Unification of Couplings in RS1 and "Bulk Holography"

LR
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INTRO

- One of the chief arguments given against extra dimensional approach to hierarchy is

  Unification of Couplings

- Must we abandon HIGH SCALE unification and LOGARITHMIC running of couplings with extra dimension

- Show not only possible, but quite natural, in RS1 with bulk gauge bosons

Difficulty with Unification and Add'l Dimensions:

- first consider LARGE EXTRA D

(1) Fundamental Scale ~ M ~ TeV

$\Rightarrow$ Highest energy ~ M ~ TeV
for field theory

(2) Large Dim $\Rightarrow$ Unnatural to have bulk gauge bosons (too weakly coupled)

(3) Not so large Xtra D and Bulk Gauge Bosons

$$\underbrace{M_Z}_{\text{no unification}} \longrightarrow \underbrace{\text{TeV}}_{\text{'unification'}}$$

- requires rapid running
- Possible; $\boxed{\text{power law}}$
- Rapid but uncontrolled

Extra D and Power Law Running

running gauge coupling (bulk gauge bosons)

$$\int^{\Lambda} \frac{d^{n+4}p}{(p^2)^2} \sim \Lambda^n$$

<u>OR</u> in terms of KK modes

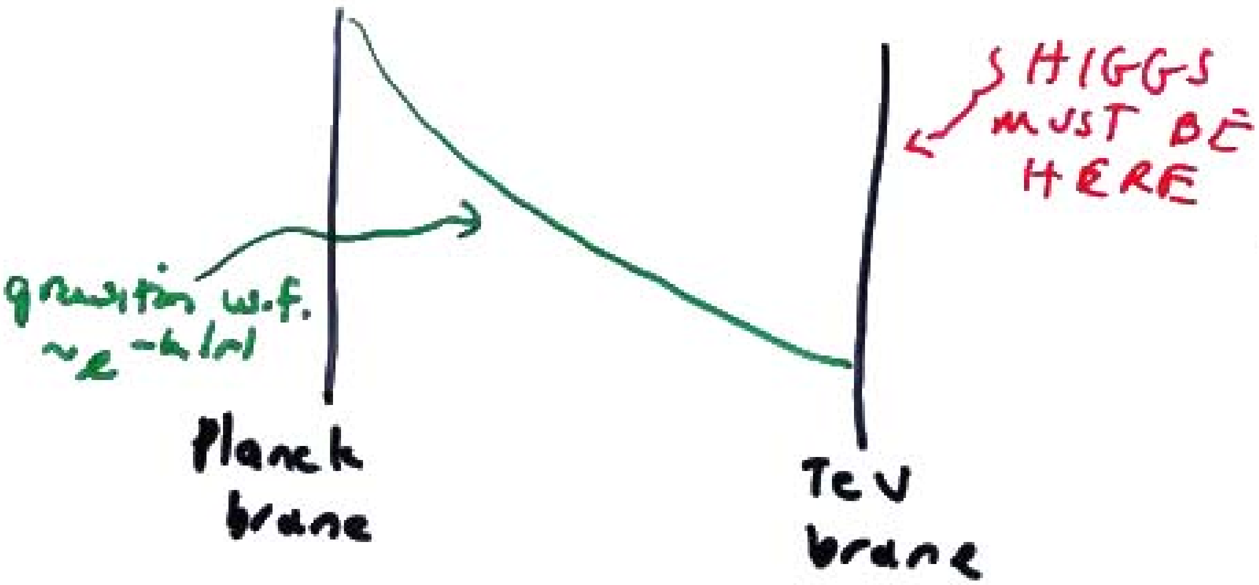
$$\sum_{j_1} \sum_{j_2} \cdots \sum_{j_n} \int^{\Lambda} \frac{d^{n+4}p}{\left[p^2 - \left(\frac{|j|}{R}\right)^2\right]^2}$$

count modes $\frac{|j|}{R} < p$

$$\int^{\Lambda} \frac{d^4p}{(p^2)^2} (pR)^n \sim \Lambda^n$$

Next consider RS1 with

BULK gauge bosons:



