Strings in Time-Dependent Orbifolds: Kinematics

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based on

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1. Introduction & Motivation
2. Geometry of some time-dependent orbifolds
3. Wave equations on the orbifold
4. Twisted sectors
5. Conclusions & Preview of Talk II

Related works:

Fabinger & McGreevy, hep-th/0206196

Horowitz & Polchinski, hep-th/0206228

Introduction & Motivation - I

The nature of time in general relativity leads to many deep – and well-known – conceptual questions.

Since string theory is a theory of quantum gravity one might hope to address some of these difficult problems in string theory, in the context of exactly solvable models.

But, in interestingly time-dependent backgrounds one encounters a host of new problems...

Among the many problems, today we focus on the crucial issue of *backreaction.*

Today I mostly discuss kinematics of certain orbifolds. Talk II, given by N. Seiberg, will discuss the dynamical issues further.

Introduction & Motivation - II

Two well-known sources of important backreaction effects:

No susy ⇒ potential tachyons and IR instabilities

CCC's ⇒ Potential divergences in loop amplitudes and $\langle T_{\mu\nu} \rangle$ △

In this talk we describe a simple model of strings in time-dependent geometry in which the above two effects are under control and yet, string perturbation theory is not well-defined.

Infinite blueshifts ⇒ divergent couplings to gravitons ⇒ string perturbation theory is out of control.

We also find some models, e.g. the "null-brane" do give examples of time-dependent geometries in string theory which can be studied in string perturbation theory ⇒ opportunity for further progress.

Time dependent orbifolds

Since time-dependent backgrounds are difficult to work we will study orbifolds:

$$\mathbb{R}^{1,9}/\Gamma, \qquad \Gamma \subset \mathrm{Poincare}(1,9)$$

This class of models was discussed about 12 years ago by Horowitz and Steif.

Recently there has been a resurgence of interest and papers on the subject

Simple example: $\mathbb{R}^{1,1}/g_0^{\mathbb{Z}}$:

$$ds^2 = -2dx^+ dx^- + \cdots$$

g_0^n acts by

$$x^+ \to e^{n\beta} x^+$$
$$x^- \to e^{-n\beta} x^-$$

Requirements on the background

$$\text{Spacetime} = \mathbb{R}^{1,9}/\Gamma \qquad\qquad \Gamma \subset \mathcal{P}(1,9)$$

What should we choose for Γ ?

- Time-dependent
- $\exists$ some unbroken SUSY
- No CTC's

Immediate implications:

Unbroken susy $\Rightarrow$ null Killing vector $\Rightarrow$

- Can use lightcone gauge: $\blacktriangle$
- First order time evolution $\Rightarrow$ No particle production
- A subclass of the pp wave backgrounds
- Results of Figueroa-O'Farrill $\Rightarrow$

$$\Gamma \subset G := \left(\mathrm{Spin}(7) \ltimes \mathbb{R}^8\right) \ltimes \mathbb{R}^{1,9}$$

Some sources of pathology in time-dependent orbifolds

Consistency conditions for orbifolds by finite subgroups of Euclidean isometries are well known (anomaly cancellation).

We have found several surprises with orbifolds by $\Gamma \subset \mathcal{P}(1,9)$ for Γ noncompact:

- Fixed points can lead to pathology
- Closed causal curves can lead to pathology
- "Nearly closed null curves"

($\exists$ closed spacelike geodesics γ_n with $\lim_{n\to\infty} L(\gamma_n) \to 0$)

- Explosive twisted sector degeneracies:

$$\sum_{\text{twisted sectors}} \mathrm{Tr}_{\mathcal{H}} q^{L_0} \bar{q}^{\bar{L}_0} = \infty$$

(Some Melvin models are ill-defined.)

- Infinite blueshifts.

We will be focusing on the last problem, which is an important and probably generic effect.

1-generator models

This talk: $\Gamma = g_0^{\mathbb{Z}}$ where $g_0 = \Lambda(\vec{v}, \vec{R})$ acts by:

$$
\begin{aligned}
x^+ &\to x^+ \\
\vec{x} &\to \vec{x} + \vec{v}x^+ + \vec{R} \\
x^- &\to x^- + \vec{v}\cdot\vec{x} + \frac{1}{2}\vec{v}^2 x^+
\end{aligned}
$$

$\vec{x}, \vec{v}, \vec{R} \in \mathbb{R}^d$, $d \leq 8$.

By conjugation in $\mathcal{P}(1, d+1)$ we can take

- $\vec{v} = v\hat{\mathsf{x}}_1$
- $\vec{R} = R\hat{\mathsf{x}}_2$.

We can set $v = 1$, but the parameter R is a modulus.

(Remark: The $\Lambda(\vec{v}, \vec{R})$, $\vec{v}, \vec{R} \in \mathbb{R}^8$, generate an Heisenberg group, present in all pp wave backgrounds.)

