SUPERSTRINGS IN A PLANE-WAVE BACKGROUND

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INTRODUCTION

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However, inability to do string calculations in this background has been a source of frustration.

Recently, Blau, Figueroa-O'Farrill, Hull, and Papadopoulos discovered a plane-wave limit of the string background that is also maximally supersymmetric (32 supercharges).

hep-th/0110242 + hep-th/0201081

Metsaev showed in hep-th/0112044 that IIB strings in this plane-wave background are described in the GS light-cone gauge formalism by free massive bosons + fermions.

Berenstein, Maldacena, + Nastase (hep-th/0202021) identified the corresponding limit of the gauge theory and carried out some checks of the duality.

Since BMN's paper this has become a very active subject. Many important results will be described in other talks.

I have been studying this subject with Curtis Callan and four Caltech students (Lee, McLoughlin, Swanson, Wu). This talk will discuss some of the issues that we have been thinking about.

THE GEOMETRY

$$ds^2_{AdS_5} = R^2\left(-\cosh^2\rho\, dt^2 + d\rho^2 + \sinh^2\rho\, d\Omega_3^2\right)$$

$$ds^2_{S^5} = R^2\left(\cos^2\theta\, d\phi^2 + d\theta^2 + \sin^2\theta\, d\tilde{\Omega}_3^2\right)$$

Let $\quad r = R\sinh\rho\,, \quad y = R\sin\theta$

$\quad\quad x^+ = t/\mu\,, \quad x^- = \mu R^2(\phi - t)$

μ is an arbitrary mass scale. x^- has period $2\pi\mu R^2$. Therefore the conjugate (angular) momentum is

$$P_- = J/\mu R^2$$

where J is an integer.

In terms of the new coordinates, the $AdS_5 \times S^5$ metric becomes

$$ds^2 = 2\left(1-\frac{y^2}{R^2}\right)dx^+dx^- - \mu^2(r^2+y^2)(dx^+)^2$$
$$+\frac{1}{\mu^2R^2}\left(1-\frac{y^2}{R^2}\right)(dx^-)^2 + ds_\perp^2$$

where

$$ds_\perp^2 = r^2\,d\Omega_3^2 + \frac{R^2}{R^2+r^2}dr^2$$
$$+ y^2\,d\tilde{\Omega}_3^2 + \frac{R^2}{R^2-y^2}dy^2$$

$R \to \infty$ gives the plane-wave limit

$$ds_{pp}^2 = 2dx^+dx^- - \mu^2(x^I)^2(dx^+)^2 + dx^Idx^I$$

$$\left(y^2 = \sum_{I=1}^{4}(x^I)^2 \text{ and } r^2 = \sum_{I=5}^{8}(x^I)^2\right)$$

We are investigating the leading $\frac{1}{R^2}$ correction to this metric, which we treat as a perturbation.

THE DUALITY

① AdS/CFT

$$SU(N) \longleftrightarrow \int_{S^5} F_5 = N$$

$$g_{YM}^2 N \longleftrightarrow R^4/(\alpha')^2$$

$$g_{YM}^2 \longleftrightarrow g_s$$

② pp-wave

U(1) R charge $\quad J \longleftrightarrow \mu R^2 P_-$

$R \to \infty \Rightarrow J, N \to \infty$ with fixed

$$\lambda' = \frac{g_{YM}^2 N}{J^2} \longleftrightarrow (\alpha' \mu P_-)^{-2}$$

This is gauge theory expansion parameter for correlation functions of (near BPS) BMN operators. It replaces the usual $\lambda = g_{YM}^2 N$.

$$g_2 = \frac{J^2}{N} \leftrightarrow g_s(\alpha' \mu P_-)^2$$

is the effective string theory expansion parameter (2D).

NB: λ' and g_2 can be small at the same time, so the two perturbative regimes overlap.

The novel thing we are exploring is the addition of one more expansion parameter

$$\eta = \frac{1}{J} \leftrightarrow (\mu R^2 P_-)^{-1}$$

We hope to compare $O(\eta)$ corrections to the gauge theory and the string theory. We do not yet have results to report.

