E_{11} and M-theory

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based on work with Peter West

hep-th/0612001

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An introduction to E_{11}

Massless maximal supergravities all arise from dimensional reduction of 11-dimensional and IIB supergravities.

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In $D = 5$ and below, G is an exceptional group

Cremmer, Julia, Marcus, Schwarz

An introduction to E_{11}

D	G
10A	$\mathbb{R}^+$
10B	$SL(2, \mathbb{R})$
9	$SL(2, \mathbb{R}) \times \mathbb{R}^+$
8	$SL(3, \mathbb{R}) \times SL(2, \mathbb{R})$
7	$SL(5, \mathbb{R})$
6	$SO(5, 5)$
5	$E_{6(+6)}$
4	$E_{7(+7)}$
3	$E_{8(+8)}$

An introduction to E_{11}

Gravity as a non-linear realisation Borisov, Ogievetsky, 1974

$$g = exp(x^a P_a)\ exp(h_a{}^b K^a{}_b)$$

where the K's are the generators of $SL(D)$

$$[K^a{}_b, K^c{}_d] = \delta^c_b K^a{}_d - \delta^a_d K^c{}_b \quad [K^a{}_b, P_c] = \delta^a_c P_b$$

Gravity is formulated as the non-linear realisation of the closure of this group with the conformal group

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The theory is invariant under

$$g \rightarrow g_0 g h^{-1}$$

where h is local $SO(D)$

An introduction to E_{11}

Maurer-Cartan form:

$$\mathcal{V} = g^{-1} dg - \omega$$

ω: spin connection. It transforms as

$$\omega \rightarrow h \omega h^{-1} + h d h^{-1}$$

As a result, $\mathcal{V}$ transforms as

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One gets

$$\mathcal{V} = dx^{\mu} (e_{\mu}{}^{a} P_{a} + \Omega_{\mu a}{}^{b} K^{a}{}_{b})$$

An introduction to E_{11}

Similar analysis for the bosonic sector of 11-dimensional supergravity:

$$[R^{abc}, R^{def}] = R^{abcdef}$$

group element:

$$g = exp(x^a P_a)\ exp(h_a{}^b K^a{}_b)\ exp(A_{abc} R^{abc} + A_{abcdef} R^{abcdef})$$

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Field equations: duality relations

West, hep-th/0005270

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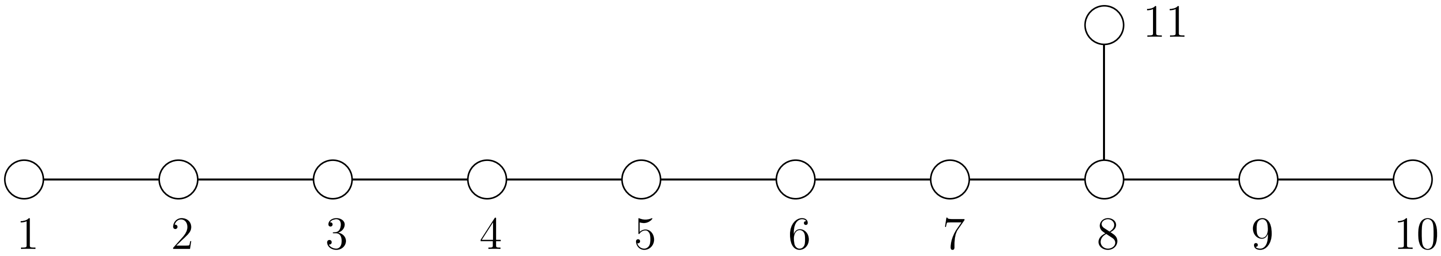
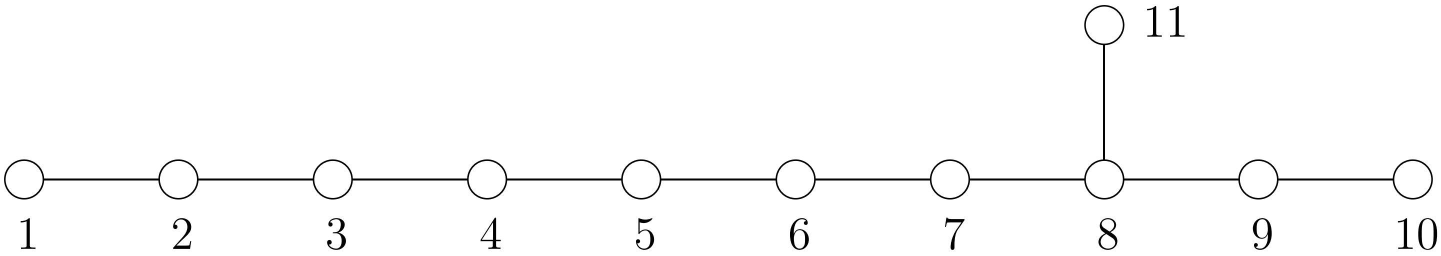
E_{11} is the smallest Kac-Moody group that contains this group

