Baryons from Instantons in holographic QCD

Shigeki Sugimoto (Nagoya Univ.)

Based on) H.Hata, T.Sakai, S.S and S.Yamato
hep-th/0701280

T.Sakai and S.S.
hep-th/0412141, hep-th/0507073

See also) D.Hong, M.Rho, H.Yee and P.Yi
hep-th/0701276, arXiv:0705.2632

Strings 2007 @ Madrid, Spain June 25-29, 2007

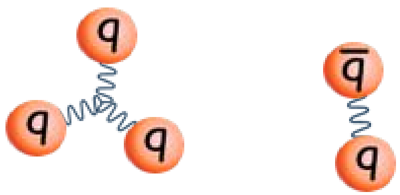
Plan

1. Introduction + brief review
2. Baryons as Instantons
3. Baryon spectrum
4. Summary and outlook

1 Introduction + brief review

- What are hadrons made of ?

QCD String theory (around 1970)



VS

fundamental effective

- Gauge/String duality suggests

Gauge theory in 4 dim $\simeq$ String theory in higher dim (QCD)

- String theory can again be a theory of hadrons!

  Problems in the old days can now be solved
  with the help of D-branes, curved space-time and holography.

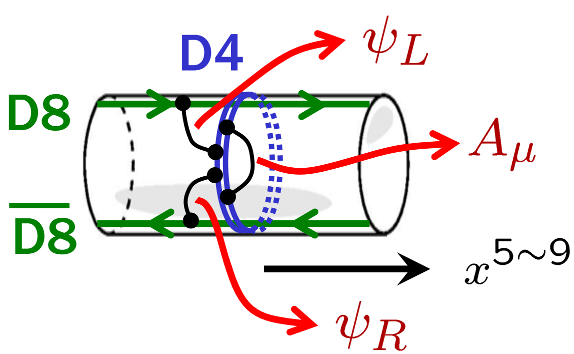
- Both QCD and string theory can be fundamental
  at the same time!

Recently, we proposed [Sakai-S.S. 2004]

Type IIA string theory in Witten's D4 background + N_f Probe D8-branes (assuming $N_c \gg N_f$) $\overset{\text{dual}}{\longleftrightarrow}$ (at low energy) 4 dim QCD with N_f massless quarks

Outline

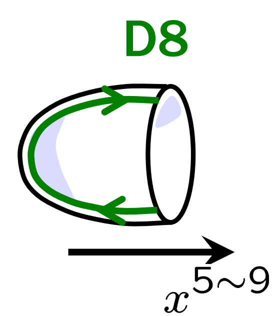
D4-D8-$\overline{\text{D8}}$ system
N_c $\underbrace{\text{D8-}\overline{\text{D8}}}_{N_f \text{ pairs}}$



QCD with N_f massless quarks (at low energy)

dual

String theory in the D4 background + N_f probe D8-branes (assuming $N_c \gg N_f$)

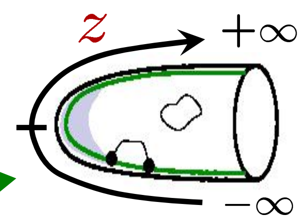


Interpretation of strings

The topology of the background is

$$\mathbf{R}^{1,3} \times \mathbf{R}^2 \times S^4$$

x^μ (y, z)



D8-branes are extended along $(x^\mu, z) \times S^4$

- **Closed strings** → **glueballs** ← **studied around 1998**
  [Csaki-Ooguri-Oz-Terning 1998, Koch-Jevicki-Mihailescu-Nunes 1998, A.Hashimoto-Oz 1998, etc etc]
- **Open strings on D8** → **mesons**

← **Reducing** S^4

(Consider only the $SO(5)$ **invariant states for simplicity)**

The effective action is a 5 dim *U(Nf)* YM-CS theory

$$S_{\text{5dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$K(z) = 1 + z^2$ **CS5-form**

$$S_{\text{YM}} = \kappa \int d^4x dz \, \text{Tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}^2 + K(z)F_{\mu z}^2\right) \quad S_{\text{CS}} = \frac{N_c}{24\pi^2}\int_5 \omega_5(A)$$

$\mu, \nu = 0 \sim 3$

Mesons

The effective action

$$S_{5\text{dim}} \simeq S_{\text{YM}} + S_{\text{CS}}$$

$K(z) = 1 + z^2$ **CS5-form**

$$S_{\text{YM}} = \kappa \int d^4x dz \, \mathrm{Tr}\left(\frac{1}{2}K(z)^{-1/3}F_{\mu\nu}^2 + K(z)F_{\mu z}^2\right) \qquad S_{\text{CS}} = \frac{N_c}{24\pi^2}\int_5 \omega_5(A)$$