WORLD SHEET ACTION

To keep story simple, I will only describe the bosonic part

$$\mathcal{L}_B = \frac{1}{2} h^{\alpha\beta} G_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu$$

$h^{\alpha\beta} \equiv g^{\alpha\beta}\sqrt{-g}$ satisfies $(h^{\tau\sigma})^2 - h^{\tau\tau} h^{\sigma\sigma} = 1$

We will use this to eliminate $h^{\sigma\sigma}$.

$$P_\mu = \frac{\delta \mathcal{L}}{\delta \dot{X}^\mu} = h^{\tau\alpha} G_{\mu\nu} \partial_\alpha X^\nu$$

$$\Rightarrow \quad \dot{X}^\mu = \frac{1}{h^{\tau\tau}} G^{\mu\nu} P_\nu - \frac{h^{\tau\sigma}}{h^{\tau\tau}} X'^\mu$$

Substituting into $\mathcal{H} = P_\mu \dot{X}^\mu - \mathcal{L}$ gives

$$\mathcal{H}_B = \frac{1}{2h^{\tau\tau}} (P_\mu G^{\mu\nu} P_\nu + X'^\mu G_{\mu\nu} X'^\nu) - \frac{h^{\tau\sigma}}{h^{\tau\tau}} X'^\mu P_\mu$$

This gives two Lagrange multiplier eqs:

$$X'^\mu P_\mu = X'^+ P_+ + X'^- P_- + X'^I P_I = 0 \qquad (1)$$

$$P_\mu G^{\mu\nu} P_\nu + X'^\mu G_{\mu\nu} X'^\nu = 0 \qquad (2)$$

LIGHT CONE GAUGE

The local reparametrization symmetry of the world sheet action allows me to set

$$x^+ = \tau \quad \text{and} \quad P_- = 1$$

The latter means uniform σ density, so that P_- is the length of the σ interval. Substituting in the previous eqs gives

$$x'^- = - x'^I p_I \qquad (1')$$

$$p_I G^{IJ} p_J + x'^I G_{IJ} x'^J + G_{--} (x'^I p_I)^2$$
$$+ G^{++} p_+^2 + 2 G^{+-} P_+ + G^{--} = 0 \qquad (2')$$

(I have assumed $G_{+I} = G_{-I} = 0$.)

Now identify $-P_+$ (obtained by solving (2')) with $\mathcal{H}_{lc}(x^I, p_I)$, the generator of $x^+ = \tau$ translations.

Eq. (2') is a linear eqn for p_+ if $G^{++}=0$. This is the case for the pp-wave metric, where one obtains

$$\mathcal{H}_{lc} = \frac{1}{2}\left[(p_I)^2 + (x'^I)^2 + \mu^2 (x^I)^2\right]$$

which is just 8 free massive bosons.

The fermionic coordinates $\theta(\sigma,\tau)$ can be added. The analysis is more complicated, but one just ends up with free fermi fields of mass μ.

The $\frac{1}{R^2}$ correction to the pp wave metric give lots of additional terms all of which are quartic in the various fields.

$$\text{Eq. } (1') \Rightarrow \oint x'^I p_I \, d\sigma = 0 .$$

QUANTIZATION OF $\mathcal{H}_{lc}$

Fourier analysis gives harmonic oscillators

$$[a_m^I, a_n^{J\,\dagger}] = \delta^{IJ}\delta_{mn} \qquad I, J = 1, \ldots, 8 \quad -\infty \le m, n \le \infty$$

with frequencies

$$\omega_n = \sqrt{1 + \lambda' n^2}$$

$$H_{lc} = \mu \sum \omega_n (a_n^{I\,\dagger} a_n^I + \text{fermions})$$

Also, $\oint x'^I p_I \, d\sigma$ + fermi term $= 0$

$$\Rightarrow \quad \sum n (a_n^{I\,\dagger} a_n^I + \text{fermions}) = 0$$

is the level-matching condition.

When $\mu = 0$, $n < 0$ and $n > 0$ correspond to left-movers and right-movers.

Still need to add string interactions to see g_2 dependence of the theory.

