# Hyperbolic compactification of M-theory and de Sitter quantum gravity

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Central problem in theoretical cosmology: understand accelerated expansion of the universe.
Need to formulate quantum gravity in cosmological space-times.

Our approach to this problem, in 2 steps:

1. construct dS/cosmo solutions in string/M theory
2. understand them as deformations of AdS/CFT

By uplifting AdS/CFT from negative to positive c.c., this would give a holographic formulation of cosmology.

Goal of this talk: present a new simple mechanism to obtain de Sitter solutions in M-theory.
This uses crucially hyperbolic manifolds and Casimir energy.

Based on arXiv:2104.13380 w/De Luca and Silverstein

See Baumann/Silverstein, Kachru/Quevedo, … for other constructions and results

# A. Overview of the framework

In string/M-theory we look for sols of the form

$$M_d \times B_n \;\begin{cases} \nearrow \; M_d \text{ max sym } \mathbb{R}^{d-1,1},\ (A)dS_d \\ \searrow \; B_n \text{ internal space} \end{cases}$$

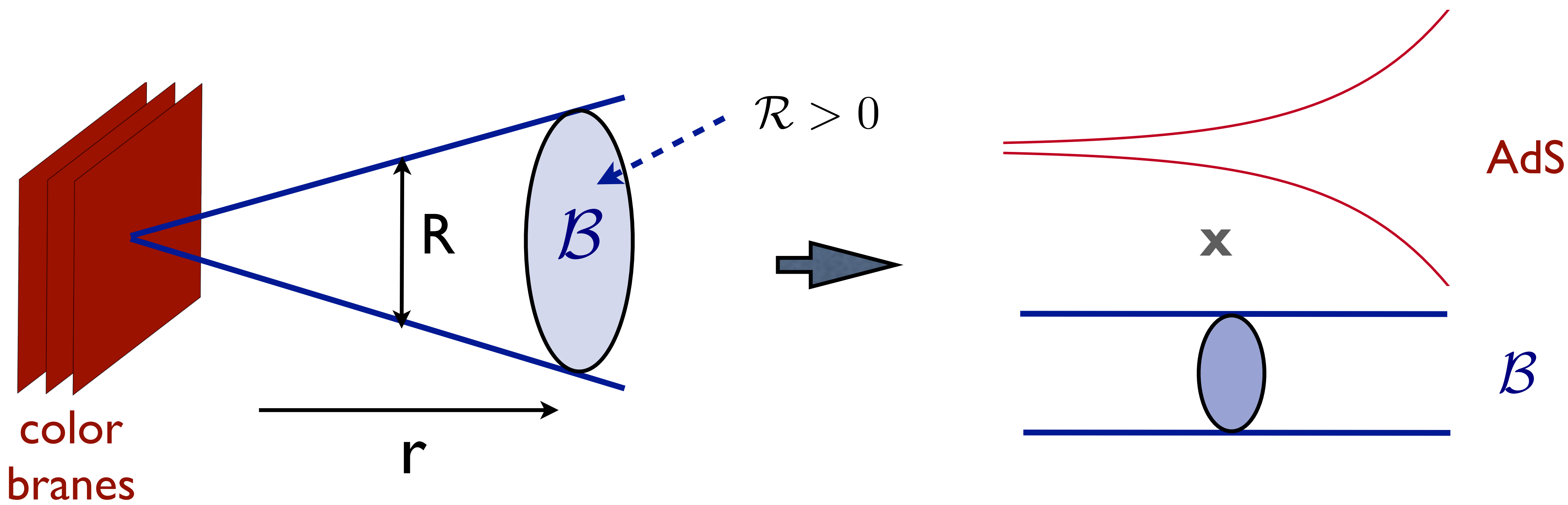
obtained by solving D=d+n dim EOMs.
Useful intuition comes from dimensionally reduced effective potential

$$V_{naive} \sim \int_B \sqrt{g^{(n)}} \left( -R^{(n)} + |F_p|^2 \right) + \dots$$