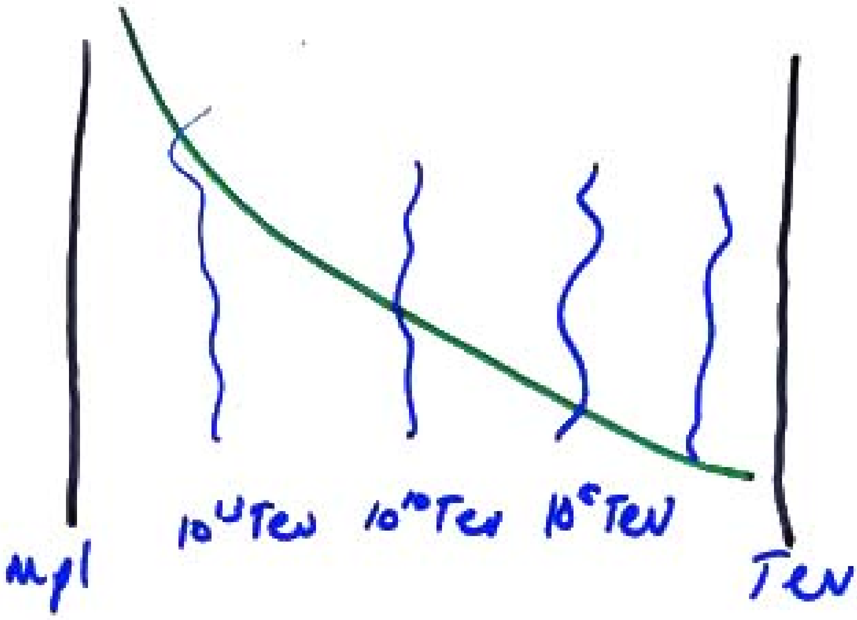
$$ds^2 = e^{-kr_c|\phi|}(-dt^2 + dx^2) + r_c^2 d\phi^2$$

$$\pi r_c \sim \ln\left(\frac{M_P}{TeV}\right) \sim 35$$

Goldberger-Wise

Diff. 1: Bulk not big $\Rightarrow$
Natural to consider bulk gauge bosons

DIFF 2: TeV NOT FUNDAMENTAL SCALE



Bulk physics accesses high energies

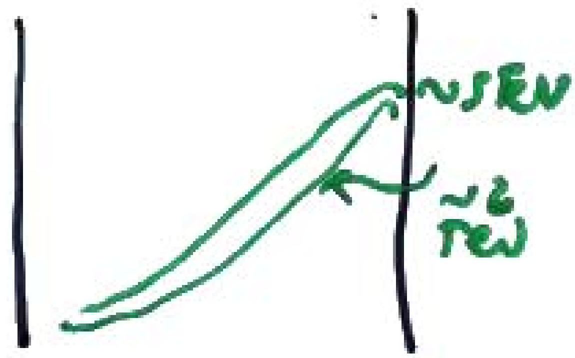
(even if Higgs, [fermions] pinned to TeV brane)

Bulk gauge bosons exist at high energy

FINAL DIFF: Logarithmic

NOT power law running

Easiest to understand in terms of KK modes

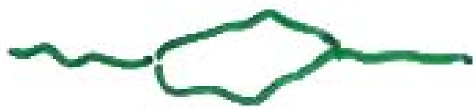


0-mode $\Rightarrow$ 4D gauge boson

masses ~ 0, TeV, 2TeV, ... $\left(\frac{\Lambda}{k}\right)$ TeV

$\underbrace{\Lambda}_{\text{cutoff} \sim \text{quantum gravity}}$

only ~ $\frac{\Lambda}{k}$ KK modes



$$\sum_{j=1}^{\Lambda/k} \int \frac{d^4 p}{(p^2)^2} \sim \frac{\Lambda}{k}\, \beta \ln\left(\frac{M_0}{TeV}\right)$$

NOT $\frac{\Lambda}{TeV}$!

Logarithmic, not power law running

If unified couplings and unified gauge group

$$\frac{1}{g_1^2} - \frac{1}{g_0^2} \sim b \log$$

but $\dfrac{1}{g^2(\mu)} - \dfrac{1}{g_0^2} \sim b\left(1 + \dfrac{\Delta}{k}\right) \log$

$\left(\frac{\Delta}{k}\right) \Rightarrow$ faster running of couplings

If no unified group

unification at lower scale

$$\frac{1}{g_1^2} - \frac{1}{g_0^2} \sim b\left(1 + \left(\frac{\Delta}{k}\right)\right) \log$$

- Can also understand log behavior as a result of existence of holographic dual.
- But strongly coupled; wavefunction renormalization
- Eventually, understand cut-off dependence in string realization or 4D dual

How to Calculate running?