West, hep-th/0104081

An introduction to E_{11}



An introduction to E_{11}



Cartan matrix has negative eigenvalues $\rightarrow$ The algebra is infinite-dimensional

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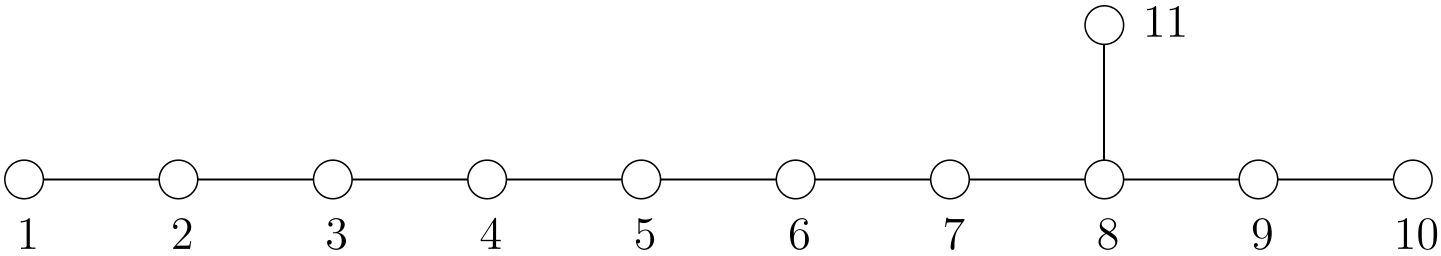

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A complete list of the generators is lacking, not to mention other representations...

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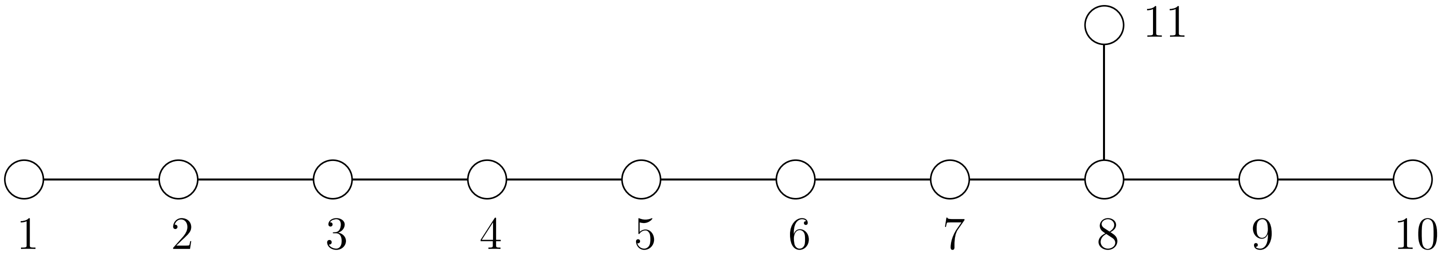

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Idea: write each positive root in terms of the simple roots of A_{10} and the simple root α_{11}

$$\alpha = \sum_{i=1}^{10} n_i \alpha_i + l\alpha_{11} \qquad l = \text{level}$$

An introduction to E_{11}

A necessary condition for the occurrence of a representation of A_{10} with highest weight $\sum_j p_j \lambda_j$ is that this weight arises in a root of E_{11}. One then gets

$$\alpha^2 = -\frac{2}{11} l^2 + \sum_{i,j} p_i (A_{ij})^{-1} p_j$$

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$$\alpha^2 = 2, 0, -2, -4 \ldots$$

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We can solve this level by level