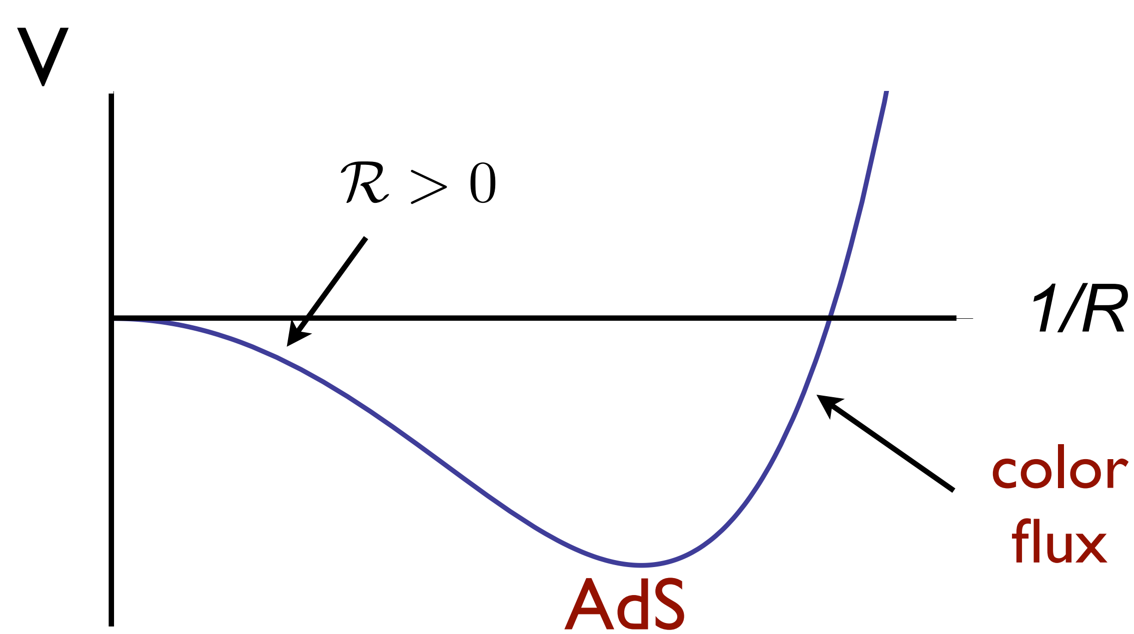
d-dim scalar fields descending e.g. from metric modes on Bn should be stabilized.

# AdS/CFT energetics

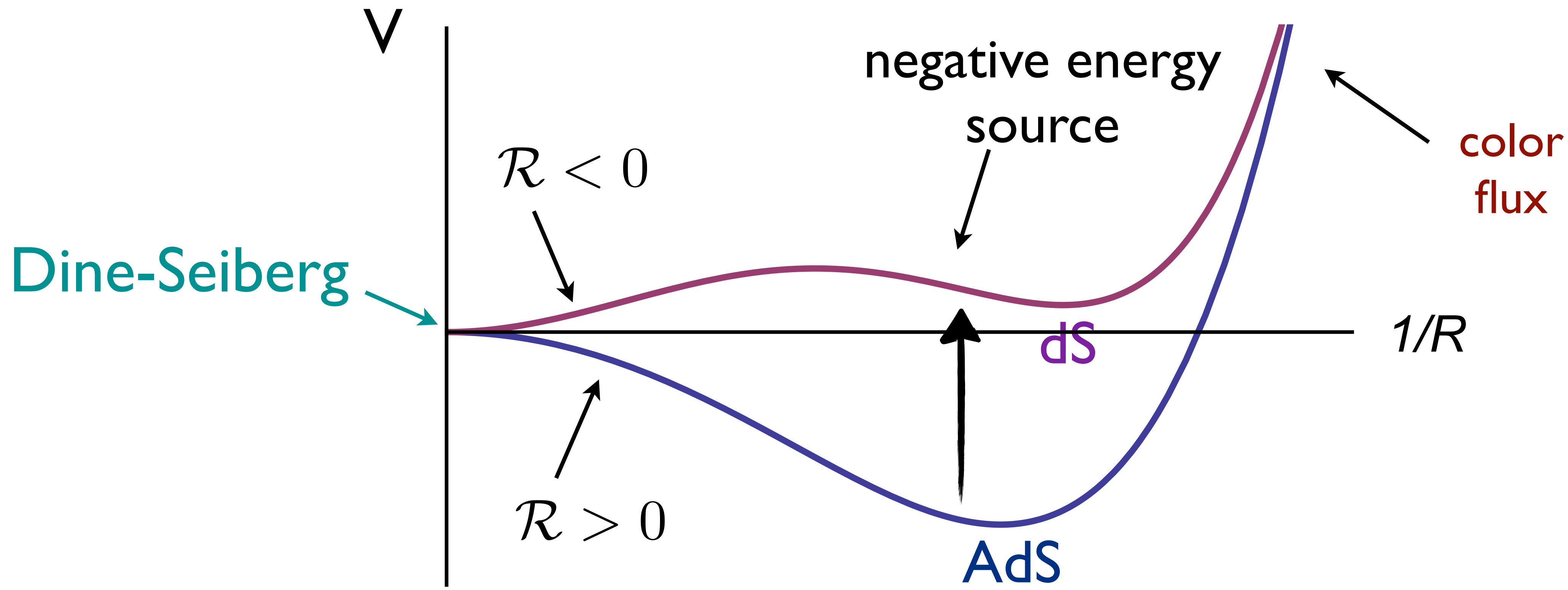
Let's summarize how AdS/CFT works from this perspective




