Quantum Phase Transition from String Theory

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Quantum Phase Transition

- As coupling parameters $\{g\}$ of a Hamiltonian is changed, ground state may undergo level crossing at $g = g^*$
- The ground-state energy $E(\{g\})$ may become nonanalytic across $g=g^*$.
- If this happens, we say that the quantum system has undergone "quantum phase transition".
- Correlation lengths, time scales and mass gaps of the ground-state may also change across $g=g^*$.

Questions for String Theory

- String theory backgrounds have a variety of effective couplings and parameters [fluxes, charges, moduli,…]. Can we find interesting "quantum phase transitions" in string theory?
- Typically, "quantum phase transitions" involve strong correlations and strong couplings. Can we understand such transitions from string theory?

A QPT from String Theory

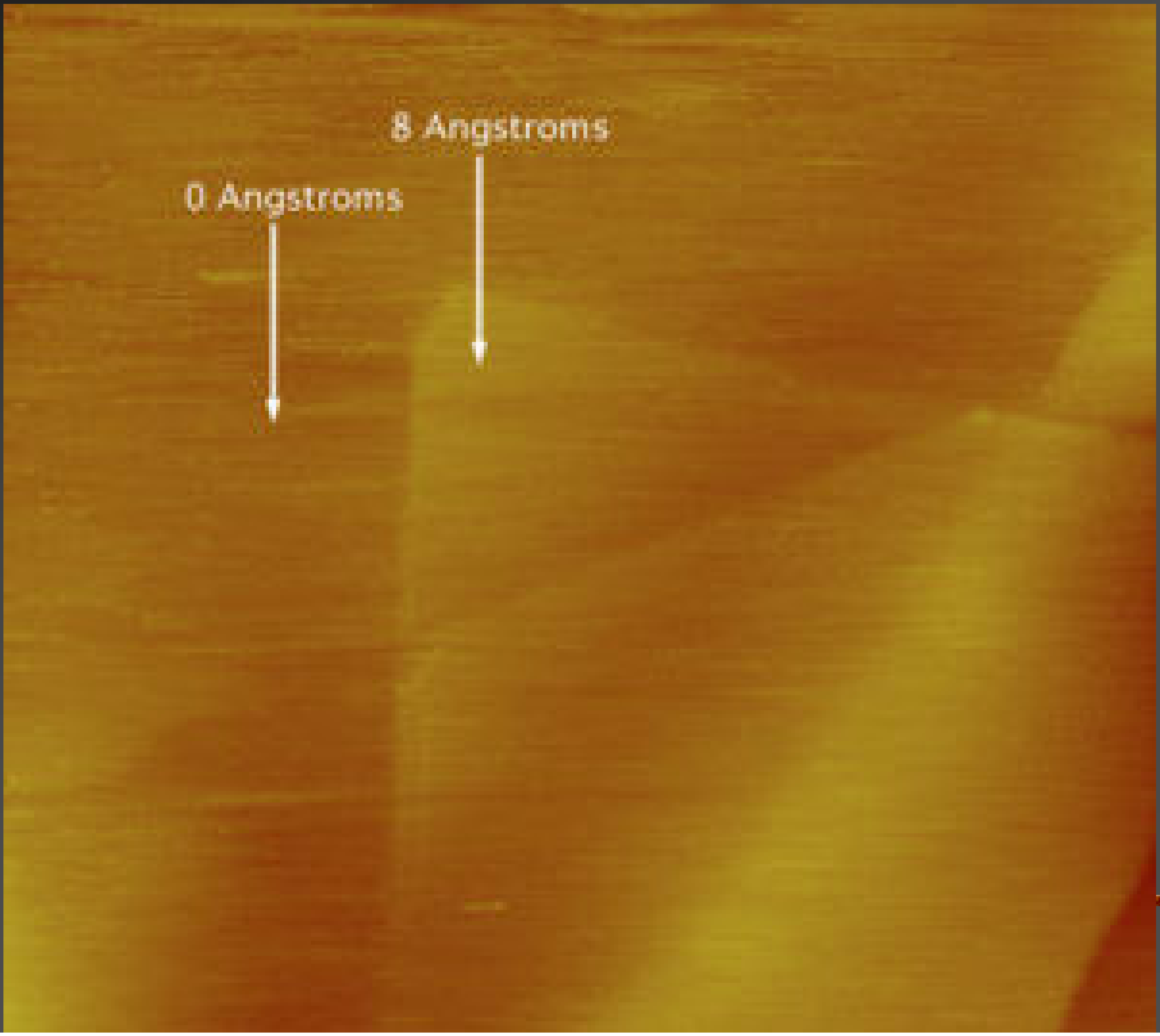
- I propose to study quantum phase transition in systems of “graphene multilayers” in terms of D7 + D3 branes.

- Why interesting? Variety of interesting phenomena are involved [*dynamical mass generation, multi-flavor symmetry breaking, parity symmetry breaking, induced (2+1)d Chern-Simons term, supercritical instability*…] with a new twist that different sectors of the system live on different spacetime dimensions.

