HIGHER SPIN

MASSLESS PARTICLES

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TWO AREAS TO GUIDE US

(preaching page)

- ANYTHING THAT INVOLVES $N=1$ SUGRA in 11-d WORTH A LOOK

SO(9) NOT AS PRETTY AS SO(8), YET...

- SUSY BREAKING 1001 WAYS.

analogy : chiral sym. breaking , spontaneous & explicit recognized early , but mechanism became natural only with QCD and quarks.

worked because π, N and quarks shared some q.n.

susy might become obvious in terms of $\boxed{\text{new}}$ variables which have supersymmetry as in M-theory?

hence search for all realizations of supersymmetry

OBSTACLE : massless $s > 2$ "particles".

$s > 2$ massless particles - a no-no ?

LOTS OF LORE :

- <u>covariant</u> conserved j_μ $\quad s > 1/2$ forbidden

  $\theta_{\mu\nu}$ $\quad s > 1$ forbidden

  D. C.G. W.W.

- spin $5/2$ massless coupled to gravity. is a no-no

  "hypergravity"

  Grisaru [illegible] Dewit + Freed. Deser, Aragone

  free spin $5/2$: gauges to eliminate bad dofs.

  couple to grav. $\delta_g(\not{D}\psi_{\mu\nu}) \neq 0 \sim \text{Ricci}(\cdots)$

  if Ricci $= 0$ in background, cannot compensate

  coupling in flat space-time not possible.

  Vasiliev

  "run out of derivative"

- May be possible for ∞ # of particles. (non-local theory)?

WHY LOOK AT THESE ?

OCCUR IN TWO PLACES :

- "CONTINUOUS SPIN REPS OF POINCARÉ GROUP"
- EULER TRIPLETS IN 11-DIMENSIONS

both in infinite number.

① CONTINUOUS SPIN REPRESENTATIONS Wigner

LIGHT-CONE POINCARE ALGEBRA

$$\boxed{P^- = \frac{p^i p_i + m^2}{2p^+}} \qquad P^+ = p^+ \qquad P_i = p_i$$

$$M_{ij} = x_i p_j - x_j p_i + S_{ij} \qquad M^{+i} = -x^i p^+ \qquad M^{+-} = -x^- p^+$$

$$\boxed{M^{-i} = x^- p^i - \frac{1}{2}\{x^i, P^-\} + \frac{1}{p^+}(T^i - p^j S^{ij})}$$

S_{ij} generate $SO(d-2)$

T^i $SO(d-2)$ vector with $[T^i, T^j] = i m^2 S^{ij}$

• if $m^2 \neq 0$ $\qquad S^i \equiv \frac{T^i}{m}$ complete $\boxed{\text{massive}}$

little group $SO(d-1)$

• if $m^2 = 0 \rightarrow [T^i, T^j] = 0$ $\qquad T^i$ are c-number, transverse vector

• $T^i = 0$ $\qquad$ usual massless representations with light-cone $SO(d-2)$ label.

• $T^i \neq 0$ $\qquad$ strange massless representations labelled by Galilean d-dim. algebra.

$$T^i T^i \equiv \Xi^2$$

CONTINUOUS SPIN REPRESENTATIONS

(Wigner)

$$P \cdot P = 0 \qquad W \cdot W = -\Xi^2 \neq 0$$

3+1 dimensions

$$| p^+, p^i ; \Xi, \alpha \rangle = \sum_\lambda e^{2\alpha i \lambda \alpha} | f_\lambda \rangle$$

$T^1 = \Xi \cos\alpha$

$T^2 = \Xi \sin\alpha$

$$M^{-i} | p^+, p^i = 0 ; \Xi, \alpha \rangle = \frac{T^i}{p^+} | p^+, p^i = 0 ; \Xi, \alpha \rangle$$

BOOSTING BY A FINITE AMOUNT GENERATES ALL HELICITIES

WITH $\Delta h = \pm 1$

TWO TYPES:

$$\cdots \quad -3 \quad -2 \quad -1 \quad 0 \quad +1 \quad +2 \quad +3 \quad + \cdots$$

$$\cdots \quad -5/2 \quad -3/2 \quad -1/2 \quad 1/2 \quad 3/2 \quad 5/2 \quad \cdots$$

