N=2 supergravity from generalized Calabi-Yau compactifications

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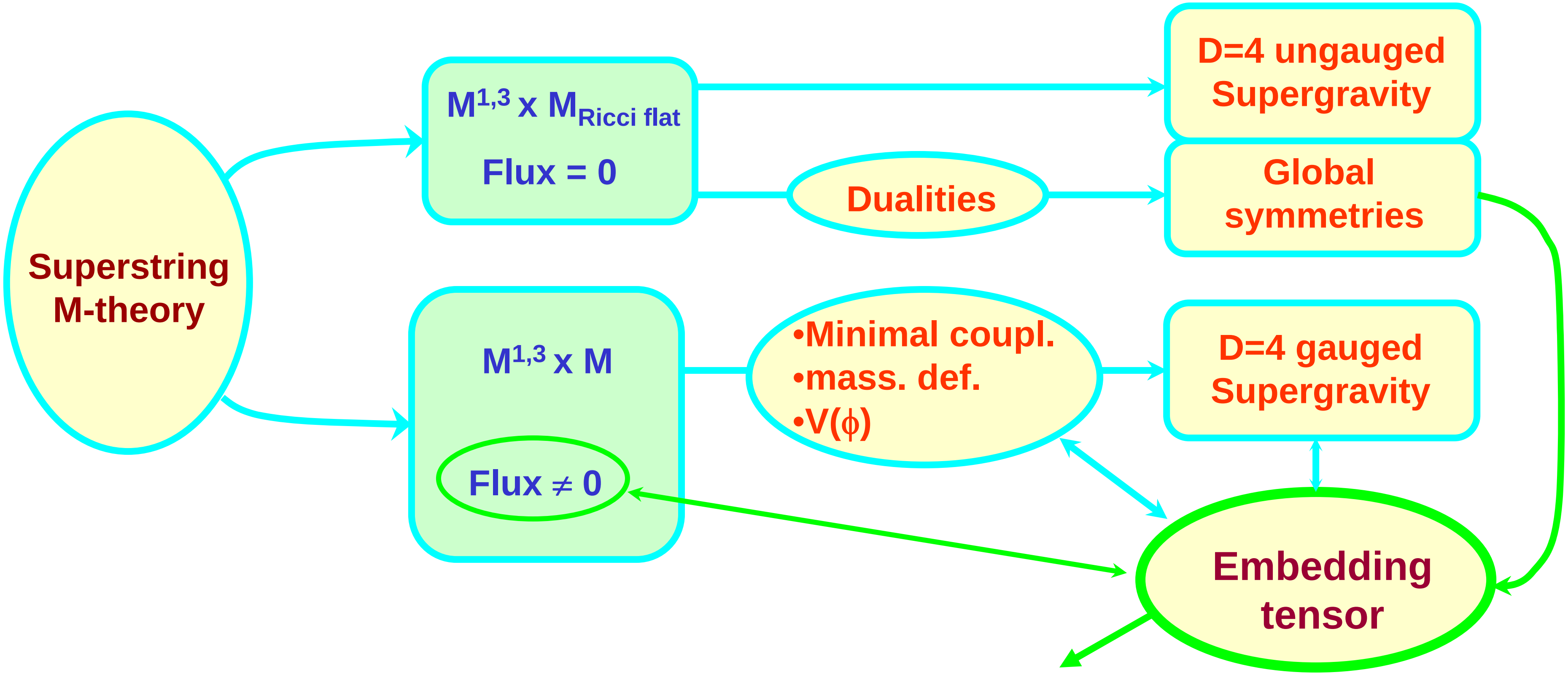
R. D’Auria, S. Ferrara and M.T. : 0701247
R. D’Auria, S. Ferrara, S. Vaula’ and M.T. : 0410290, 0412063

Introduction

• Superstring/M-theory flux compactifications: moduli fixing, spontaneous Susy breaking, cosmological constant etc… [Graña 0509003; Douglas, Kachru 0610102]

• D=4 Supergravity from Superstring/M-theory:



• Issue: Unveil web of dualities underlying flux vacua

Type IIA on CY_3 $\longleftrightarrow$ **Mirror symmetry** $\longleftrightarrow$ Type IIB on $\tilde{CY}_3$

Equivalent low-energy ungauged N=2 SUGRAS in D=4

- Extend mirror symmetry to the presence of fluxes.