An introduction to E_{11}

Solutions, using $q_j = p_{11-j}$:

$$K^a{}_b \;\; l = 0$$

$$R^{abc} \;\; l = 1, \; q_3 = 1$$

$$R^{a_1 \ldots a_6}, \;\; l = 2, \; q_6 = 1$$

$$R^{a_1 \ldots a_8, b}, \;\; l = 3, \; q_1 = 1, \; q_8 = 1$$

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The (8,1) generator is associated to the dual graviton

All the generators arise from multiple commutators of R^{abc}

The level is the number of times R^{abc} occurs

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Non-linear realisation: To each positive level generator we associate a gauge field

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At level 4 one gets the solution $q_{10} = 1, q_1 = 2$ corresponding to the gauge field

$$A_{10,1,1}$$

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Dimensional reduction $\rightarrow A_9$, that is Romans theory!

Schnakenburg and West, hep-th/0204207, West, hep-th/0402140

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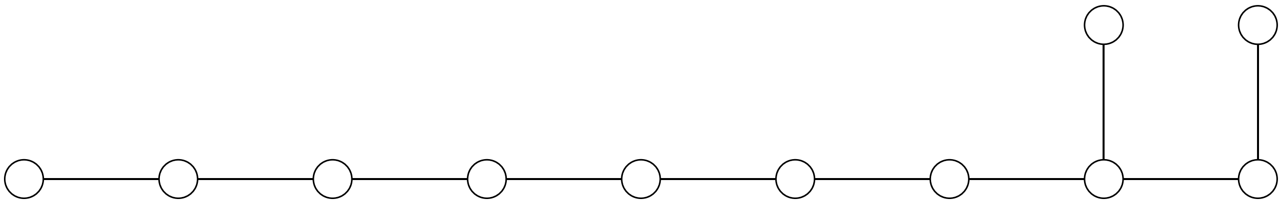
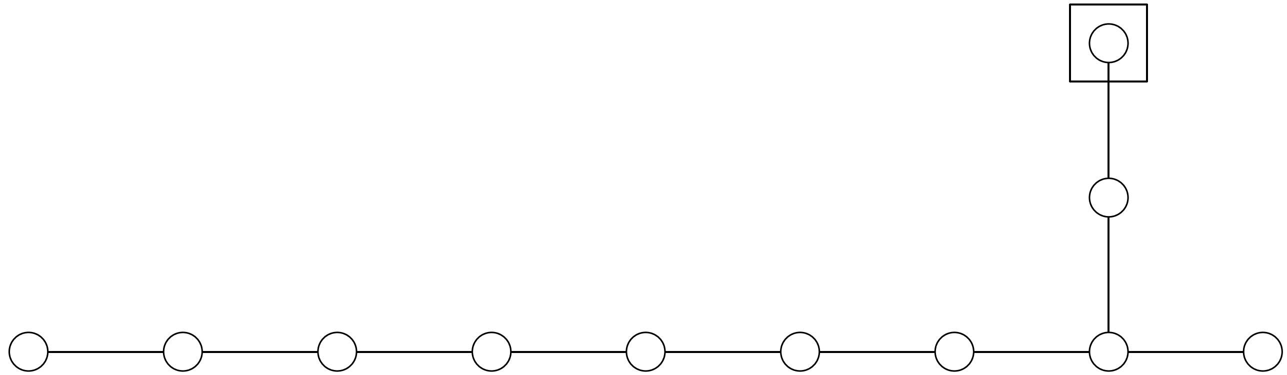
The theory is unique, gravity emerges from the choice of the background