SUSY

HIGHER DIMENSIONS   MANY MORE REPRESENTATIONS

fix direction of T^i

find $SO(d-3)$ that leave $\vec{T}$ invariant

example 10+1 dimensions   CSR labelled by $SO(8)$

DIMENSIONAL REDUCTION $d-1,1 \rightarrow d-2,1$

T^i vector in $d-2$ space

transformations $\perp$ to $\vec{T}$ $SO(d-3)$ little group

can get to one less space dimension.

Meaning of T^i ? can it be dynamical ?

transformations in space orthogonal to T^i look normal.

SUPERSYMMETRY

easy to implement. $[T^i, Q] = 0$

in (3+1) unique CSR of SuperPoincaré algebra.

example in 10+1 dimensions

the N=1 susy charge in 10+1 split into $\underset{\sim}{8}_s, \underset{\sim}{8}'_s$

for the CSR. This leads to a label like IIA

$$(\underset{\sim}{1} + \underset{\sim}{28} + \underset{\sim}{35} + \underset{\sim}{8} + \underset{\sim}{56})_{CSR,v} + (8' + 56' + 8 + 56)_{CSR,s}$$

but each CSR has infinite number of states.

infinite multiplet of massless particles (with IIA label)!

HOW DO THESE FIT IN GROUPS THAT CONTAIN THE POINCARE GROUP?

FINAL CURIOSITY.

If T^i IS NILPOTENT THEN GET

FINITE NUMBER OF SPINS, BUT

SUPERCHARGES CONTAIN CENTRAL CHARGES

but...

Central Charge + Massless particle $\rightarrow$ negative and/or zero norm states

- Wigner thought the CSR would be useless
- A variety of people have tried to use them Abbott, Hirata, Zoller

(higher derivative theory.)

see also Vasiliev ...

② SUPERSYMMETRY AND SO(9)

if SO(8) is the Mercedes of groups, SO(9) looks like the Trabant

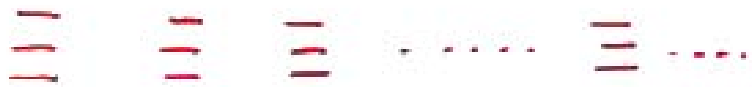
(E_8 is the DELAHAYE !)

WILL FIND THAT THERE IS AN INFINITE NUMBER OF

TRIPLETS OF SO(9) IRREPS THAT GENERALIZE N=1 SUGRA

SPIN-STATISTICS PRESERVED IF #F = #B

NOT SUPERSYMMETRIC LEVEL BY LEVEL



IF RELATIVISTIC, ALL MASSLESS, AND HIGHER SPINS

N = 1 SUSY IN 10+1 DIMENSION

LIGHT-CONE FORMULATION

Q_+^a kinematic $\quad Q_-^a$: dynamic $\quad a = 1, \dots 16$ spinor

$$\{ Q_+^a , Q_+^b \} = \sqrt{2} p^+ \delta^{ab} \qquad \{ Q_+^a , Q_-^b \} = -(\gamma_i)^{ab} p^i \qquad i = 1, \dots 9 \text{ vector}$$

$$\{ Q_-^a , Q_-^b \} = \frac{1}{\sqrt{2}} \delta^{ab} \frac{\vec{p} \cdot \vec{p}}{p^+}$$

solved for $\quad Q_-^a = - \frac{p^i}{p^+} (\gamma^i Q_+)^a$

$\theta^\alpha \quad \alpha = 1, \dots 8 \quad$ complex Grassmann variables such that

$$\frac{1}{\sqrt{2}} (Q_+^\alpha + i Q_+^{\alpha+8}) = \mathcal{Q}_+^a = \frac{\partial}{\partial \bar{\theta}^\alpha} + \frac{1}{\sqrt{2}} p^+ \theta^\alpha$$

