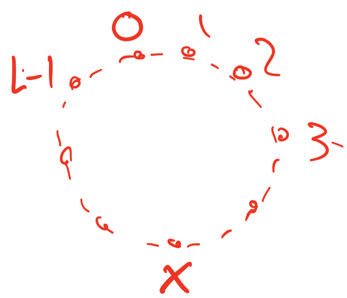
Problem Set II

(1) We showed that on the infinite 1D lattice ($\mathbb{Z}$) the "Propagator" for transition from 0 to x in time n is given by

$$P(x,n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} dq\, (\cos q)^n e^{-iqx}$$

* Show that on a finite lattice with L sites, with periodic boundary conditions, we have

$$P(x,n) = \frac{1}{L}\sum_{s=0}^{L-1} (\cos q)^n e^{-iqx}$$

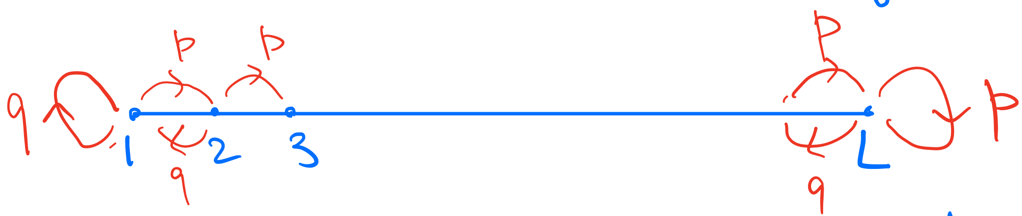


where $q = \frac{2s\pi}{L}$

Hence $$G(x,z) = \frac{1}{L}\sum_{s=0}^{L-1} \frac{e^{-iqx}}{1-z\cos q}.$$

* What is the corresponding result for $G(x,z)$ for a biased walk with jump probabilities p and $q=(1-p)$?

* Lattice with reflecting walls



Show that the transition rates above correspond to reflecting walls or zero-current boundary conditions

$$p \cdot P(0,n) = q\, P(1,n) \quad \text{— A}$$

$$p \cdot P(L,n) = q\, P(L+1,n) \quad \text{— B}$$

At all sites we have

$$P(x,n) = p\, P(x-1,n) + q\, P(x+1,n) \quad \text{— C}$$

Find the propagator for transitions from y to x in time n, $P(x,n|y,0)$.

Hint: This should be of the form $P(x,n|y,0) = \sum_{s=0}^{L-1} \lambda_s^n \phi_s(x) \chi_s(y)$

Find $G(x,z|y) = \sum z^n P(x,n|y,0)$

(2) The relation $F(x,z|x) = 1 - \frac{1}{G(x,z|x)}$ gives the generating function for the first return from x to x in n steps.

Find if the following is true in all the above cases (in Q1)

$$\sum_{n=1}^{\infty} n P_1(n|x \to x) = L$$

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Hint: Prove and use the relations

$$\langle n \rangle = z\,\partial_z F \Big|_{z=1}$$

$$\langle n^k \rangle = (z\,\partial_z)^k F \Big|_{z=1}$$

3 What is $\langle n \rangle$ for a persistent random walker for returning to the initial state (x,σ) on a periodic lattice of size L?

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4 Simulate the random walk (biased and un-biased) on a ring with L sites and compute $P_1(n)$, the first return time distribution to a site. Plot $P_1(n)$ for

$L = 64, 256, 1024$.