$\mu, \nu = 0 \sim 3$

- **mode expansion**

$$A_\mu(x^\mu, z) = \sum_{n\geq 1} B_\mu^{(n)}(x^\mu)\psi_n(z)$$

$$A_z(x^\mu, z) = \sum_{n\geq 0} \varphi^{(n)}(x^\mu)\phi_n(z)$$

Some complete sets

- **We interpret**

$\varphi^{(0)} \sim$ pion $\quad B_\mu^{(1)} \sim \rho$ meson $\quad B_\mu^{(2)} \sim a_1$ meson $\quad \cdots$

($\varphi^{(n)}$ $(n = 1, 2, \cdots)$ **are eaten by** $B_\mu^{(n)}$)

$\pi, \rho, a_1, \cdots$ **are unified in the 5 dim gauge field !**

Caution !

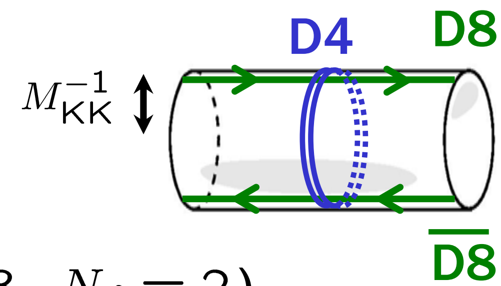
Our analysis is reliable when:

- large N_c
- large $\lambda = g_{\rm YM}^2 N_c$

} ... Supergravity approximation

($\rightarrow$ difficult to make $M_{\rm KK}$ large)

- $E \ll M_{\rm KK}$ $\leftarrow$ scale of 5th dimension
- $N_c \gg N_f$ ... probe approximation
- $m_q = 0$ (We mainly consider $N_c = 3,\ N_f = 2$)



We should not be too serious in the quantitative comparison with the experiments.

Nevertheless we find fairly good agreement.

Highlights

- Geometric realization of the chiral symmetry breaking

$$U(N_f)_L \times U(N_f)_R \longrightarrow U(N_f)_V$$

- Naturally reproduces various old ideas.

  Vector meson dominance, Gell-Mann -Sharp-Wagner model, Skyrme model, Hidden local symmetry, Wess-Zumino-Witten term, Witten-Veneziano formula etc...

- Numerical estimate of the masses and couplings roughly agrees with the various experimental data

Quantitative tests

mass

mass	ρ	a_1	ρ'	(a_1')	ρ''
exp.(MeV)	776	1230	1465	(1640)	1720
our model	[776]	1189	1607	2023	2435
ratio	[1]	1.03	0.911	(0.811)	0.706

↑ input $(M_{\mathrm{KK}} \simeq 949\ \mathrm{MeV})$

coupling

coupling		fitting m_ρ and f_π	experiment
f_π	$1.13 \cdot \kappa^{1/2} M_{\mathrm{KK}}$	[92.4 MeV]	92.4 MeV
L_1	$0.0785 \cdot \kappa$	0.584×10^{-3}	$(0.1 \sim 0.7) \times 10^{-3}$
L_2	$0.157 \cdot \kappa$	1.17×10^{-3}	$(1.1 \sim 1.7) \times 10^{-3}$
L_3	$-0.471 \cdot \kappa$	-3.51×10^{-3}	$-(2.4 \sim 4.6) \times 10^{-3}$
L_9	$1.17 \cdot \kappa$	8.74×10^{-3}	$(6.2 \sim 7.6) \times 10^{-3}$
L_{10}	$-1.17 \cdot \kappa$	-8.74×10^{-3}	$-(4.8 \sim 6.3) \times 10^{-3}$
$g_{\rho\pi\pi}$	$0.415 \cdot \kappa^{-1/2}$	4.81	5.99
g_ρ	$2.11 \cdot \kappa^{1/2} M_{\mathrm{KK}}^2$	$0.164\ \mathrm{GeV}^2$	$0.121\ \mathrm{GeV}^2$
$g_{a_1\rho\pi}$	$0.421 \cdot \kappa^{-1/2} M_{\mathrm{KK}}$	4.63 GeV	$2.8 \sim 4.2$ GeV

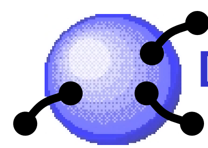
Much better than expected!

