Something from "Nothing"

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hep-th/0204158
+ gr-qc

Summary

"Bubbles of nothing"

- are smooth, weakly coupled t-dependent solutions in which we can reliably study phenomena such as cosmological horizons & particle production

squeezed states

↕

nonlocal string theory (NLST)

- exhibit features of dS, Rindler, moving mirror spacetimes but are good (& simple) solutions of string theory
- generically stop accelerating
- generically stabilized quantum mechanically by absorbing quantum-generated stress energy in radial variation of metric & dilaton

Consider D-dimensional Schwarzchild solution

$$ds^2 = -\left(1-\left(\frac{r_0}{r}\right)^{D-3}\right)dt^2 + \left(1-\left(\frac{r_0}{r}\right)^{D-3}\right)^{-1}dr^2$$

$$+ r^2\left(d\theta^2 + \sin^2\theta \, d\Omega^2_{D-3}\right)$$

let $\chi \equiv it$

$i\tau \equiv \theta - \frac{\pi}{2}$

(Gott 1974
Witten 1982 : instanton solution
⋮
Fabinger/Horava
⋮)

$$\Rightarrow ds_s^2 = \left(1-\left(\frac{r_0}{r}\right)^{D-3}\right)d\chi^2 + \left(1-\left(\frac{r_0}{r}\right)^{D-3}\right)^{-1}dr^2$$

$$+ r^2\underbrace{\left(-d\tau^2 + \cosh^2\tau \, d\Omega^2_{D-3}\right)}_{ds^2_{dS_{D-2}}}$$

$r \geq r_0$

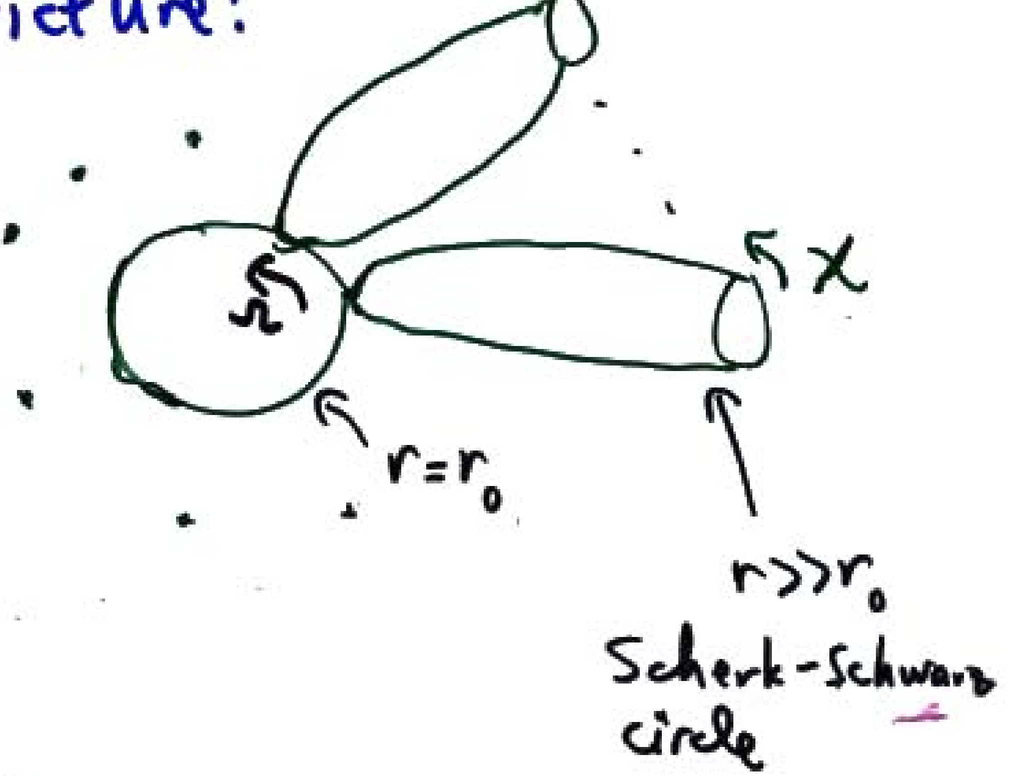
regularity at $r = r_0$

$\Rightarrow \chi = \chi + 2\pi\left(\frac{2r_0}{D-3}\right)$, with antiperiodic boundary conditions for fermions