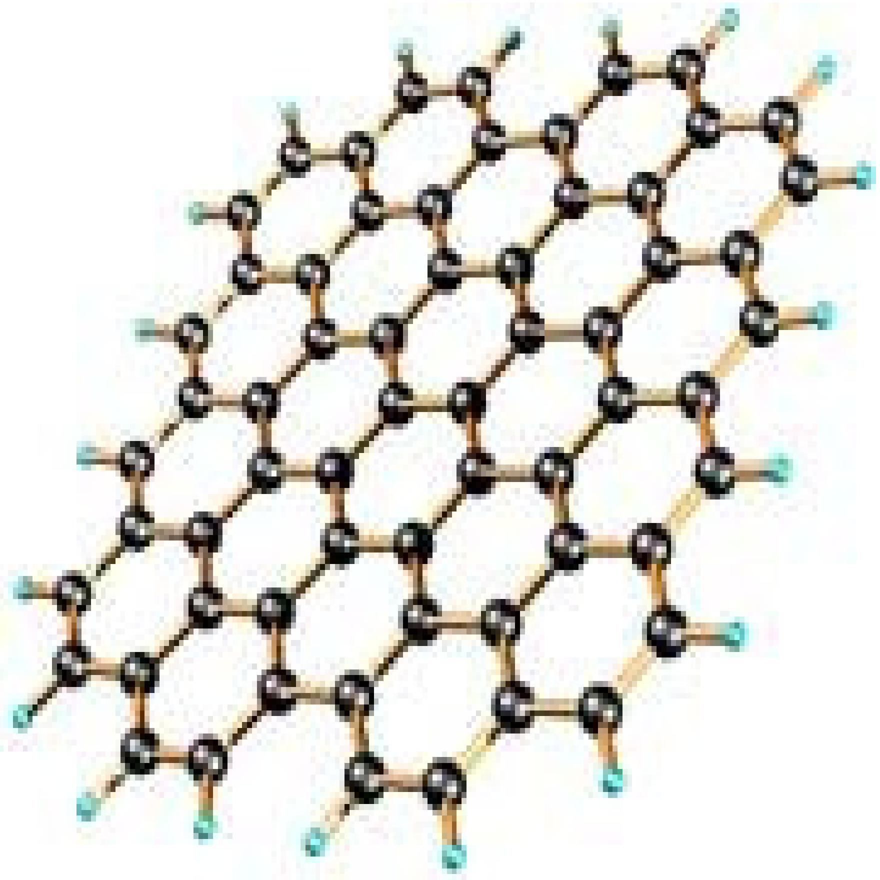
Graphenes and D-branes

- graphene = an example of gapless semiconductor
- system of (2+1)-dim. 4-component Dirac fermion
- QED of graphene multilayers with dielectric substrate – concrete setup for quantum phase transition (QPT)
- string theory realization of graphene QPT via D-branes
- study at weak coupling and strong coupling limits
- phase diagram (both T=0 and T > 0)
- laboratory exploration of AdS/CFT potentially feasible

What is Graphene?



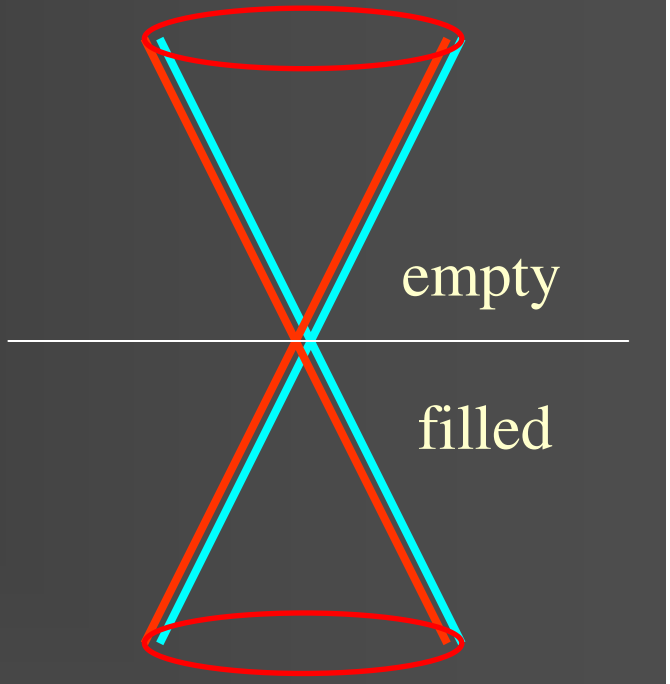
- monolayer of pure carbons
- hexagonal lattice bonding
- band structure [Wallace, 1948] – valence, conduction bands meet at two points (gapless semiconductor)
- at half filling, the valence band is filled (metal)
- energy independent dispersion [Haldane, Semenoff, Guinea …]
- (hetero)multi-layers can realize multitudes of vanishing band gaps.