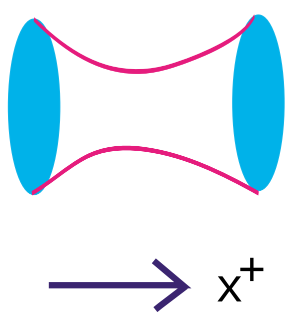
Geometry of the 1-generator models

What does $\mathbb{R}^{1,3}/\Gamma$ *look like?*

$$X := \begin{pmatrix} x^+ \\ x_1 \\ x_2 \\ x^- \end{pmatrix} \quad \to \quad g_0^n \cdot X = \begin{pmatrix} x^+ \\ x_1 + (nv)x^+ \\ x_2 + nR \\ x^- + (nv)x_1 + \frac{1}{2}(nv)^2 x^+ \end{pmatrix}$$

Compute distance to the image point:

$$(X - g_0 \cdot X)^2 = (vx^+)^2 + R^2$$

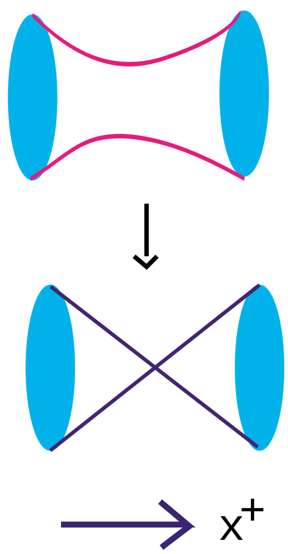


• This geometry was called the "null-brane" by Figueroa-O'Farrill & Simón (it can be interpreted as a generalized fluxbrane)

The parabolic orbifold: $R = 0$

The $R \to 0$ limit of the null brane defines the *parabolic orbifold*: (Horowitz and Steif, 1990, Tseytlin & Klimcek, 1994)

$$X := \begin{pmatrix} x^+ \\ x_1 \\ x^- \end{pmatrix} \quad \to \quad g_0^n \cdot X = \begin{pmatrix} x^+ \\ x_1 + (nv)x^+ \\ x^- + (nv)x_1 + \frac{1}{2}(nv)^2 x^+ \end{pmatrix}$$



$$(X - g_0 \cdot X)^2 = (vx^+)^2$$

This is an accurate picture away from $x^+ = 0$.

However, $(\mathbb{R}^{1,2}/\Gamma) \times \mathbb{R}^7$ is in fact a non-Hausdorff space near $x^+ = 0$.

Nevertheless, the string orbifold procedure constructs string theory on the group quotient, and not on the double-cone.

Wave Equation on the Orbifolds

First quantized wave equation on the orbifold:

$$\Big(-2\frac{\partial}{\partial x^+}\frac{\partial}{\partial x^-} + (\frac{\partial}{\partial \vec{x}})^2\Big)\Psi = m^2\Psi$$

Solutions become untwisted vertex operators in the quantized string.

Project onto states invariant under $\mathcal{U}(g_0)$

$$\Psi \in \mathcal{H}^{\mathrm{orbifold}} \qquad \Leftrightarrow \qquad \mathcal{U}(g_0)\cdot\Psi = \Psi$$

Write solutions of the wave equation in a basis of plane waves

$$\phi_{p^+,p}(x^+,\vec{x},x^-) = \exp\big(-ip^+x^- - ip^-x^+ + i\vec{p}\cdot\vec{x}\big)$$

The states are on-shell for

$$p^- = \frac{\vec{p}^2 + m^2}{2p^+}$$

Invariant Wavefunctions

General solution of the wave equation (for fixed p^+):

$$U_\chi(X) := \int_{\mathbb{R}^d} d\vec{p}\,\chi(\vec{p})\; e^{i(P,X)}$$

$$X = (x^+, \vec{x}, x^-) \qquad \text{with} \qquad \vec{x} \in \mathbb{R}^d$$

$$P = \left(p^+, \vec{p}, p^- = \frac{\vec{p}^2 + m^2}{2p^+}\right),$$

U_χ are invariant under $g = \Lambda(\vec{v}, \vec{R})$ *iff*

$$\chi(\vec{p} + \vec{v}p^+) = e^{-i(\vec{p}+p^+\vec{v})\cdot\vec{R}}\chi(\vec{p})$$

i.e., the wavefunction must be *quasiperiodic* in transverse momentum space.

The parabolic orbifold and null brane

$$\chi(\vec{p} + \vec{v}p^+) = e^{-i(\vec{p}+p^+\vec{v})\cdot\vec{R}}\chi(\vec{p})$$

Specialize to the parabolic orbifold ($d = 1$):

$$\chi(p + vp^+) = \chi(p)$$

$\Rightarrow$ χ is periodic in p $\Rightarrow$ use Fourier series $\Rightarrow$ basis:

$$\chi_n(p) = e^{-ip\xi_n} \qquad \xi_n := \frac{2\pi}{vp^+}n \qquad n \in \mathbb{Z}$$

(These will be the "J-eigenstates" in talk II)

Specialize to the null brane ($d = 2$):

$$\chi\big(p_1 + vp^+, p_2\big) = e^{-iRp_2}\chi\big(p_1, p_2\big)$$

$\Rightarrow$ χ is quasi-periodic in p_1 $\Rightarrow$ basis of wavefunctions:

$$\chi_n(p_1, p_2) = e^{-i\frac{R}{p^+v}p_1p_2}e^{-ip_1\xi_n}\tilde{\chi}(p_2)$$

With $\tilde{\chi}(p_2)$ arbitrary.

