Progress in Open String Field Theory

Strings 2002, Cambridge

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Based on work with
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Open string field theory (OSFT)
(here *cubic bosonic*, Witten '86):

• In the '80's:
OSFT reproduces the *perturbative* on-shell amplitudes.

Remarkably, closed string poles correctly arise at one loop without the need for explicit closed string variables.
Closed strings as certain singular open string functionals: gauge invariant operators of the theory.

• Crucial new clue (Sen $\sim$ '99):
D-branes are solitons of the open string tachyon

$\rightarrow$ The classical eom's of OSFT must have solitonic solutions corresponding to D-branes. They do!

• String theory as a 2^{nd}-quantized theory of open strings?

In principle, path integral over the string field Ψ could define the theory non-perturbatively. BUT still missing a consistent definition of the allowed space of Ψ's.

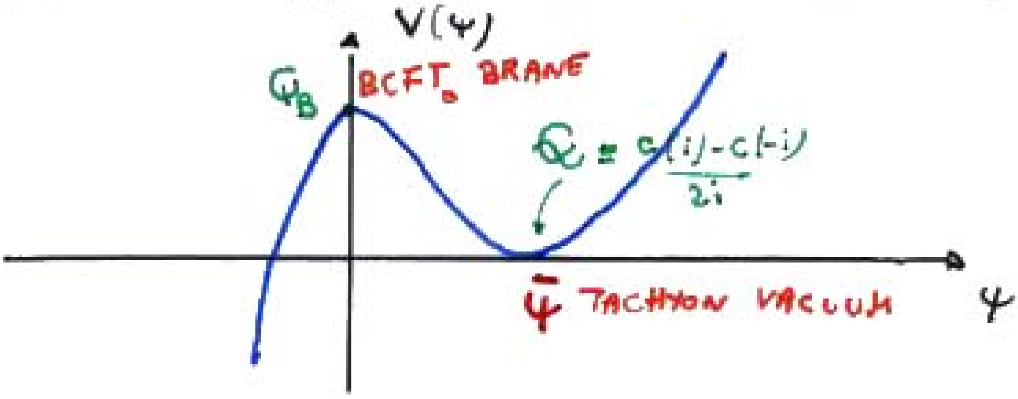
Focus for now on the *classical* dynamics of OSFT.

- OSFT action (Witten '86)

$$S[\Psi] = -\frac{1}{g_o^2}\left(\frac{1}{2}\langle\Psi, Q_B\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right)$$

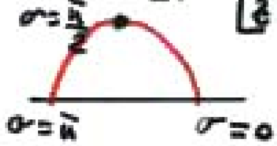
Worldvolume action on a D-brane defined by some BCFT_0 (*e.g.* a D-p brane in flat space).
$\Psi \in \mathcal{H}_{\mathrm{BCFT}_0}$ and Q_B is the BRST operator of BCFT_0.

Numerical solutions (level truncation).
No analytic solutions so far. Technical problem: *either* choose a basis for Ψ where Q_B is simple *or* a basis where $*$ is simple.



- Vacuum SFT proposal: around $\bar{\Psi}$ (after field redef.)

$$S[\Psi] = -\frac{\mathcal{N}}{g_o^2}\left(\frac{1}{2}\langle\Psi, \mathcal{Q}\Psi\rangle + \frac{1}{3}\langle\Psi, \Psi * \Psi\rangle\right), \quad \mathcal{Q} \equiv \frac{c(i) - c(-i)}{2i}$$



$\Psi \in \mathcal{H}_{\mathrm{BCFT}_0}$, but choice of BCFT_0 immaterial.
By construction, no perturbative open string states.

VSFT much simpler. Eom's essentially $\Psi * \Psi = \Psi$: *exactly solvable*!
Symmetry enhancement: *linearly* realized "$U(\infty)$" .

VSFT somewhat singular: $\mathcal{N}$ formally infinite.
Regulation possible (*e.g.*, level truncation).

Is VSFT $\cong$ OSFT?