act on chiral superfield $\quad \frac{\partial}{\partial \bar{\theta}} - \frac{1}{\sqrt{2}} p^+ \theta \approx 0$

$$\Phi(y^-, \vec{x}, \theta) = \varphi(y^-, \vec{x}) + \theta \psi(y^-, \vec{x}) + \cdots + (\theta)^8 \psi_8 (y^-, \vec{x})$$

$$y^- = x^- - i \frac{\bar{\theta}\theta}{\sqrt{2}}$$

256 fields : describe superparticle

WRITE IN TERMS OF HIGHEST WEIGHT STATES

$$\Phi = h\,\theta^1\theta^8 + \psi\,\theta^1\theta^4\theta^8 + A\,\theta^1\theta^4\theta^5\theta^8$$

SO(9) $\underset{\sim}{44}$ $\underset{\sim}{128}$ $\underset{\sim}{84}$

SO(9) GENERATORS

$$S^{ij} = -\frac{i}{4}(\gamma^{ij})^{ab}\,\Gamma^{ab} \qquad a,b = 1,\dots 16$$

VECTOR INDEX!

$$\{\gamma^i, \gamma^j\} = 2\delta^{ij}$$

SPINOR INDEX!

$$\{\Gamma^a, \Gamma^b\} = 2\delta^{ab}$$

$$[S^{ij}, \Gamma^a] = \frac{i}{2}(\gamma^{ij}\Gamma)^a \quad \dots$$

$$\equiv i\, f^{[ij]ab}\,\Gamma^b$$

FOR ANY SPINOR χ^a.

structure function of Lie algebra

$$\left.\begin{array}{ll} S^{ij} & \chi^a \\ 36 & 16 \end{array}\right\} F_4 \quad = 52$$

$$[\chi^a, \chi^b] = \frac{i}{2}(\gamma^{ij})^{ab}S^{ij}$$

THREE IRREPS OF SO(9) SATISFY STRANGE RELATIONS

higher Dynkin index

$$I^{(k)} = \sum_{\text{irrep}} w^k \qquad I^{(0)} = \text{dimension}$$

	128	44	84
$I^{(0)}$			
$I^{(2)}$	256	88	168
$I^{(4)}$	640	232	408
$I^{(6)}$	1792	712	1080
$I^{(8)}$	5248	2440	3000

$$I^{(N)}_{128} = I^{(N)}_{44} + I^{(N)}_{84} \qquad N = 0, 2, 4, 6$$

FAILS FOR $N = 8$
(embed in SO_{16})

lack of renormalizability of $SUGRA_{11}$.

FOUND SUCH RELATIONS SHARED BY ∞ #
OF TRIPLETS OF SO(9) IRREPS !!

STEM FROM CHARACTER FORMULA

$F_4 \supset SO_9$

$c \equiv$ element of Weyl group which maps

Weyl Chamber of F_4 into that of SO_9

3 × times bigger

$$\boxed{V_\lambda \otimes S^+ - V_\lambda \otimes S^- = \sum_c \mathrm{sgn}(c)\, \mathcal{U}_{c\bullet\lambda}}$$

$V_\lambda =$ irrep of F_4 as sum of SO_9 irreps

$S^\pm =$ sps of SO_{16}

$c\bullet\lambda = c(\lambda + \rho_{F_4}) - \rho_{SO_9}$ $\rho = \Sigma$ fund. weights

in Dynkinese : $[a_1\ a_2\ a_3\ a_4]$ = irrep of F_4

$$\mathcal{U}_{c\cdot\lambda} = \begin{cases} (2+a_2+a_3+a_4,\ a_1,\ a_2,\ a_3) \\ (a_2,\ a_1,\ 1+a_2+a_3,\ a_4) \\ (1+a_2+a_3,\ a_1,\ a_2,\ 1+a_3+a_4) \end{cases}$$

Euler Triplet

three $SO(9)$ irreps

last entry odd for spinors

$\# F = \# B$: a_3, a_4 even :

