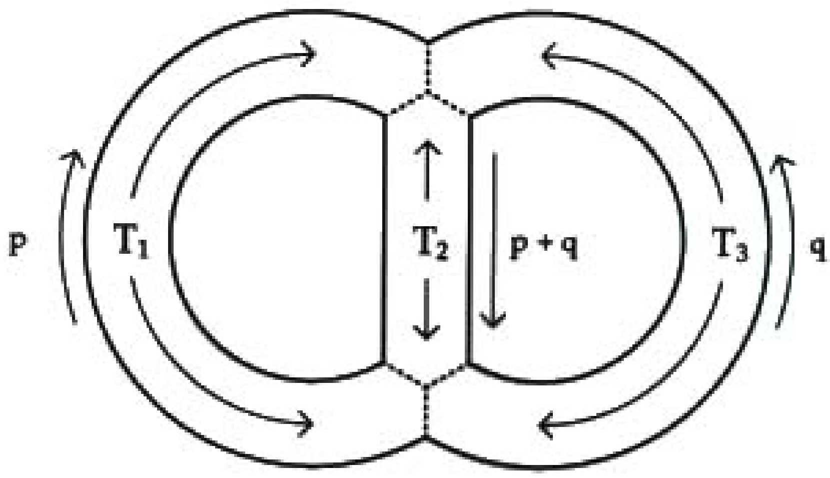
Computing Tree and Loop Amplitudes in String Field Theory

hep-th/0207132



Strings 2002

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W. Taylor (MIT)

String Field Theory

String Field Theory (SFT) is a formulation of string theory in target space

- Theory has infinite number of space-time fields
- Action is highly nonlocal

SFT goes beyond perturbative world-sheet formalism:

A) Organizes perturbative expansion

B) Off-shell nonperturbative formalism

C) DOF encode different backgrounds

Recent work on Sen's tachyon condensation conjectures involves features B and C.

This talk deals with feature A:

How to describe perturbative string amplitudes in SFT

Perturbative SFT Amplitudes

Previous analysis used CFT, conformal maps

- 4-tachyon tree amplitude (Giddings, Sloan, Samuel)
- 1-loop 2-point function (Freedman/Giddings/Shapiro/Thorn)

—Analysis is quite complicated

—Intractable at high loop/genus level

Main results of this talk

1. Closed form expression for any diagram (tree/loop) in open bosonic SFT (in terms of infinite-size matrices) using oscillator/squeezed state methods
2. Evidence that level truncation on oscillators gives rapidly converging approximation scheme for general amplitude

Ingredients of SFT

String field

$$|\Phi\rangle = \int d^{26}p \; \left[\phi(p)\,|0;p\rangle + A_\mu(p)\,\alpha^\mu_{-1}|0;p\rangle + \cdots\right] .$$

Witten action

$$S = -\frac{1}{2}\langle V_2|\Phi, Q\Phi\rangle - \frac{g}{3}\langle V_3|\Phi,\Phi,\Phi\rangle$$

$Q = \mathrm{BRST}$ operator

$$\langle V_2| \approx \langle 0|^2 \exp\left(-a^{(1)}\cdot C\cdot a^{(2)} - c^{(1)}\cdot C\cdot b^{(2)} - c^{(2)}\cdot C\cdot b^{(1)}\right)$$

$$\langle V_3| \approx \langle 0|^3 \exp\left(-\frac{1}{2}a^{(i)}\cdot N^{ij}\cdot a^{(j)} - c^{(i)}\cdot X^{ij}\cdot b^{(j)}\right)$$

Feynman-Siegel gauge: $b_0|\Phi\rangle = 0$

$$Q = c_0 L_0 = c_0(p^2 + n a_{-n}a_n + n c_{-n}b_n + n b_{-n}c_n - 1) .$$

Propagator

$$\frac{1}{L_0} = \int_0^\infty dT\; e^{-TL_0} .$$

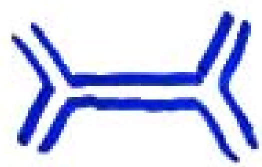
Note: V_2, V_3, propagator are squeezed states