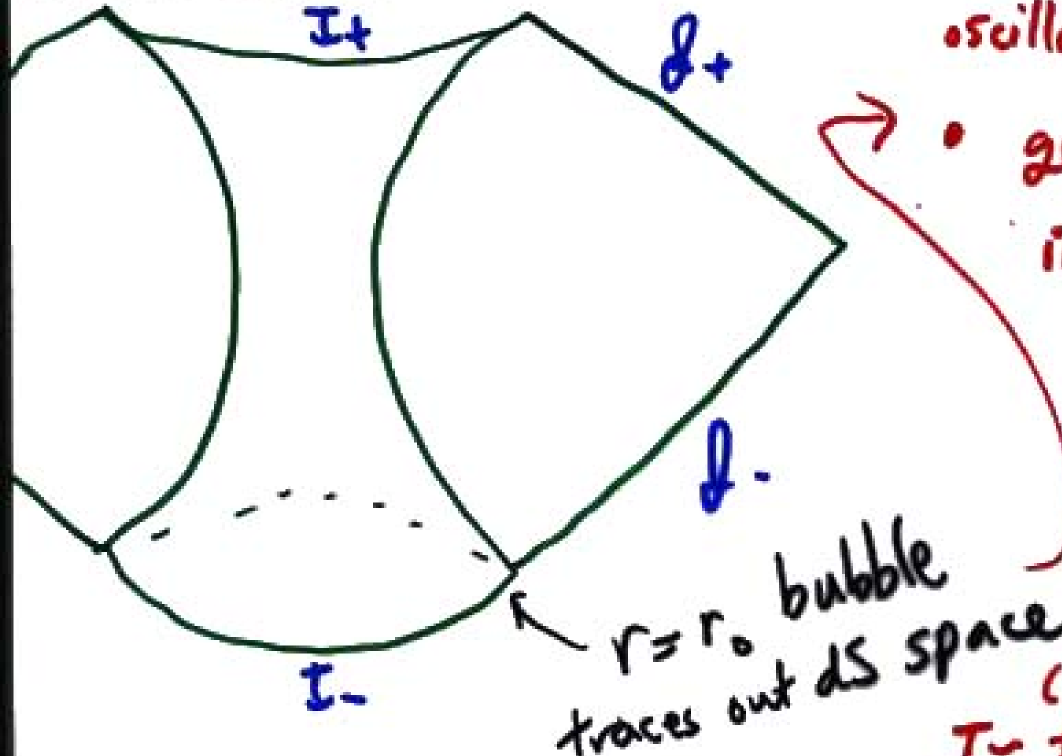
This is a smooth, g_s = constant, solution classically

Schematic picture:

fixed time:



evolution, suppressing X direction:



- massive geodesics oscillate near the bubble
- get horizons like in dS space, but horizon area (= entropy) is infinite.
- $\exists$ thermal (Euclidean) state with $T \sim \frac{1}{r_0}$ thermal bubble bath

These are smooth solutions to GR $\Rightarrow$ solve classical string theory to leading order in $\frac{\alpha'}{r_0^2}$

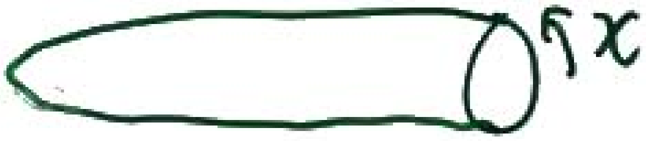
Moreover,

- The Euclidean continuation is an <u>isolated</u> solution $\Rightarrow \exists$ worldsheet RG fixed point order by order in α' shifted a small amount from GR (leading α') solution

- the Lorentzian bubble solution has no massless (or tachyonic) modes localized near the bubble $\Rightarrow$ again $\exists$ nearby solution including α' corrections

cf Gibbons & Hartnoll

b) quantum mechanically, the Schwarzchild bubble is unstable because of the Scherk-Schwarz circle at $r \to \infty$:



Rohm

$$\Lambda_{D-1} \sim -\frac{\alpha'^4}{r_x^{10}} g_s^2$$

$\Rightarrow$ potential for r_x, driven toward tachyonic regime at small radius.

