D-instanton Contributions to String Amplitudes

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Motivation

1. String perturbation theory can in principle generate perturbative contribution to string amplitudes to any given order in string coupling

– resummation can give the contribution from path integral over the steepest descent contour / Lefschetz thimble associated with the perturbative saddle point.

A similar analysis for other saddle points could generate the contribution from the Lefschetz thimbles associated with these saddle points.

If we could do this for every saddle point then by taking the appropriate weighted union of these contributions we may be able to get the full non-perturbative amplitudes (in principle).

Our goal will be to understand the perturbative contribution from a class of saddle points – D-instantons or more generally Euclidean D-branes wrapped on compact cycles.

2. D-instanton contributions play an important role in studying many aspects of string phenomenology

– moduli stabilizations via KKLT or LVS

– computation of Yukawa couplings and gauge field kinetic terms

However before we get into these computations we should test the procedure on examples where the results are known from various conjectured duality relations

– two dimensional string theory (discussed earlier)

Balthazar, Rodriguez, Yin; A.S.

– ten dimensional type IIB string theory (this talk)

– type IIA/IIB string theory compactified on Calabi-Yau manifolds (work in progress)

Alexandrov, A.S., Stefansky

The issues

The dynamics of D-instantons (including euclidean D-branes on compact cycles) is described by open strings with ends stuck on the D-instanton

– do not carry continuous momenta.

Therefore the infrared divergences associated with the zero modes cannot be treated by analytic continuation.

In the world-sheet description these divergences show up as divergences from the boundaries of the moduli spaces of Riemann surfaces associated with open string degeneration.

Strategy:

1. Represent the world-sheet contribution as coming from sum over Feynman diagrams of string field theory (SFT)

2. Explicitly remove the zero mode contribution from the internal propagators

3. Carry out the integral over the zero modes at the end

– generates unambiguous, divergence free results

However computing the overall normalization, associated with one loop determinant of the open string fields in the instanton background, requires a somewhat different approach

– subject of this talk.

We shall illustrate this procedure for a particular class of problems

– D-instanton contribution to IIB string theory amplitudes in 10 dimensions.

However the method is quite general and applies to any euclidean D-brane in any string theory.

The problem

Consider four graviton scattering amplitude in type IIB string theory.

At tree level, it is given by the supergravity result and an additional R^4 contact interaction: Gross, Witten

$$\frac{i}{4}\,\kappa^2\,K_c\,\left[\frac{64}{stu}+2\zeta(3)\right]\,(2\pi)^{10}\,\delta^{(10)}(p_1+p_2+p_3+p_4)$$

in $\alpha' = 1$ unit.

$$\kappa^2 \equiv 8\pi G = 2^6\pi^7 g_s^2$$

g_s: string coupling, normalized so that the D-string tension is $1/(2\pi g_s)$.

K_c: A kinematic factor that depends on the polarizations and momenta of the external gravitons (carries 8 powers of momentum).

$$\frac{i}{4}\,\kappa^2\,K_c\,\left[\frac{64}{stu}+2\zeta(3)\right]\,(2\pi)^{10}\,\delta^{(10)}(p_1+p_2+p_3+p_4)$$

The supergravity contribution, given by the first term, is S-duality invariant.

The second term is not, but there is an S-duality invariant completion containing one loop and non-perturbative terms.

Green, Gutperle

$$\frac{i}{4}\,\kappa^2\,K_c\,\left[\frac{64}{stu}+2\zeta(3)+\frac{2\pi^2}{3}\,g_s^2+4\,\pi\,g_s^{3/2}\,\sum_{k=1}^{\infty}\sqrt{k}\,\left(\sum_{d|k}d^{-2}\right)\,\left\{e^{2\pi ik\tau}+e^{-2\pi ik\tau^*}\right\}\,\{1+O(g_s)\}\right]$$

$\tau=a+i/g_s$, a: vev of RR scalar field

$e^{2\pi ik\tau}$: k D-instanton contribution,

$e^{-2\pi ik\tau^*}$: k anti-D-instanton contribution

$O(g_s)$ contains higher powers of g_s

This gives the following prediction for the leading term in the k D-instanton contribution to the four graviton amplitude:

$$i\, 2^6 \pi^8\, g_s^{7/2}\, e^{2\pi i k a}\, e^{-2\pi k/g_s}\, K_c\, k^{1/2} \sum_{d|k} d^{-2}$$

Our goal: Reproduce this from direct D-instanton computation.