(At least) *classically*, the answer seems **yes**:

- $\forall$ D-brane configuration, $\exists$ corresponding VSFT solution:
  – general BCFT construction in arbitrary background (RSZ);
  – explicit algebraic solutions in flat space (RSZ)
  (also with constant $B_{\mu\nu}$, Bonora Mamone Salizzoni).

- Tensions of branes correctly reproduced (up to *overall* coeff.):
  – BCFT proof that $T \sim Z_{BCFT}$, matter partition function (RSZ);
  – explicit exact computations in algebraic approach (using spectroscopy of Neumann matrices) (RSZ, Okuyama, Okuda)

- Found tachyon fluctuation around D-brane (RSZ, Okawa):
  Okawa's computation: tachyon 3pt function $\rightarrow \mathcal{N} \rightarrow$ *overall* D-25 brane tension reproduced!

- Proposal for all open string states on D-branes (RSZ, Okawa).
  (Work still needed: *e.g.* decoupling of null states?)

- Tentative mechanism for *purely closed* string amplitudes (GRSZ).
  Somewhat *ad hoc* regularization needed.

Less singular version of VSFT may be necessary for more subtle issues (quantum theory, closed strings).

Outline of the talk

1. Introduction: OSFT $\leftrightarrow$ VSFT, checks of VSFT

2. D-branes as projectors in VSFT:

• Boundary CFT point of view (surface states)

• Non-commutative geometry point of view (GMS solitons)

How do we understand moduli?

3. Some surprising numerical results in OSFT

Assuming matter × ghost factored ansatz $\Psi = \Psi^{(g)} \otimes \Psi^{(m)}$, VSFT eom's factorize:

$$\Psi^{(g)} *_g \Psi^{(g)} = \mathcal{Q}\Psi^{(g)}$$

$$\Psi^{(m)} *_m \Psi^{(m)} = \Psi^{(m)} \quad \text{projector equations}$$

In fact, the *full* VSFT eom's (including ghosts) correspond to projector equations $\Psi' *' \Psi' = \Psi'$ in the theory obtained by *twisting* the standard bc system (c=-26) to the $b'c'$ system with c'=-2.

• In the following, pick a ghost solution $\bar{\Psi}^{(g)}$ and consider string fields $\Psi = \bar{\Psi}^{(g)} \otimes \Psi^{(m)}$ as we vary $\Psi^{(m)}$. Indeed this ansatz will give all the expected solutions.

Gauge transformations preserving this ansatz:

$$\delta\Psi^{(m)} = \Lambda^{(m)} *_m \Psi^{(m)} - \Psi^{(m)} *_m \Lambda^{(m)}$$

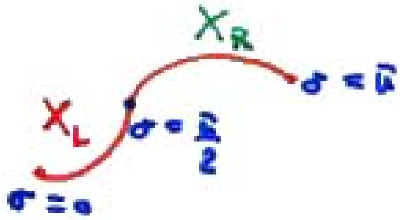
$$\Psi^{(m)} \to \exp(\Lambda^{(m)}) *_m \Psi^{(m)} *_m \exp(-\Lambda^{(m)})$$

"U(∞)" gauge symmetry.

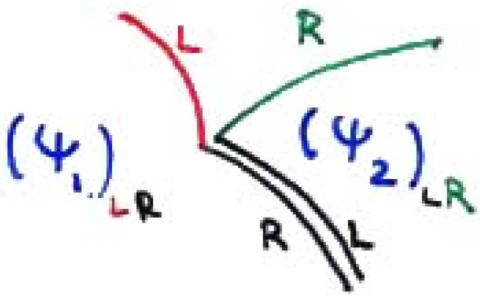
(From now, drop superscript $^{(m)}$).

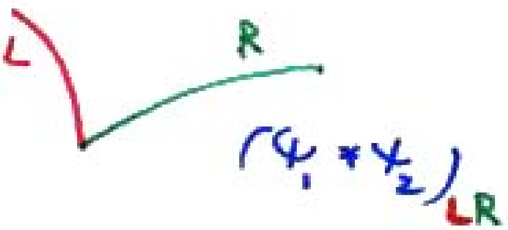
In which space is Ψ a "projector"?