- Mirror symmetry as T-duality along T^3-fiber in Y_3

[Strominger, Yau, Zaslow, 9606040]

$$R-R: \quad F_p \longleftrightarrow F_{p\pm1}$$

$$NS-NS: \quad H_3 \longleftrightarrow \begin{cases} \text{geometric fluxes} \\ \text{non-geometric fluxes} \end{cases}$$

[Micu, Palti,Tasinato , 0701173]
[Graña,Louis,Waldram , 0612237]
[Kashani-Poor,Minasian , 0611106]
[Graña,Louis,Waldram , 0505264]
[Fidanza,Minasian,Tomasiello , 0311122]
[Gurrieri,Micu , 0212278]
[Louis and A.~Micu , 0202168]

- Constructing corresponding mirror-covariant N=2 gauged low energy supergravity

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Fluxes = Embedding tensor

Type II superstring theories

- Type II Theories in D=10: 32 supercharges

- Bosonic field content:
  - NS-NS sector: $g_{\hat{\mu}\hat{\nu}},\ \hat{B}_{(2)},\ \varphi$
  - R-R sector:
    - $\hat{C}_0,\ \hat{C}_2,\ \hat{C}_4$ — II B
    - $\hat{C}_1,\ \hat{C}_3$ — II A

- Field strengths

II B:

$$\begin{aligned} \hat{H}_3 &= d\hat{B}_2 \\ \hat{F}_3 &= d\hat{C}_2 - C_0\, d\hat{B}_2 \\ \hat{F}_5 &= d\hat{C}_4 + \hat{B}_2 \wedge d\hat{C}_2 = {}^\star\hat{F}_5 \end{aligned}$$

II A:

$$\begin{aligned} \hat{H}_3 &= d\hat{B}_2 \\ \hat{F}_2 &= d\hat{C}_1 \\ \hat{F}_4 &= d\hat{C}_3 - \hat{B}_2 \wedge d\hat{C}_1 \end{aligned}$$

Low-energy N=2 SUGRA

- Theory has 8 supercharges and bosonic sector consists in:

Grav. mult.	h_2 vector mult.			
$\begin{pmatrix} g_{\mu\nu} \\ A^0_\mu \end{pmatrix}$	$\begin{pmatrix} A^i_\mu \\ w^i \end{pmatrix}$	$\begin{pmatrix} \tilde{\zeta}_0 \\ \zeta^0 \\ a + i\phi \end{pmatrix}$	$\begin{pmatrix} \tilde{\zeta}_a \\ \zeta^a \\ z^a \end{pmatrix}$	h_1 +1 hyper-mult.

II B:
- **h_1=$h_{1,1}$, h_2=$h_{2,1}$**; $w^i = \tau^i$ (complex struct.); z^a=t^a (*Kaehler mod.*)

II A:
- **h_1=$h_{2,1}$, h_2=$h_{1,1}$**; w^i = t^i (Kaehler mod.); z^a=τ^a (*complex struct.*)
- $(\zeta^\Lambda) = (\zeta^0, \zeta^a),\ (\tilde{\zeta}_\Lambda) = (\tilde{\zeta}_0, \tilde{\zeta}_a)$, Λ =0,…, h_1, R-R axions
- **a :** scalar dual to $B_{\mu\nu}$; ϕ **:** D=4 dilaton.

- Scalar fields described by non-linear σ-model on $\mathcal{M}_{scal} = \mathcal{M}_{SK\,2} \times \mathcal{M}_{QK}$
- Quaternionic scalars z^a span submanifold $\mathcal{M}_{SK\,1} \subset \mathcal{M}_{QK}$
- $\text{Isom}(\mathcal{M}_{scal}) = \text{global symmetry of } S_{scal}$

$\mathcal{M}_{SK\,2}$ h_2 – dim. *Special Kähler* manifold locally described by

- $\{w^i\} \quad i = 1, \ldots, h_2$
- $\Omega_2(w) = (\Omega_2^r) = \begin{pmatrix} X^I(w) \\ F_I(w) \end{pmatrix}$; $I = 0, \ldots, h_2$

(Holomorphic section of **flat Sp(2h_2+2)** bundle)

$\Downarrow$

- -*Kähler potential*: $K_2(w, \bar{w}) \;=\; -\log\left[i\,(\bar{X}^I F_I - X^I \bar{F}_I)\right]$
- -*Covariantly holomorphic section*: $V_2(w, \bar{w}) = e^{\frac{K_2}{2}}\,\Omega_2(w) = \begin{pmatrix} L_2^I \\ M_{2I} \end{pmatrix}$

- Vector Lagrangian: $\mathscr{L}_{vec} \;=\; \mathrm{Im}(\mathcal{N}_2)_{IJ}\, F^I \wedge \star F^J + \frac{1}{2}\mathrm{Re}(\mathcal{N}_2)_{IJ}\, F^I \wedge F^J$

The (h_2+1)x (h_2+1) matrix $\mathcal{N}_2(w, \bar{w})$ defined in terms of V_2, rV_2

