Reading between the lines of four-dimensional gauge theories

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Ofer Aharony, NS, Yuji Tachikawa, arXiv:1305.0318

Main point

- The local operators and their correlation functions do not uniquely specify a quantum field theory (not original).
- We need additional information:
  - line, surface (and even higher dimension) operators
  - behavior on non-trivial topology (the lines, surfaces, etc. create topology)

- Studying these line operators leads to new insights about the dynamics (phases) and electric/magnetic duality.

More concretely

- First choice: the gauge group, e.g. $SU(N)$, or $SU(N)/\boldsymbol{Z}_N$.

  This determines

  - the allowed Wilson lines – massive probe particles in representations of the gauge group
  - the allowed representations of matter fields
- Second choice: the 't Hooft lines (restricted by mutual locality – Dirac quantization)
  - They represent massive probe magnetic (or dyonic) particles.
  - Several different choices are possible.
- Additional freedom with surfaces, 3-dim. observables, …

A simple special case $su(2)$

- Gauge group is $SU(2)$
  - Basic Wilson line *W* in fundamental of $SU(2)$
  - 't Hooft lines have integer magnetic charge H^2, …
- Gauge group is $SO(3)$
  - No Wilson line in fundamental – only W^2, …
  - Basic 't Hooft line has half integer magnetic charge, but there are two choices [Gaiotto, Moore, Neitzke]:
    - $SO(3)_+$ the basic 't Hooft line *H* is electrically neutral
    - $SO(3)_-$ the basic 't Hooft line *HW* has half unit of electric charge

A simple special case $su(2)$

- $SU(2)$: W, H^2, …
- $SO(3)_+$: W^2, H, …
- $SO(3)_-$: W^2, HW, …

Witten effect: magnetic particles acquire electric charges under shift of Θ [Gaiotto, Moore, Neitzke]

$$SU(2)^{\theta} = SU(2)^{\theta+2\pi}$$

$$SO(3)_{+}^{\theta} = SO(3)_{-}^{\theta+2\pi}$$

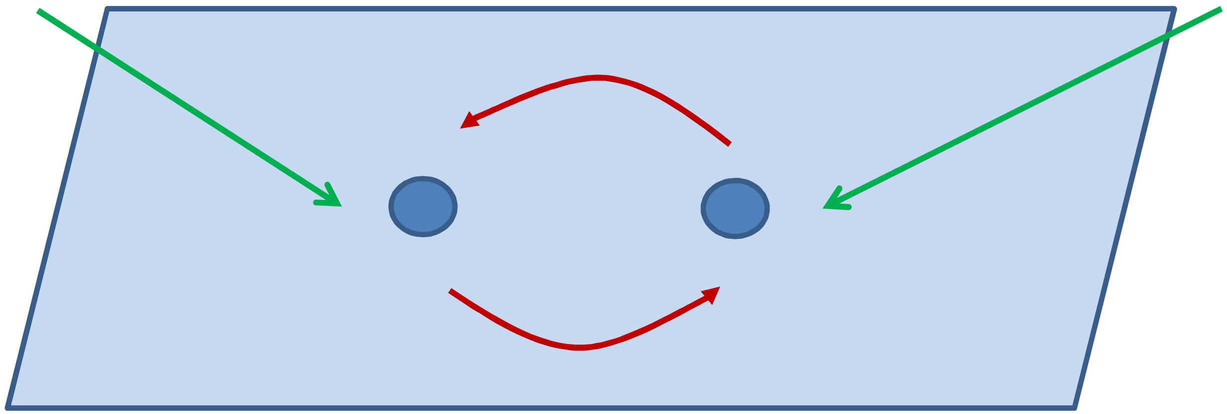
This is not typical.