Once this is done, all the higher order terms in power series expansion in g_s can be obtained using the differential equation implied by supersymmetry

Green, Sethi; Green, Vanhove; $\cdots$, Wang, Yin

– determines the function completely without help of S-duality

Nevertheless it is important to develop techniques for evaluating the higher order terms by direct perturbative computation

– work in progress by ABCRY who also determined the g_s dependence for k=1 earlier.

Agmon, Balthazar, Cho, Rodriguez, Yin

$$i\,2^6\pi^8\,g_s^{7/2}\,e^{2\pi i k a}\,e^{-2\pi k/g_s}\,K_c\,k^{1/2}\sum_{d|k} d^{-2}$$

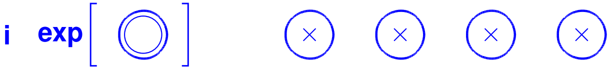
The $e^{2\pi i k a}\,e^{-2\pi k/g_s}$ comes from exponential of the action of k instantons.

We shall not discuss these any further.

Our goal: Reproduce the $2^6\pi^8\,g_s^{7/2}\,K_c\,k^{1/2}\sum_{d|k} d^{-2}$ factor.

Single instanton

The leading contribution comes from the product of four disk one point functions and arbitrary number of annulus zero point functions.



Define N = i exp[A], A: Annulus zero point function

Our first task will be to calculate N.

At this order, all the subtleties reside in the calculation of N.

Note: we include disconnected world-sheet since individual world-sheets do not conserve momentum

– restored at the end after integration over zero modes

$$\mathbf{N} = \mathbf{i}\, \mathbf{e}^{\mathbf{A}}$$

For type IIB D-instantons:

$$\mathbf{A} = \int_0^\infty \frac{dt}{2t} \left[\frac{1}{2}\,\eta(it)^{-12}\left\{\vartheta_3(0,it)^4 - \vartheta_4(0,it)^4 - \vartheta_2(0,it)^4 + \vartheta_1(0,it)^4\right\}\right]$$

ϑ_α **are Jacobi theta functions.**

A = 0 by theta function identity

– result of cancellation between NS and R sector open string states.

The first two terms come from NS sector and the last two terms come from the R sector.

The general structure of A:

$$A = \int_0^\infty \frac{dt}{2t}\, \mathrm{Tr}\left[e^{-2\pi t L_0}(-1)^F\right]$$

The vanishing of A shows that the contribution from the positive L_0 states cancel in Bose-Fermi pair.

For these there are no subtleties and the cancellation is genuine.

However the cencellation cannot be trusted for the zero modes and we need to treat them carefully.

For this it will be useful to regulate the system by introducing a small non-zero L_0 value h for the zero modes at the intermediate steps

– put slightly shifted boundary condition on the two boundaries

– preserves conformal and BRST invariance on the world-sheet

In this regulated system

$$A = \int_0^\infty \frac{dt}{2t}\left(8\,e^{-2\pi th} - 8\,e^{-2\pi th}\right) = \int_0^\infty \frac{dt}{2t}\left(10\,e^{-2\pi th} - 2\,e^{-2\pi th} - 8\,e^{-2\pi th}\right)$$

$\downarrow$ absence of UV divergence as $t \to 0$

$$\Rightarrow N = i\,e^A = i\sqrt{\frac{h^8\,h^2}{h^{10}}} = i\int\left\{\prod_{\mu=0}^{9}\frac{d\xi_\mu}{\sqrt{2\pi}}\right\}dp\,dq\,\exp\left[-\frac{1}{2}h\sum_{\mu=0}^{9}\xi_\mu\xi^\mu - h\,p\,q\right]$$

$$\int\prod_{\alpha=1}^{16}d\chi_\alpha\,\exp\left[\frac{1}{2}g_{\alpha\beta}\chi_\alpha\chi_\beta\right]$$

ξ_μ: 10 grassmann even modes related to D-instanton position

p,q: 2 grassmann odd modes representing ghosts

χ_α: 16 grassmann odd modes representing fermion zero modes

$g_{\alpha\beta}$: an anti-symmetric, 16×16 hermitian matrix satisfying:

$$g^2 = h\,I_{16}\,, \quad I_{16}: 16 \times 16 \text{ identity matrix}$$

We now proceed as follows.

1. First we interpret N as the Siegel gauge fixed path integral of open string field theory on the instanton

– fixes the normalization of the integration measure.

2. In this interpretation, the modes p and q represent Faddeev-Popov ghosts in the NS sector.

3. Then we show that the Siegel gauge becomes singular in the h $\to$ 0 limit, and this is the reason why the coefficient of the p q term, representing the ghost kinetic operator, vanishes.