INDEX FORMULA FOR KOSTANT'S OPERATOR

F_4/SO_9 COSET : 16-dim. projective Cayley plane

introduce Clifford Γ^a $a=1,\dots 16$

$$\{\Gamma^a, \Gamma^b\} = 2\delta^{ab}$$

T^a : generators of F_4 in F_4/SO_9 $a=1,\dots 16$

<u>KOSTANT</u> EQUATION

$$\boxed{\Gamma^a T^a \Psi = 0 \equiv \not{K} \Psi}$$

conformal gen. for F_4/SO_9

$$[T^a, T^b] = i f^{abij} T^{ij}$$

$\nwarrow$ generate B_4.

$$[\not{K}, L^{ij}] = 0$$

$$L^{ij} = T^{ij} - \frac{i}{2}(\gamma^{ij})^{ab}\Gamma^{ab}$$ generate B_4

SOLUTIONS ARE EULER TRIPLETS.

EULER # OF F_4/SO_9 : 3 = ratio of Weyl groups.

TO GENERATE F_4 REPRESENTATIONS, IT IS CONVENIENT TO USE OSCILLATORS (Fulton et al)

START FROM $SO(26) \supset F_4$

$$26 = 1 + 9 + 16$$

$$A_0^{\dagger} \qquad A_i^{\dagger} \qquad B_a^{\dagger}$$

u_0 u_i ζ_a coordinates

$$[A_0, A_0^{\dagger}] = 1 \quad ; \quad [A_i, A_j^{\dagger}] = \delta_{ij} \quad ; \quad [B_a, B_b^{\dagger}] = \delta_{ab}$$

B_a, $B_a^{\dagger}$ HAVE BOSE COMMUTATOR BUT SPINOR INDEX

SO(9) GENERATORS

$$T_{ij} = -i\,(A_i^{\dagger}A_j - A_j^{\dagger}A_i) - \frac{i}{2}\, B_a^{\dagger}(\gamma^{ij})_{ab}\, B_a$$

$F_4/SO(9)$ GENERATORS

$$T_a = -\frac{i}{2}\,(\gamma_i)_{ab}\,(A_i^{\dagger}B_b - B_b^{\dagger}A_i) + i\sqrt{\frac{3}{2}}\,(B_a^{\dagger}A_0 - B_a A_0^{\dagger})$$

(works because of special Fierz)

- $a_3 \neq 0$ ONLY

$$\theta^1\theta^8\,[\zeta_1+i\zeta_9,\ \zeta_8+i\zeta_{16}]^{a_3} + \theta^1\theta^8\theta^4\,[u_1+iu_2,\ \zeta_1+i\zeta_9]^{a_3} + \theta^1\theta^8\theta^4\theta^5\,[u_1+iu_2,\ \zeta_1+i\zeta_9]^{a_3}$$

$$[x,y] = x^{(1)}y^{(2)} - x^{(2)}y^{(1)} = \det\begin{pmatrix} x^{(1)} & x^{(2)} \\ y^{(1)} & y^{(2)} \end{pmatrix}$$ NEED TWO COPIES

SAME CONCLUSION : NEED a_3 EVEN FOR SPIN-STATISTICS

$$\boxed{\text{SPIN-STATISTICS} \longleftrightarrow \#F = \#B}$$

- $a_2 \neq 0$ ONLY NEED THREE COPIES

$$\theta^1\theta^8(1+\theta^4+\theta^4\theta^5)\,[u_1+iu_2,\ \zeta_1+i\zeta_9,\ \zeta_8+i\zeta_{16}]^{a_2}$$

$[x,y,z]$ = volume .