$\forall g_2 = \mathrm{Isom}(\mathcal{M}_{SK2}) \hookrightarrow Sp(2h_2 + 2, \mathbb{R})$:

$$\begin{cases} V_2(w, \bar{w}) \to g_2 \cdot V_2(w', \bar{w}') \\ \begin{pmatrix} F^I_{\mu\nu} \\ G_{\mu\nu I} \end{pmatrix} \to g_2 \cdot \begin{pmatrix} F^I_{\mu\nu} \\ G_{\mu\nu I} \end{pmatrix} \end{cases}$$

$\Downarrow$

Electric/magnetic duality symmetry of f.eqs. and Binachi ids.

- On h_1-dim. *Special Kähler* manifold $\mathcal{M}_{SK\,1}$ spanned by $\{z^a\}$ there is **flat Sp(2 h_1+2)** – structure. Define $\Omega_1(z),\, K_1(z,\bar z),\, V_1(z,\bar z),\, \mathcal{N}_1(z,\bar z)$

- Scalar Lagrangian:

$$\mathscr{L}_{scal} = -\overbrace{K_{i\bar\jmath}\, dw^i \wedge \star\, d\bar w^{\bar\jmath}}^{SK_2} - \overbrace{K_{a\bar b}\, dz^a \wedge \star\, d\bar z^{\bar b}}^{SK_1} - \frac{1}{4\phi^2}\, d\phi \wedge \star d\phi - \frac{1}{4\,\phi^2}\,(da - Z^A \mathbb{C}_{AB}\, dZ^B) \wedge \star (da - Z^A \mathbb{C}_{AB}\, dZ^B) + \frac{1}{2\,\phi}\, dZ^A\, \mathscr{M}(\mathcal{N}_1)_{AB} \wedge \star dZ^B$$

Sp(2 h_1+2)-covariant notation: $Z^A \equiv (\zeta^\Lambda, \tilde\zeta_\Lambda)$ $\quad \Lambda = 0, \dots, h_1$ $\quad \mathbb{C}_{AB} = \begin{pmatrix} 0 & 1\!\!1 \\ -1\!\!1 & 0 \end{pmatrix}$

$$\mathscr{M}(\mathcal{N}_1) = \begin{pmatrix} I + R I^{-1} R & -R I^{-1} \\ -I^{-1} R & I^{-1} \end{pmatrix} \xrightarrow{g_1 \in \mathrm{Sp}(2h_1+2)} g_1^T\, \mathscr{M}(\mathcal{N}_1)\, g_1$$

$I \equiv \mathrm{Im}\mathcal{N}_1$; $R \equiv \mathrm{Re}\mathcal{N}_1$

- ***Heisenberg algebra*** of quaternionic isometries $\xi^A t_A = \xi^\Lambda t_\Lambda + \xi_\Lambda t^\Lambda\,,\ \xi\, \mathscr{Z}$

$$\left.\begin{aligned} \delta\zeta^\Lambda &= \xi^\Lambda\,;\ \ \delta\tilde\zeta_\Lambda = \xi_\Lambda \ \Rightarrow\ \delta Z^A = \xi^A \\ \delta a &= \xi + \xi^\Lambda \tilde\zeta_\Lambda - \xi_\Lambda \zeta^\Lambda = \xi + \xi^A \mathbb{C}_{AB} Z^B \end{aligned}\right\} \Rightarrow \quad [t_A, t_B] = 2\,\mathbb{C}_{AB}\, \mathscr{Z}$$

Gauging: Introduce *abelian* local symmetry group $\mathscr{G} = e^G$

$$G = \{X_I\} \subset Heis = \{t_A, \mathscr{Z}\} :$$
$$[X_I, X_J] = 0$$

Global symmetry of $\mathscr{L}$ → **Local symmetry of $\mathscr{L}_g$**

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- Parameters of gauged $\mathscr{L}_g$ encoded in a single tensor θ (***embedding tensor***)

de Wit, Samtleben, M.T. : 0212239, 0507289

$$G \overset{\theta}{\hookrightarrow} Heis : \qquad X_I = \theta_I{}^A t_A + \theta_I \mathscr{Z} \quad \longleftrightarrow \quad A^I_\mu$$

- ***G abelian:***

$$\boxed{\theta_I{}^A \theta_J{}^B \mathbb{C}_{AB} = 0 \;\Leftrightarrow\; \theta_I{}^\Lambda \theta_{J\Lambda} - \theta_J{}^\Lambda \theta_{I\Lambda} = 0}$$