Baryons

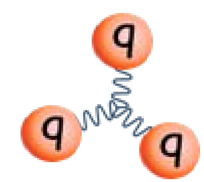
- We can apply the construction of baryons proposed in the AdS/CFT context. [Witten 1998, Gross-Ooguri 1998]

→ Baryon $\simeq$ D4-brane wrapped on the S^4

- Relation to other descriptions

meson	baryon
 F1	 D4
fundamental	soliton

→ quark model



low energy →

↓ ↓

5 dim:	gauge field	instanton

← Topic of this talk

↓ ↓

4 dim:	pion	Skyrmion

← Skyrme model

Summary of the rest of the talk

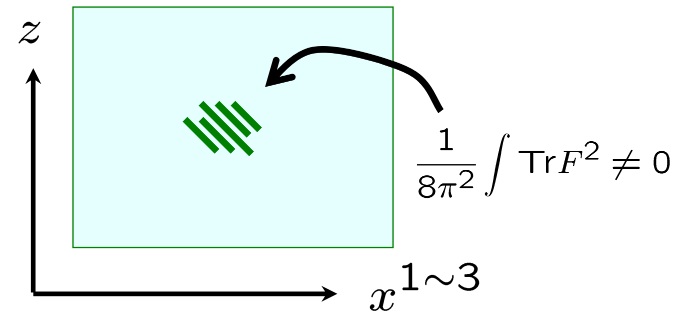
- We propose a new way to analyze baryons that extends Skyrme's old idea including contributions from vector mesons.
- Baryons are described as (4 dim) instantons in the 5 dim gauge theory.
- Quantum mechanics on the instanton moduli space, gives the baryon spectrum.
- The quantitative tests are not good enough yet. Please be generous !

② Baryons as instantons

- Consider an instanton config. in $x^M = (\vec{x}, z) \in \mathbf{R}^4$ ($M = 1,2,3,z$)

  ➡ behaves as a point-like particle

  ➡ Interpreted as a baryon



- In fact,

$$\boxed{\underbrace{N_B}_{\text{baryon \#}} = \underbrace{\frac{1}{8\pi^2}\int \mathrm{tr}F\wedge F}_{\text{Instanton \#}}}$$

- <u>Note</u>

D4 wrapped on S^4 $\simeq$ instanton on D8 $\simeq$ Skyrmion

[Witten, Gross-Ooguri 1998] [Atiyah-Manton 1989] [Skyrme 1961]

Realization of Atiyah-Manton:

$$\underbrace{U(x^\mu)}_{\text{Skyrmion}} \equiv P\exp\left\{-\int_{-\infty}^{\infty} dz\, \underbrace{A_z(x^\mu, z)}_{\text{Instanton}}\right\}$$

Classical solution

- The instanton solution for

$K(z) = 1 + z^2$

$$S_{\mathrm{YM}} = \kappa \int d^4x dz \, \mathrm{Tr} \left(\frac{1}{2} K(z)^{-1/3} F_{\mu\nu}^2 + K(z) F_{\mu z}^2 \right)$$

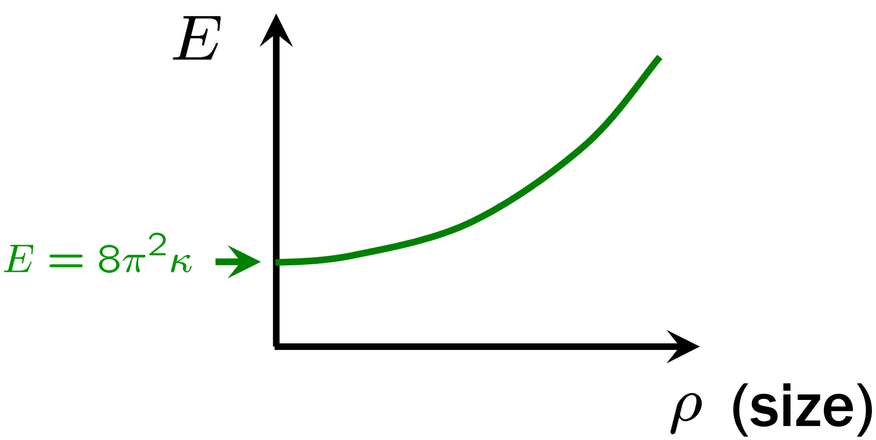
$(M_{\mathrm{KK}} = 1 \text{ unit})$

$$\kappa = \frac{\lambda N_c}{216\pi^2}$$

λ : 't Hooft coupling (assumed to be large)

shrinks to zero size !