4. The remedy is to work with the original gauge invariant path integral before gauge fixing.

This gets rid of the zero modes p and q from the integral.

Final result (after setting h=0)

$$\mathbf{N} = \mathbf{i} \int \left\{ \prod_\mu \frac{\mathbf{d}\xi^\mu}{\sqrt{\mathbf{2}\pi}} \right\} \left\{ \prod_\alpha \mathbf{d}\chi_\alpha \right\} \mathbf{d}\phi \, \mathbf{e}^{-\phi^2/4} \bigg/ \int \mathbf{d}\theta$$

ϕ**: The mode wrongly fixed to 0 in the Siegel gauge**

θ**: The gauge transformation parameter used for the wrong gauge fixing**

We can now do the ϕ integral and write:

$$\mathbf{N} = \mathbf{i}\,(\mathbf{2}\pi)^{-\mathbf{5}}\, \mathbf{2}\sqrt{\pi} \int \left\{ \prod_\mu \mathbf{d}\xi^\mu \right\} \left\{ \prod_\alpha \mathbf{d}\chi_\alpha \right\} \bigg/ \int \mathbf{d}\theta$$

ξ^μ's are related to the location x^μ of the instanton in space-time.

Precise relation may be found by comparing

– the explicit coupling of ξ^μ to a string amplitude from world-sheet computation, and

– the expected coupling of x^μ via $e^{ip.x}$ factor

Result:

$$\xi^\mu = x^\mu/(g_o\,\pi\,\sqrt{2}) \;\Rightarrow\; \prod_\mu d\xi^\mu = (\pi\sqrt{2}g_o)^{-10}\prod_\mu dx^\mu$$

g_o: open string coupling = $2^{-1}\pi^{-3/2}g_s^{1/2}$

The gauge transformation parameter θ is related to the rigid U(1) gauge transformation parameter α on the D-instanton.

If we take an open string connecting the D-instanton to a neighboring instanton, this state picks up a phase $e^{i\alpha}$.

Compare the infinitesimal transformation by α to the string field theory gauge transformation generated by θ.

This gives:

$$\theta = 2\,\alpha/g_o \quad \Rightarrow \quad \int d\theta = (2/g_o)\int d\alpha = 4\pi/g_o$$

since α has period 2π.

$$\mathbf{N} = \mathbf{i}\,(\mathbf{2}\pi)^{-5}\,\mathbf{2}\sqrt{\pi}\,(\pi\sqrt{\mathbf{2}}\mathbf{g_o})^{-10}\,\mathbf{g_o}/(\mathbf{4}\pi)\,\int\left\{\prod_\mu \mathbf{dx}^\mu\right\}\left\{\prod_\alpha \mathbf{d}\chi_\alpha\right\}$$

Integrations over the collective modes x^μ and χ_α have to be done at the end after computing the full amplitude, since the other world-sheet components also have x^μ and χ_α dependence.

x^μ integral eventually generates the $(2\pi)^{10}\delta^{(10)}(\sum_j p_j)$ factor.

Integration over the grassmann odd variables χ_α will vanish unless there are 16 insertions of χ_α in the rest of the amplitude.

Only non-vanishing contribution comes from inserting 4 fermion zero modes on each of the four disks

The χ_α integrals generate a factor of $\epsilon^{\alpha_1\cdots\alpha_{16}}$ multiplying the product of the four disk amplitudes with $\chi_{\alpha_1},\cdots,\chi_{\alpha_{16}}$ insertions.

– precisely reproduces Green-Gutperle prediction for k=1

k-instantons

The center of mass degrees freedom give the same integral as a single instanton except for some powers of k from Chan-Paton factors

The relative degrees of freedom have to be analyzed similarly by gauge 'unfixing' the Siegel gauge and explicitly integrating over the out of Siegel gauge modes.

The remaining part is a supersymmetric matrix integral which had already been computed while studying the index of multiple D0-branes Krauth, Nicolai, Staudacher; Moore, Nekrasov, Shatashvili

Putting these results together we reproduce precisely the leading term in the k-instanton amplitude as predicted by Green and Gutperle.

Conclusion

String (field) theory gives a systematic procedure for computing D-instanton contribution to the amplitudes.

We should be able to apply this procedure to calculate D-instanton contribution in situations where S-duality may not be of help

e.g. semi-realistic string compactifications $\cdots$

$\cdots$ and other problems where D-instanton corrections might play a dominant role.