- Only scalars among $\{a, \zeta^\Lambda, \tilde{\zeta}_\Lambda\} = \{a, Z^A\}$ are charged under G
- Local G-invariance:

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu - A^I_\mu X_I \qquad \begin{cases} \partial_\mu Z^A \longrightarrow D_\mu Z^A = \partial_\mu Z^A - A^I_\mu \theta_I{}^A \\ \partial_\mu a \longrightarrow D_\mu a = \partial_\mu a - A^I_\mu \theta_I - A^I_\mu \theta_I{}^A \mathbb{C}_{AB} Z^B \end{cases}$$

$$\mathscr{L}_g^Q = -K_{a\bar{b}}\, dz^a \wedge \star d\bar{z}^{\bar{b}} - \frac{1}{4\,\phi^2}(Da - Z^A \mathbb{C}_{AB}\, DZ^B) \wedge \star(Da - Z^A \mathbb{C}_{AB}\, DZ^B) + \\ + \frac{1}{2\,\phi} DZ^A \mathscr{M}(\mathscr{N}_1)_{AB} \wedge \star DZ^B - \frac{1}{4\phi^2} d\phi \wedge \star d\phi$$

- Minimal couplings break SUSY.

Restoring SUSY ⟹

- Fermion/gravitino SUSY shifts
- Fermion/gravitino mass terms
- **$V(\phi) \neq 0$** (bilinear in f. shifts)

D'Auria, Ferrara, S. Vaula' and M.T. : 0412063

- Generalize construction:

$$\theta_I{}^\alpha = \{\theta_I{}^A, \theta_I\} \longrightarrow \theta_r{}^\alpha = \{\theta_I{}^\alpha, \theta^{I\,\alpha}\}$$

$Sp(2\,h_1+2)$ covariance

$Sp(2\,h_2+2)$ covariance X **$Sp(2\,h_1+2)$** covariance

de Wit, Samtleben, M.T. : 0507289

- Introduce magnetic components by dualizing **only** Peccei-Quinn scalars transforming under G

$h_2 < h_1$
rank $\theta_I{}^A = h_2$

$$Z^A = \theta_I{}^A Z^I + \hat{Z}^A$$

Covered by derivatives

$$G : \begin{cases} Z^I \to Z^I + \xi^I \\ \hat{Z}^A \to \hat{Z}^A \end{cases}$$

- Dualization $\begin{cases} a \longrightarrow B_{\mu\nu} \\ Z^I \longrightarrow B_{\mu\nu\, I} = \theta_I{}^A\, B_{\mu\nu\, A} \end{cases}$

- Introduce magnetic charges by replacing $F^I_{\mu\nu} \to \hat{F}^I_{\mu\nu} = F^I_{\mu\nu} + \theta^{IA}\, B_{\mu\nu A} + \theta^I\, B_{\mu\nu}$

  And modify the topological term into $-(\theta_I{}^A B_A + \theta_I\, B) \wedge (\hat{F}^I - \frac{1}{2}\theta^{IA} B_A - \frac{1}{2}\theta^I B)$

Dall'Agata, D'Auria, Sommovigo, Vaula', 0312210
D'Auria, Sommovigo, Vaula', 0409097

- Gauge invariance

$$
\begin{aligned}
A^I_\mu &\to A^I_\mu + d\xi^I - \theta^{IA}\,\Xi_{\mu A} - \theta^I\,\Xi_\mu \\
B_{\mu\nu I} &\to B_{\mu\nu I} + \partial_{[\mu}\Xi_{\nu]I} \;;\; B_{\mu\nu} \to B_{\mu\nu} + \partial_{[\mu}\Xi_{\nu]}
\end{aligned}
$$

Provided: $\boxed{\theta^{I[A}\,\theta_I{}^{B]} = \theta^{IA}\,\theta_I = 0} \Leftrightarrow \mathbb{C}^{rs}\,\theta_r{}^A\,\theta_s{}^B = \mathbb{C}^{rs}\,\theta_r{}^A\,\theta_s = 0$ *(locality)*

- **The Scalar potential:** $\Theta = (\theta_r{}^A) \equiv (\theta_I{}^A,\, \theta^{IA}) \;;\; \theta = (\theta_r) = (\theta_I,\, \theta^I)$

$$
\begin{aligned}
\mathscr{V} = & -\frac{1}{8\,\phi^2}(\theta + 2\,\Theta\,\mathbb{C}\,Z)^T\,\mathbb{C}^T\,\mathscr{M}(\mathscr{N}_2)\,\mathbb{C}\,(\theta + 2\,\Theta\,\mathbb{C}\,Z) - \\
& -\frac{2}{\phi}\overline{V}_1{}^T\tilde{\Theta}^T\mathscr{M}(\mathscr{N}_2)\,\tilde{\Theta}V_1 - \frac{2}{\phi}\overline{V}_2{}^T\,\Theta\,\mathscr{M}(\mathscr{N}_1)\,\Theta^T\,V_2 - \\
& -\frac{8}{\phi}\overline{V_1}{}^T\,\mathbb{C}^T\,\Theta^T\,(V_2\overline{V}_2^T + \overline{V}_2V_2^T)\,\Theta\,\mathbb{C}\,V_1
\end{aligned}
$$