(Even though the pion effective action contains the Skyrme term !)



The BPST instanton configuration with $\rho \to 0$ is the minimum energy configuration.

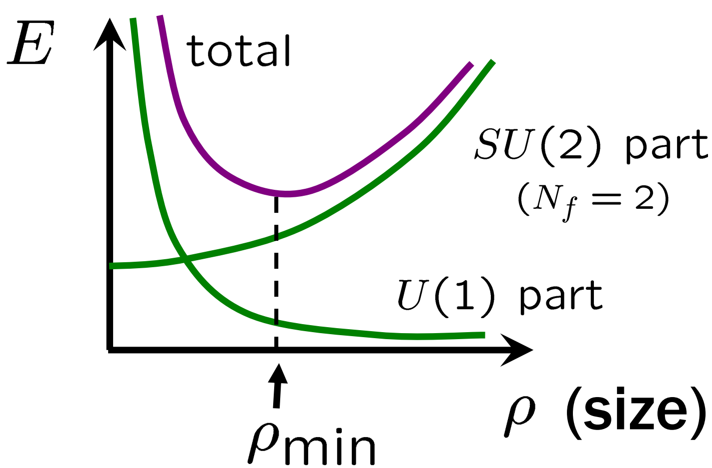
The effect of the Chern-Simons term:

$$S_{CS} = \frac{N_c}{24\pi^2}\int_5 \omega_5(A) = \frac{N_c}{16\pi^2}\int d^4x dz\, \underset{\uparrow\ U(1)\ \text{part}}{A_0^{U(1)}}\, \underbrace{\epsilon^{ijk}\mathrm{Tr} F_{ij}F_{kz}}_{\text{Non-zero for instanton}} + \cdots$$

➡ **source of the U(1) charge**

➡ **point-like charge costs energy**

➡ **The size will be stabilized with a non-zero finite value.**



This is the same mechanism as the stabilization of Skyrmions via ω meson. [Adkins-Nappi 1984]

- We can show $\rho_{\text{min}} \sim \mathcal{O}(\lambda^{-1/2})$

  It is convenient to rescale as

$$x^M \to \lambda^{-1/2} x^M \qquad A_M \to \lambda^{1/2} A_M \qquad (M = 1, 2, 3, z)$$

  Then, we have

$$\mathcal{L}_{\text{YM}} \sim \kappa \, \text{Tr}\left(\frac{1}{2} F_{MN}^2 + \mathcal{O}(\lambda^{-1}) \right)$$

  YM in flat space

➡ The leading order classical solution is
  the BPST instanton with $\rho = \rho_{\text{min}}$ and $Z = 0$

$$A_M^{\text{cl}} = -i \frac{\xi^2}{\xi^2 + \rho^2} g \partial_M g^{-1}$$

$$g = \frac{(z - Z) - i(\vec{x} - \vec{X}) \cdot \vec{\tau}}{\xi} \qquad \xi = \sqrt{(\vec{x} - \vec{X})^2 - (z - Z)^2}$$

ρ : size $\qquad (\vec{X}, Z)$: position of the instanton

③ Baryon spectrum

- Consider a slowly moving (rotating) baryon configuration.
  Use the moduli space approximation method :

Instanton moduli $\mathcal{M} \ni (X^\alpha) \rightarrow (X^\alpha(t))$ $(\alpha = 1, 2, \cdots, \dim \mathcal{M})$

time

$$\boxed{A_M(t,x) \sim A_M^{\text{cl}}(x; X^\alpha(t))}$$

$S_{5\text{dim}} \rightarrow$ Quantum Mechanics for $X^\alpha(t)$

- For SU(2) one instanton,

$$\mathcal{M} \simeq \{(\underbrace{\vec{X}, Z}_{\text{position}}, \rho)\} \times SU(2)/\mathbf{Z}_2 \qquad \mathbf{Z}_2 : \mathbf{a} \rightarrow -\mathbf{a}$$

size

$SU(2) \ni \mathbf{a}$ ← *SU(2)* orientation

$$\rightarrow \quad L_{\text{QM}} = \frac{G_{\alpha\beta}}{2}\dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) \qquad U(X^\alpha) = 8\pi^2\kappa\left(1 + \lambda^{-1}\left(\frac{\rho^2}{6} + \frac{3^6\pi^2}{5\,\rho^2} + \frac{Z^2}{3}\right) + \mathcal{O}(\lambda^{-2})\right)$$

<u>Note</u> $(\vec{X}, \mathbf{a})$: genuine moduli (same as in Skyrme model)

(ρ, Z) : new degrees of freedom, added since they are light compared with the other massive modes.