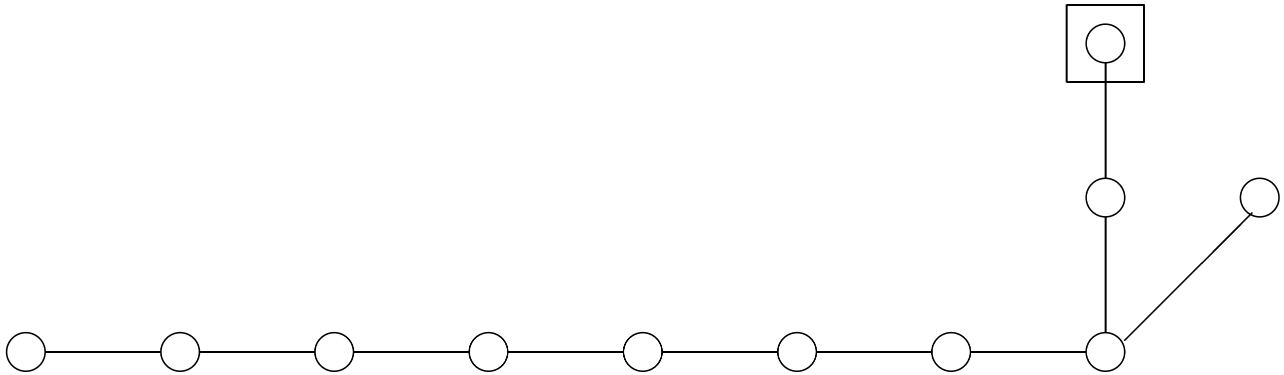
$D = 10A$



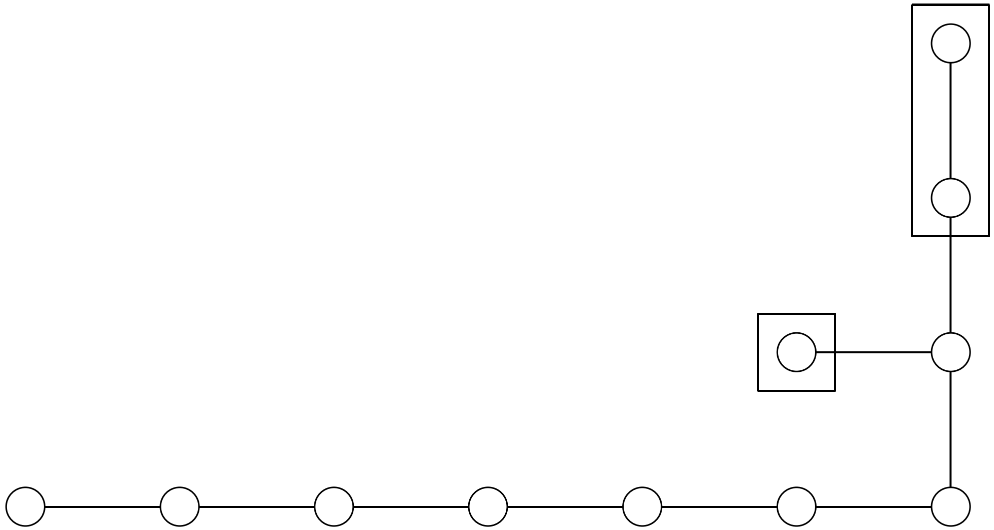
$D = 10B$



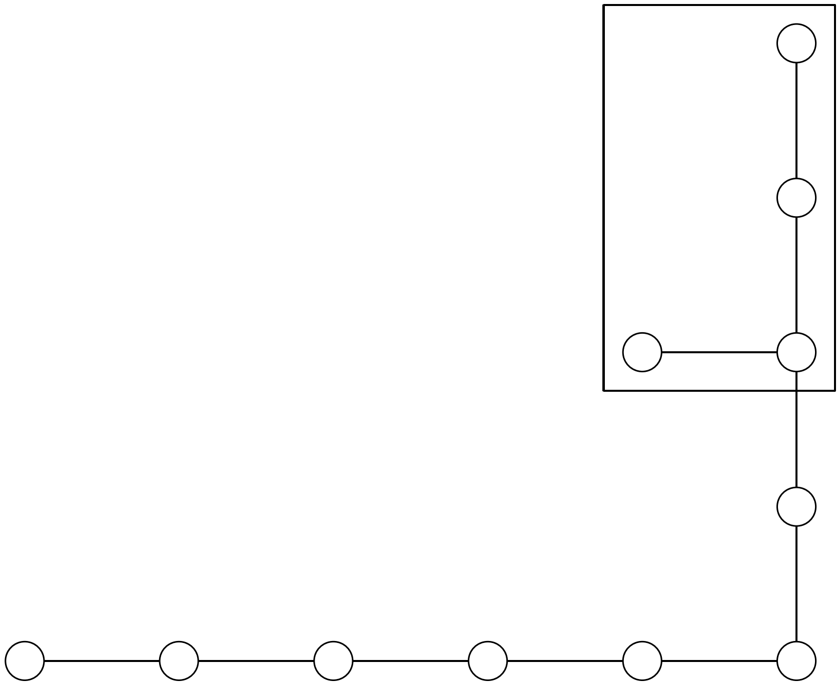
$D = 9$



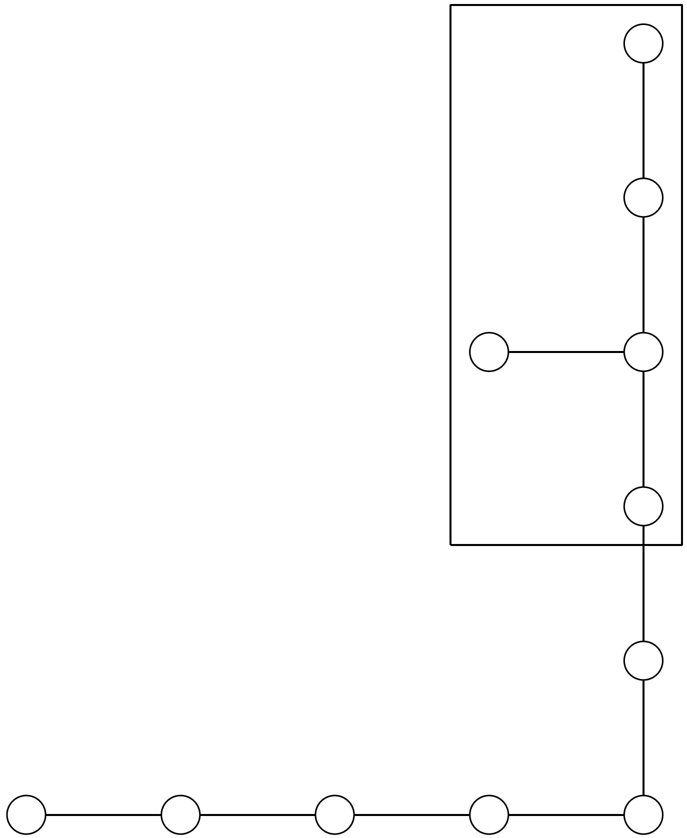
$D = 8$



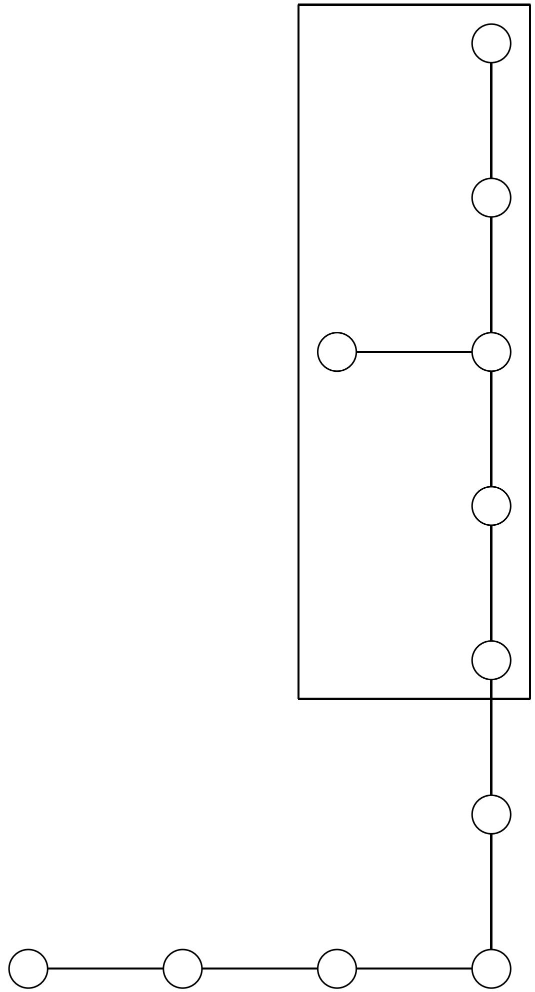
$D = 7$



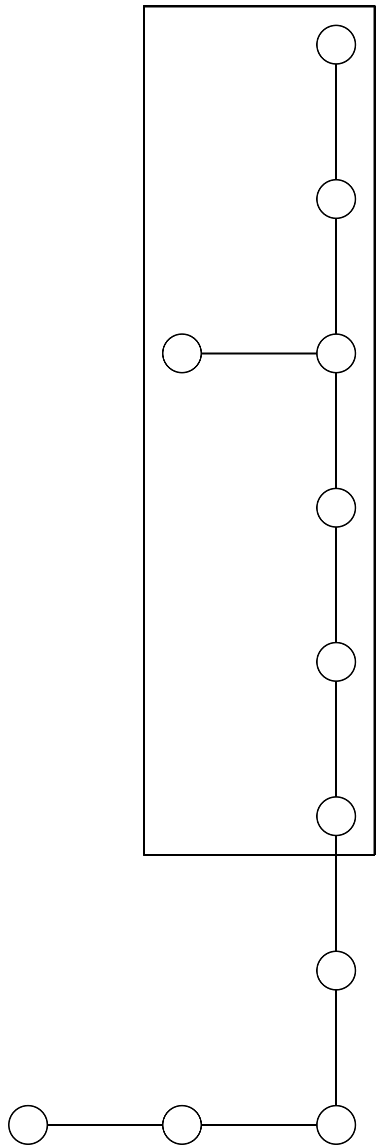
$D = 6$



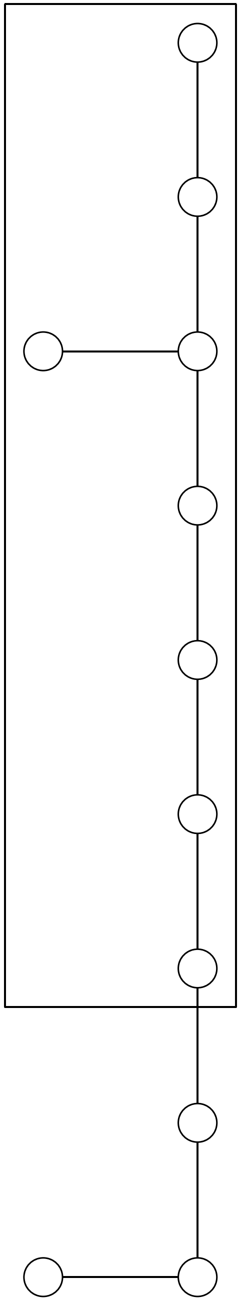
$D = 5$



$D = 4$



$D = 3$



E_{11} and supergravities

E_{11} predicts for IIB the following fields at low levels:

$$A_2^{\alpha} \qquad A_4 \qquad A_6^{\alpha} \qquad A_8^{(\alpha\beta)} \qquad A_{10}^{(\alpha\beta\gamma)} \qquad A_{10}^{\alpha}$$

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Supersymmetry algebra of IIB: democratic formulation. All the fields appear together with their magnetic duals