From dimensionally reduced theory:



# Uplifting AdS/CFT




✓ Internal space of negative curvature
✓ Negative energy for the intermediate term in the potential
✓ May need other ingredients in place to stabilize all light fields

... can lead to metastable de Sitter solutions

Concrete brane construction for $dS_3$ given in Dong, Horn, Silverstein, GT arXiv:1005.5403

We will present $dS_4$ sols with minimal ingredients

# B. Setup and stabilization mechanism

We consider 11d SUGRA/M-theory

- ➡ compactified on finite volume hyperbolic manifold $\mathbb{H}_7/\Gamma$
- ➡ negative Casimir energy from small circles (antiperiodic boundary conditions for fermions)
- ➡ N units of F7 flux (M2 branes)

Finite volume hyperbolic manifold obtained from $\mathbb{H}_7$

$$ds^2_{\mathbb{H}} = \ell^2 \frac{dz^2 + d\vec{y}^{\,2}}{z^2}$$

by modding out by freely acting discrete subgroup $\Gamma \subset SO(1,n)$

✓ Mostow rigidity: hyperbolic structure is uniquely fixed by $\pi_1(B)$

All metric fluctuations (except for conformal mode) are gapped!

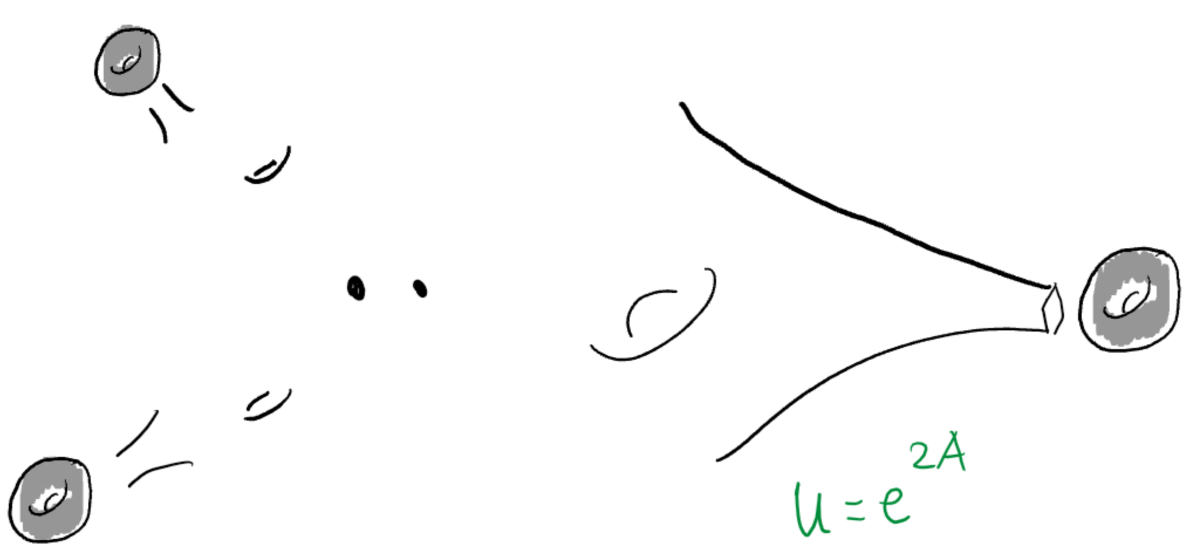
# Casimir energy

Source of negative energy: we use hyperbolic manifolds with small cycles. We put antiperiodic boundary conditions for fermions, so Casimir energy is net negative due to bosons.