So this case is not a good t-dependent background after all. However certain generalizations of this are:

Kerr Bubbles

instantons: Dowker Gauntlett Giddings Horowitz
Tseytlin / Russo
Costa Gutperle
Gutperle Strominger

4D case (after double analytic continuation)

$$ds^2_{\text{kerr}_4} = -(r^2+\beta^2\sinh^2\tau)\,d\tau^2$$

$$+\,d\chi^2 - \frac{r_0 r}{r^2+\beta^2\sinh^2\tau}\big(d\chi + \underbrace{\beta}_{=\,i\alpha}\cosh^2\tau\, d\varphi\big)^2$$

angular momentum of Kerr BH

$$+\cosh^2\tau\,(r^2-\beta^2)\,d\varphi^2$$

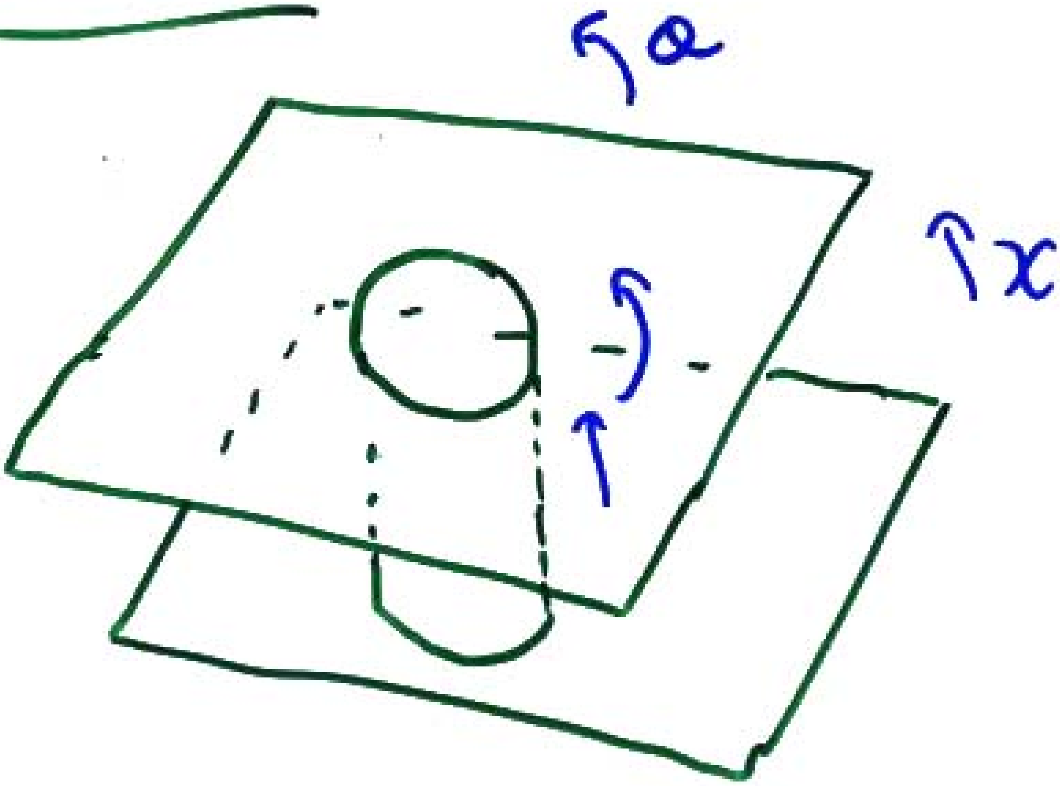
$$+\frac{(r^2+\beta^2\sinh^2\tau)}{r^2-\beta^2-r_0 r}\,dr^2$$