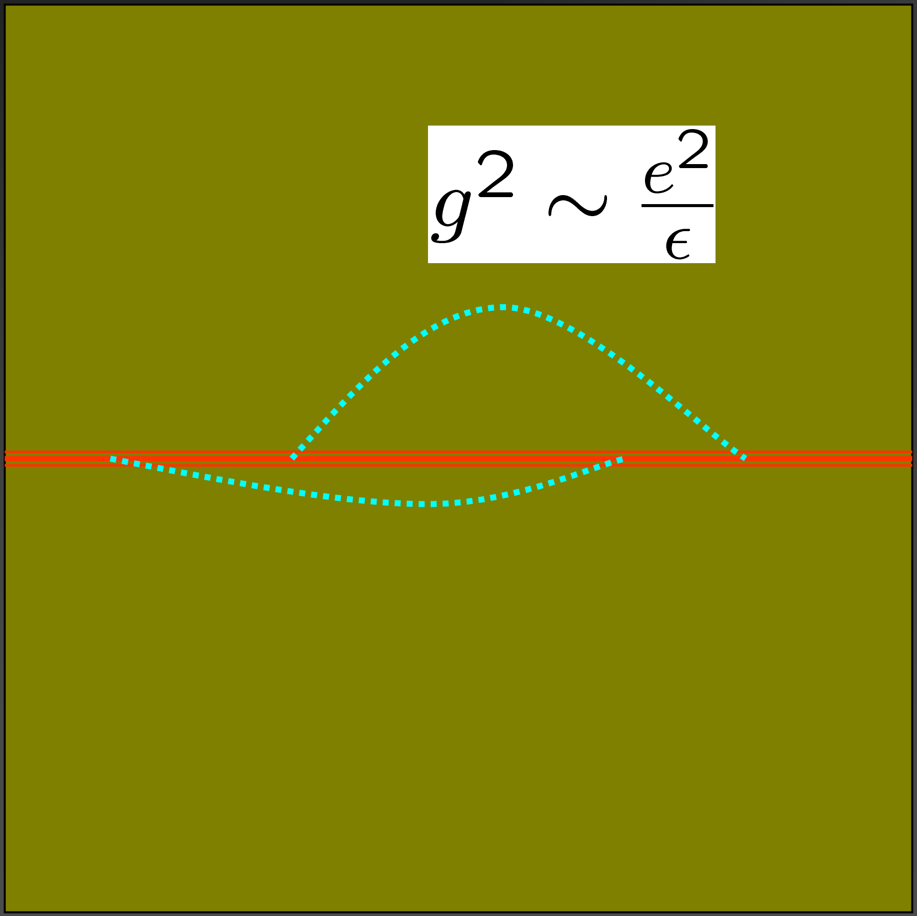
Why Graphene Interesting to us?

Graphenes provide ideal realization of

- (2+1)d relativistic 2-flavor fermions
- speed of light ~ 1/300
- at half-filling, realize the Dirac vacuum
- *N* multilayer as flavor symmetry *U(2N)*
- (3+1)d electromagnetic interactions with tunable fine structure constant by change of substrate material