ζ AUTOMATICALLY QUADRATIC ! CHECK $\#B = \#F$

(1010) of SO(9)

Gr-23

$a_1 \neq 0$ ONLY NEED TWO COPIES

$$\theta^1\theta^8(1+\theta^4+\theta^5\theta^4)\Big([u_1+i\bar{u}_2,\, u_3+iu_4]+[S_1+iS_9,\, S_6-iS_{14}]$$
$$+\,[S_8+iS_{16},\, S_3-iS_{11}]\Big)^{a_1}$$

5 QUADRATIC SPIN-STAT O.K. # F = # B ✓

LOOKS LIKE INTERNAL VARIABLES FORM AREA

HAVE SO(9) (0100)

2-FORM

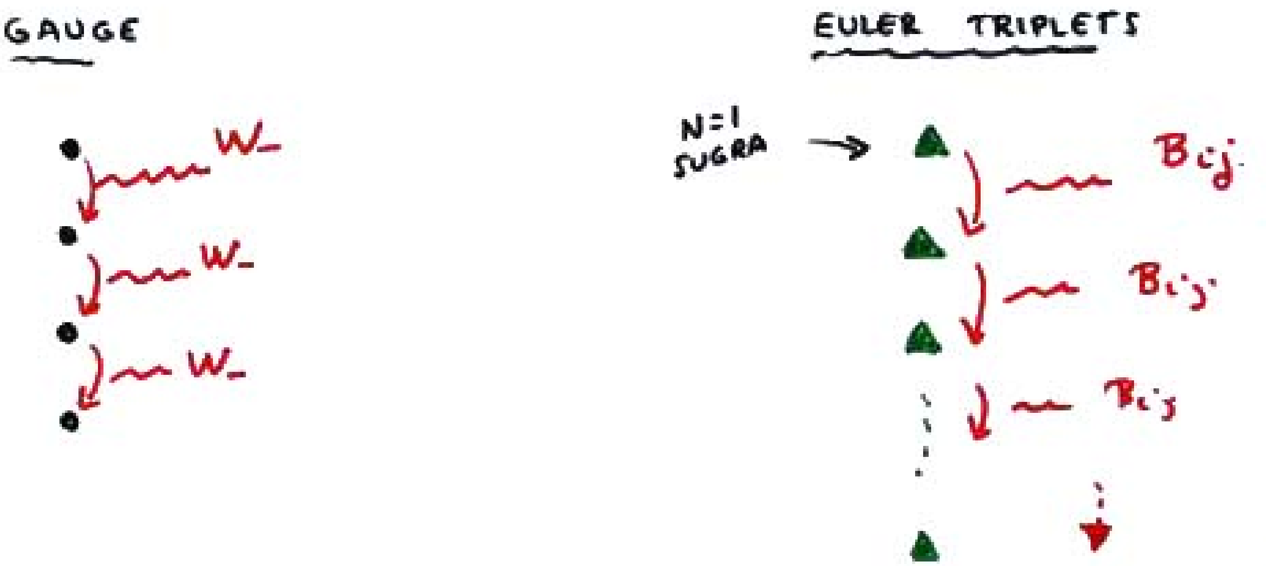
- NONE OF THESE STATES CAN HAVE MASS AND POINCARE INV.
- ONLY $a_1 = a_2 = a_3 = a_4$ HAS SUPERSYMMETRY
- SPIN-STATISTICS $\longleftrightarrow$ NO. FERMIONS = NO. BOSONS

WHAT DO WE DO WITH THEM?

SPECULATIONS

SYMMETRY LINKS INFINITE NUMBER OF EULER TRIPLETS

(REPRESENTATION IS INFINITE-DIMENSIONAL)



TO TEST THE IDEA:

COUPLE TO EXTERNAL FIELD B_{ij} ~ 2-form.

AS STEP DOWN OPERATOR ($\Delta a_1 = 1$ series)

SUPERPARTICLE IN 11-D COUPLED TO 2-FORM ...

ARE THERE REPRESENTATIONS OF SUPERSYMMETRY

WITH INFINITE NUMBER OF REPRESENTATIONS ?

$\Delta I^{(8)} = \dim F_4 \ \#$?

BUILD A 2-FORM CURRENT

$$\underbrace{(ET)^{\dagger}(T^{-})_{ij}(ET)}_{j_{ij}} \cdot B_{ij}$$

CAN IT BE OBTAINED FROM M NOETHER ?

UNDER INVESTIGATION

B_{ij} coupled to superparticle in (10+1)

not supersymmetric, but is there more ?