Split the string into left and right:



$$(\Psi_1 * \Psi_2)[X_L, X_R] = \int \mathcal{D}Y \;\; \Psi_1[X_L, Y]\, \Psi_2[Y, X_R]$$





$*$ product = operator multiplication in the space of half-string curves (Witten, Bordes et al., RSZ, Gross-Taylor).

- Basic idea:
A rank N projector is a configuration of N D-branes.

- Puzzle:
where are the D-brane moduli?

Naively all rank N projectors are gauge-equivalent!

The resolution must lie in subtle mid-point issues (Gross-Taylor).

Two points of view that make these subtleties more transparent:

- BCFT formalism
- $*$ spectroscopy

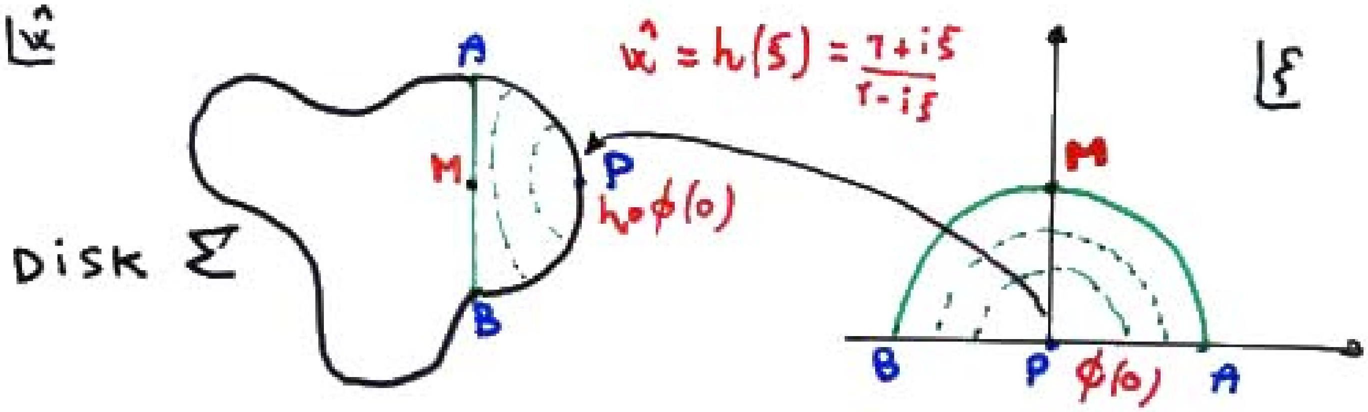
BCFT approach

$$\Psi * \Psi = \Psi$$

Look for solutions in the subalgebra of *surface states*.

Punctured disk $\Sigma \to$ surface state $|\Sigma\rangle$:

$$\langle\Sigma|\phi\rangle \equiv \langle h \circ \phi(0)\rangle_\Sigma \qquad \forall\, |\phi\rangle \in \mathcal{H}_{\mathrm{BCFT}_0}$$



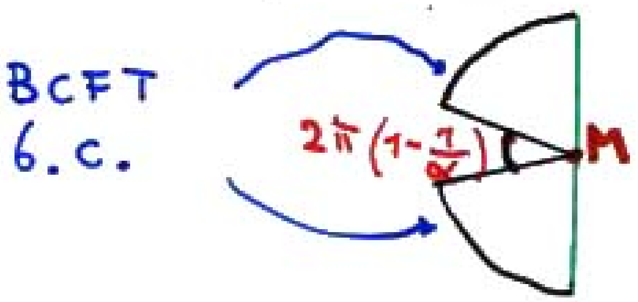
• If the boundary of Σ touches the string midpoint,

then $|\Sigma\rangle$ corresponds to a left/right split functional:

$\Psi_\Sigma[X_L, X_R] = \Phi_L[X_L]\Phi_R[X_R]\,, \qquad \Psi_\Sigma[X(\sigma)] \equiv \langle X(\sigma)|\Sigma\rangle\,.$

This is a *rank one* projector: $|\Sigma\rangle * |\Upsilon\rangle * |\Sigma\rangle = \langle\Sigma|\Upsilon\rangle\,|\Sigma\rangle \quad \forall\,|\Upsilon\rangle\,.$

Some simple surface state projectors, the *butterflies*:



As $\alpha \to 0^+$, get an infinite helix: the *sliver*.

Varying the boundary conditions outside the coordinate patch, we get a state $|\mathcal{B}_\alpha^{\text{BCFT}}\rangle$ for each BCFT.

Sandwiching $|\mathcal{B}_\alpha^{\text{BCFT}}\rangle$ with a generic $|\phi\rangle \in \mathcal{H}_{\text{BCFT}_0}$, we can express $|\mathcal{B}_\alpha^{\text{BCFT}}\rangle$ in the *reference* state-space $\mathcal{H}_{\text{BCFT}_0}$.

- Proposal:

'pinched' surface state with BCFT b.c. (*e.g.* $|\mathcal{B}_\alpha^{\text{BCFT}}\rangle$ = VSFT solution for *one* D-brane with BCFT b.c.

For example, a D-24 brane in flat space can be described by a sliver with Dirichlet b.c. for one coordinate and Neumann b.c. for the other coordinates.

For this to make sense, it must be that the *shape* of the projector is a gauge artifact, but the b.c. cannot be changed by a gauge transformation.

Naively *all* such surface state projectors, being rank one, are gauge-equivalent.

Witten's $*$ → Moyal's $*$ in κ-basis

Computing $*$-products hard in standard Fock basis, complicated Neumann matrices.

Neumann matrices can be exactly diagonalized! (RSZ)

- Consider for simplicity the zero-momentum subspace.

Continuous non-degenerate eigenbasis v_κ, $-\infty < \kappa < +\infty$. κ is the eigenvalue of $K_1 \equiv L_1 + L_{-1}$. v_κ and $v_{-\kappa}$ twist-conjugate pairs. For $\kappa = 0$, $v_{\kappa=0}$ unpaired and twist *odd.*

Change variables to this diagonal basis:
from standard *discrete* basis

$$X(\sigma) = \sum_{n=1}^{\infty} x_n \cos(n\sigma)$$

to the continuous basis

$$x(\kappa) = \sum_{n=1}^{\infty} \tilde{v}_{2n}(\kappa) x_{2n}, \qquad y(\kappa) = \sum_{n=1}^{\infty} \tilde{v}_{2n-1}(\kappa) p_{2n-1}.$$

The string field $\Psi[\{x_n\}] \to \Psi^M[x(\kappa), y(\kappa)]$ is $*$-multiplied using the Moyal structures

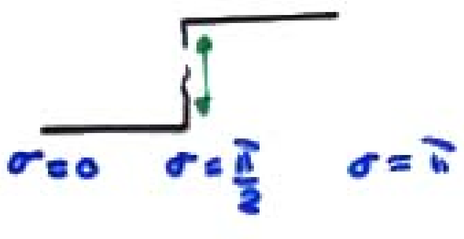
$$[x(\kappa), y(\kappa')]_* = i\theta(\kappa)\delta(\kappa - \kappa'), \quad \theta(\kappa) = 2\tanh\left(\frac{\pi\kappa}{4}\right).$$

Extra unpaired *commutative* coordinate $y(\kappa = 0)$.

(Douglas Liu Moore Zwiebach; earlier work by Bars, Bars-Matsuo).

- $y(\kappa = 0)$ twist *odd.*

Geometrically:
(Moore Taylor)



Immediate to construct projectors in this language.