with identifications

$$\begin{pmatrix}\chi\\ \varphi\end{pmatrix} = \begin{pmatrix}\chi + 2\pi n_1 R\\ \varphi + 2\pi n_1 R\Omega + 2\pi n_2\end{pmatrix}\quad [(-1)^F]^{n_1}$$

$$R = \frac{1}{\kappa}\ \text{surface gravity} \qquad \Omega = \frac{\beta}{r_0^2-\beta^2}\ \text{angular momentum}$$

Fixed time :



Bubble excises region near origin of "twisted line" $(\mathbb{C}\times\mathbb{R})/\Gamma$

Bubble evolution:

Near-bubble metric:

$$ds_b^2 = -\left(r_0 r_b + \beta^2 \cosh^2 \tau\right) d\tau^2 + \frac{\cosh^2 \tau}{r_0 r_b + \beta^2 \cosh^2 \tau} \left(\beta\, d\chi - r_0 r_b\, d\varphi\right)^2$$

For $\beta^2 \cosh^2 \tau \ll 1$, get dS behavior near bubble.

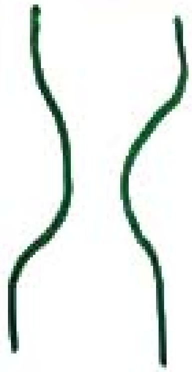
For $\tau \to \infty$, (suppressing χ)

$$ds_b^2 \to -\beta^2 \cosh^2 \tau\, d\tau^2 + \frac{r_0 r_b}{\beta^2} d\varphi^2$$

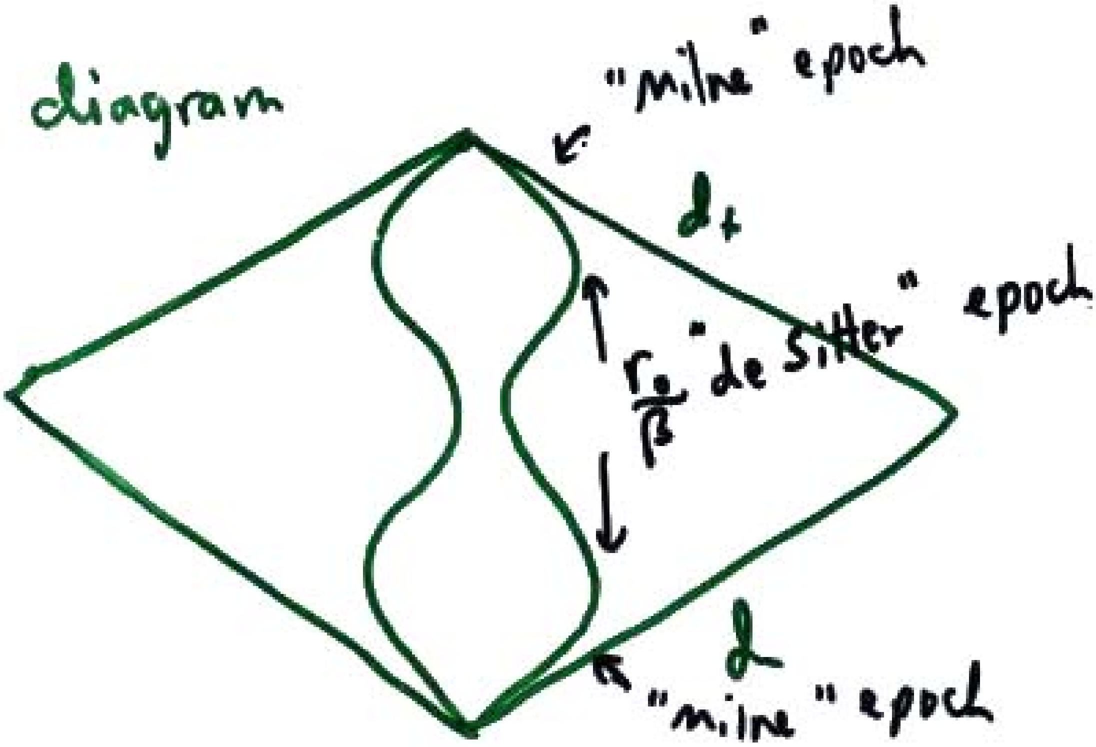
$$= -\beta^2\, d(e^\tau)^2 + \frac{r_0 r_b}{\beta^2} d\varphi^2$$

Bubble stops!