Graphene Configuration



$$g^2 \sim \frac{e^2}{\epsilon}$$

substrate dielectric constant
= gauge coupling g

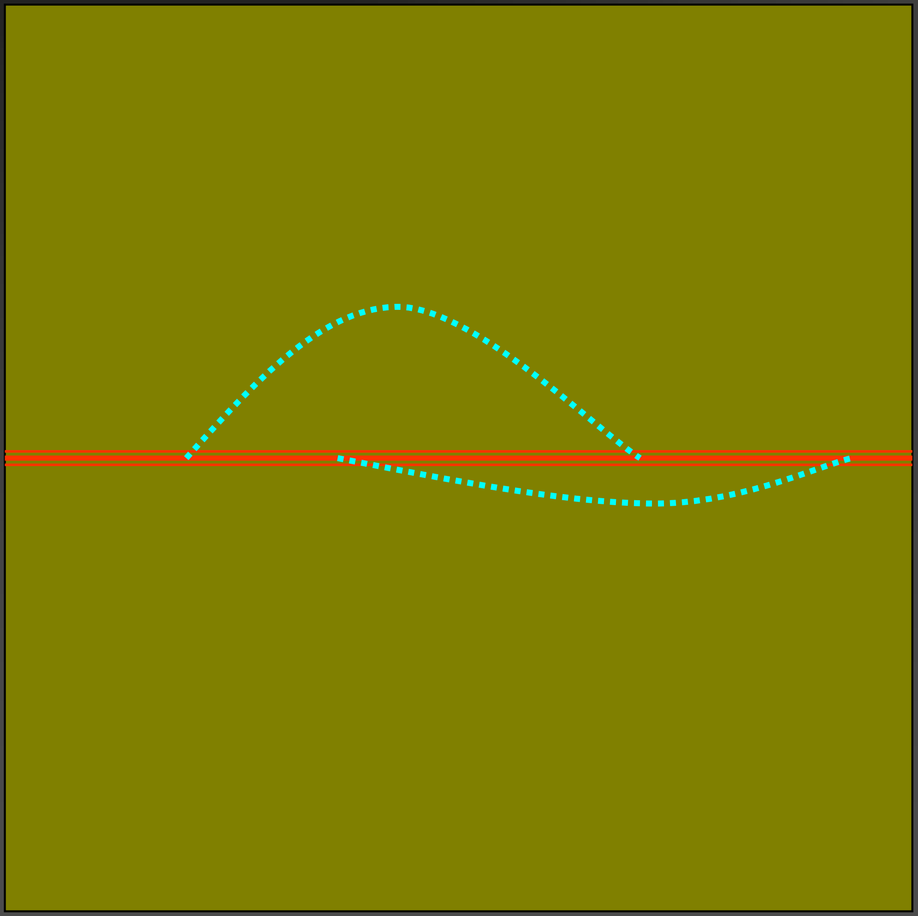
Graphene multilayers N_g

temperature T

D-brane Setup

- 4d gauge interaction: N_3 D3 branes (0123)
- 3d graphene layers: $N_7 = 2N_g$ D7 branes (01245678)
- overall transverse direction (9)
- D3–D7: NS = massive, RR = massless fermions
- $N_3 \rightarrow$ infinity, $g N_3$ = fixed >> 1, N_7 = fixed

D-brane Configuration



(123)-plane

D3-brane

gauge interactions

D7-brane

Weak Coupling Analysis

- $2N_g$ fermions in 3d coupled to 4d N=4 SYM of tunable g
- 4d CFT coupled to 3d "impurity" multiflavor fermions

$$I = \int d^4x [\frac{1}{4g^2} F_{mn}^2 + \frac{1}{2\xi}(\partial^m A_m)^2 + \delta(x_3)\,\mathcal{L}_f]$$

$$L_I = [\overline{\psi} i \partial\!\!\!/ \psi + A_m \overline{\psi} \gamma^m \psi]$$

- We study multi-flavor and parity symmetry breakings, dynamical mass generation and QPT phase transition.

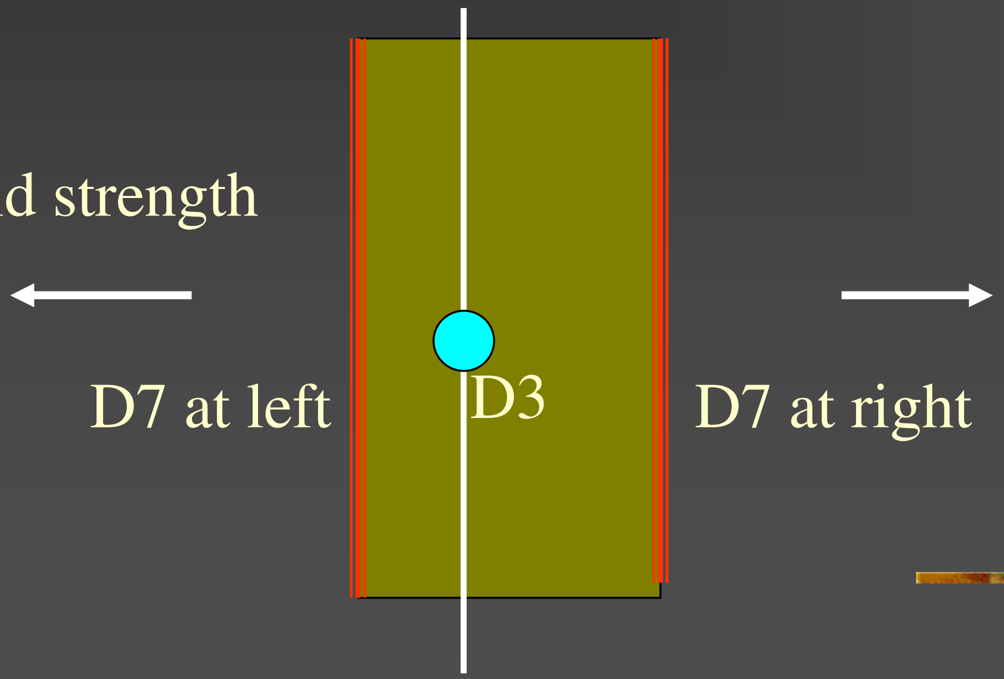
- In 3d description, we have novel conformal field theory
- 3d effective action of conformal QPT

$$I_3 = \int d^3x \Big[\; - \; \frac{1}{g^2} F_{mn} \frac{1}{\sqrt{-\partial^2}} F_{mn} + \frac{1}{1-\xi} (\partial_m A^m) \frac{1}{\sqrt{-\partial^2}} (\partial_m A^m) \\ + \; \overline{\psi} (i\partial_m + A_m) \gamma^m \psi \Big]$$

- There is NO built-in dimensionful coupling parameter.
- Any dynamically generated mass gap in QPT have to be proportional to UV cutoff.
- This is an important difference from 3d gauge theory.
- This will bear important implication to gauge-gravity analysis.

