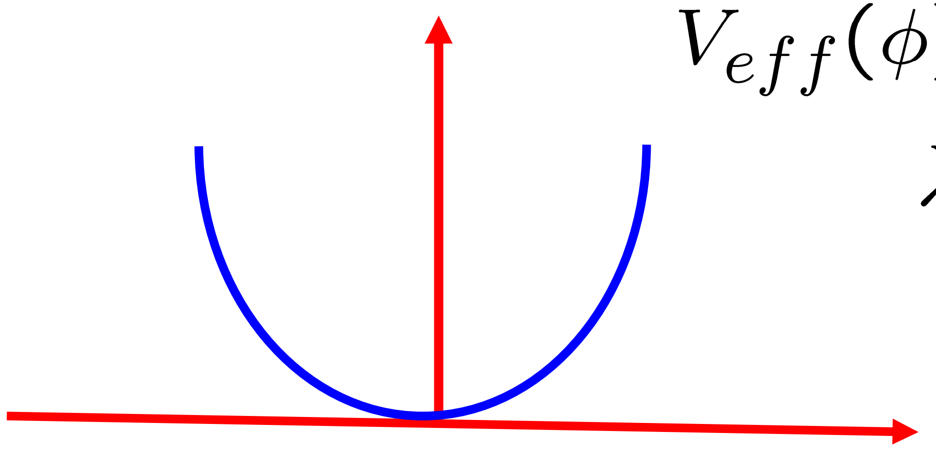
$$R^2_{AdS} = R^2 = V_{eff}(\phi_{i0})$$

- Entropy $S = \pi V_{eff}(\phi_{i0})$

Conditions for an Attractor Cont'd

If there are zero eigenvalues of M_{ij} :

- Critical point must be a minimum.
- Flat directions can be present.



$$V_{eff}(\phi) = V_{eff}(\phi_0) + \lambda(\delta\phi)^4$$

$$\lambda > 0$$

Analysis:

The essential complication is that the equations of motion are non-linear second order equations. Difficult to solve exactly.

Attractor Solution:

- If Scalars take attractor value at infinty they can be set to be constant everywhere.

$$\partial_r(a^2 b^2 \partial_r \phi_i) = \frac{\partial_i V_{eff}}{2b^2}$$

- Resulting solution: Extremal Reissner Nordstrom Black Hole, with near-horizon

$$AdS_2 \times S^2$$

Small Parameter: $\varepsilon = \phi_\infty - \phi_0$

Equations are second order but Linear in perturbation theory.

$\phi \approx 1/r, const.$

$$\phi_1(r) = \epsilon\left(\frac{r - r_H}{r}\right)^{\gamma_\pm}$$

$$\gamma_\pm = \frac{1}{2}\left(\pm\sqrt{1 + \frac{4M^2}{r_H^2}} - 1\right)$$

$$M^2 = V''(\phi_0)$$

Essential Point: For $M^2 > 0$ there is one solution which is well defined at the horizon and it vanishes there.

Conditions for an Attractor Cont'd

- This first order perturbation in moduli sources a second order perturbation in the metric, and so on.
- A solution can be constructed to all orders in perturbation theory.
- It shows attractor behaviour as long as the two conditions mentioned above are met.