- 4D EFT?

  EFT breaks down at ~TeV

Do 5D Calculation : weakly coupled

- Gauge invariant eg Pauli Villars regulator?
  - Cannot take PV mass $\to \infty$
  - Light ghosts (beyond physical spectrum)

- 5D Calculation with spatially varying cutoff

  $z \leftrightarrow \Lambda/z$

  - required by general covariance
  - will effectively yield reduced number of states

Conclude: FULL 5-D Calculation

BONUS → INSIGHT INTO CURVED SPACE

$$dS_5^2 = \frac{1}{k^2 z^2}\left(dt^2 - dx^2 - dz^2\right)$$

Planck brane: $z = 1/k$

TeV brane: $z = \frac{1}{T}$ $\quad T = k e^{-k\pi r_c}$

$$M_{pl}^2 = M^3/k\left(1 - T^2/k^2\right)$$

$$S_{5D} = \int d^4x\, dz \sqrt{G}\left[-\frac{1}{4} F_{MN} F^{MN}\right]$$

$$L = \frac{1}{2kz}\left[A_\mu\left(\partial^2 \eta^{\mu\nu} - z\,\partial_z\left(\frac{1}{z}\partial_z\right)\eta^{\mu\nu} - \partial^\mu \partial^\nu\right)A_\nu\right] + A_5 \text{-terms}$$

Gauge Fix

$$\Delta L = \frac{1}{2\xi k z}\left[\partial_\mu A^\mu - \xi z\,\partial_z\left(\frac{1}{z} A_5\right)\right]^2$$

$$\langle A^\mu A^\nu \rangle = -i G_p(z,z')\left(\eta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right) + p^\mu p^\nu\text{-terms}$$

$p_\mu = i\partial_\mu$ 4 mtm

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + p^2\right] G_p(z,z') = zk\,\delta(z-z')$$

$$\left[\partial_z^2 - \frac{1}{z}\partial_z + p^2\right] G_p(z,z') = zk\,\delta(z-z')$$

$u = \min(z,z')$ $\quad$ $v = \max(z,z')$

Homogeneous Soln involves Bessel Fcns

$$G_p(u,v) = u\left(A J_1(pu) + B Y_1(pu)\right)$$
$$= v\left(C J_1(pv) + D Y_1(pv)\right)$$

BC:
$$\partial_u G_p(\tfrac{1}{k}, v) = \partial_v G_p(u, \tfrac{1}{T}) = 0$$

match over δ-fcn

$$G_p(u,v) = \frac{\pi}{2}\,\frac{k\,uv}{AD-BC}\left(A J_1(pu) + B Y_1(pu)\right)\left(C J_1(pv) + D Y_1(pv)\right)$$

$A = -Y_0(p/k)$ $\quad$ $C = -Y_0(p/T)$

$B = J_0(p/k)$ $\quad$ $D = J_0(p/T)$

For later, massive vector field:

J_ν, Y_ν $\quad$ $\nu = \sqrt{1+m^2}$

Edifying to consider LIMITS of Greens fcn

1) $q \ll T$ $\qquad R = r_c \pi$

$$G_q(u,v) \to -\frac{1}{R q^2}$$

Low energy:
only 0-mode ✓

2) $q \gg T, \quad qu \ll 1, \quad qv \ll 1$

$$G_q(u,v) \to \frac{-k}{q^2\left(\log \frac{2k}{q} - \gamma\right)}$$

eg 

Holographic answer
Σ KK modes

3) $q \gg T$ on TeV brane

$$G_q\left(\tfrac{1}{T}, \tfrac{1}{T}\right) \to -\frac{k}{qT}$$

· Flat Space Regime

$$\frac{1}{q^2} \cdot \underbrace{\frac{qk}{T}}_{\#\text{modes}}$$

· $\frac{k}{z} \to \frac{\Lambda}{z}$

· Need to cut off at Λ/z

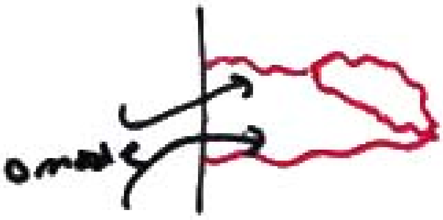
Limits

4) $q \gg T$, $qu, qv \gg 1$

$$G_q(u,v) \to \frac{-k}{2q}\sqrt{uv}\, e^{-q(v-u)}$$

$\Rightarrow v-u$ small

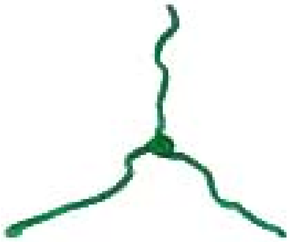
- source of UV divergence
- largest contribution $\lambda \gg k$ from flat space region