Flavor-Parity Symmetry Breaking

- flavor symmetry $U(2N_g)$
- (2+1)d parity: (x, y) → (x, - y) + flavor pairwise exchange
- [Vafa+Witten]: $U(2N_g)$ → $U(N_g) \times U(N_g)$, parity unbroken
- D-brane configuration: RR-field strength energetics → the same



RR field strength

D7 at left

D3

D7 at right

Schwinger - Dyson

- U(1) Schwinger-Dyson for dynamical mass
- solve in rainbow + nonlocal gauge [Georgi]
- solving differential equation for mass gap

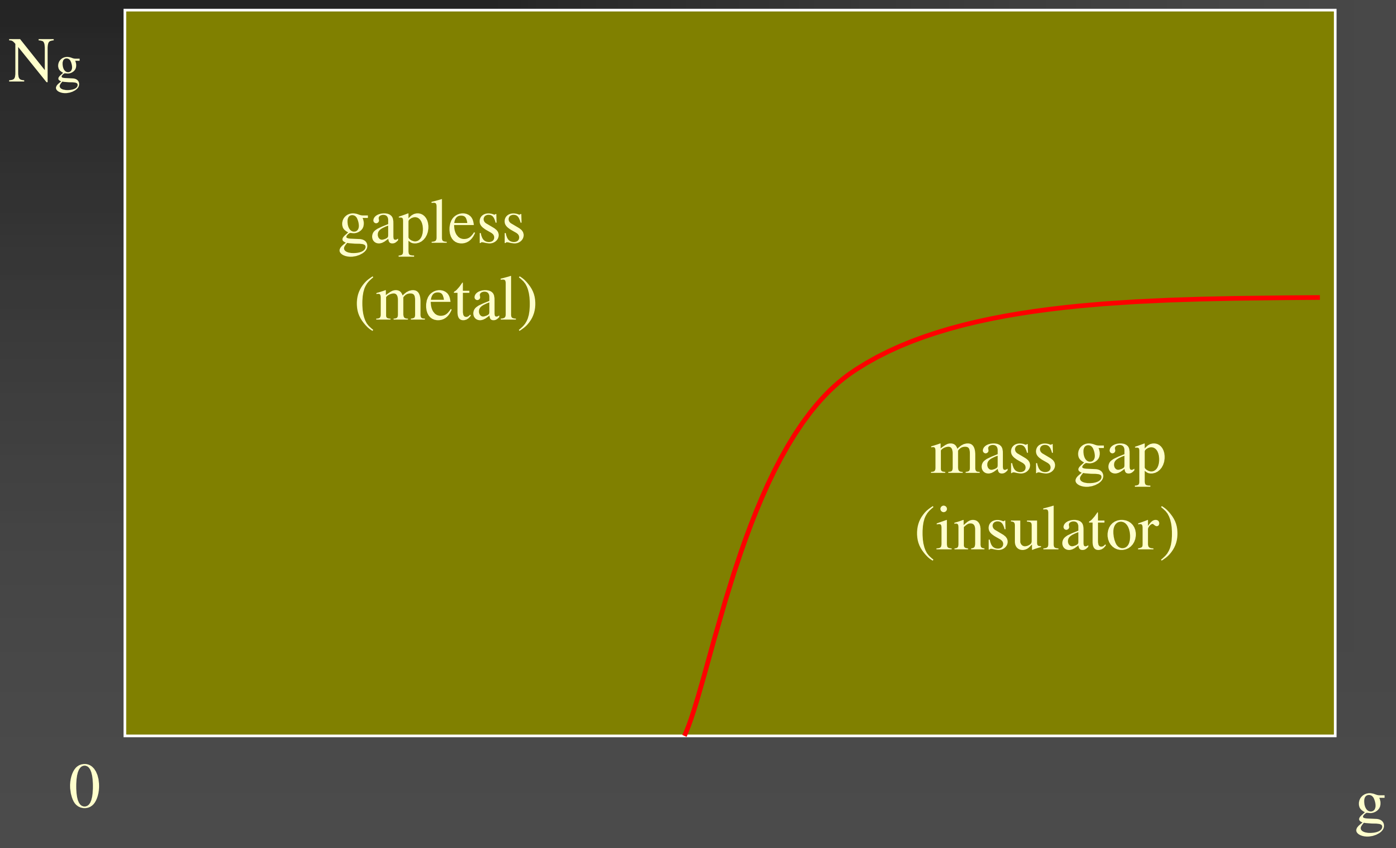
$$S(p) = 1/[\not{p} + \Sigma(p^2)]$$

$$\boxed{\Sigma(p^2) \sim \theta(\kappa^2)\sqrt{\frac{M^3}{p}}\sin\left(\frac{\kappa}{2}\log\frac{p}{M}\right)}$$

$$\text{where} \quad M \equiv \Sigma(M^2), \qquad \kappa^2 \equiv \frac{8g^2/3\pi^2}{1+g^2N_g/8} - 1$$

- supercritical instability above critical gauge coupling
- mass-gap generation at IR as back-reaction to instability
- metal-insulator QPT across critical line in (g, Ng) space

QPT Phase Diagram (schematic)