$SU(2)$ with ***N*=2** SUSY [NS, Witten]

The theory has a continuous space of vacua with two singular points with additional massless particles…

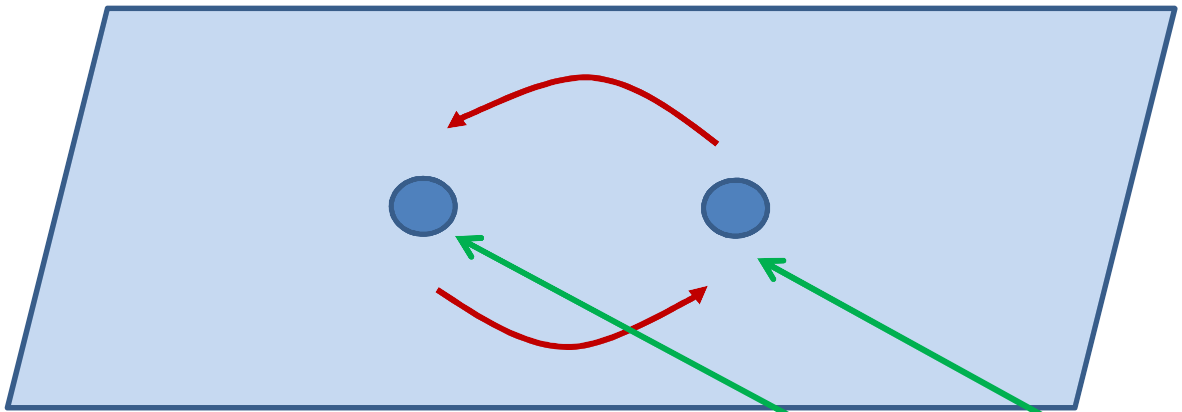
Dyon with magnetic charge one and electric charge one

Monopole with magnetic charge one and no electric charge



$$\theta \rightarrow \theta + 2\pi$$

$SO(3)$ with ***N*=2 SUSY**



$$\theta \rightarrow \theta + 2\pi$$

$SO(3)_+$: the basic line *H* has half the charges of this monopole

$SO(3)_-$: the basic line *HW* has half the charges of this dyon

No global symmetry relating the vacua. The theory with Θ is the same as with Θ + 4 π (not Θ + 2 π).

$SU(2)$ with ***N*=*1*** SUSY [NS, Witten]

- Upon breaking supersymmetry to ***N*=*1***, most of the vacua disappear and we are left with two vacua associated with the condensation of these monopoles.
- The theory confines.
  - The Wilson loop W has an area law.
  - The ‘t Hooft line H^2 has a perimeter law

$SO(3)$ with ***N=1*** SUSY

- Upon breaking supersymmetry to ***N**=1*, most of the vacua disappear and we are left with two vacua associated with the condensation of these monopoles.
- $SO(3)_+$: the basic line *H* has a perimeter law in one vacuum and an area law in the other.
- $SO(3)_-$: the basic line *HW* has an area law in one vacuum and a perimeter law in the other.
- There is an unbroken $\boldsymbol{Z_2}$ gauge symmetry in the vacuum with a perimeter law.
- Despite the mass gap, this $\boldsymbol{Z_2}$ gauge symmetry can be detected as long range (topological) order!

$su(2)$ gauge theories without SUSY

Conjectures:

- For every gauge group there is a single vacuum with a mass gap (a Clay problem).
- $SU(2)$: *W* exhibits confinement for every Θ (periodicity 2π, level crossing at $|\Theta| = \pi$).
- $SO(3)_+$: *Θ* periodicity is 4π, phase transition at $|\Theta| = \pi$
  - $|\Theta| \leq \pi$: *H* has a perimeter law, unbroken $\mathbf{Z}_2$ gauge symmetry
    - The particle spectrum is gapped, but there is long range topological order!
  - $\pi \leq |\Theta| \leq 2\pi$: *H* has an area law.
- $SO(3)_-$: same as $SO(3)_+$, but the phases are exchanged.

$su(2)$ with ***N=4*** SUSY [Vafa, Witten]

S-Duality:

$$\begin{tikzcd} SU(2) \arrow[loop below, "T"] \arrow[r, leftrightarrow, "S"'] & SO(3)_{+} \arrow[r, leftrightarrow, "T"'] & SO(3)_{-} \arrow[loop below, "S"] \end{tikzcd}$$

As we said above, this is not typical.

Usually the orbits are more complicated...