Bergshoeff, de Roo, Kerstan, F.R., hep-th/0506013

One finds exactly the same forms

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Besides, it turns out E_{11} reproduces the same bosonic algebra encoded in the supersymmetric theory

West, hep-th/0511153

E_{11} and supergravities

Same analysis for IIA

Bergshoeff, Kallosh, Ortin, Roest, Van Proeyen, hep-th/0103233

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Again, precise agreement with E_{11}

E_{11} and supergravities

In a series of papers, all the gauged maximal supergravities in $D = 7, 6, \ldots, 3$ have been classified

de Wit, Samtleben and Trigiante, hep-th/0212239, hep-th/0412173, hep-th/0507289

Samtleben and Weidner, hep-th/0506237 Nicolai and Samtleben, hep-th/0010076

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Gauging:

$$D_\mu = \partial_\mu - A_\mu^M \Theta_M{}^\alpha t_\alpha$$

The embedding tensor Θ belongs to a reducible representation of G

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Jacobi identities, as well as supersymmetry, pose constraints on Θ

E_{11} and supergravities

D	G	Θ
7	$SL(5,\mathbb{R})$	$\mathbf{15} \oplus \mathbf{40}$
6	$SO(5,5)$	$\overline{\mathbf{144}}$
5	$E_{6(+6)}$	$\overline{\mathbf{351}}$
4	$E_{7(+7)}$	$\mathbf{912}$
3	$E_{8(+8)}$	$\mathbf{1} \oplus \mathbf{3875}$