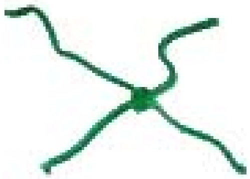
Need • vertices
• range of integration
• regularization scheme

Vertices :

$-i\, G_p(z, z')\, \eta^{\mu\nu}$



$$g_{5d}\ \frac{1}{kz}\, f^{abc} \big[\eta^{\mu\nu}(k-p)^{\rho} + \eta^{\nu\rho}(p-q)^{\mu} + \eta^{\rho\mu}(q-k)^{\nu} \big]$$

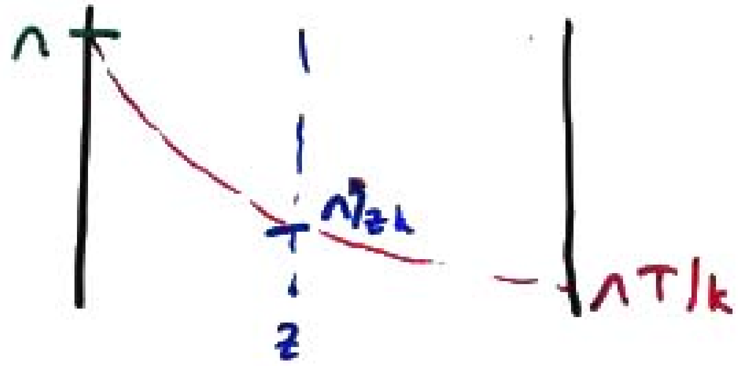


$$-i\ g_{5d}^2\ \frac{1}{kz}\ \underbrace{N^{\mu\nu,\rho\sigma}_{abcd}}_{\text{std}}$$

$+ A_5$ diagrams

+ ghost

Heart of the matter: Regularization



General covariance

$$\boxed{\Lambda \to \frac{\Lambda}{zk}}$$

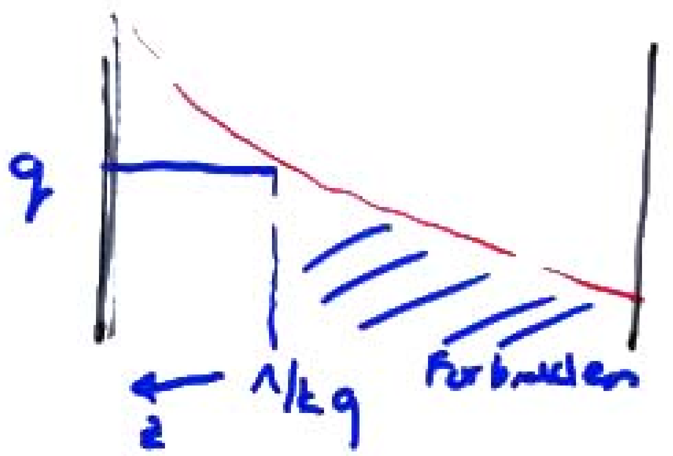
- eg TeV brane

  cutoff ~ TeV, not ~ M_{pl}

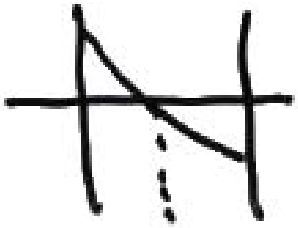
- Λ a mass scale

  $\Rightarrow \Lambda/zk$

$$\boxed{qz < \Lambda/k} \qquad \boxed{z < \frac{\Lambda}{kq}}$$



Subtlety:



$qz < \Lambda$

Consider

$$\sim g_{5D}^2 \int d^4q \int \frac{du}{ku}\, G_p(z,u)\, G_p(u,z')\, G_{p+q}(u,u)$$