Simplest construction:

$$\Xi[x(\kappa), y(\kappa)] \sim \exp\left(-\int_0^\infty d\kappa\, \frac{x^2(\kappa)+y^2(\kappa)}{\theta(\kappa)}\right),$$

tensor product over κ of the lowest GMS soliton ($\sim \exp(-r^2/\theta)$).
This is just the sliver! (Chen and Lin)

Small generalization: the canonical transformation

$$x(\kappa) \to f(\kappa)x(\kappa), \quad y(\kappa) \to \frac{y(\kappa)}{f(\kappa)}$$

should give a new projector (for any 'reasonable' $f(\kappa)$). Indeed $f_\alpha(\kappa) = \tanh(\kappa\pi(2-\alpha)/(4\alpha))$ gives the butterflies $|\mathcal{B}_\alpha\rangle$! (Fuchs Kroyter Marcus).

This can be generalized to more general transformations mixing κ's. Also higher GMS solitons (Bonora Mamone Salizzoni).

This makes it transparent that butterflies $|\mathcal{B}_\alpha\rangle$ for different α are gauge-equivalent.

- In the above, implicitly assumed $\kappa \neq 0$.

Heuristically, dependence of Ψ on non-commutative coordinates x_κ, y_κ ($\kappa \neq 0$) can be changed by a unitary transformation. But dependence on $y_0 \equiv y(\kappa = 0)$ *cannot*.

Proposal:
dependence of Ψ on commutative coordinate accounts precisely for the D-brane moduli (L.R.).

Example: one D-25 brane in flat space.
In VSFT, described by a rank one projector, *e.g.* $|\mathcal{B}_\alpha\rangle$.

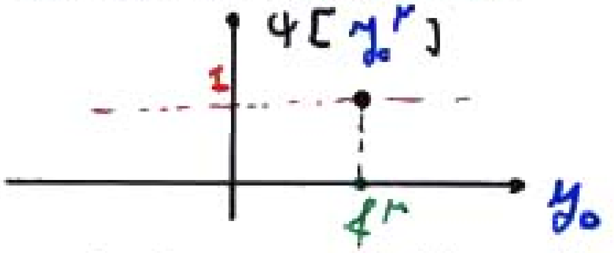
The usual butterfly has support at $y_0^\mu = 0$ ($\mu = 0 \ldots 25$),

$$\hat{y}_0^\mu \, |\mathcal{B}_\alpha\rangle = 0 \, .$$

'Deform' the state by changing the dependence on y_0^μ,

$$(\hat{y}_0^\mu - f^\mu)|\mathcal{B}_\alpha^{f}\rangle = 0 \, .$$

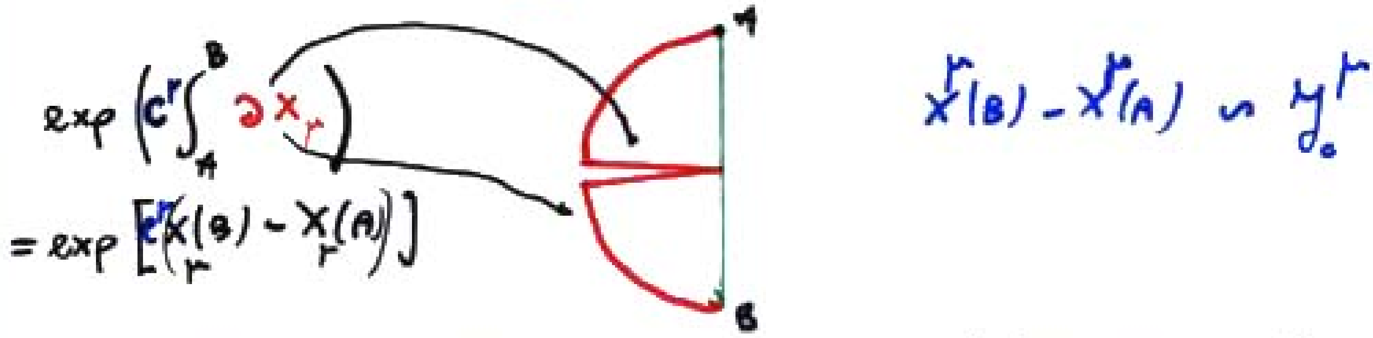
Butterflies $|\mathcal{B}_\alpha^{f}\rangle$ with different f^μ are *not* gauge-equivalent (also an explicit algebraic check, Imamura).