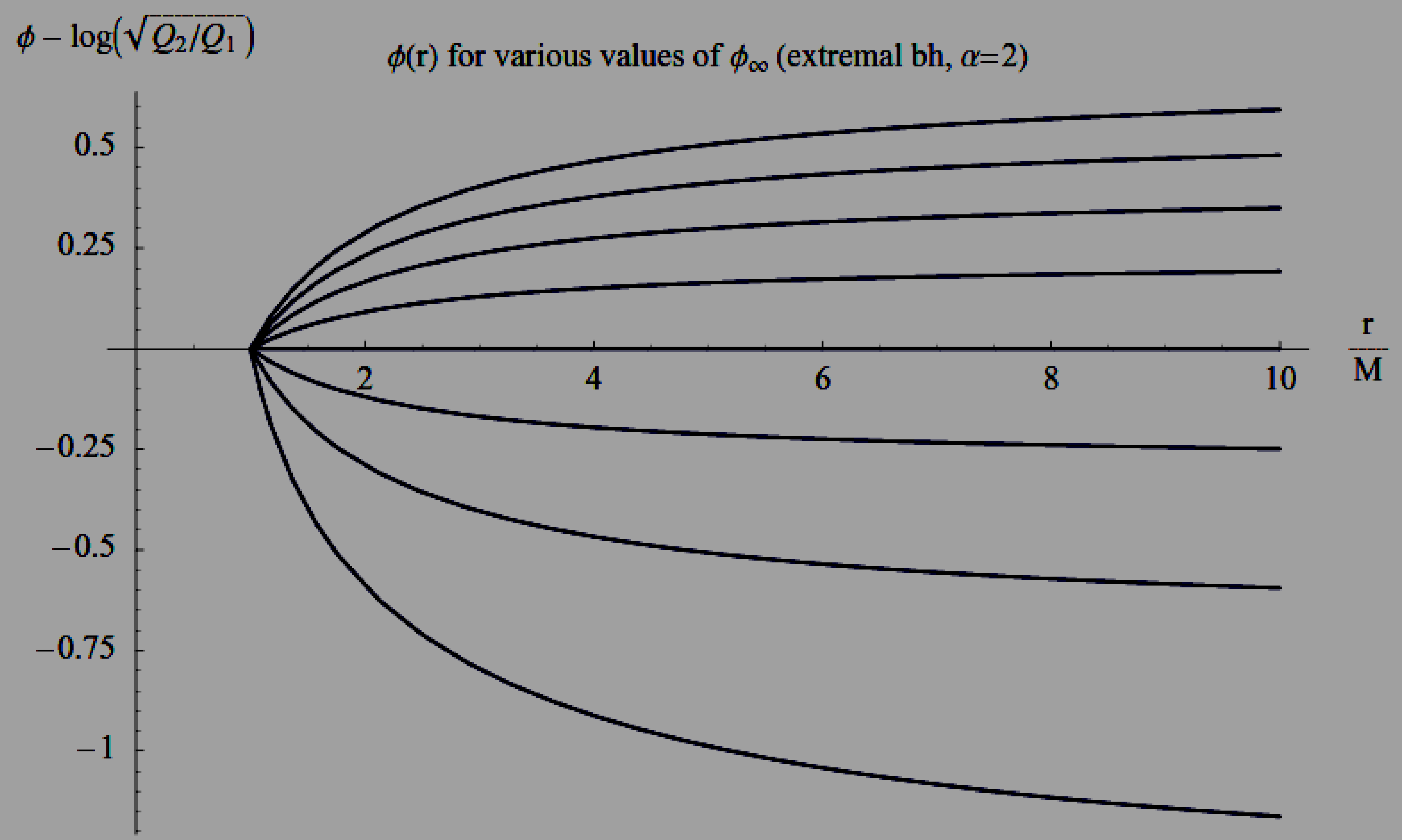
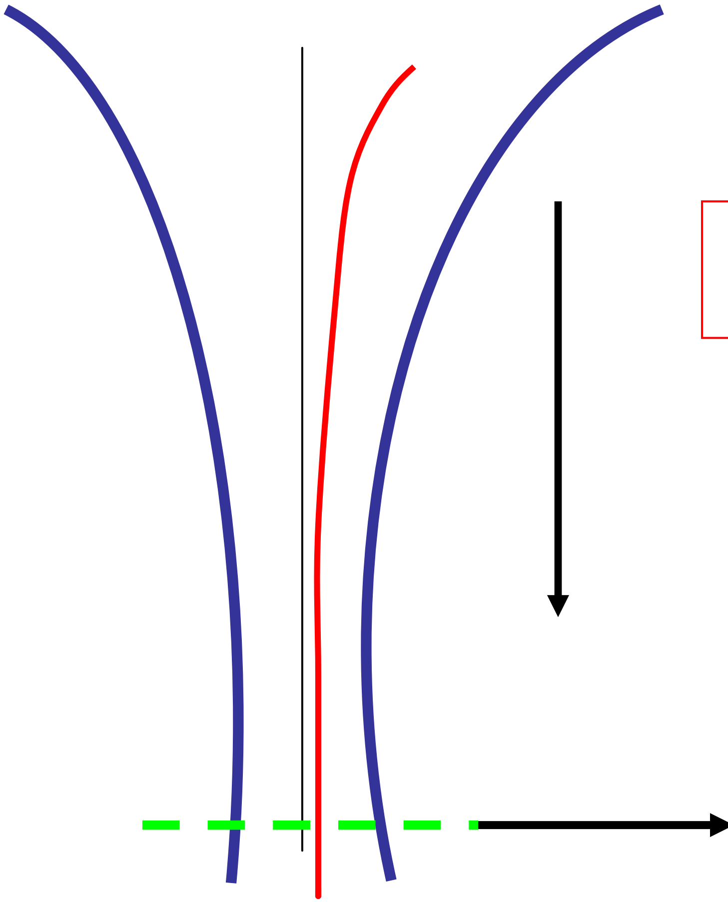