- Solving the Schrodinger equation for this Quantum mechanics, we obtain the baryon spectrum

→ Generalization of Adkins-Nappi-Witten [Adkins-Nappi-Witten1983]
including vector mesons and ρ , Z modes

<u>Results</u>

- Only $I = J$ states appear. (Just as in the ANW)

  isospin ↑ ↑ spin

- Parity odd states appear. (Unlike in the ANW!)

$$\text{parity} = (-1)^{n_z}$$

n_z : excitation of the Z mode

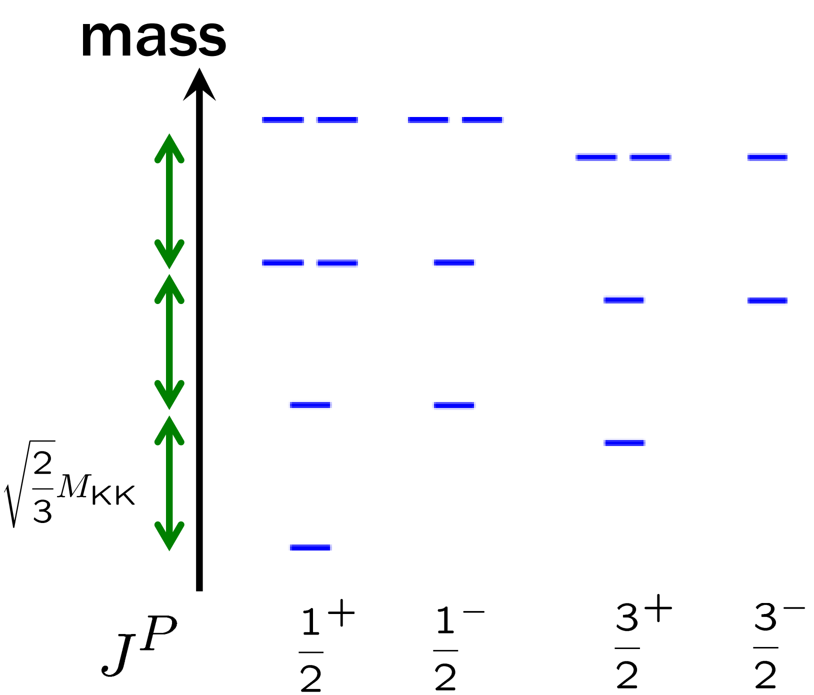
- Mass spectrum

$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2} + \sqrt{\frac{2}{3}} (n_\rho + n_z) \right) M_{\mathrm{KK}}$$

$$l = 2I = 2J = 1, 3, 5, \cdots \quad n_\rho = 0, 1, 2, \cdots \quad n_z = 0, 1, 2, \cdots$$

Theory

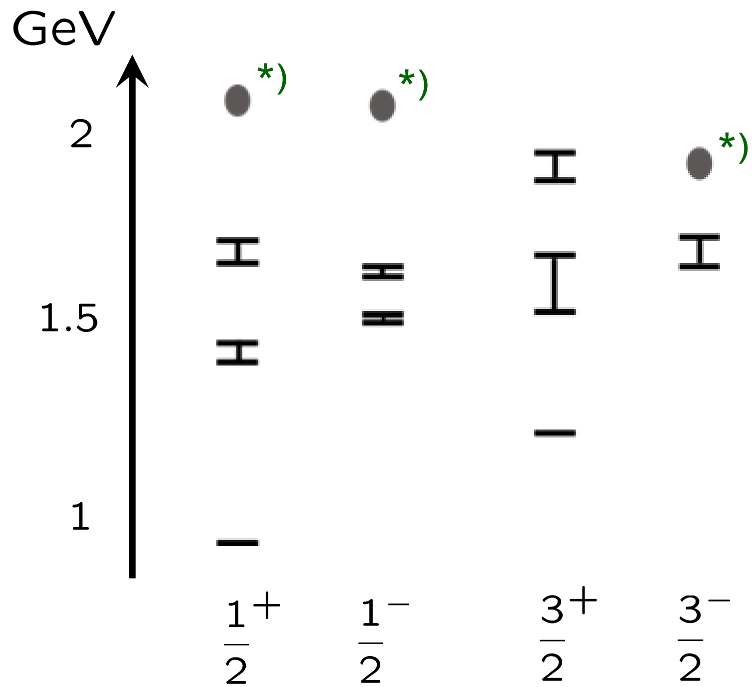
$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2} + \sqrt{\frac{2}{3}}(n_\rho + n_z) \right) M_{\text{KK}}$$