Modulus f^μ: constant gauge field on the brane (the only exactly marginal deformation for a D-25).

Check this explicitly by going back to the BCFT formalism.

To describe a Wilson line deformation, integrate the exactly marginal operator ∂X^μ on the boundary of the butterfly.



One finds *precisely* that the support of y_0^μ (and only of y_0^μ) ia shifted.

- N D-branes $\sim$ rank N projector,

$$\Psi = \Psi_1 + \Psi_2 + \cdots + \Psi_N\,, \qquad \Psi_i * \Psi_j = \delta_{ij}\Psi_i$$

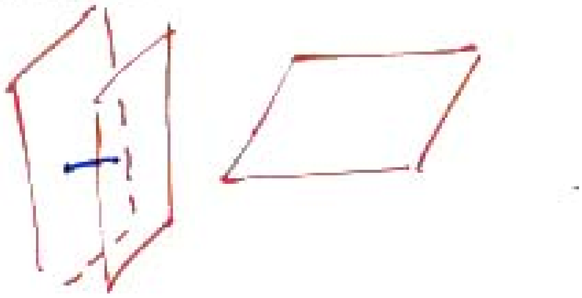
with each Ψ_i of rank one.

Dependence on y_0^μ of each of the Ψ_i's gives the expected moduli.

• These ideas generalize for non-zero momentum.

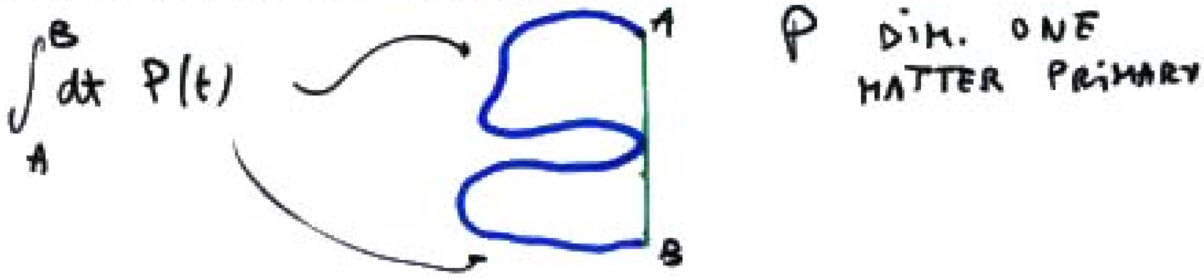
Neumann spectroscopy with momentum (Feng He Moeller) → Moyal formulation (Belov).

One twist even *commutative* coordinate $x^i(\kappa = 0) = X^i(\pi/2)$ for each transverse direction $i = p+1 \dots 25$ ↔ translational modulus of the D-p brane.



• Open string states around a D-brane?

Natural idea: integrate dimension one matter primaries along the boundary of the surface state (RSZ).



–Okawa's computation: this definition of the tachyon → non-zero 3pt function → correct absolute D-brane tension!

–Similar puzzle: if $\bar{\Psi}$ is projector, formally all solutions of the linearized eom's

$$\delta\Psi * \bar{\Psi} + \bar{\Psi} * \delta\Psi = \delta\Psi$$

are pure gauge. There must be a similar resolution involving the commutative coordinate.

–Extend this to higher open string modes. Are *null states* pure gauge?

The universal tachyon condensate in OSFT: new numerical results

(D. Gaiotto, L.R, work in progress)

$$\mathcal{T} = t\, c_1|0\rangle + u\, L^m_{-2} c_1|0\rangle + v\, c_{-1}|0\rangle + \cdots$$

$\mathcal{T}$ represents the vacuum with no branes.
Universal form (ghost + matter Virasoro's).
Choice of Siegel gauge: SU(1,1) symmetry, intriguing analytic patterns (GRSZ).