Equal x^+ commutation relations:

$$[\Phi(1), \Phi(2)] = \frac{1}{\alpha_1}\, \delta(\alpha_1 + \alpha_2)\, \Delta^8[x_1(\sigma) - x_2(\sigma)] \cdot \Delta^8[\theta_1(\sigma) - \theta_2(\sigma)]$$

or, equivalently, for the F. transform fields

$$[\tilde{\Phi}(1), \tilde{\Phi}(2)] = \frac{1}{\alpha_1}\, \delta(\alpha_1 + \alpha_2)\, \Delta^8[p_1(\sigma) + p_2(\sigma)] \cdot \Delta^8[\lambda_1(\sigma) + \lambda_2(\sigma)]$$

- Transcribe 1st-quantized operators to second-quantized ones:

$$H_2 = i\int dx^- D^8x(\sigma)\, D^8\theta(\sigma)\, \partial\Phi\, H_{2.c.}\, \Phi \quad , \text{etc.}$$

- Add interactions:

$$H = H_2 + H_3 + \cdots \quad , \quad Q = Q_2 + Q_3 + \cdots \quad \text{etc.}$$

Require that $p^I(\sigma)$ and $\lambda^a(\sigma)$ are conserved locally on the world-sheet and that the superalgebra is preserved.

LIGHT CONE GAUGE STRING FIELD THEORY

This was worked out for bosonic strings in 1973-4 by Mandelstam, Kaku + Kikkawa, Cremmer + Gervais. In 1982-3 it was generalized to superstrings by Green, Brink, & JHS. The papers are reprinted in Vol. II of "Superstrings, the First 15 Years".

The generalization to include mass parameter μ was initiated by Spradlin + Volovich (hep-th/0204146).
0206073

Type IIB superstring field:

$$\Phi(\alpha, x^I(\sigma), \theta^a(\sigma)) \qquad (\alpha \equiv 2p_-)$$

THREE STRING VERTEX

In the multi-Fock-space description it has the structure

$$|V_3\rangle = G\, E_a E_b |0\rangle$$

where

$$E_a = \exp\left(\frac{1}{2}\sum_{r,s=1}^{3}\sum_{m,n=-\infty}^{\infty} \bar{N}^{rs}_{mn}\, a^{(r)\dagger}_m a^{(s)\dagger}_n\right)|0\rangle$$

E_b is a similar fermionic expression and G is polynomial in the various oscillators. E_a and E_b correspond to the Δ functionals and G to the interaction point operator.

The $\bar{N}^{rs}_{mn}$ arise as products and inverses of various infinite matrices. Some cleverness is needed to evaluate them.

The flat space ($\mu=0$) formulas are

$$\bar{N}^{rs}_{mn} = - \frac{mn\,\alpha_1\alpha_2\alpha_3}{n\alpha_r + m\alpha_s}\,\bar{N}^r_m \bar{N}^s_n$$

$m,n = 1,2,\ldots$

$$\bar{N}^r_m = \frac{1}{\alpha_r\, m!}\,\frac{\Gamma(m\gamma_r)}{\Gamma(1-m+m\gamma_r)}\,e^{m\tau_0/\alpha_r}$$

$$\tau_0 = \sum \alpha_r \log|\alpha_r| \,, \quad \gamma_r = -\frac{\alpha_{r+1}}{\alpha_r}$$

The $\mu \neq 0$ formulas are not known. However, Klebanov, Spradlin, + Volovich (hep-th/0206221) have evaluated the leading large μ behavior, which is needed for comparison with the leading (small λ') correlation functions in the gauge theory.

CONCLUDING REMARKS

- The success with pp-wave background raises the hope that the results could be extended to the full $AdS_5 \times S^5$ case. This is one motivation for studying the leading $1/R^2$ corrections to the plane wave geometry.

- Various authors (starting with Thorn) have suggested that for finite J, the continuum string should be replaced by J discrete "bits". (The most recent proposal is by H. Verlinde hep-th/0206059). I would like to know whether this is just a kind of regulator or whether it is more fundamental.

- The BMN duality identifies limits of sequences of operators in the gauge theory with string states. It would be nice if, instead, one could identify a holographically dual theory on the conformal boundary. Berenstein + Nastase (hep-th/0205048) have shown that the conformal boundary of the plane wave geometry is a null curve.

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