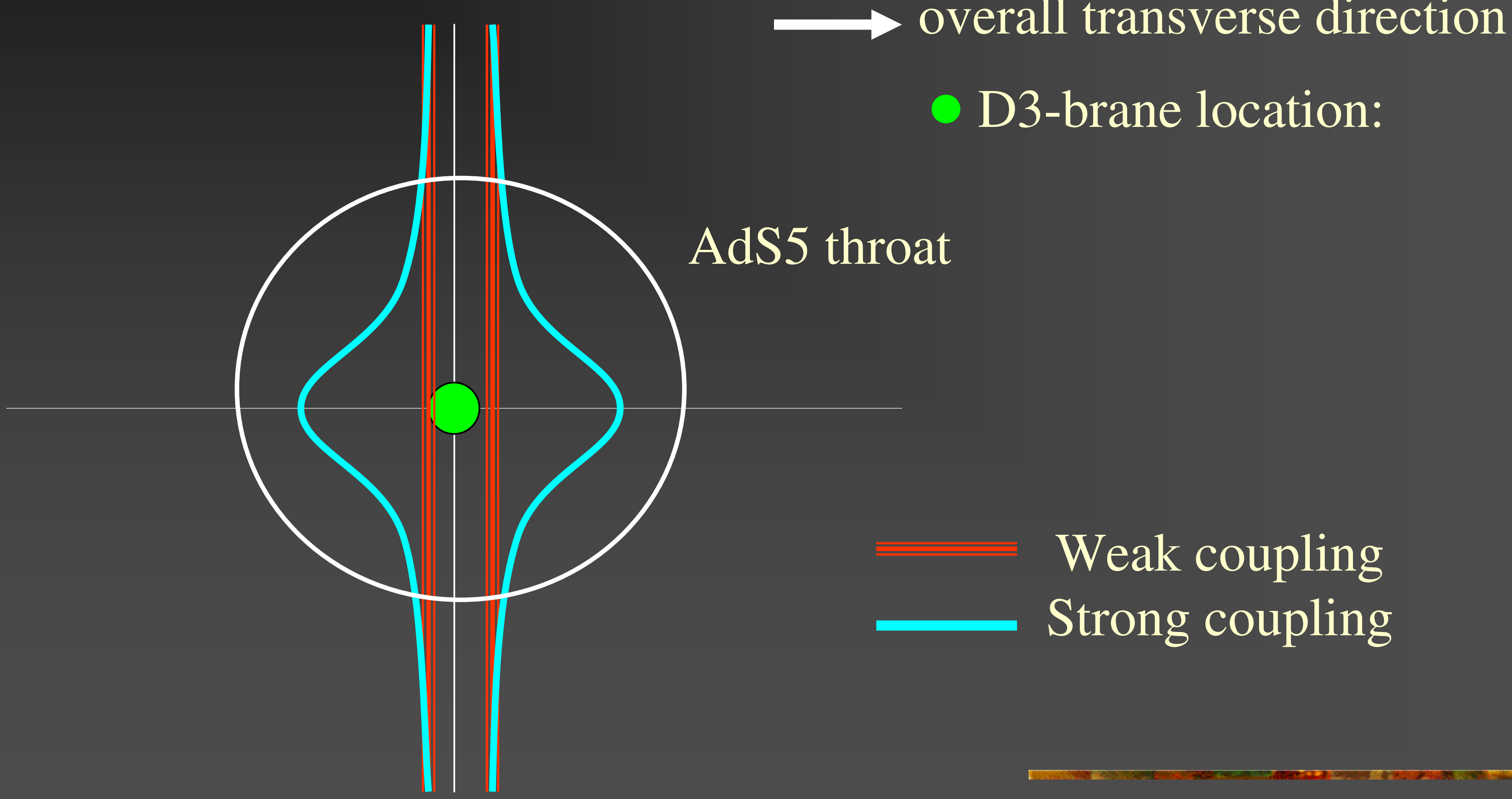
Bethe-Salpeter

- I also studied 2-particle channel Bethe-Salpeter kernel
- Aforementioned novel “conformal invariance” plays a very important role --- both SD and BS kernel exhibits conformal invariance
- The result is in agreement with Schwinger-Dyson and onset of supercritical instability phenomena.
- Goldstone boson on Grassmannian $U(2N_g)/U(N_g)xU(N_g)$

Strong Coupling Analysis

- replace D3-brane by supergravity background
- D7 branes in (partially) quenched approximation
- ground-state configuration of D7-brane profile
- study X9 profile for 3d coupling: $$\sum_{a,b=1}^{2N_g} \overline{\psi}_a {X_9}^{ab} \psi_b$$
- flavor symmetry breaking and mass generation

Effect of Strong Coupling



→ overall transverse direction

● D3-brane location:

AdS5 throat

Weak coupling

Strong coupling

Flavor-Parity Symmetry Breaking

- RR 1-form field energy in D3 supergravity background
- found symmetry breaking patter

  $U(2N_g) \rightarrow U(N_g) \times U(N_g)$, P(3d) unbroken
- partially quenched extension also studied and found

  $U(2N_g \mid 2M_g) \rightarrow U(N_g \mid M_g) \times U(N_g \mid M_g)$

  [cf. Okuda+Takayanagi]