- Hyperbolic manifolds can contain near-cusps. Described by

$$ds^2_{cusp} = dy^2 + e^{-2y/\ell} ds^2_{T^6} \ , \quad y_0 < y < y_c$$

Joined to the bulk part at $y_0$. Radial direction should be stopped at finite $y_c$ to get compact space. Achieved via Anderson/Dehn filling.




Let $\lambda_c$ be size of smallest circle in $T^6$. Proper size

$$R_c(y) = e^{-y/\ell}\lambda_c$$

shrinks slowly $\dfrac{dR_c}{dy} \sim \dfrac{\lambda_c}{\ell} e^{-y/\ell} \ll 1$

- In this situation, the Casimir energy becomes

$$\rho_C(R_c) \sim -\frac{1}{R_c^{11}} \sim -\frac{e^{11y/\ell}}{\lambda_c^{11}}$$

and semi-classical EOMs $$-\frac{2}{\sqrt{-g^{11}}}\frac{\delta S_{cl}}{\delta g^{MN}} = \langle T_{MN}^{(Cas)} \rangle$$

- Casimir strongly localized near thin regions. Will need to include backreaction.
- This quantum effect will have to be large enough to compete with classical sources. Will use a combination of small cycles and warping.
- At the same time, other quantum corrections are suppressed if

$$\ell \gg R_c \gg \ell_{11}$$

# Homogeneous potential

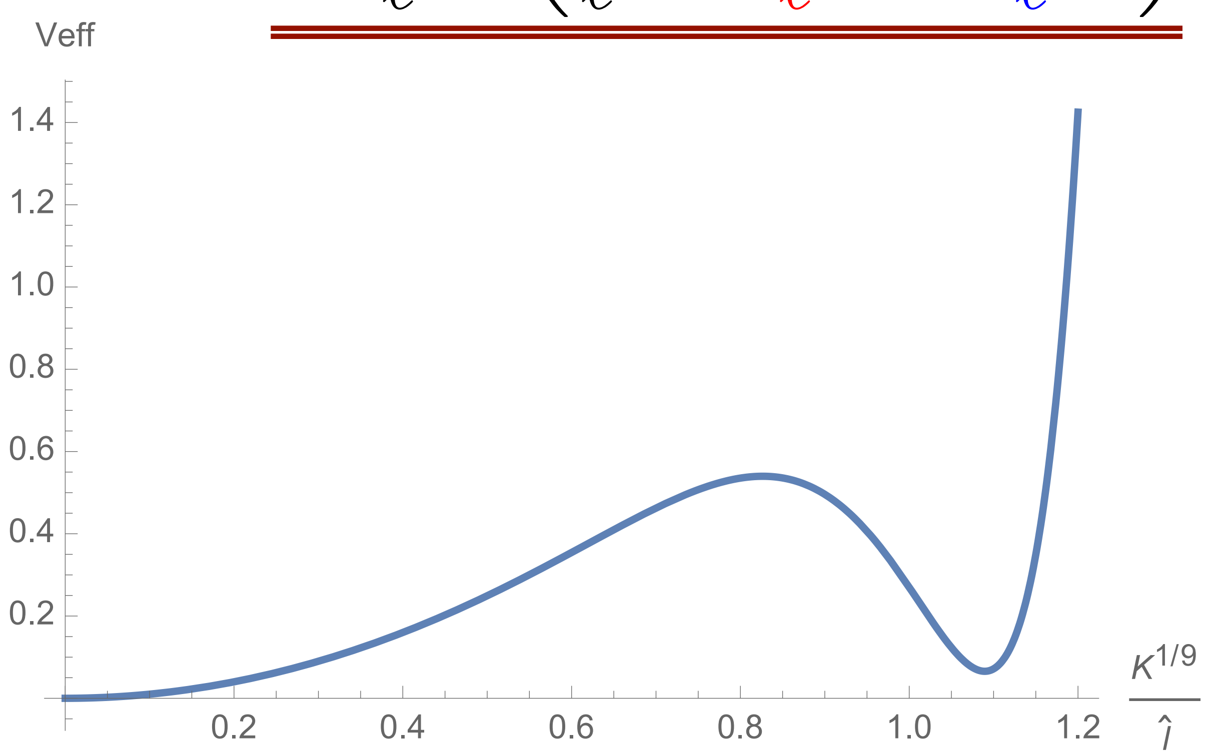
Stabilization mechanism in homogeneous approx.