Manifestly **Sp(2 h_2+2) x Sp(2 h_1+2)** invariant

$[\tilde{\Theta} = \mathbb{C}^T\Theta\mathbb{C}]$

Examples

II A on Y_3:

$i = 1, \ldots, h_{1,1}$
$I = 0, \ldots, h_{1,1}$
$\Lambda = 0, \ldots, h_{2,1}$

Form-fluxes:

(R-R)

$$\hat{F}_0 = c^0 \; ; \quad \hat{F}_2 = \ldots + c^i \omega_i$$
$$\hat{F}_4 = \ldots + c_i \tilde{\omega}^i \; ; \quad \hat{F}_6 = c_0 \,\mathrm{Vol} \qquad (c_r)$$

(NS-NS)

$$\hat{H}_3 = \ldots + e_0{}^\Lambda \alpha_\Lambda + e_{0\Lambda} \beta^\Lambda = \qquad ({e_0}^A)$$
$$= \ldots + e_0{}^A \alpha_A$$

Geometric fluxes:

$$d\alpha_A = \mathbb{C}_{AB}\, e_i{}^B \, \tilde{\omega}^i \qquad (e_i{}^A)$$
$$d\omega_i = e_i{}^A \alpha_A \; ; \quad d\tilde{\omega}^i = 0$$

Inspection of minimal couplings In l.e. theory:

$$d^2\omega_i = 0 = d\hat{H}_3 = d\hat{F}_2$$

$\Rightarrow$ $\theta_I{}^A = e_I{}^A \; ; \quad \theta_r = c_r$

$\Rightarrow$ $\theta_I{}^A \theta_J{}^B \mathbb{C}_{AB} = 0 = \theta^I \theta_I{}^A$

Special case: $e_{i\,0} \neq 0 \Leftrightarrow$ Half-flat manifold

[Gurrieri,Micu , 0212278]

II B on $\tilde{Y}_3$:

$$\hat{H}_3 = \ldots + e_{0\,i}\beta^i + m_0{}^i \alpha_i$$

$\rightarrow$ magnetic couplings

• In hep-th/0612237 low-energy modes on *SU(3)xSU(3)-structure* manifold Y described in terms of 2 b_-+2 odd forms α and 2 b_++2 even forms ω:

II B: $I = 0, \ldots, b_-$, $\Lambda = 0, \ldots, b_+$

$$(\alpha_r) = (\alpha_I, \alpha^I) \; ; \; (\omega_A) = (\omega_\Lambda, \omega^\Lambda)$$

$$d\alpha_r = Q^{(\mathrm{IIB;Y})}{}_r{}^A \omega_A$$

$$d\omega_A = \left[\tilde{Q}^{(\mathrm{IIB;Y})\ \mathrm{T}}\right]_A{}^r \alpha_r$$

$$Q^{(\mathrm{IIB;Y})}{}_r{}^A = \begin{pmatrix} e_I{}^\Lambda & e_{I\Lambda} \\ m^{I\Lambda} & m^I{}_\Lambda \end{pmatrix}$$

$$\tilde{Q} = \mathbb{C}^T Q\, \mathbb{C}$$

Relation to embedding tensor $\theta^{(\mathrm{IIB;Y})}{}_r{}^A = Q^{(\mathrm{IIB;Y})}{}_r{}^A \; ; \quad \theta_r = c_r$ (R-R flux)

II A: $I = 0, \ldots, b_+$, $\Lambda = 0, \ldots, b_-$

$$(\alpha_A) = (\alpha_\Lambda, \alpha^\Lambda) \; ; \; (\omega_r) = (\omega_I, \omega^I)$$

$$d\alpha_A = Q^{(\mathrm{IIA;Y})}{}_A{}^r \omega_r$$

$$d\omega_r = \left[\tilde{Q}^{(\mathrm{IIA;Y})\ \mathrm{T}}\right]_r{}^A \alpha_A$$

$$Q^{(\mathrm{IIA;Y})}{}_A{}^r = \begin{pmatrix} e_\Lambda{}^I & e_{\Lambda I} \\ m^{\Lambda I} & m^\Lambda{}_I \end{pmatrix}$$