Diagrams in SFT

Basic squeezed state relation

$$\langle 0| \exp\left(-\frac{1}{2} a \cdot S \cdot a\right) \exp\left(-\mu \cdot a^\dagger - \frac{1}{2} a^\dagger \cdot \tilde{N} \cdot a^\dagger\right) |0\rangle$$
$$= \frac{1}{\det(1 - S\tilde{N})^{1/2}} \exp\left(-\frac{1}{2} \mu \cdot (1 - S\tilde{N})^{-1} S \cdot \mu\right)$$

General diagram:

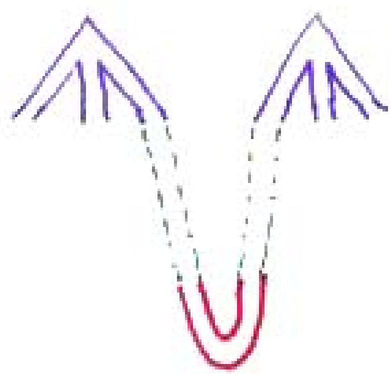


v vertices, e internal edges

$$|V\rangle = |V_3\rangle^v \in \mathcal{H}^{3v}$$

$$P = \prod_{i=1}^{e} \int dT_i e^{-\frac{1}{2} T_i (L_0^{l_i} + L_0^{r_i})}$$

$$\langle D| = \langle V_2|^e \in (\mathcal{H}^*)^{2e}$$



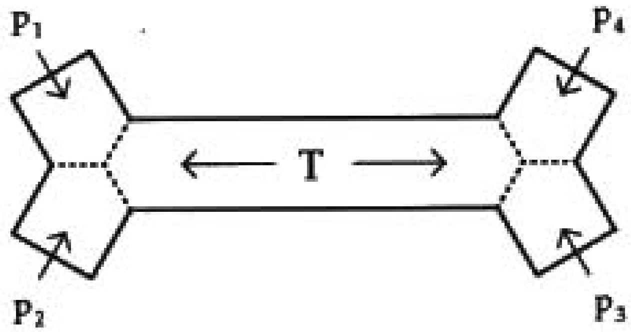
General amplitude

$$\mathcal{A} = \langle D|P|V\rangle$$

is a squeezed state in $\mathcal{H}^{3v-2e}$

True for any number of loops, higher genus, etc.

Example: $p = 0$ four-tachyon amplitude



Take $p_i = 0$ on all external edges, drop external oscillators

$$P|V\rangle = \exp\left(-\frac{1}{2}a^{(i)} \cdot \tilde{N}^{ij} \cdot a^{(j)} - c^{(i)} \cdot \tilde{X}^{ij} \cdot b^{(j)}\right)|0\rangle^2$$

$$\tilde{N}(T) = \begin{pmatrix} \hat{N}^{11}_{nm}(e^{-T}, e^{-T}) & 0 \\ 0 & \hat{N}^{11}_{nm}(e^{-T}, e^{-T}) \end{pmatrix},$$

$$\hat{N}^{11}_{nm}(x, y) = x^{n/2} N^{11}_{nm} y^{m/2}.$$

$$\langle D| = \langle 0|^2 \exp\left(-\frac{1}{2}a^{(i)} \cdot S^{ij} \cdot a^{(j)}\right), \qquad S = \begin{pmatrix} 0 & C \\ C & 0 \end{pmatrix}$$

$$\mathcal{A}_4 = \int_0^\infty dT\, e^T \frac{\det(1 + S\tilde{X}(T))}{\det(1 - S\tilde{N}(T))^{13}}.$$

Compute c_4 = coefficient of ϕ^4 in tachyon potential

$$V(\phi) = -\frac{1}{2}\phi^2 + g\kappa\phi^3 + c_4(g\kappa)^2\phi^4 + \cdots$$

Analytic computation $c_4 = -1.742 \pm 0.001$ (Samuel)

$$c_4 = \frac{9}{2}\int_0^\infty dT\, e^T \left[\frac{\det(1+S\tilde{X}(T))}{\det(1-S\tilde{N}(T))^{13}} - 1\right].$$