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In $D=9$ all the gauged supergravities have been classified via a case-by-case analysis
Mass deformations in $\mathbf{2} \oplus \mathbf{3}$ of $SL(2,\mathbb{R})$

Bergshoeff, de Wit, Gran, Linares, Roest, hep-th/0209205

The fields of E_{11}

Supersymmetry relates gauging and mass deformations, in the same representation of the embedding tensor

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Following the IIA case, we assume that these are dual to $D-1$ forms in D dimensions

We want to classify all the forms that arise in E_{11} in D dimensions

Basic idea: the sum of the indices of each field has to be equal to $3l$:

$$11n + \sum_j j q_j = 3l$$

The fields of E_{11}

We substitute $11n + \sum_j j q_j = 3l$ into

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We get

$$\alpha^2 = \frac{1}{9}\sum_{j=1}^{10} j(9-j)q_j^2 + \frac{2}{9}\sum_{i<j} i(9-j)p_i p_j$$

$$-\frac{4}{9}n\sum_i ip_i - \frac{2\cdot 11}{9}n^2 = 2, 0, -2, \ldots$$

The fields of E_{11}

Propagating fields have $n = q_{10} = 0$. One gets

$$A_{9,9,\ldots,9,3} \qquad A_{9,9,\ldots,9,6} \qquad A_{9,9,\ldots,9,8,1}$$

That is we get infinitely many dual descriptions of the same fields. The propagating fields in dimension D arise from the propagating fields in $D = 11$

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Remarkably, there are only a finite number of 11-dimensional fields that give rise to forms in any dimension above two

The fields of E_{11}

D	field
10	$\hat{g}^1{}_1$ $\hat{A}_3$ $\hat{A}_6$ $\hat{A}_{8,1}$
8	$\hat{A}_{9,3}$
5	$\hat{A}_{9,6}$
3	$\hat{A}_{9,8,1}$

The fields of E_{11}

D	field
10	$\hat{A}_{10,1,1}$
7	$\hat{A}_{10,4,1}$
5	$\hat{A}_{10,6,2}$
4	$\hat{A}_{10,7,1}$ $\hat{A}_{10,7,4}$ $\hat{A}_{10,7,7}$
3	$\hat{A}_{10,8}$ $\hat{A}_{10,8,2,1}$ $\hat{A}_{10,8,3}$ $\hat{A}_{10,8,5,1}$ $\hat{A}_{10,8,6}$ $\hat{A}_{10,8,7,2}$ $\hat{A}_{10,8,8,1}$ $\hat{A}_{10,8,8,4}$ $\hat{A}_{10,8,8,7}$

The fields of E_{11}

D	field	μ
10	$\hat{A}_{11,1}$	1
8	$\hat{A}_{11,3,1}$	1
7	$\hat{A}_{11,4}$	1
	$\hat{A}_{11,4,3}$	1
6	$\hat{A}_{11,5,1,1}$	1
5	$\hat{A}_{11,6,1}$	2
	$\hat{A}_{11,6,3,1}$	1
	$\hat{A}_{11,6,4}$	1
	$\hat{A}_{11,6,6,1}$	1
4	$\hat{A}_{11,7}$	1
	$\hat{A}_{11,7,2,1}$	1
	$\hat{A}_{11,7,3}$	2
	$\hat{A}_{11,7,4,2}$	1
	$\hat{A}_{11,7,5,1}$	1
	$\hat{A}_{11,7,6}$	2
	$\hat{A}_{11,7,6,3}$	1
	$\hat{A}_{11,7,7,2}$	1
	$\hat{A}_{11,7,7,5}$	1

E_{11} and dimensional reduction

Consider the 7-dimensional example

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6-forms:

$$\hat{A}_6 \rightarrow \mathbf{1} \qquad \hat{A}_{8,1} \rightarrow \overline{\mathbf{4}} \oplus \overline{\mathbf{20}}$$

$$\hat{A}_{9,3} \rightarrow \mathbf{6} \oplus \overline{\mathbf{10}} \qquad \hat{A}_{10,1,1} \rightarrow \mathbf{10} \qquad \hat{A}_{10,4,1} \rightarrow \mathbf{4}$$

of $SL(4, R)$. This is $\overline{\mathbf{15}} \oplus \overline{\mathbf{40}}$ of $SL(5, R)$