Take $p=0$

Assuming WI

$$\frac{1}{R_p^2}\, g_{4D}^2 \int q\, dq \int \frac{du}{ku}\, G_q(u,u)$$

$G_q = G_{0\text{-mode}} + G_{KK\text{ modes}}$

↑ Focus on THIS

$$\int_m^{\Lambda} q\, dq \int_{1/k}^{\Lambda/kq} \frac{du}{ku}\, \frac{1}{q^2 R} \sim \frac{1}{2kR} \log^2 \frac{\Lambda}{m}$$

However, would like $\log \Lambda/m$ ⟺ 0-mode contribution

Integration over full space → $\int_m^{\Lambda} \int_{1/k}^{\Lambda} \frac{du}{ku}\, \frac{1}{q^2 R} = \frac{1}{kR} \log \Lambda/m$

Conclude: cutoff Λ/kq

→ Renormalized Greens fcn

BC at $u = \Lambda/kq$

(not $1/k$)

⇒ entire 0-mode contribution

~ holography

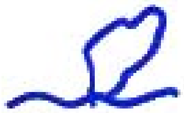
So 2 Aspects to Regularizatn

I: $qz < \Lambda/k$

II. Apply new boundary conditions.

IR branenot at T, but at z_q.

Calculation



$$-1/2 \int \frac{d^4q}{(2\pi)^4} \; \frac{p^2\eta_{\mu\nu} - p^\mu p^\nu}{(q^2)^2} \; 4\, \overset{\substack{\text{Dynkin}\\ \text{index}\\ \shortparallel}}{C_r}\; (\overset{\substack{2\ \text{vectors}\\ \shortparallel}}{j}) \qquad \times$$

$$(q^2)^2 \int_{1/k}^{\Lambda/kq} \frac{du}{ku} \int_{u}^{\Lambda/kq} \frac{dv}{kv} \; G_q(u,v)\, G_q(u,v)$$

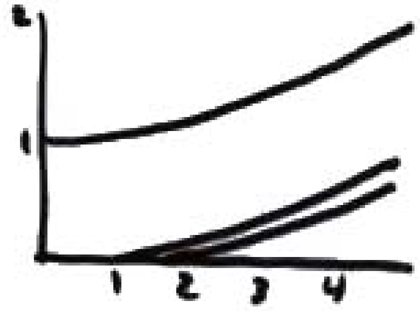
$y = qu \;,\; z = qv \;,\; k = 1$

$$I(\Lambda, q) = q^4\, 2 \int_q^\Lambda \frac{dy}{y} \int_y^\Lambda \frac{dz}{z} \; G_q^2\left(\frac{y}{q}, \frac{z}{q}\right)$$

$$I(\Lambda, q) = I_0(\Lambda) + I_1(\Lambda)\, q/k + \cdots$$

$q < k \;, I \simeq I_0 \approx$ constant

$I_0(\Lambda/k)$



$\beta(g_{4D}) =$

$$-\frac{g_{4D}^3}{4\pi^2} C_2(G)\left(\frac{11}{3} I_0^{1,0} - \frac{1}{6} I_0^{\overset{\Lambda_r}{\wr}\,1,1}\right)$$