Truncation including level 1 oscillators

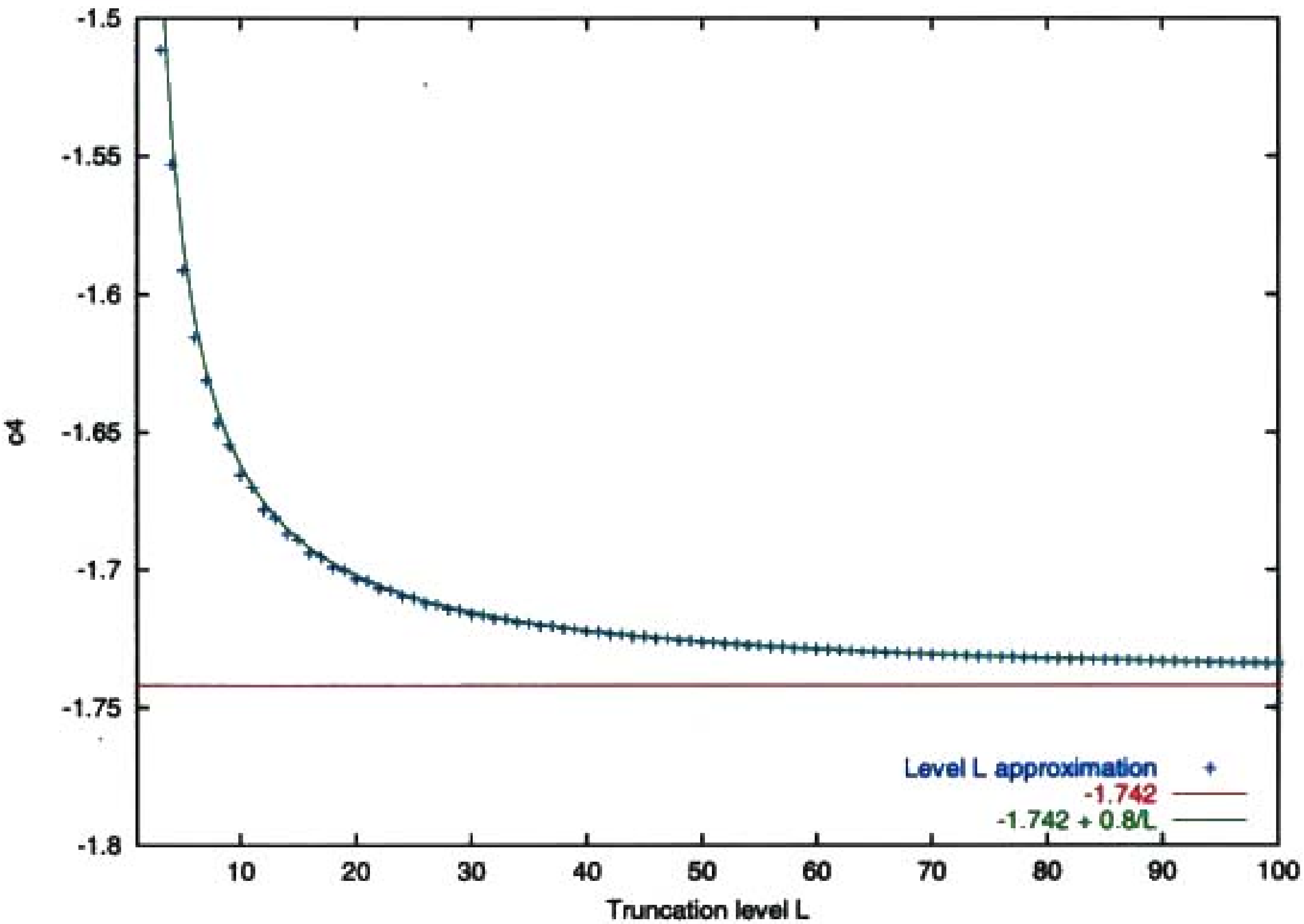
$$S = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

$$\tilde{N}(T) = \begin{pmatrix} \frac{5}{27}e^{-T} & 0 \\ 0 & \frac{5}{27}e^{-T} \end{pmatrix}, \quad \tilde{X}(T) = \begin{pmatrix} -\frac{11}{27}e^{-T} & 0 \\ 0 & -\frac{11}{27}e^{-T} \end{pmatrix}$$

$$c_4^{(1)} = -\frac{9}{2}\int_0^1 \frac{dx}{x^2}\left[\frac{1-\frac{121}{729}x^2}{(1-\frac{25}{729}x^2)^{13}} - 1\right] \approx -1.309$$