Time evolution of Kerr bubble (4D)



Penrose diagram



At late times, the bubble stops accelerating, & the surrounding region expands linearly in proper time (similarly to the Milne universe)

The pattern so far is that the bubbles will accelerate into locally non-SUSY regions, but not into locally SUSY regions. (or bosonically, regions in which compact dimension grows to ∞)

This pattern continues in higher dimensions:

- If we put generic rotation angles into Myers-Perry-Kerr BH, get a bubble which stops accelerating in all directions
- If we rotate some but not all planes, the bubble stops accelerating in some but not all directions:

Stability of Kerr case

Myers
Perry

* Classically Kerr BHs in $D>4$ have not been shown explicitly to be stable under small perturbations (difficult to separate eqns of motion for $\delta g_{\mu\nu}$), nor have we done so for Kerr bubbles, but we expect both to be stable. cf Empavan Reall

* Quantum mechanically :

1)

Quantum-induced $\langle T_{\mu\nu} \rangle$ is localized near $r = r_{bubble}$, and falls like $\frac{1}{r^{10}}$ for large r since the susy-breaking identifications relate points separated by $\sim r$

$\Rightarrow$ can be absorbed in radial variation of Φ, $g_{\mu\nu}$

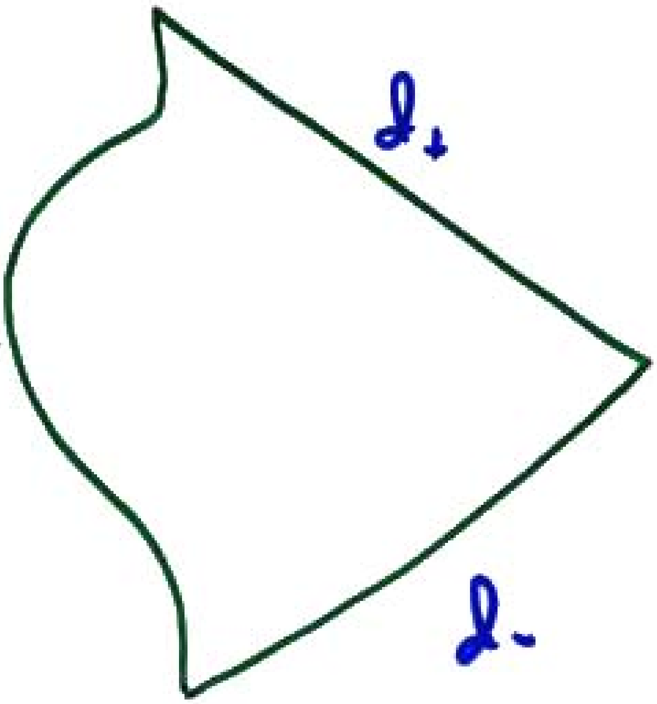
2) • <u>No</u> additional bubbles nucleate in case with all angles rotated