$+$ matter

$\approx \propto \beta_{SM}$

$I_0 \sim 1 \qquad \Lambda \lesssim k$

$I_0 \sim \Lambda/k \qquad \Lambda \gtrsim k$

Λ/k Dependence

Consider

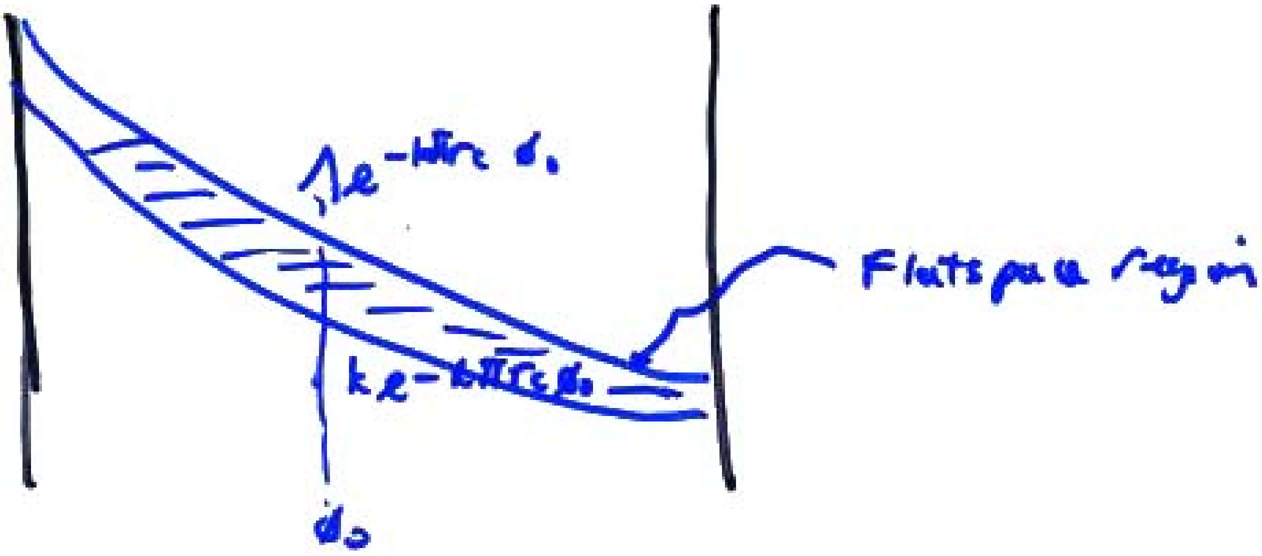
$$\int_k^{\Lambda} d^4q \int^{\Lambda/kq} du \int_u^{\Lambda/kq} dv \; \frac{e^{q(u-v)}}{q^2}$$

flatspace contribution

$$\int d^4q \int^{\#} d(u+v) \int d(u-v) \; \frac{e^{k(u-v)}}{q^2}$$

$$\int d^4q \; \overbrace{\log \frac{M_P}{TeV}}^{\# = R_c} \; \frac{1}{q^3}$$

$$\Rightarrow (\Lambda/k) \log M_P/TeV \quad \checkmark$$



- $\hat{k}$ $\underbrace{\log}_{\text{size of space}}$
- counterterm in 5D of same form
- Exact model, exact dual would permit investigation of FLAT SPACE regime
- Alternative interpretation of 1/k dependence is contribution of 1/k KK modes (due to regularization)

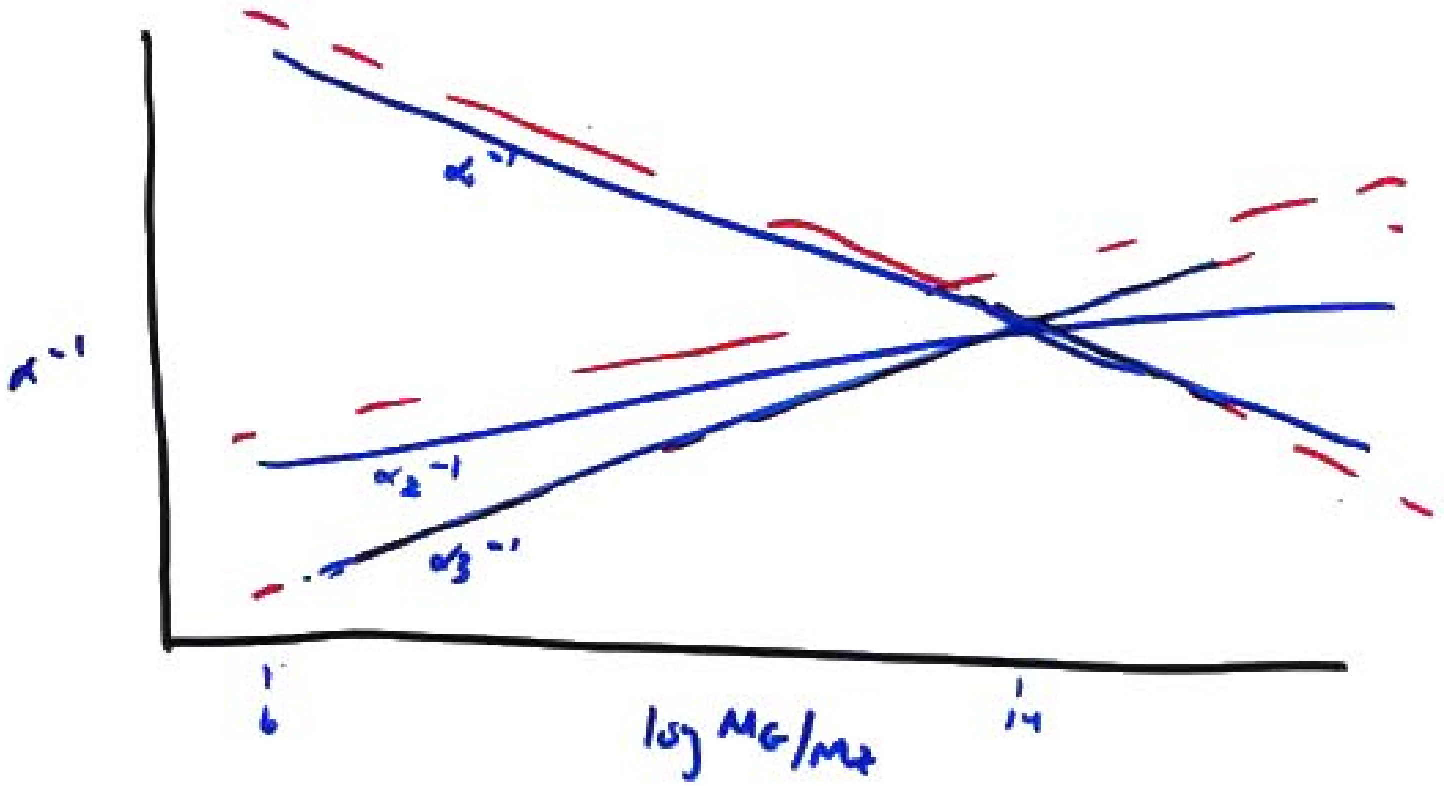
Results:

$$\alpha_1^{-1}(M_G) = \alpha_1^{-1}(M_t) - \frac{6}{\pi}\left(\frac{n_g}{3} + \frac{3}{5}\frac{n_s}{24}\right)\log\left(\frac{M_{GUT}}{M_t}\right)$$

$$\alpha_2^{-1}(M_G) = \alpha_2^{-1}(M_t) - \frac{6}{\pi}\left(-\frac{11}{6} I_0^{1,0}(\Lambda) + \frac{1}{12} I_0^{1,1}(\Lambda) + \frac{n_g}{3} + \frac{n_s}{24}\right)\log\left(\frac{M_G}{M_t}\right)$$

$$\alpha_3^{-1}(M_G) = \alpha_3^{-1}(M_t) - \frac{6}{\pi}\left(-\frac{11}{4} I_0^{1,0}(\Lambda) + \frac{1}{8} I_0^{1,1}(\Lambda) + \frac{n_g}{3}\right)\log\left(\frac{M_G}{M_t}\right)$$

- $I_0 = 1 \Rightarrow$ 4-D running
- bulk scalars $n_s \to n_s I_0^{0,1}(\Lambda)$
- tension $n_s \to 2 n_f I_0^{1/2,0}$
- n_g # complete multiplets: irrelevant to unification



- - - - SM

——— $\Lambda = k$, 4 majorana fermions in bulk doublets

- Clear <u>unification</u> at level of SM
- $\Lambda > k \Rightarrow$ unification scale $\sim M_Z\, 10^{14/\log_2 \Lambda/k}$
- X, Y completion of multiplet $\Rightarrow$ ~std scale

Comments

Uncertainties :

- ∄ counterterm absorbs Λ dependence
- subleading contributions

- Model Dependence
  - X, Y contribution even if heavy unified group vacuum/tree coupling ⇒ unification scale
  - SM: bulk or brane unknown Higgs-related bulk contribution

- PV (5D) gives same answer

  Recall $J_\nu(kt)$ $\nu = \sqrt{1 + (\ldots)}$

  $kt \gg m$ cancellation

- our regularization gives insight into spatial dependence

- EXPLICIT REALIZATION OF A UV / IR correspondence
  spatial

* RS1 with bulk gauge bosons consistent with high scale unification

* True for other curved spaces as well

current work deconstruct + generalize
Arkani-Hamed
LR
Sundrum
Warner

- Notable that BULK CALCULATION gave 4D behavior: Log 'running'

- Gives insight into counting DOF
- Motivates version of UV/IR
- Find area law behavior
  AdS, BH, de S