Figure 1: Attractor behaviour for the case $\gamma = 1$; $\alpha, -\tilde{\alpha} = 2$

Non-extremal Black Holes : Not attractors

I)



τ :Proper distance

Non-extremal horizon

Extremal case horizon at $\tau \longrightarrow \infty$

An Example: Type IIA On Calabi-Yau Threefold.

Vector multiplet moduli fixed by attractor mechanism.

Hypermultiplets not fixed (at two derivative level).

Tripathy, S.P.T., hep-th/051114

$$V_{eff} = e^K [g^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} + |W|^2]$$

$$F = D_{abc} \frac{X^a X^b X^c}{X^0}, a = 1, \cdots, N$$

$$W = q_0 X^0 + q_a X^a - P^a \partial_a F - P^0 \partial_0 F$$

$$K = -\ln((X^A)^* \partial_A F - X^A (\partial_A F)^*)$$

q_0, q_a, p^a, p^0 are the $D0, D2, D4, D6$ Brane charges carried by the black hole.

IIA On CY_3 Cont'd

- E.g: D0-D4 Black Hole, Charges, q_0, p^a
- Susy Solution: $$\frac{X^a}{X^0} = ip^a\sqrt{\frac{q_0}{D}}$$
- Non-susy solution $$\frac{X^a}{X^0} = ip^a\sqrt{\frac{-q_0}{D}}$$
- Entropy: $$S = 2\pi\sqrt{|Dq_0|}$$ $$D = D_{abc}p^a p^b p^c$$

Stability

Nampuri, Tripathy, S.P.T. 0705.4554

- N Vector multiplets
- N+1 (Real) directions have positive $(mass)^2$
- N-1 (Real) Directions have vanishing mass. Stability along these needs to be examined further.
- Hypermultiplets are flat directions.

Stability Cont'd

•Quartic Terms are generated along the N-1 massless directions.

•These consist of two terms with opposite signs. As the charges (and intersection numbers) vary, the attractor can change from stable to unstable.

•Rich Structure of resulting attractor flows.

• Similar Story when D6-brane charge included

Stability Cont'd

$$V_4 \propto [-(D_{lm}\delta\xi^l\delta\xi^m)^2 + (\frac{-D}{4q_0})g^{a\bar{b}}(D_{alm}\delta\xi^l\delta\xi^m)(D_{bpq}\delta\xi^p\delta\xi^q)]$$

$$D = D_{abc}p^a p^b p^c$$

$$D_{ab} = D_{abc}p^c$$

Rotating Black Holes : Entropy Function

•Basic Idea: Focus on the near horizon region.

• Assume it has $AdS_2 \times S^2$ symmetry.

• This imposes restrictions on metric and moduli.

• Resulting values obtained by extremising, $\mathcal{E}(\vec{\phi}, \vec{v}, \vec{Q_e}, \vec{Q_m})$

Sen '05

• Shows horizon region universal.

Rotating Black Holes : Entropy Function

$$dS^2 = v_1(-r^2 dt^2 + \frac{dr^2}{r^2}) + v_2 d\Omega_2^2$$

$$\phi_i = u_i$$

$$f(\vec{e}, \vec{Q}_m, \vec{\phi}, \vec{v}) = \int d\theta d\phi \sqrt{-g}\ \mathcal{L}$$

$$\mathcal{E}(\vec{\phi}, \vec{v}) = \vec{e} \cdot \vec{Q}_e - f(\vec{e}, \vec{Q}_m)$$

Entropy Function Cont'd

At the horizon $\partial_i \mathcal{E}(\vec{\phi}_0, \vec{v}) = 0$

The entropy is the critical value:

$$\mathcal{E}(\vec{\phi}_0, \vec{v})$$

•In The Two derivative case:

$$\mathcal{E} = \pi[v_2 - v_1) + \frac{v_1}{v_2} V_{eff}(\phi_i)]$$

•Main Advantage: Higher derivatives can be included and give Wald's entropy.