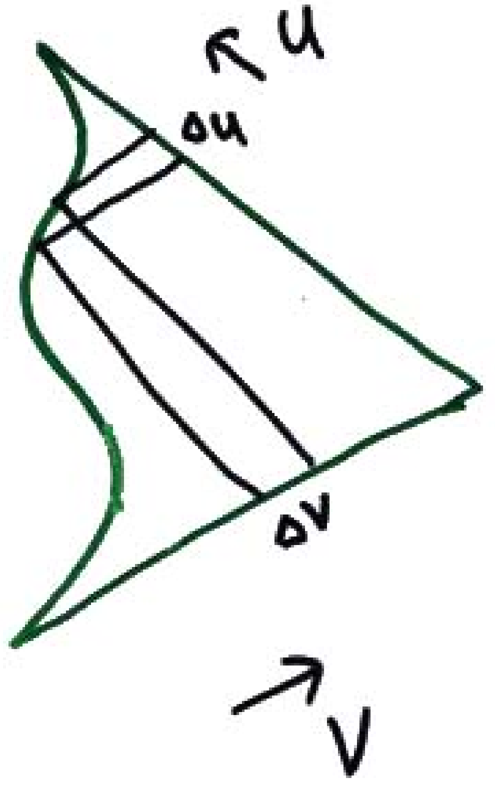
• Additional bubbles may nucleate in hybrid case with not all angles rotated

Particle creation by squeezing in the bubble bath

The bubble solutions provide a textbook example of particle creation (in fact the examples in textbooks – Rindler space, moving mirrors, BHs, FRW, non-solutions, – are not immediately applicable in string theory whereas the bubbles are)

Bubbles are similar to Rindler, moving mirror except they are solutions to Einstein eqs with no external forces





Massless particle production:

consider high-frequency $(\omega \gg \frac{1}{r_0})$ mode

$\phi \sim e^{i\omega \Delta U}$ at $\mathcal{J}+$ (pure positive frequency wrt U)

Using geometric optics (constant phase), trace this back to a mode on $\mathcal{J}-$ by studying the geodesics $\Rightarrow$

$\phi \sim e^{i\omega \Delta U(\Delta V)}$ at $\mathcal{J}-$ (contains both positive & negative frequency wrt V)

4D Kerr:

S-wave null geodesics: $t = \sinh \tau$

$$ds^2 = -\frac{r^2+\beta^2 t^2}{1+t^2}\,dt^2 + \frac{r^2+\beta^2 t^2}{r^2-\beta^2-r_0 r}\,dr^2$$

$$+\, d\chi,\ d\alpha \text{ terms } \stackrel{!}{=} 0$$

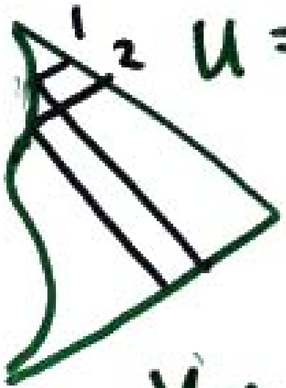
$$\Rightarrow \int \frac{dt}{\sqrt{1+t^2}} = \pm \int \frac{dr}{\sqrt{r^2-\beta^2-r_0 r}}$$

outgoing vs incoming wave

$$\Rightarrow \cdots \Rightarrow\ e^{\tau} = e^{-C}\left[r - \frac{r_0}{2} \pm \sqrt{r^2-\beta^2-r_0 r}\right]$$

where $t = \sinh \tau$

integration constant (labels geodesic)



$$U = -re^{-\tau} + \frac{\beta^2}{4}\frac{e^{\tau}}{r} = T - X$$

$$V = re^{\tau} - \frac{\beta^2}{4}\frac{e^{-\tau}}{r} = T + X$$

$ds^2 \to -dU\,dV$ for $r \gg r_0, r_b, \beta$

$|\tau| \to \infty$

$dX = 0 = d\varphi$

Plug in geodesic relation b/w e^{τ} & r $\Rightarrow$

$U \to \frac{1}{2}\left(-e^{c} + \beta^2 e^{-c}\right)$ outgoing on $\mathcal{I}^+$

$V \to e^{-c}\Big(\underbrace{\frac{\beta^2}{2} + \frac{r_0^2}{8}}_{K_0^{\text{in}}}\Big) - \frac{\beta^2}{4}\frac{e^{c}}{\frac{\beta^2}{2} + \frac{r_0^2}{8}}$ incoming on $\mathcal{I}^-$