$\mathcal{N}$=4 S-duality with $so(N)$ with odd N

There are two different $SO(N)$ theories with odd $N > 3$
$SO(N)_{\pm}$ (not related by extending the range of Θ).

$$
\begin{tikzcd}
SO(2n+1)_{+} \arrow[r, leftrightarrow, "S"'] \arrow[loop below, "T"] & Sp(n) \arrow[loop below, "T"]
\end{tikzcd}
$$

$$
\begin{tikzcd}
Spin(4n+1) \arrow[r, leftrightarrow, "S"'] \arrow[loop below, "T"] & \left(\frac{Sp(2n)}{\mathbf{Z}_2}\right)_{+} \arrow[loop below, "T"] & \left(\frac{Sp(2n)}{\mathbf{Z}_2}\right)_{-} \arrow[r, leftrightarrow, "S"'] \arrow[loop below, "T"] & SO(4n+1)_{-} \arrow[loop below, "T"]
\end{tikzcd}
$$

$$
\begin{tikzcd}
Spin(4n+3) \arrow[r, leftrightarrow, "S"'] \arrow[loop below, "T"] & \left(\frac{Sp(2n+1)}{\mathbf{Z}_2}\right)_{+} \arrow[r, leftrightarrow, "T"'] & \left(\frac{Sp(2n+1)}{\mathbf{Z}_2}\right)_{-} \arrow[r, leftrightarrow, "S"'] & SO(4n+3)_{-} \arrow[loop below, "T"]
\end{tikzcd}
$$

***N*=4** S-duality

This example is typical (all simple gauge groups were analyzed).

- New theories – not merely extending the range of Θ
- New weak coupling limits (or new theories in known weak coupling limits)
- New orbits of the modular group

N=1 duality in $so(N)$ with matter

- This theory with N_f vector chiral superfields is dual to $so(N_f - N + 4)$ (with additional particles) [NS; Intriligator, NS].
- In special cases it was clearly identified as EM duality.
- Strassler argued that this duality exchanges $Spin(N)$ with $SO(N_f - N + 4)$.
- The more precise statement

$$\begin{aligned} Spin(N) &\longleftrightarrow SO(N_f - N + 4)_- \\ SO(N)_+ &\longleftrightarrow SO(N_f - N + 4)_+ \end{aligned}$$

- The line operators are exchanged as in EM duality.

Analogy with 2d orbifolds

2d orbifolds

- Keep only invariant operators
- Add twisted sector operators – restricted by mutual locality
- Demand completeness – modular invariance
- Different choices of twisted operators – discrete torsion

4d gauge theories

- Keep only Wilson lines of representations of the group
- Add ‘t Hooft lines – restricted by mutual locality (Dirac quantization)
- Demand completeness – modular invariance
- Different choices of ‘t Hooft lines – new theories

Both are associated with a discrete gauge symmetry.

A Euclidean path integral description

- The configuration space of gauge theories splits to different topological sectors (different bundles).
- The choice of gauge group determines the allowed bundles.
- We need a rule how to sum over them.
- The standard Θ-angle is related to the instanton number.
- The choice of lines depends on ${w_2}^2$ of the gauge bundle (more precisely, need Pontryagin square). It is a new discrete Θ-like parameter.

Restricting the range of Θ [NS 2010]

- Similarly, we can restrict the range of Θ by coupling a standard gauge theory to a $\boldsymbol{Z}_p$ gauge theory of forms (associated with 3-dimensional observables)

$$\frac{p}{2\pi}\Phi F^{(4)} \quad \text{with} \quad \Phi \sim \Phi + 2\pi \quad ; \quad \int F^{(4)} \in 2\pi\mathbf{Z}$$

$$\cdots + \frac{\theta}{16\pi^2}\mathrm{Tr}F\tilde{F} + \frac{p}{2\pi}\Phi F^{(4)} + \frac{\Phi}{16\pi^2}\mathrm{Tr}F\tilde{F}$$

- The integral over Φ forces the topological charge to be a multiple of *p*. *Hence*, $\theta \sim \theta + 2\pi/p$.
- Φ is a "discrete axion."
- Note, this is consistent with locality and clustering!

Conclusions

- The global part of the gauge group is essential in defining the theory.
- In addition, there are different choices of line operators.
- Using these operators as order parameters, we find new information about the phase diagram.
- New results about duality in theories with various amounts of supersymmetry.

Conclusions

- The choice of lines is related to a new discrete ϴ-like parameter.
- Coupling to other topological theories, we can even change the rules about the standard ϴ-angle.
- More generally, new nontrivial phenomena by coupling a gauge theory to a topological field theory.