$$S_{11} = \frac{1}{\ell_{11}^9} \int d^{11}x \sqrt{-g^{(11)}} \left( R^{(11)} - \frac{1}{2} |F_7|^2 \right) + S_{quantum}$$

$$\Rightarrow V \sim \frac{1}{\ell_{11}^9} \mathrm{Vol}_7 \left( \frac{a}{\ell^2} + \ell_{11}^{12} \frac{N^2}{\mathrm{Vol}_7^2} \right) + \int_{\mathbb{H}_7/\Gamma} d^7y \sqrt{g^{(7)}}\, \rho_C$$

$$\propto \frac{M_4^4}{\hat{\ell}^7} \left( \frac{a}{\hat{\ell}^2} - \frac{K}{\hat{\ell}^{11}} + \frac{N^2}{\hat{\ell}^{14}} \right)$$

$$\hat{\ell} \equiv \ell / \ell_{11} \qquad K \sim \left( \frac{\ell}{R_c} \right)^{11} \frac{\mathrm{Vol}_C}{\mathrm{Vol}_7}$$




Supports a dS minimum

$$\hat{\ell} = \frac{\ell}{\ell_{11}} \sim \left( \frac{K}{a} \right)^{1/9} \gg 1$$

with $\ell_{11} \ll \ell \leq \ell_{dS}$

# C. Inhomogeneities and backreaction

Casimir contributions localized in thin regions. Need to determine the effect of these inhomogeneities.
The metric will be deformed to a warped product

$$ds^2 = e^{2A(y)} ds^2_{dS_4} + e^{2B(y)} (g_{\mathbb{H}\,ij} + h_{ij}) dy^i dy^j$$

warp factor $u(y) \equiv e^{2A(y)}$ conformal mode

EOMs will be PDEs in 7 variables; possible numerical approach using NNs described at the end.
Here instead we combine analytic estimates with numerical approximations in the cusp region.

# Warped effective potential

[Douglas, 2009]

In the 4d EFT, it is useful to organize 11d EOMs in terms of an off-shell effective potential that includes warping effects:

$$V_{eff} = \frac{1}{2\ell_{11}^9} \int d^7y \sqrt{g^{(7)}} u^2 \left( -R^{(7)} - \frac{1}{4}\ell_{11}^9 T^{(Cas)\mu}{}_{\mu} + \frac{1}{2}|F_7|^2 - 3\left(\frac{\nabla u}{u}\right)^2 \right)$$
$$+ \frac{C}{2}\left( \frac{1}{G_N} - \frac{1}{\ell_{11}^9}\int \sqrt{g^{(7)}} u \right)$$

C: Lagrange multiplier $\frac{\delta V_{eff}}{\delta C} = 0 \Rightarrow \frac{1}{G_N} = \frac{1}{\ell_{11}^9}\int \sqrt{g^{(7)}} u$ $\quad C \sim R_{dS}^{(4)}$

GR constraint: $\frac{\delta V_{eff}}{\delta u} = 0 \Rightarrow \left( -\nabla^2 - \frac{1}{3}\left( -R^{(7)} - \frac{1}{4}\ell_{11}^9 T^{(Cas)\mu}{}_{\mu} + \frac{1}{2}|F_7|^2 \right) \right) u = -\frac{C}{6}$