Moeller and Taylor computed $\mathcal{T}$ up to level (10,20) using a basis with bosonic oscillators. They found that 99.91% of the orginal brane tension is cancelled by the negative potential energy.

We implemented the level truncation algorithm using ghost and Virasoro conservation laws.

We reached level (18,36): 3985 fields and $\sim 10^{10}$ cubic interaction terms.

L	$E_{[L,3L]}$	$E_{[L,2L]}$
2	-0.9593766	-0.9485534
4	-0.9878218	-0.9864034
6	-0.9951771	-0.9947727
8	-0.9979302	-0.9977795
10	-0.9991825	-0.9991161
12	-0.9998223	-0.9997907
14	-1.0001737	-1.0001580
16	-1.0003754	-1.0003678
18	-1.0004937	

← MOELLER AND TAYLOR
EXACT AGREEMENT WITH THEIR RESULT

Values of the energy for the Siegel tachyon condensate in the $[L,3L]$ and $[L,2L]$ approximation schemes.

L	$\Delta_L \equiv E_L - E_{L+2}$	Δ_{L+2}/Δ_L
2	.0284452	
4	.0073553	.2585779
6	.0027531	.3743015
8	.0012523	.4548690
10	.0006398	.5108999
12	.0003514	.5492341
14	.0002017	.5739897
16	.0001183	.5865146

Behavior of the differences Δ_L of two consecutive approximations for the energy of the tachyon condensate (here $E_L \equiv E_{[L,3L]}$).

The tachyon field t (coefficient of $c_1|0\rangle$) contributes by itself most of the vacuum energy.

At level (2,4), t alone gives 98.8% of the brane tension!

Could it be that in the exact solution *all* of the energy comes from t? (Hata Shinohara). This would require

$$t = t_c \equiv \frac{\sqrt{3}}{\pi} \cong 0.551329$$

Numerical results do not seem to support this idea.

- Could the problem with the vacuum energy and the failure of $t^{(L)}$ to approach t_c be somehow related?

Ad hoc 'wavefunction renormalization': at level L, we multiply $\mathcal{T}^{(L)}$ by the overall factor $t_c/t^{(L)}$, so that the renormalized field has $t = t_c$.

The 'renormalized' energies

$$\widetilde{E}_L \equiv \left[3\left(\frac{t_c}{t_{(L)}}\right)^2 - 2\left(\frac{t_c}{t^{(L)}}\right)^3\right] E_L$$

appear to converge beautifully to -1!

- Interpretation?

L	$\tilde{E}_{[L,3L]}$	$\tilde{E}_{[L,2L]}$
2	-0.9588789	-0.9476223
4	-0.9877368	-0.9863044
6	-0.9950619	-0.9946579
8	-0.9977462	-0.9975984
10	-0.9989228	-0.9988599
12	-0.9994901	-0.9994619
14	-0.9997757	-0.9997630
16	-0.9999186	-0.9999136
18	-0.9999847	

Values of the *renormalized* energy for the Siegel tachyon condensate in the $[L,3L]$ and $[L,2L]$ approximation schemes.

L	$1+\tilde{E}_L$
2	0.041121
4	0.012263
6	0.004938
8	0.002253
10	0.001077
12	0.000509
14	0.000224
16	0.000081
18	0.000015

Conclusions

• New insights and new technical tools in open string field theory.

Very direct connections with non-commutative field theory.

• VSFT: a simple ansatz which has passed many tests. Remarkably, despite being somewhat singular, it seems to correctly reproduce (at least) *classical* open string physics.

A simple picture: D-branes as projectors.

• Some unexpected numerical results in OSFT.

• Can we make analytic progress directly in the original Witten's OSFT?

Can the structures found in VSFT be somehow extended to OSFT?

Search for a closed analytic form for the tachyon condensate $\mathcal{T}$.