NB:
- We are working in curved spaces
- Not incorporating back reaction
- We are studying only reduction ~~in~~ DOF attributable to curvature
- Counting is intact what should be expected with SPATIAL CUTOFF

Prescription for Counting: field in curved space

$$ds^2 = -v(r)dt^2 + \frac{dr^2}{v(r)} + r^2 d\Omega^2$$

$$g(E) \sim \int^{r_E} \left(\sqrt{g^{tt}}\, E\right)^{n-1} r^{n-2} \sqrt{g_{rr}}\, dr$$

$$\sim E^{n-1} \int^{r_E} \frac{r^{n-2}\, dr}{[v(r)]^{n/2}}$$

Naive energy dependence E^{n-1}
but

$r_E(E)$ from $E < \sqrt{v(r_E)}\,\Lambda$

can change E-dependence

Global AdS

$r = R$ regulator brane

$$g(E) \sim E^4 \int \frac{r^3 dr}{V(r)^{5/2}}$$

$$\sim E^4 \int_{r_E}^{R} \frac{r^3 dr}{k^5 r^5}$$

$$r_E \sim \frac{E}{k\Lambda}$$

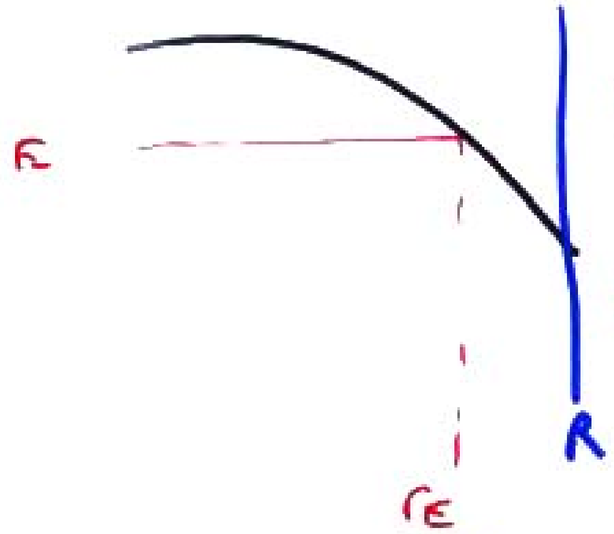
$$g(E) \sim \left(\frac{E}{k}\right)^4 \left(\frac{\Lambda}{E}\right) \sim \left(\frac{E}{k}\right)^3 \frac{\Lambda}{k}$$

Example: Static patch of de Sitter space

$V(r) = 1 - r^2/R^2$

$E < \sqrt{1-\frac{r^2}{R^2}}\,\Lambda \Rightarrow r < r_E = R\sqrt{1-\frac{E^2}{\Lambda^2}}$

$V(r_E) = R^2 \frac{E^2}{\Lambda^2}$



Finite E does not reach horizon

Regulates calculation

$$g(E) \sim E^3 R^4 \int^{r_E} \frac{r^2\,dr}{(R^2-r^2)^2} \sim E^3 R^3 \frac{\Lambda^2}{E^2}$$

enhancement

$$\sim E \Lambda^2 R^3$$

$E \sim T \sim 1/R$

$$\Rightarrow \frac{\Lambda^2 R^2}{T}$$

see also 't Hooft mgen et al

Surprisingly large # states (temp T)

States near R go up to local cutoff

∃ much more energetic states; not excited at low T

Comments

- works for AdS, Global AdS, RNBH, BH
- Not boundary theory, but bulk theory with fewer DOF, perhaps concentrated on bdy [horizon]
- meaning of Λ? Above BH, String's
- BH, string contribution should have same Λ-dependence
- Suggests add'l "stored" "bound" DOF at cut-off ; eg change k
- Coordinate dep'ce?

CONCLUSIONS

- **RS1**:
  - Possibility of unification makes idea more compelling
  - Should apply to other curved spaces as well: TeV accident of location?

- RS1 as example(s) with which to study generalizations FT and gravity:
  - eg AdS4 brane in AdS5: massive graviton, local 4D gravity
  - RS1: AdS/CFT with UV, IR cutoffs
  - 5D calculation gives ~ 4D result
  - INSIGHT into ST requirements in other curved spaces to compensate cut-off dependence
  - AdS/CFT motivates more general 'UV/IR' prescription
  - useful for bulk/bdy correspondence
  - ([illegible]? [illegible])