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6-forms:

$$\hat{A}_6 \to \mathbf{1} \qquad \hat{A}_{8,1} \to \overline{\mathbf{4}} \oplus \overline{\mathbf{20}}$$

$$\hat{A}_{9,3} \to \mathbf{6} \oplus \overline{\mathbf{10}} \qquad \hat{A}_{10,1,1} \to \mathbf{10} \qquad \hat{A}_{10,4,1} \to \mathbf{4}$$

of $SL(4,R)$. This is $\overline{\mathbf{15}} \oplus \overline{\mathbf{40}}$ of $SL(5,R)$

7-forms:

$$\hat{A}_{8,1} \to \mathbf{6} \oplus \mathbf{10} \qquad \hat{A}_{9,3} \to \mathbf{4} \oplus \mathbf{20}$$

$$\hat{A}_{10,1,1} \to \mathbf{4} \oplus \mathbf{36} \qquad \hat{A}_{10,4,1} \to \mathbf{1} \oplus \mathbf{15}$$

$$\hat{A}_{11,1} \to \mathbf{4} \qquad \hat{A}_{11,3,1} \to \mathbf{15} \qquad \hat{A}_{11,4} \to \mathbf{1} \qquad \hat{A}_{11,4,3} \to \overline{\mathbf{4}}$$

that is $\mathbf{5} \oplus \mathbf{45} \oplus \mathbf{70}$ of $SL(5,R)$

E_{11} and dimensional reduction

D	G	1-forms	2-forms	3-forms	4-forms	5-forms	6-forms	7-forms	8-forms	9-forms	10-forms
10A	$\mathbb{R}^+$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$		$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$ $\mathbf{1}$
10B	$SL(2,\mathbb{R})$		$\mathbf{2}$		$\mathbf{1}$		$\mathbf{2}$		$\mathbf{3}$		$\mathbf{4}$ $\mathbf{2}$
9	$SL(2,\mathbb{R}) \times \mathbb{R}^+$	$\mathbf{2}$ $\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{1}$	$\mathbf{3}$ $\mathbf{2}$	$\mathbf{4}$ $\mathbf{2}$ $\mathbf{2}$	
8	$SL(3,\mathbb{R}) \times SL(2,\mathbb{R})$	$(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{3},\mathbf{1})$	$(\mathbf{1},\mathbf{2})$	$(\overline{\mathbf{3}},\mathbf{1})$	$(\mathbf{3},\mathbf{2})$	$(\mathbf{8},\mathbf{1})$ $(\mathbf{1},\mathbf{3})$	$(\mathbf{6},\mathbf{2})$ $(\overline{\mathbf{3}},\mathbf{2})$	$(\mathbf{15},\mathbf{1})$ $(\mathbf{3},\mathbf{3})$ $(\mathbf{3},\mathbf{1})$ $(\mathbf{3},\mathbf{1})$		
7	$SL(5,\mathbb{R})$	$\overline{\mathbf{10}}$	$\mathbf{5}$	$\overline{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{24}$	$\overline{\mathbf{40}}$ $\overline{\mathbf{15}}$	$\mathbf{70}$ $\mathbf{45}$ $\mathbf{5}$			
6	$SO(5,5)$	$\mathbf{16}$	$\mathbf{10}$	$\overline{\mathbf{16}}$	$\mathbf{45}$	$\mathbf{144}$	$\mathbf{320}$ $\overline{\mathbf{126}}$ $\mathbf{10}$				
5	$E_{6(+6)}$	$\mathbf{27}$	$\overline{\mathbf{27}}$	$\mathbf{78}$	$\mathbf{351}$	$\overline{\mathbf{1728}}$ $\overline{\mathbf{27}}$					
4	$E_{7(+7)}$	$\mathbf{56}$	$\mathbf{133}$	$\mathbf{912}$	$\mathbf{8645}$ $\mathbf{133}$						
3	$E_{8(+8)}$	$\mathbf{248}$	$\mathbf{3875}$ $\mathbf{1}$	?							

Conclusions

- 3-forms in 3 dimensions: $\mathbf{248} \oplus \mathbf{3875} \oplus \mathbf{147250}$ of E_8

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- E_{11} provides a completely unified description of all supergravities and it encodes all their dynamical features