Strong Coupling to Schwinger-Dyson

- study D7-brane coordinate $X_9(r)$

  (r = radius of 5d transverse to D3 and X_9)
- nonlinear DBI equation of motion solved
- large r asymptotics in AdS5 throat for all N_g

$$\boxed{X_9(r) \sim \sqrt{\frac{L^3}{r}} \sin(\frac{\kappa_s}{2} \log \frac{r}{L}) \qquad \kappa_s \equiv \sqrt{3}}$$

- at small r, X_9 remain stable
- full D3-brane geometry cuts off supercritical instability

  $X_9(r) \sim \frac{1}{r^3}$ --- intuitively, a point source on D7

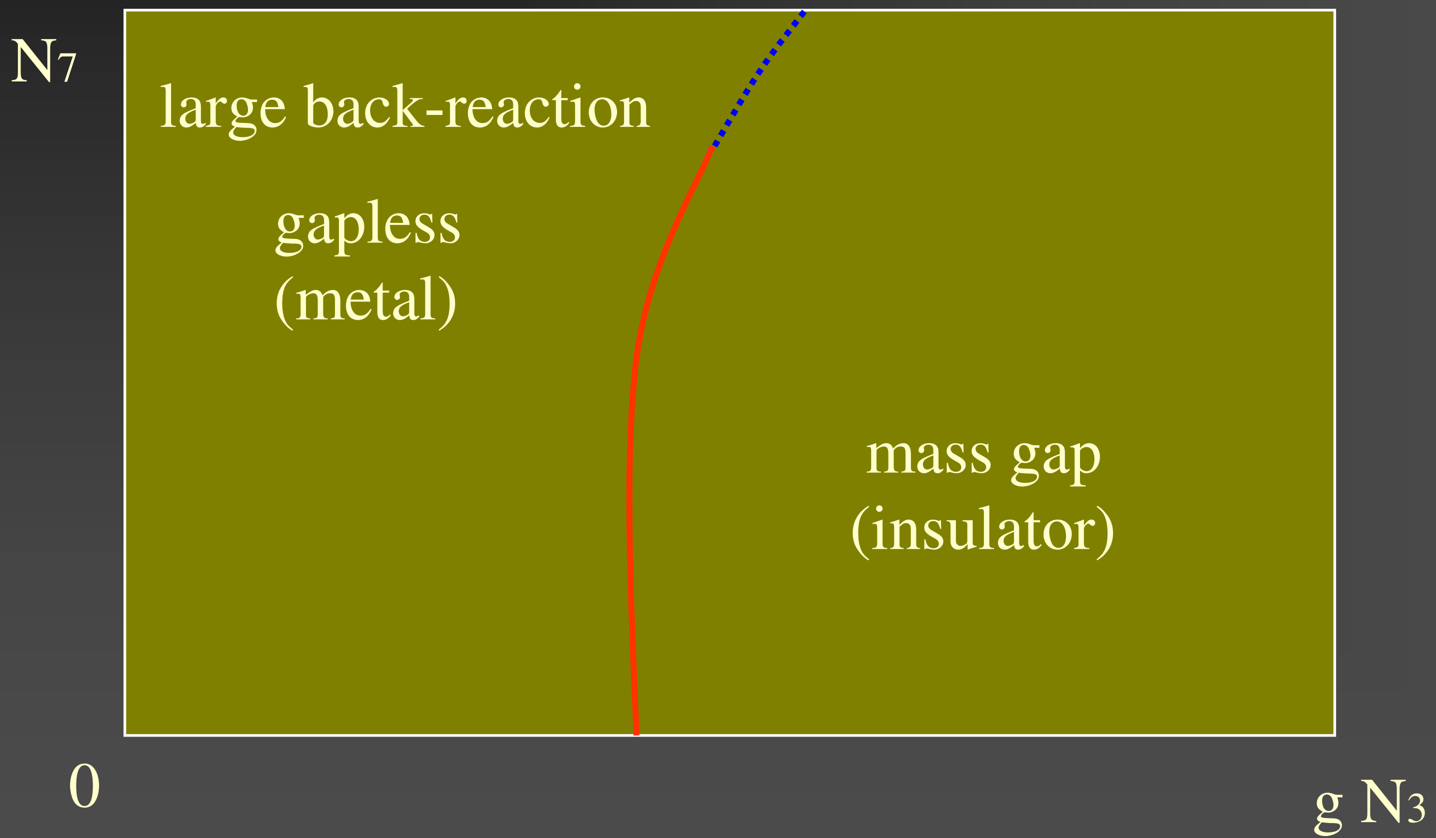
Results of Strong Coupling Analysis

- We found
- D-brane setup as a new approach to conformal QPT
- flavor/parity symmetry breaking pattern unmodified
- dynamical mass generation from AdS5 throat region

$$M \ll r \ll R_{\mathsf{AdS}}$$

- geometric interpretation of supercritical instability
- universal stable IR mass gap
- extends results to partially quenched fermions – can be compared with lattice simulations