Tune C<<1, then this is a Schrodinger problem $-\nabla^2 u(y) + U_{Schr} u(y) = 0$

Other eoms reproduced by $\frac{\delta V_{eff}}{\delta B} = 0 = \frac{\delta V_{eff}}{\delta h} = \frac{\delta V_{eff}}{\delta C_6}$

## General properties can be established using warped Veff:

- $a \ll 1$ tuning, from $\int(-u^2 R^{(7)} - 3(\nabla u)^2)$

- Naive conformal factor instability stabilized by warping [Douglas, 2009]

- Bound on negative energy, since u decays in classically forbidden region

- Positive contribution to mass matrix from warp factor

# de Sitter solution in near-cusp region

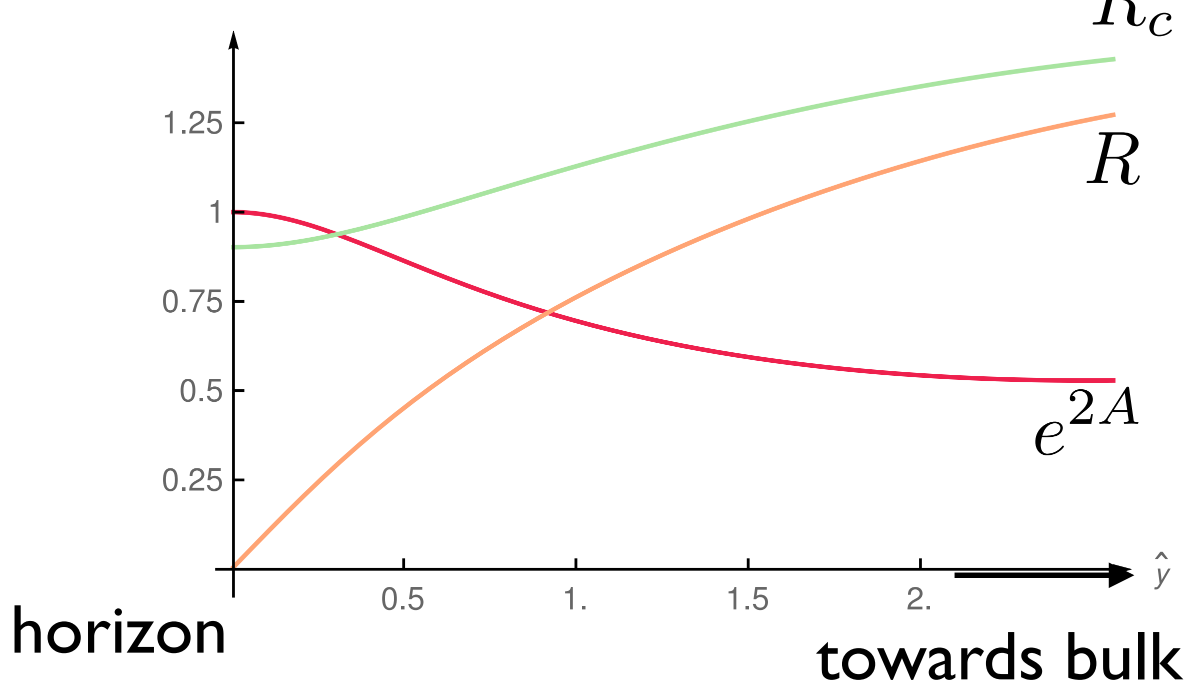
Casimir energy concentrates inhomogeneously in near-cusp regions. Focus on one of these regions, keeping only radial y-dependence. Angular variations become important when we glue onto the bulk.