•Stability conditions with higher derivative terms not well understood.

Entropy Function Cont'd

- The entropy function can have flat directions. The entropy does not change along these flat directions.

Rotating Attractors

- In rotating case we still assume that for an extremal black hole the near horizon geometry has an $AdS_2 \times U(1)$ symmetry .

- Now there is less symmetry, fields depend on the polar angle, $\phi(\theta)$

- Entropy function is functional of fields,

$$\mathcal{E}[\phi(\theta), \vec{v}(\theta), \vec{Q}_e, \vec{Q}_m, J]$$

- Horizon values are given by extremising $\mathcal{E}$.

Astefanesci, Goldstein, Jena, Sen, S.P.T., 0606244

- Essential point is the AdS_2 symmetry.

$$\phi(\theta) = \sum_l \phi_l P_l(\cos(\theta))$$

- Attractor mechanism fixes the ϕ_l 's and thus $\phi(\theta)$

- A similar argument should apply for extremal rings as well.

Goldstein, Jena;

Connections with Microscopic Counting

Interesting Features/Puzzles:

1. Huge ground state degeneracy (for large charge) without supersymmetry.

(Degeneracy approx.) $\frac{\delta M}{M} \sim \frac{1}{Q^{\#}}$

2. Attractor mechanism :Degeneracy not renormalised as couplings varied.

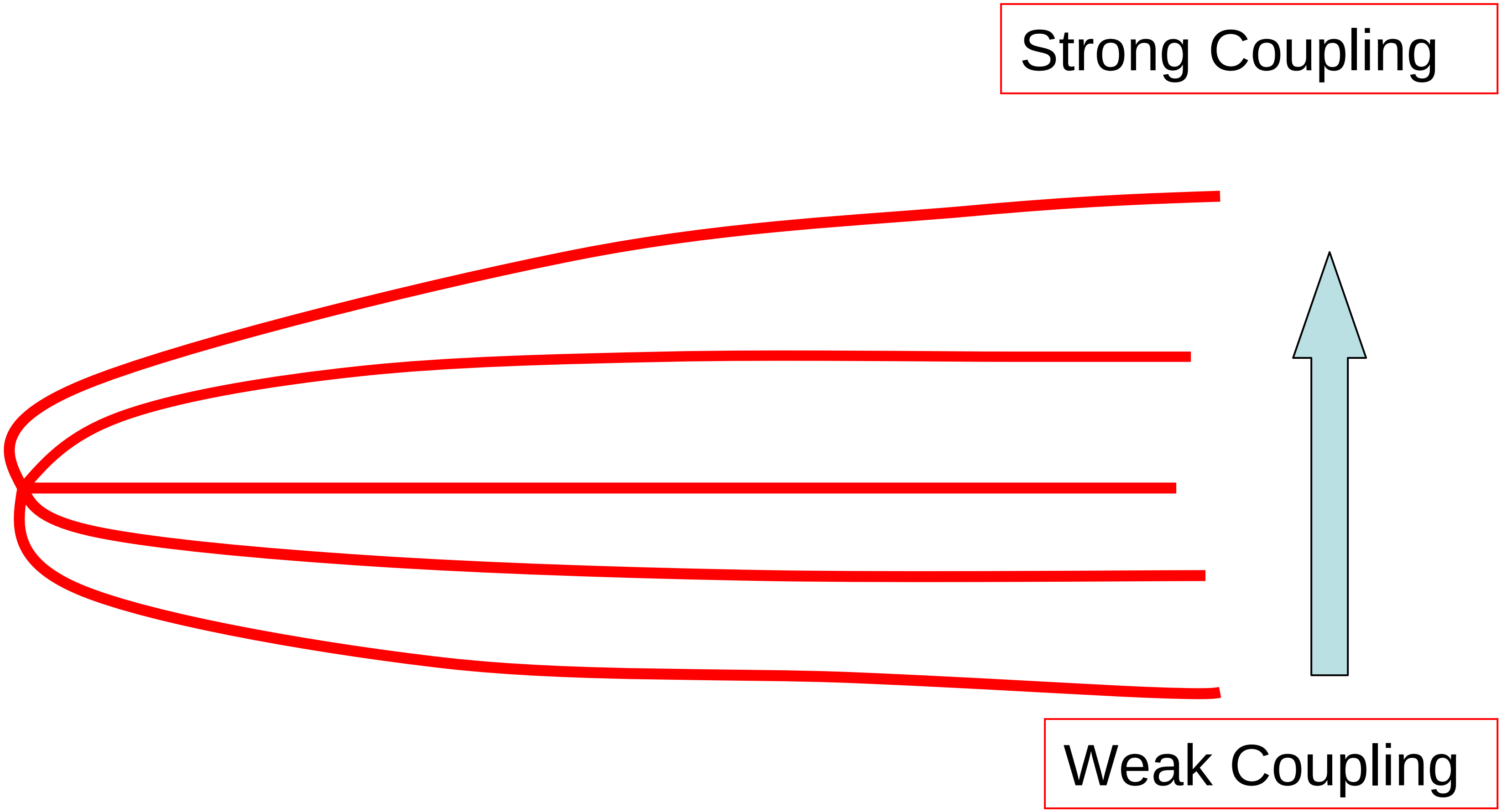
Connections with Microscopic Counting

3. In many non-susy cases weak coupling calculation of entropy agrees with Beckenstein-Hawking entropy.

What light can the attractor mechanism shed on this agreement?

Dabholkar, Sen, S.P.T., hep-th/0611143

Essential Idea



If Strong and Weak Coupling Regions lie in same basin of attraction, entropy will be the same.

Supersymmetric Case:

A) Microscopic calculation: $g_s Q \ll 1$