Embedding tensor: $\theta^{(\mathrm{IIA;Y})}{}_r{}^A = \left[\tilde{Q}^{(\mathrm{IIA;Y})\ \mathrm{T}}\right]_r{}^A \; ; \quad \theta_r = c_r$ (R-R flux)

Integrability: $d^2=0$ $\Rightarrow$ $\mathbb{C}^{rs}\,\theta_r{}^A\,\theta_s{}^B = 0$ **(locality)** $\quad \mathbb{C}_{AB}\,\theta_r{}^A\,\theta_s{}^B = 0$ **(abelian G)**

Mirror symmetry

$$\Theta^{(\mathrm{IIA};Y)} = \Theta^{(\mathrm{IIB};\tilde{Y})} \quad \Leftrightarrow \quad Q^{(\mathrm{IIA};Y)} = \tilde{Q}^{(\mathrm{IIB};\tilde{Y})T}$$

Scalar potential:

$$\begin{aligned}\mathscr{V} &= -\frac{1}{8\,\phi^2}(c+2\,Q\,\mathbb{C}\,Z)^T\,\mathbb{C}^T\,\mathscr{M}(\mathscr{N}_2)\,\mathbb{C}\,(c+2\,Q\,\mathbb{C}\,Z) - \\ &\quad -\frac{2}{\phi}\overline{V}_1^T\tilde{Q}^T\mathscr{M}(\mathscr{N}_2)\,\tilde{Q}V_1 - \frac{2}{\phi}\overline{V}_2^T\,Q\,\mathscr{M}(\mathscr{N}_1)\,Q^T\,V_2 - \\ &\quad -\frac{8}{\phi}\overline{V_1}^{\,T}\,\mathbb{C}^T\,Q^T\,(V_2\overline{V}_2^T + \overline{V}_2V_2^T)\,Q\,\mathbb{C}\,V_1\end{aligned}$$

At the minimum $\frac{\partial\mathscr{V}}{\partial Z^A} = 0 \Rightarrow c_r + Q_r{}^A\mathbb{C}_{AB}\,Z^B = c_r + Q_r{}^A\mathbb{C}_{AB}\,\hat{Z}^B = 0$

$\hat{Z}_A \longrightarrow$ h_2+1massive, 2(h_1-h_2) flat directions $\Downarrow$

$$\mathscr{V}_{eff} = e^{K_s}\,(g^{\sigma\bar{\delta}}\,D_\sigma W\,D_{\bar{\delta}}\overline{W} - 3|W|^2)$$

$$W = V_2^T Q\mathbb{C}V_1 \;\; ; \quad e^{K_s} = \frac{4}{\phi}$$

Manifestly mirror symmetric: $1 \leftrightarrow 2$; $Q \leftrightarrow \tilde{Q}^T$ [Berglund,Mayr , 0504058]

Conclusions

- Proposed gauging of N=2 supergravity which describes Type II compactification on SU(3)xSU(3)-structure manifold with general fluxes.
  Gauged abelian subalgebra of Heisenberg algebra of quaternionic isometries.

- Geometric, non-geometric and form-fluxes enter the theory through the embedding tensor

$$(\theta_r{}^\alpha) = (\theta_r{}^A, \theta_r) \;\; ; \;\; \theta_r{}^A \theta_s{}^B \mathbb{C}_{AB} = \theta_r{}^A \theta_s{}^B \mathbb{C}^{rs} = \theta_r{}^A \theta_s \mathbb{C}^{rs} = 0$$

forming a mirror covariant picture.

- More general characterization of internal geometric structure allowing non abelian gauging in the low-energy D=4 N=2 supergravity .

- Understand what happens to the gauged theory when perturbative and non-perturbative corrections are taken into account.

[Kashani-Poor, Tomasiello, 0505208]

IIA on T^n ⟷ IIB on T'^n

T-duality along S^1

$$R_z \to R'_z = \frac{\alpha'}{R_z}$$

$$\gamma_y{}^z \equiv G_{yz} G^{zz} \quad \leftrightarrow \quad B_{yz} \quad (y \neq z) \ldots$$

[Buscher '85]

- Natural extension to the corresponding fluxes:

T- duality T^z :

$$H_{xyz} \equiv \partial_x B_{yz} \longleftrightarrow \partial_x \gamma_y{}^z \equiv T_{xy}{}^z$$

Torsion: Topology change

Scale of mass deformation

$$\frac{\alpha'}{R_x R_z R_y} \xrightarrow{R_z \to \frac{\alpha'}{R_z}} \frac{R_z}{R_x R_y}$$

- Further action of T-duality along y,x yield non-geometric fluxes $Q_x{}^{yz}$, R^{xyz}