$$ds_{11}^2 = e^{2A(y)} ds_4^2 + dy^2 + R_c(y)^2 ds_{T^5}^2 + R(y)^2 d\theta^2$$

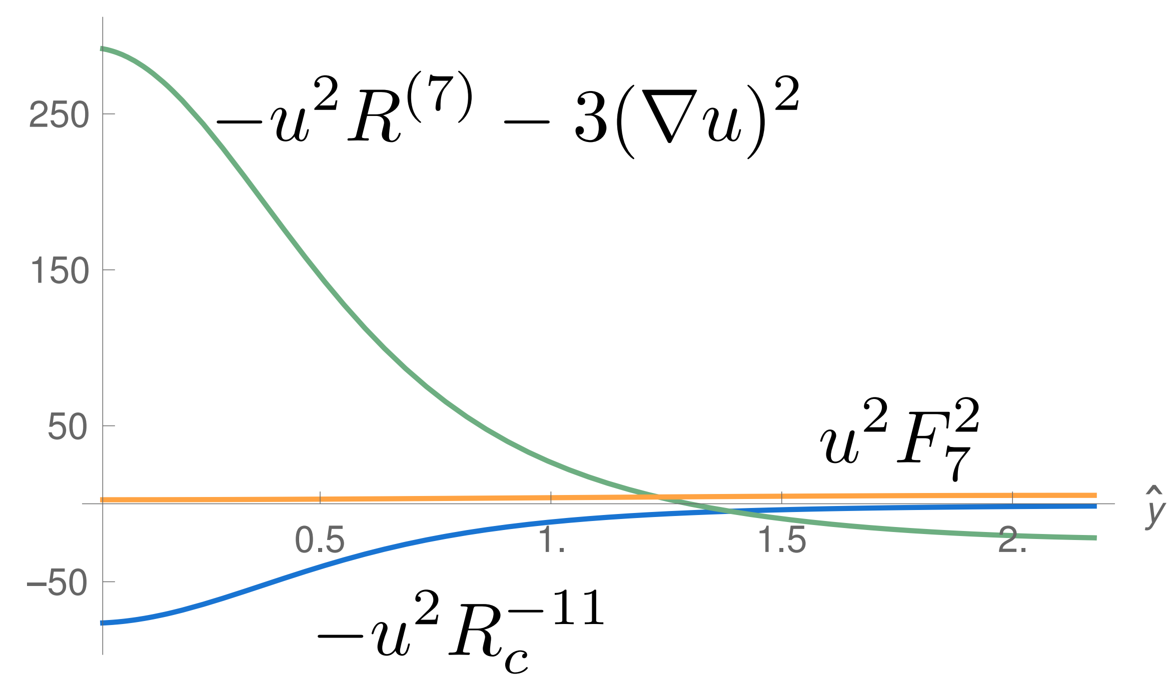
$R_c$: Casimir circles  $R$: Anderson/Dehn-filling

We find numerical smooth solutions with dS4:

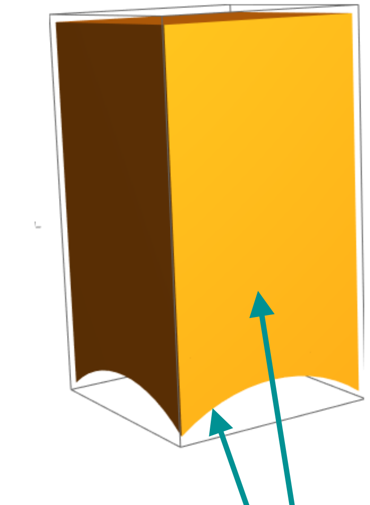
solution




energy sources



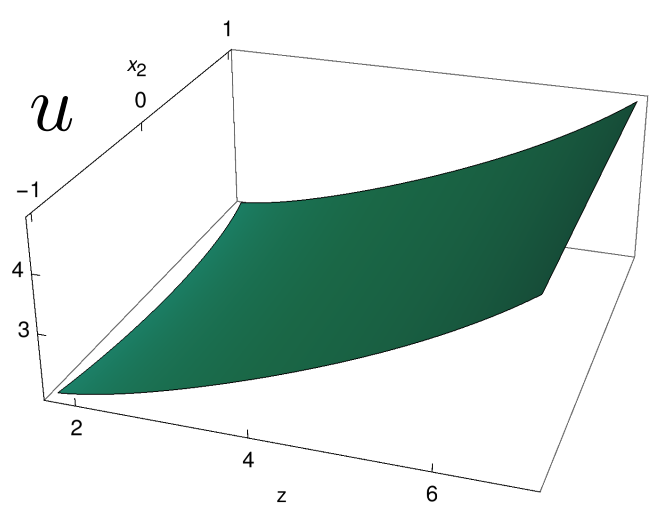
# Matching onto the bulk

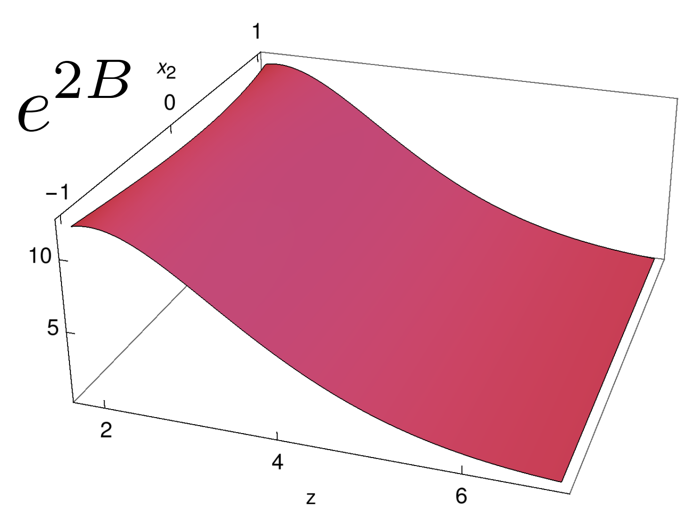


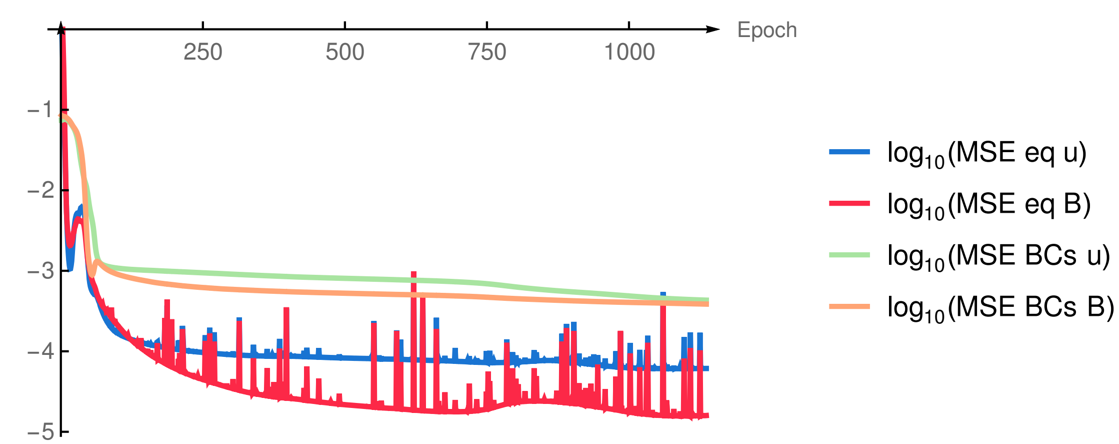

Previous approx. cusp joined to the bulk of the hyperbolic manifold at a totally geodesic face of polygon. Boundary conditions involve angular variables and more intense numerics.

Promising direction: use neural network methods to approximate PDE sol, flowing down a loss function $\mathrm{Loss} = \sum (\mathrm{eqs})^2$

Preliminary results in $\mathbb{H}_3/\Gamma$




- Current analytic estimates and general constraints suggest a positive mass matrix

# D. Discussion & other directions

We have found a simple mechanism for obtaining dS sols in M-theory: uplift of the large N M2 brane theory using negative curvature and automatically generated Casimir energy.

- We gave arguments for the existence of sol's beyond the homogeneous level. Explicit sol's will require solving PDEs but seems doable.
- Holography: the uplifts provide a microscopic realization of dS/dS correspondence. Combines with recent progress on TTbar-type deformations in field theory. Add matter fields & uplift.
- Can also look for more general cosmological solutions w/accelerated expansion (slow-roll functionals in the landscape)

*Thanks!*