Fix C_1 (reference geodesic), solve for $\Delta U(V_2)$ by first solving for $e^{C_2} = 2\frac{K_0}{\beta^2}\left(-V_2 + \sqrt{V_2^2 + \beta^2}\right)$

$\to U_2^{(V_2)} = \frac{1}{2}\left(-e^{C_2} + \beta^2 e^{-C_2}\right)$

Project onto Fourier modes in V:

$$\frac{1}{\sqrt{\omega}} e^{i\omega \Delta u} = \int d\omega' \left(\alpha^*_{\omega'\omega} \frac{e^{-i\omega' V_2}}{\sqrt{\omega'}} - \beta_{\omega'\omega} \frac{e^{i\omega' V_2}}{\sqrt{\omega'}} \right)$$

i.e.

$$\alpha_{\omega'\omega} = \frac{1}{2\pi} \sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{\infty} dV \, e^{i\omega' V} e^{i\omega \Delta u(V)}$$

* $$\beta_{\omega'\omega} = \frac{1}{2\pi} \sqrt{\frac{\omega'}{\omega}} \int_{-\infty}^{\infty} dV \, e^{-i\omega' V} e^{i\omega \Delta u(V)}$$

$$dN_\omega = d\omega \int_{\frac{1}{r_0}}^{M_p} d\omega' |\beta_{\omega'\omega}|^2$$

Stationary phase point:

$$V_s^2 = \pm \frac{K_0}{2} \frac{\omega}{\omega'} \quad \text{for } \begin{cases} \alpha \\ \beta \end{cases}$$

where $\frac{\beta^2}{r_0^2} \ll \frac{\omega}{\omega'}$ for all ω, ω' in our range of validity $(\frac{1}{r_0}, M_p)$

* "de Sitter" phase dominates

Plug in the stationary phase →

$$\alpha \sim e^{i \frac{r_0}{2} \sqrt{\omega' \omega}}$$

$$\beta \sim e^{- \frac{r_0}{2} \sqrt{\omega' \omega}}$$

This agrees with an exact calculation for the Schwarzschild bubble (with an eternal "de Sitter" phase):

$$|\alpha_{\omega'\omega}| = \frac{r_0}{8} H^{(1)}_{-1}\left(\tfrac{1}{2} r_0 \sqrt{\omega\omega'}\right)$$

$$|\beta_{\omega'\omega}| = \frac{r_0}{4\pi} K_1\left(\tfrac{1}{2} r_0 \sqrt{\omega\omega'}\right)$$

Exponentially suppressed at high frequencies; because of the flat space surrounding the bubble, get vanishing particle number density ⟹ no back-reaction issue

Particle Creation produces squeezed states:

$|in\rangle$ vacuum satisfies

$$a|in\rangle = 0$$

where $Q \to a e^{i\omega t} + a^{+} e^{-i\omega t}$

on initial slice.

$$Q \to b e^{i\omega t'} + b^{+} e^{-i\omega t'}$$

on final slice.

These are related by Bogolubov coefficients

$$a = \alpha b + \beta b^{+}$$

$$\Rightarrow \alpha b + \beta b^{+} |in\rangle = 0$$

$$\Rightarrow |in\rangle = N e^{-\frac{\beta}{2\alpha} b^{+2}} |out\rangle \qquad b|out\rangle = 0$$

→ How do we describe spacetime squeezed states in perturbative string theory? cf Lawrence Martinec SFT

Firstly, recall that coherent states (infinitesimal shift of background) correspond to perturbations of the world sheet action by the corresponding vertex operator:

$$\delta S_{ws} = c \int d^2\sigma \, V$$

this suggests a generalization to squeezed states

$$\delta S_{ws} = -\frac{\beta}{2g} \int_{\Sigma} V_+ \int_{\Sigma} V_+$$

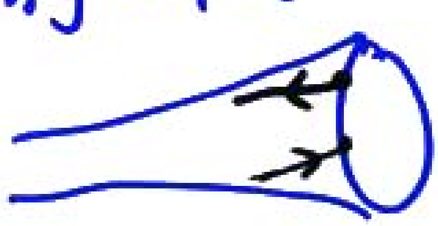
positive frequency modes

Such nonlocal worldsheet theories (NLST) arise in a very similar way in considering multitrace deformations of AdS/CFT