[Hull, 0406102; Dabholkar, Hull, 0512005; Shelton, Taylor, Wecht, 0508133, 0607015]

Reduction on Calabi-Yau manifold Y_3 with SU(3)-holonomy

- Topology:

$$\{\alpha_\Lambda, \beta^\Lambda\} \equiv H^3(Y) \;\; ; \;\; \int_Y \alpha_\Lambda \wedge \beta^\Sigma = \delta_\Lambda^\Sigma \, ; \;\; \int_Y \alpha_\Lambda \wedge \alpha_\Sigma = 0 = \int_Y \beta^\Lambda \wedge \beta^\Sigma$$

$$(\Lambda, \Sigma = 0, \ldots, h_{2,1})$$

$$\{\omega_i\} \equiv H^{1,1}(Y) \;\; ; \;\; \{\tilde{\omega}^i\} \equiv H^{2,2}(Y) \;\; ; \;\; \int_Y \omega_i \wedge \tilde{\omega}^j = \delta_i^j$$

$$(i, j = 1, \ldots, h_{1,1})$$

- Reduction on Y_3 yields N=2, D=4 (ungauged) SUGRA

- Matter content: Expand D=10 fields in Y_3 harmonics

$$\left.\begin{aligned} \hat{B}_2 &= B_2 + b^i \omega_i \\ J &= v^i \omega_i \end{aligned}\right\} \longrightarrow \text{h}_{1,1} \text{ Complex Kaehler moduli} \qquad t^i = b^i + i\, v^i$$

$$\Omega = X^\Lambda \alpha_\Lambda - F_\Lambda \beta^\Lambda \longrightarrow \text{h}_{2,1} \text{ Complex structure moduli} \qquad \tau^a = \frac{X^a}{X^0}$$

$$(a, b = 1, \ldots, h_{2,1})$$

D=4 tensor B_2 dualized to axion a

D=10 dilaton and Vol(Y_3) combine in D=4 dilaton ϕ

- The R-R sector:

II B:
$$\begin{cases} C_0 = \zeta^0 \\ \hat{C}_2 = C_2 + \zeta^a \omega_a \\ \hat{C}_4 = A^I \wedge \alpha_I + \tilde{\zeta}_a \tilde{\omega}^a + \dots \end{cases}$$

Dualize C_2 into scalar $\tilde{\zeta}_0$

II A:
$$\begin{cases} C_1 = A^0 \\ \hat{C}_3 = A^i \wedge \omega_i + \zeta^\Lambda \alpha_\Lambda + \\ \quad + \tilde{\zeta}_\Lambda \beta^\Lambda \end{cases}$$

- D=4, N=2 supergravity in which we choose to dualize tensor fields into scalars

II B:

Grav. mult.	$h_{2,1}$ vec. mult.			
$\begin{pmatrix} g_{\mu\nu} \\ A^0_\mu \end{pmatrix}$	$\begin{pmatrix} A^i_\mu \\ \tau^i \end{pmatrix}$	$\begin{pmatrix} \tilde{\zeta}_0 \\ \zeta^0 \\ a + i\phi \end{pmatrix}$	$\begin{pmatrix} \tilde{\zeta}_a \\ \zeta^a \\ t^a \end{pmatrix}$	→ $h_{1,1}$ +1 hyper-mult.

II A:

Grav. mult.	$h_{1,1}$ vec. mult.			
$\begin{pmatrix} g_{\mu\nu} \\ A^0_\mu \end{pmatrix}$	$\begin{pmatrix} A^i_\mu \\ t^i \end{pmatrix}$	$\begin{pmatrix} \tilde{\zeta}_0 \\ \zeta^0 \\ a + i\phi \end{pmatrix}$	$\begin{pmatrix} \tilde{\zeta}_a \\ \zeta^a \\ \tau^a \end{pmatrix}$	→ $h_{2,1}$ +1 hyper-mult.

$$\mathscr{L}^{gauge}_{DQ} = \Big[\mathscr{L}^{gauge}_{Q}(da' \to \eta,\, dZ^I \to U^I) + + dB \wedge (\eta - da') + dB_I \wedge (U^I - dZ^I)\Big]\Big|_{\frac{\delta\mathscr{L}}{\delta\eta}=\frac{\delta\mathscr{L}}{\delta U^I}=0}$$