String Prediction of Phase Diagram



N_7

large back-reaction

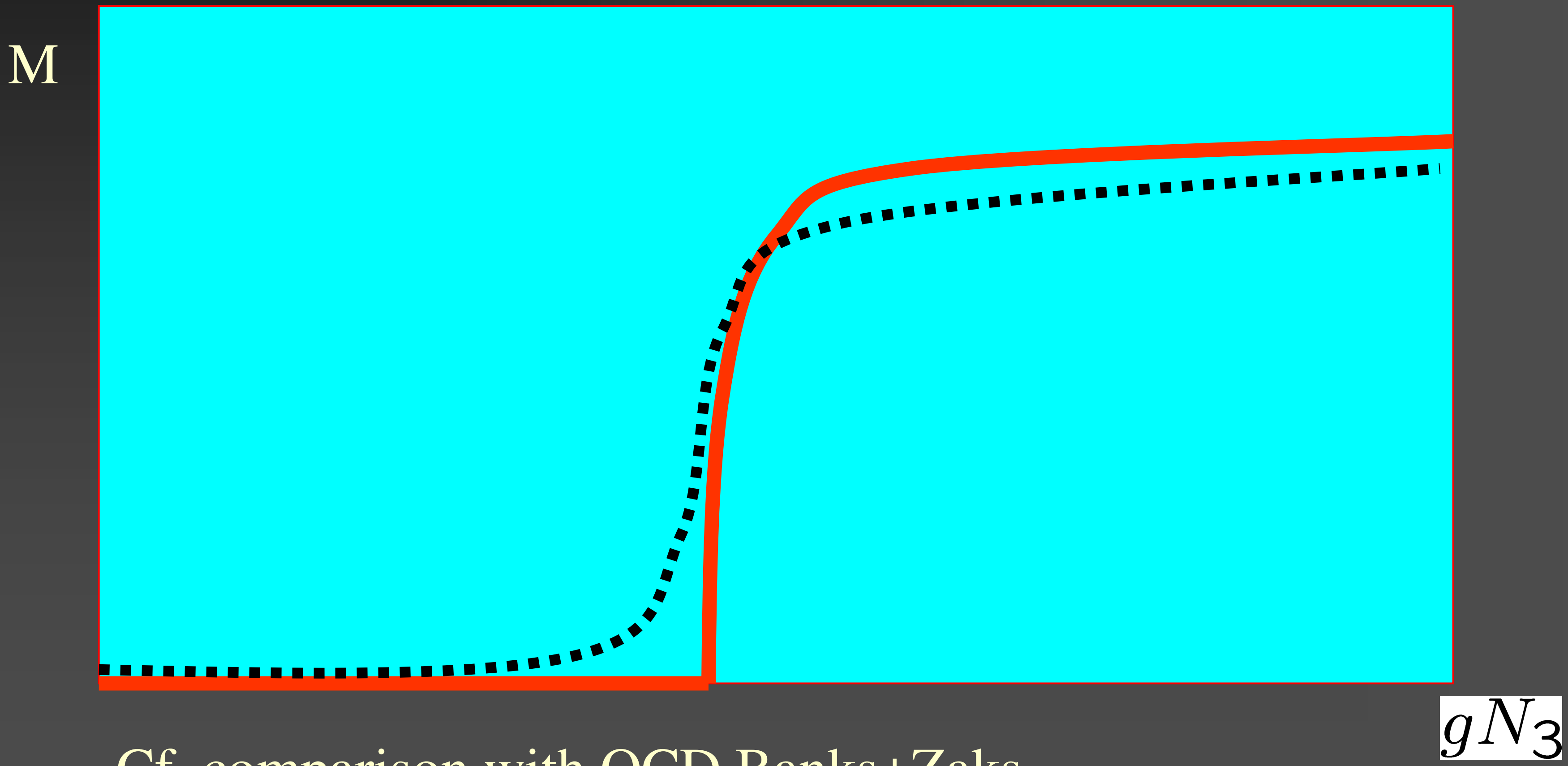
gapless
(metal)

mass gap
(insulator)

0

$g\ N_3$

Non-analyticity



Cf. comparison with QCD Banks+Zaks

Other results on

- finite tempeature extension and new phase boundaries
- 2-, 4-dimensional counterpart studied --- considerable difference and no conformal QPT found
- compactify z-direction of D3 with APBC [Witten; Sakai+Sugimoto] and study graphene / D3-D7 away from conformality (akin to standard 3d gauge theory)
- electrical transport properties with(out) B are obtained
- deformation of graphene lattice → odd number of fermion flavors and induced Chern-Simons term

Conclusion

- from string theory, we found novel 4d-3d coupled system exhibiting conformal quantum phase transition
- relevant for graphenes on substrates, gauge interaction
- laboratory experiment test exciting – can change gauge coupling (substrates of different dielectric parameter)
- potential new avenue for string theory application-test
- there are other potential quantum phase transitions in string theory contexts. One example I am currently studying is decay of ¼-BPS states [cf. Sen, Moore]

The Theorists of Everything

[circa 1991, at Santa Barbara lunch table]

- Laughlin “I found an incredibly beautiful wave function of strongly correlated many electrons in a semiconductor !”.
- Hartle “Sometime ago (with Hawking) I found the wave function of the Universe. Isn’t yours a **tiny part of mine**??”
- Laughlin “….”

--- WYSWYG ---

What You See is What You Get!