B) Supergravity: $g_s Q >> 1$

- If g_s flat direction of the entropy function. Then we can go from A) to B) Without changing entropy.

- Note: This argument applies to the entropy and not the index.

Some Assumptions:

1. No phase transitions: i.e., the same basin of attraction

2. Assume that extremal black holes correspond to states with minimum mass for given charge.

Non-Supersymmetric Case

- Do not expect exactly flat directions.
- However there can be approximately flat directions. This can explain the agreement of entropy upto some order.
- E.g. $D1 - D5 - p$ System in Type I.

Susy breaking $\frac{1}{Q}$ effect.

Dabholkar,'97

Non-Supersymmetric Case 'Contd

- Other Possibilities: Attractor Mechanism for fixed scalars.

- E.g.:Microscopic description arises by branes wrapping circle of radius R

- If R big states lie in 2-Dim conf. field theory

- Dual description is BTZ black hole in AdS_3

- Entropy can be calculated reliably in Cardy limit (Kraus and Larsen).

Non-Supersymmetric Case Con'td

- As asymptotic size of circle R reduced, states do not lie in the field theory anymore, dual description is not a black hole in AdS_3.
- But if R is not a flat direction, attractor mechanism tells us that the entropy cannot have changed in the process and must be the same.

Conclusions

1. Extremal Non-supersymmetric Black Holes generically exhibit attractor behaviour.

2. The resulting attractor flows have a rich and complicated structure.

3. There are interesting connections with microstate counting.

4. More Progress to come.

Moduli and metric approach attractor values determined by exponent:

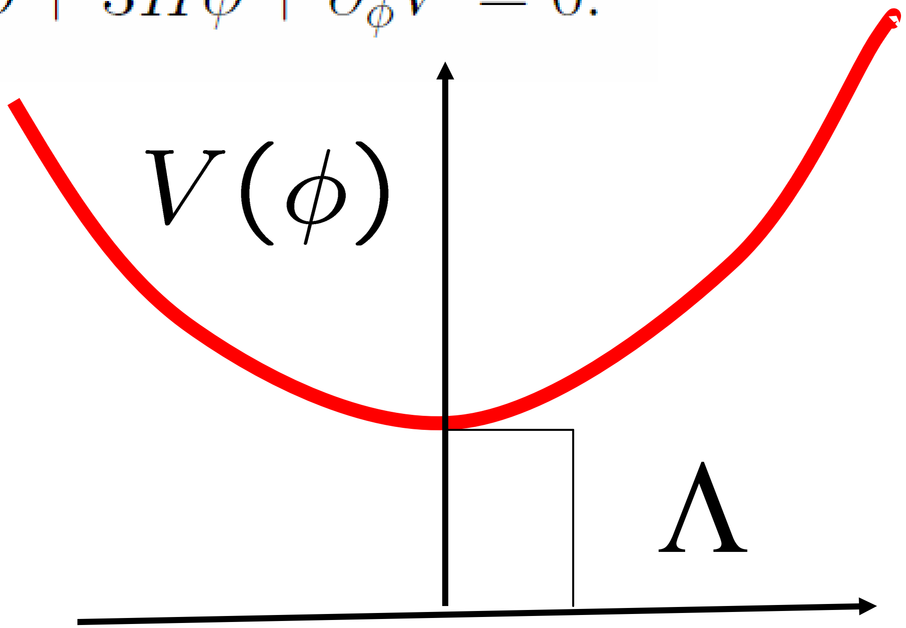
$$\delta\phi = Ae^{\gamma t}$$

$$\gamma = \frac{1}{2}(\sqrt{1+4M^2} - 1)$$

$$M^2 = \frac{\partial^2 V_{eff}(\phi_0)}{r_H^2}$$

II)Cosmology in an expanding universe:

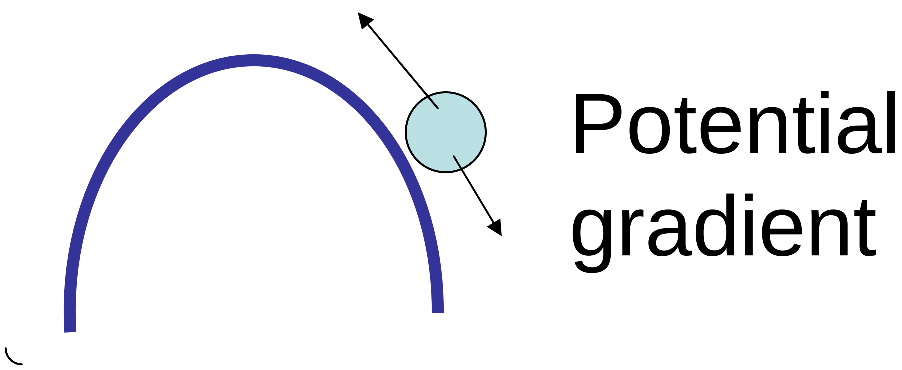
$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V = 0.$$



Friction term means system will settle to bottom of potential regardless of initial conditions This is the attractor.

Black Hole Attractor :

$$\delta\ddot{\phi} - \delta\dot{\phi} - \frac{m^2}{r_h^2}\delta\phi = 0$$



By choosing initial conditions we can arrange so that the field just comes to rest at the top in far future. This is attractor solution.

- As $r \longrightarrow r_H$ one solution is well behaved and for it $\phi \longrightarrow \phi_0$

- As $r \longrightarrow \infty$, both solutions are well behaved,

$$\phi \rightarrow C_0 + \frac{C_1}{r}$$

- In the attractor solution one tunes C_1 as a function of C_0 to match on to the well behaved solution near the horizon.

$$
\begin{aligned}
ds^2 &= v_1(\theta)(r^2 dt^2 + \frac{dr^2}{r^2}) + \beta^2 d\theta^2 \\
&\quad + \beta^2 v_2(\theta)(d\phi - \alpha r dt)^2
\end{aligned}
$$

$$
\Phi^a(\theta) = u_s^a(\theta)
$$

$$
\begin{aligned}
F^i &= (e_i - \alpha b_i(\theta)) dt \wedge dr \\
&\quad + \partial_\theta b_i(\theta) d\theta (d\phi - \alpha r dt)
\end{aligned}
$$

Rotating Attractors Cont'd

Extremising $\mathcal{E}$ gives attractor values.

$$\frac{\partial \mathcal{E}}{\partial \beta} = 0, \qquad \frac{\partial \mathcal{E}}{\partial e_i} = 0\,,$$

$$\frac{\delta \mathcal{E}}{\delta v_1(\theta)} = 0\,, \qquad \frac{\delta \mathcal{E}}{\delta v_2(\theta)} = 0,$$

$$\frac{\delta \mathcal{E}}{\delta u_s(\theta)} = 0, \qquad \frac{\delta \mathcal{E}}{\delta b_i(\theta)} = 0\,.$$