Computed to level $L = 100$

Converges as $c_4^{(L)} \approx -1.742 + 0.80/L + \cdots$



Level truncation approximations

to C_4 coefficient of ϕ^4 in $p=0$ tachyon potential

Example: Veneziano amplitude

Include momenta,

$$\mathcal{A}_4(p_1, p_2, p_3, p_4) = I(s,t) + I(t,s)$$

$$I(s,t) = \frac{3^9}{2^{12}} \int_0^\infty dT\, e^T \,\frac{\det(1 + S\tilde{X}(T))}{\det(1 - S\tilde{N}(T))^{13}} \exp\left(-\frac{1}{2} p_i Q_{ij} p_j\right)$$

$$Q \sim N_{00}^{11} + T + \hat{N}_{0\cdot} \frac{1}{1 - S\tilde{N}} S\hat{N}_{\cdot 0}$$

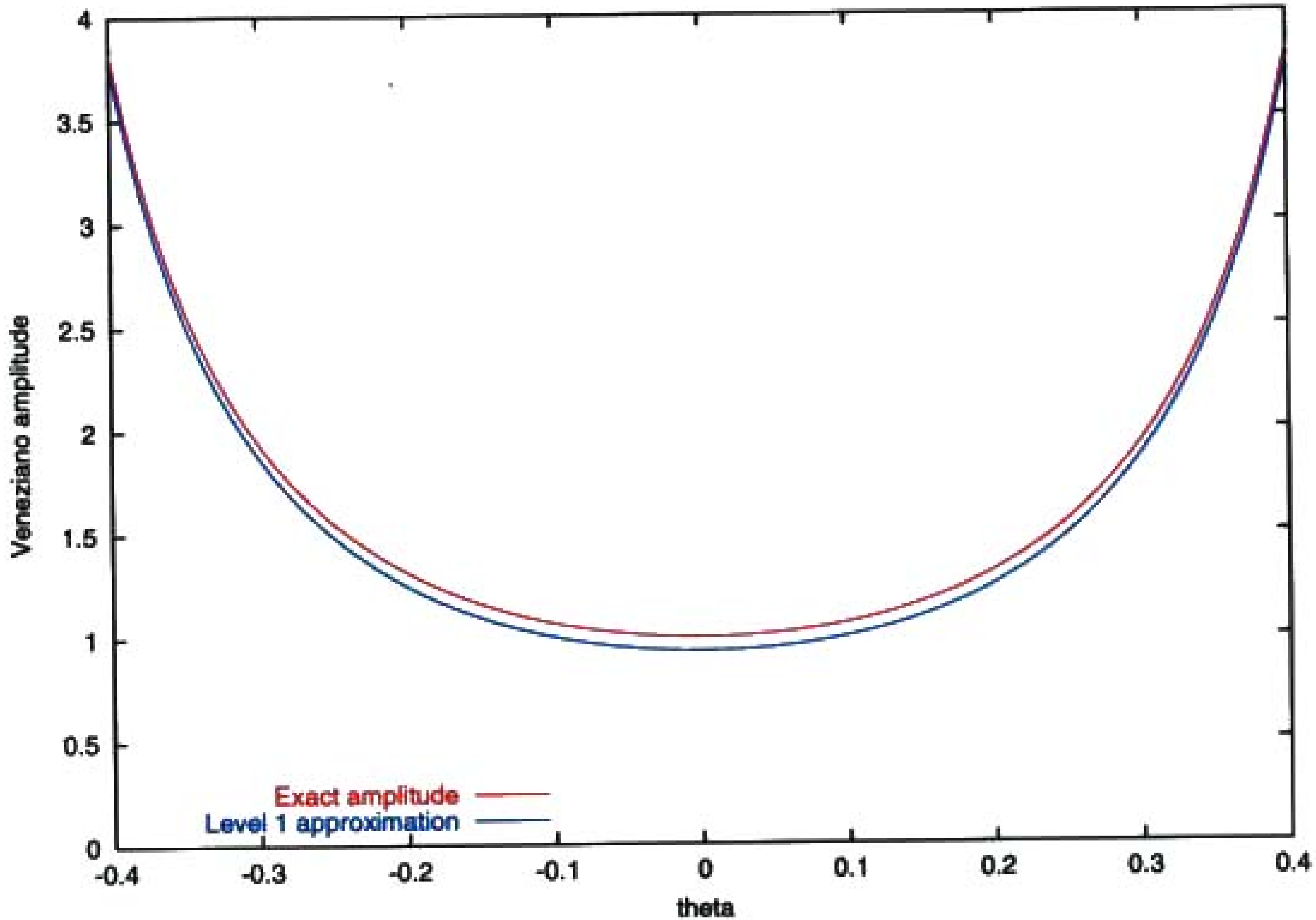
Computed for

$$s = -2 - 2\cos\theta, \qquad t = -2 + 2\cos\theta$$

Compared with

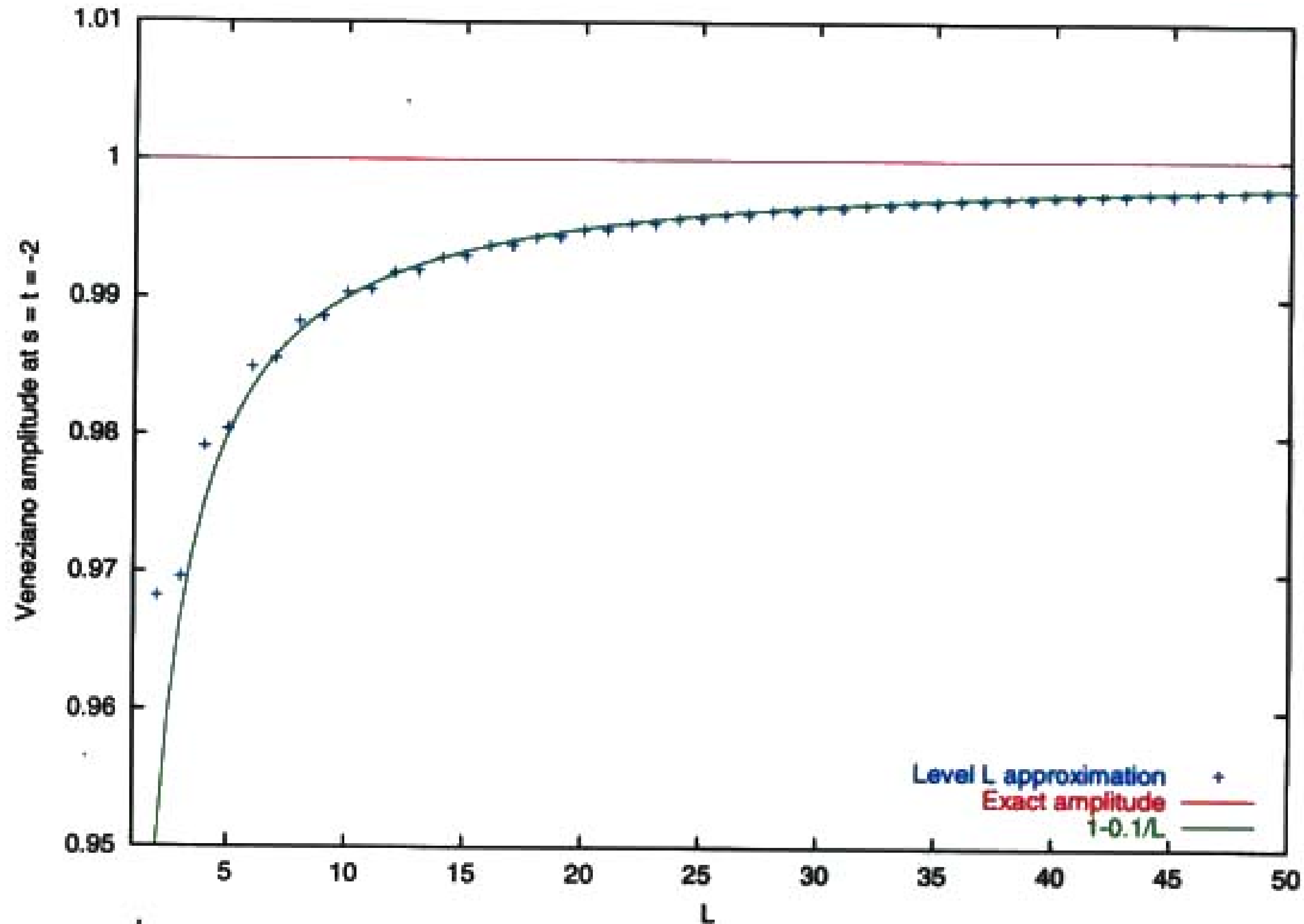
$$\mathcal{A}_4^{[V]} = B(-s-1, -t-1) = \int_0^1 d\xi\, \xi^{-s-2}(1-\xi)^{-t-2}$$