O. Aharony, M. Berkooz, E.S.
Berkooz, Sever, Shomer
Witten
Muck, Minces, Petkou, ...

AdS/CFT

$\delta \mathcal{L}_{CFT} \sim \tilde{h} \int \mathcal{O}_1 \mathcal{O}_2$



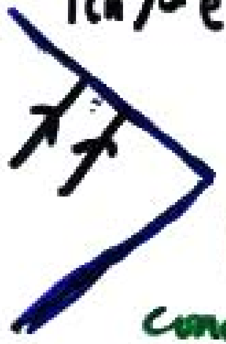
- SUGRA on AdS: deformation of boundary condition
- worldsheet (perturbatively in $\tilde{h}$):

$$\delta S_{ws} \sim \tilde{h} \int V_{\mathcal{O}_1} \int V_{\mathcal{O}_2}$$

$|in\rangle \sim e^{-\frac{\beta}{2\nu} a_{out}^{+2}} |out\rangle$

$\Updownarrow$

boundary condition in path integral on $\mathcal{I}_+$



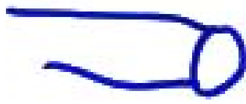
particle creation

- nontrivial (squeezed) state $\Rightarrow$ deformation of boundary condition
- worldsheet

$$\delta S_{ws} \sim \frac{\beta}{2\nu} \int V_+ \int V_+$$

Comments on Bubbles as Instantons

1. Schwarzchild case:



$R \ll r_0$

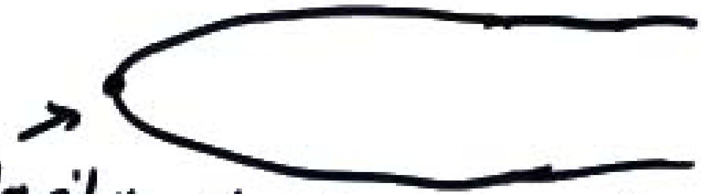
a) perturbative instability $\Lambda_1 \sim -\frac{g_s^2}{R^{10}}$

dominates over bubble nucleation $\sim e^{-\frac{r_0^2}{\lambda_0^2}}$

b) At least some methods of stabilizing R perturbatively also remove the smooth instanton:

e.g. flux $\int dC^{(0)} = Q$

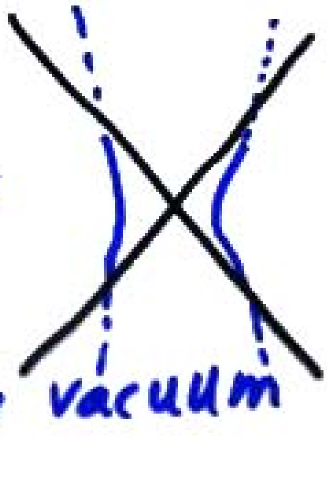
$\Rightarrow$



($C^{(0)}$ twisted)

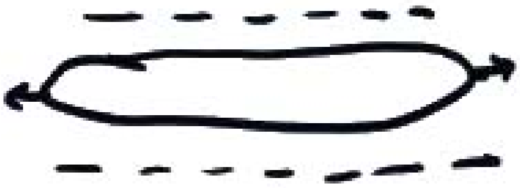
2. Kerr case:

As we saw, bubble stops accelerating $\Rightarrow$ timelike observers can avoid the bubble, so the instanton does not destabilize this vacuum



Future Directions

- compactify transversely to obtain gravity on bubble



- study vacuum ambiguity, particle creation for massive modes localized near bubble in dS phase
- Horowitz & Maeda : multiple bubbles & their behavior in collision
- Birmingham & Rinaldi
- Balasubramanian, Ross ; ... + de Boer, Levi, Minic

  AdS BHs → AdS bubbles → boundary is $dS \times S^1$
- Buchel solutions

- dS from noncritical strings   Maloney, ES, Strominger