- Dual vector and scalar gauged Lagrangian

$$\begin{aligned}\mathscr{L}^{gauge}_{D} = {} & \mathrm{Im}(\mathcal{N}_2)_{IJ}\,\hat{F}^I \wedge \star \hat{F}^J + \frac{1}{2}\mathrm{Re}(\mathcal{N}_2)_{IJ}\,\hat{F}^I \wedge \hat{F}^J - K_{i\bar{\jmath}}\,dw^i \wedge \star d\bar{w}^{\bar{\jmath}} - \\ & -K_{a\bar{b}}\,dz^a \wedge \star d\bar{z}^{\bar{b}} - (\phi^2 - \Delta^{IJ}\,Z_I\,Z_J)\,H \wedge \star H + \frac{1}{4}\Delta^{IJ}\,H_I \wedge \star H_J - \Delta^{IJ}\,H \wedge \star H_I\,Z_J - \\ & -(H_I - 2\,H\,Z_I)\,\Delta^{IJ}\,\theta_J{}^A\,\Delta_{AB} \wedge d\hat{Z}^B + H \wedge \hat{Z}^A\,\mathbb{C}_{AB}\,d\hat{Z}^B - \\ & -(B_I + \theta_I\,B) \wedge (\hat{F}^I - \frac{1}{2}\theta^{IA}\,B_A - \frac{1}{2}\theta^I\,B) + d\hat{Z}^A\,\tilde{\Delta}_{AB} \wedge \star d\hat{Z}^B\end{aligned}$$

$$\begin{aligned}Z_I &\equiv \theta^A_I\mathbb{C}_{AB}Z^B = \theta^A_I\mathbb{C}_{AB}\hat{Z}^B \\ \Delta_{AB} &= \frac{1}{2\phi}\mathscr{M}(\mathcal{N}_1)\ ;\ \ \Delta_{IJ} = \theta_I{}^A\,\theta_J{}^B\,\Delta_{AB} \\ \tilde{\Delta}_{AB} &= \Delta_{AB} - \Delta^{IJ}\,\theta_I{}^C\,\theta_J{}^D\,\Delta_{CA}\,\Delta_{DB}\end{aligned}$$

- Scalar potential for the electric gauging

$$\mathscr{V} = -\frac{1}{2}(\mathrm{Im}\mathcal{N}_2)^{-1\,IJ}\,k^u_I k^v_J\,\omega^x_u\,\omega^x_v + 4\,(h_{uv} - \omega^x_u\,\omega^x_v)\,k^u_I\,k^v_J\,L^I\,\overline{L}^J$$

- Where the Killing vectors coincide with the embedding tensor

$$k^u_I = (\theta_I{}^A,\, \theta_I)$$

and the moment maps are: $\mathcal{P}^x_I = k^u_I\,\omega^x_u$

Use results from Ferrara, Sabharwal to write

$$(h_{uv} - \omega_u^x \omega_v^x) dq^u dq^v|_{Heis} = \frac{1}{\phi}\left(-\frac{1}{2} dZ^A \mathscr{M}(\mathscr{N}_1)_{AB} dZ^B - 4\left|V_1^A \mathbb{C}_{AB} dZ^B\right|^2\right)$$

$$\omega_u^x \omega_v^x dq^u dq^v|_{Heis} = \frac{1}{2\,\phi}\left[\frac{1}{2\phi}\left(d\phi^2 + (da + dZ^A \mathbb{C}_{AB} Z^B)^2\right) + 8\left|V_1^A \mathbb{C}_{AB} dZ^B\right|^2\right]$$

Which are manifestly $Sp(2h_1+2)$- invariant

In the presence of magnetic charges, the potential is obtained from the previous Expression for the electric gauging by replacing

$$(\mathrm{Im}\mathscr{N}_2)^{-1} \to \mathscr{M}(\mathscr{N}_2)$$

$$L\bar{L}^T \to V_2 \bar{V}_2^T$$

So we find

$$\begin{aligned}\mathscr{V} \;=\; & -\frac{1}{8\,\phi^2}(\theta + 2\,\Theta\,\mathbb{C}\,Z)^T\,\mathbb{C}^T\,\mathscr{M}(\mathscr{N}_2)\,\mathbb{C}\,(\theta + 2\,\Theta\,\mathbb{C}\,Z) - \\ & -\frac{2}{\phi}\,\overline{V}_1^{\,T}\,\tilde{\Theta}^T\,\mathscr{M}(\mathscr{N}_2)\,\tilde{\Theta} V_1 - \frac{2}{\phi}\,\overline{V}_2^{\,T}\,\Theta\,\mathscr{M}(\mathscr{N}_1)\,\Theta^T\,V_2 - \\ & -\frac{8}{\phi}\,\overline{V_1}^{\,T}\,\mathbb{C}^T\,\Theta^T\,(V_2\,\overline{V}_2^T + \overline{V}_2 V_2^T)\,\Theta\,\mathbb{C}\,V_1\end{aligned}$$