Experiment

($I = J$ states from PDG)

*) Evidence for existence is poor



Note:

We only consider the mass difference, since $\mathcal{O}(N_c^0)$ term in M_0 is not known,

$$M_0 = (\text{classical soliton mass}) + \mathcal{O}(N_c^0)$$
$$\sim \mathcal{O}(N_c)$$

numerical values (just for illustration !)

$$M \simeq M_0 + \left(\sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2} + \sqrt{\frac{2}{3}} (n_\rho + n_z) \right) M_{\mathrm{KK}}$$

$$l = 2I = 2J = 1, 3, 5, \cdots \quad n_\rho = 0, 1, 2, \cdots \quad n_z = 0, 1, 2, \cdots \quad \text{parity} = (-1)^{n_z}$$

- **If we choose** $M_{\mathrm{KK}} \simeq 500$ MeV **and use nucleon mass** ($\simeq 940$ MeV) **to fix the constant** M_0 **,** **(we only consider the mass difference), we obtain**

(n_ρ, n_z)	$\boxed{(0,0)}$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	$[940]^+$	1348^+	1348^-	1756^-	$1756^+, 1756^+$	$2164^-, 2164^-$	$2164^+, 2164^+$
$\Delta\,(l=3)$	1240^+	1648^+	1648^-	2056^-	$2056^+, 2056^+$	$2464^-, 2464^-$	$2464^+, 2464^+$

States appeared in the Skyrme model

($\pm$: parity)

- $I = J$ **states from Particle Data Group look like....**

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	940^+	1440^+	1535^-	1655^-	1710^+, ?	2090^-_*, ?	2100^+_*, ?
$\Delta\,(l=3)$	1232^+	1600^+	1700^-	1940^-_*	1920^+, ?	?, ?	?, ?

(? : not found, $*$ **: evidence of existence is poor)**

Some interesting features captured in the model (?)

- $I = J$ states from Particle Data Group

Roughly 300～400 MeV?

(n_ρ, n_z)	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$	$(2,0)/(0,2)$	$(2,1)/(0,3)$	$(1,2)/(3,0)$
$N\,(l=1)$	940^+	$\boxed{1440^+}$	$\boxed{1535^-}$	1655^-	1710^+, ?	2090^-_*, ?	2100^+_*, ?
$\Delta\,(l=3)$	1232^+	1600^+	1700^-	1940^-_*	1920^+, ?	?, ?	?, ?

Roughly 200～300 MeV?

nearly degenerate
between parity even and odd states?

<u>Note</u> The phenomenological quark model predicts

$$M_{N'(1440)} - M_N \sim 2 \times (M_{N^*(1535)} - M_N)$$

Recent lattice study suggests [K.Sasaki-S.Sasaki-Hatsuda 2005]

$$M_{N'(1440)} \sim M_{N^*(1535)}$$

Caution !

The predicted baryon spectrum looks nice,

but there are a lot of reasons that
you should NOT trust these values.

- $1/\lambda$ expansion may not work well.
- Higher derivative terms are neglected.
- $N_c = 3$ is not large enough especially for $l \geq 3, \;\; n_\rho + n_z \geq 3$
- The model deviates from real QCD at high energy $\sim M_{\rm KK}$
- $M_{\rm KK} \simeq 950$ MeV is the value consistent with ρ meson mass

➡ Need more investigation for the quantitative tests.

4 Summary and outlook

- Baryons are described as (4 dim) instantons in a 5 dim gauge theory.

- We proposed a new way to analyze baryons that extends Skyrme's old idea including contributions from vector mesons.

- There are a lot more to do to improve the analysis.
  (solve EOM numerically, include higher derivative terms, α', loop, N_f/N_c corrections etc.)

- It would be interesting to investigate other static properties of baryons.
  (couplings, charge radii, magnetic moments etc.)

[See Hong-Rho-Yee-Yi 2007]