- Excellent agreement at low levels
- Converges with error $1/L$ at fixed s, t
- Integrand compares well with analytic form of Samuel
- Level truncation acts as UV cutoff

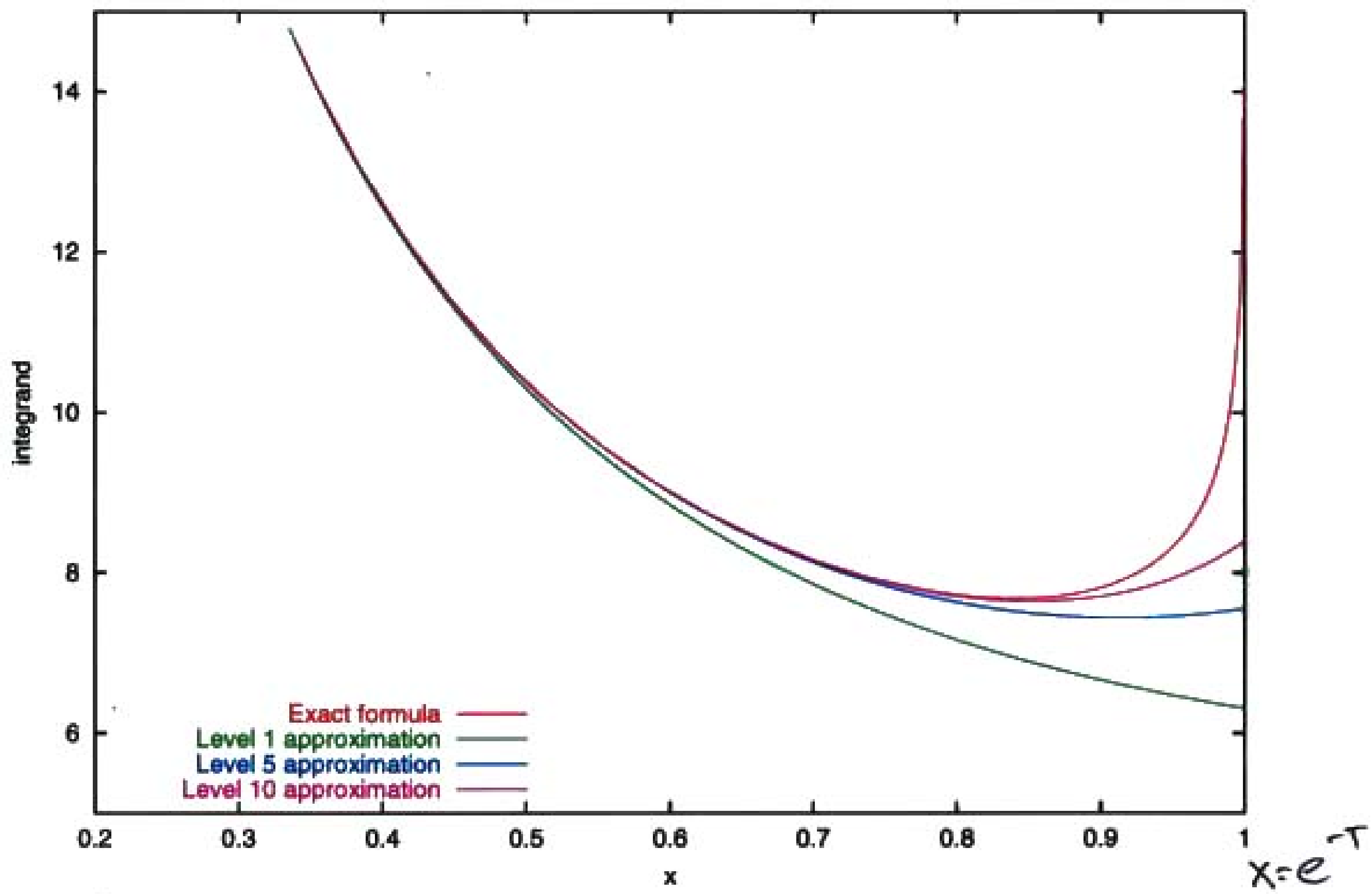


Level 1 oscillator truncated approximation to Veneziano

$s = -2 - 2\sin\theta \qquad t = -2 + 2\sin\theta \qquad u = 0$



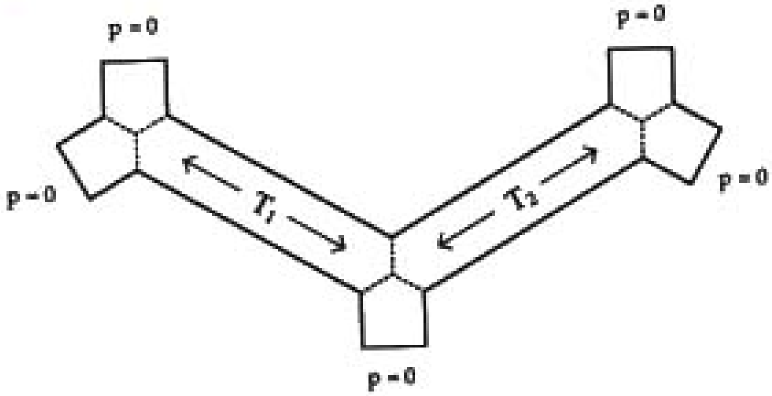
Level truncation approximations to Veniziano at $s = t = -2$, $u = 0$



Level truncation approximations to analytic 4-point tree integrand of Samuel

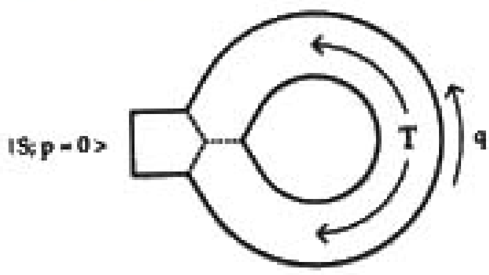
Other examples

$p = 0$ tachyon 5-point function (c_5)



- Converges as $1/L$ again

$p = 0$ one-loop tadpole



Form is

$$|S\rangle = \int_0^1 \frac{dx}{x^2} \frac{\det(1 + S\tilde{X})}{\det(1 - S\tilde{N})^{13} Q^{13}} \exp\left(-\frac{1}{2} a \cdot G \cdot a - c \cdot R \cdot b\right) |0\rangle$$

- Integrand diverges as $T \to 0, \infty$ (closed/open tachyons)
- Quadratic terms in exponent $\to C$ as $T \to 0$, gives form of Shapéro-Thorn closed string states

Conclusions

Summary

- Closed form expressions for perturbative amplitudes from SFT
- Numerical approximation from truncation in oscillator level

Further directions

- Exact analysis: "diagonalize" $\hat{N}_{nm} = x^{n/2} N_{nm} y^{m/2}$?
- Understand/prove rate of convergence
- Generalize to superstring
- Understand closed strings from loop diagrams
- Generalize analysis to other vacua
- Derive nonabelian Born-Infeld, tachyon-BI action