Two sources of trouble

- $U_\chi(X)$ can be *singular*
- $\chi(\vec{p})$ can have "too much" support at large energy:

$$p^- = (\vec{p}^2 + m^2)/(2p^+)$$

Talk II will show how these can lead to pathologies in string perturbation theory, and infinities even at string tree level.

Amplification of item 1:

$$U_\chi(X)|_{x^+=0} = e^{-ip^+x^-} \int_{\mathbb{R}^d} d\vec{p}\, \chi(\vec{p})\, e^{i\vec{p}\cdot\vec{x}}$$

- Parabolic orbifold ($d = 1$):

$$\chi_n(p) = e^{-ip\xi_n} \quad \Rightarrow \quad U_\chi = e^{-ip^+x^-}\delta(\xi_n - x)$$

- Null brane ($d = 2$): (put $p^+ v/R = 1$)

$$\chi_n(p_1, p_2) = e^{-ip_1 p_2} e^{-ip_1\xi_n} \tilde{\chi}(p_2) \quad \Rightarrow$$

$$U_\chi \sim e^{-ip^+x^-} e^{ix_1 x_2} \tilde{\chi}(x_1 - \xi_n)$$

Conclusion: Wavefunctions are smooth on the null brane and singular on the parabolic orbifold.

High Energy Support

Amplification of item 2:

Momentum space wavefunction $\chi(\vec{p})$ is quasiperiodic.

Since $p^- = (\vec{p}^2 + m^2)/2p^+$ the vertex operators involve states of arbitrarily large energy. Gravitons couple to the energy ⇒potential *trouble* ...

- Parabolic orbifold $(d = 1)$:

$$\chi(p + vp^+) = \chi(p)$$

⇒ $\mathcal{O}(1)$ support at arbitrarily high energy.

- Null brane $(d = 2)$:

$$\chi(p_1, p_2) = e^{-i \frac{R}{p^+ v} p_1 p_2} \psi(p_1) \tilde{\chi}(p_2)$$

with $\psi(p_1 + vp^+) = \psi(p_1)$.

⇒ can take $\tilde{\chi}(p_2)$ of rapid decrease ⇒ sufficient suppression of the high energy component that one can define finite amplitudes.

Quantization of Twisted Sectors

Main results:

- Interesting exchange algebra in the twisted sectors
- There are physical states in the twisted sectors

Return to the covariant action

$$S = \frac{1}{4\pi\alpha'} \int_{-\infty}^{\infty} d\tau \int_0^{2\pi} d\sigma \; \eta_{\mu\nu} \left(\partial_\tau x^\mu \partial_\tau x^\nu - \partial_\sigma x^\mu \partial_\sigma x^\nu \right)$$

Twisted sector conditions in sector $w \in \mathbb{Z}$:

$$X(\sigma + 2\pi, \tau) = e^{2\pi w \mathcal{J}} X(\sigma, \tau)$$

$$\mathcal{J} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$x^+(\sigma + 2\pi, \tau) = x^+(\sigma, \tau)$$
$$x(\sigma + 2\pi, \tau) = x(\sigma, \tau) + 2\pi w x^+(\sigma, \tau)$$
$$x^-(\sigma + 2\pi, \tau) = x^-(\sigma, \tau) + 2\pi w x(\sigma, \tau) + \frac{1}{2}(2\pi w)^2 x^+(\sigma, \tau)$$

Equation of motion:

$$(-\partial_\tau^2 + \partial_\sigma^2) X^\mu = 0$$

Covariant oscillators

Solve the equations of motion in the twisted sector in terms of oscillators:

$$\hat{x}_L^\mu := i \sum_{n \neq 0} \frac{\alpha_n^\mu}{n} e^{-inu^+}$$

$$\hat{x}_R^\mu := i \sum_{n \neq 0} \frac{\tilde{\alpha}_n^\mu}{n} e^{inu^-}$$

General solution in the twisted sector:

$$X(\sigma, \tau) = \exp(w\sigma\mathcal{J})X_z(\tau) +$$

$$+ \exp(wu^+\mathcal{J})\hat{X}_L(u^+) + \exp(wu^-\mathcal{J})\hat{X}_R(u^-)$$

Zeromodes are given by

$$X_z(\tau) := \begin{pmatrix} x_0^+ + \alpha' p^+ \tau \\ x_0 + \alpha' p \tau \\ x_0^- + \alpha' p^- \tau + w^2 \left(\alpha' p^+ \frac{\tau^3}{6} + x_0^+ \frac{\tau^2}{2} \right) \end{pmatrix}$$

Symplectic form is standard

$$\Omega = \frac{1}{2\pi\alpha'} \int d\sigma \; \delta x^\mu \eta_{\mu\nu} \partial_\tau \delta x^\nu$$

Noncommuting coordinates?

Surprise in terms of oscillators:

$$[\alpha_n^\mu, \alpha_m^\nu] = n\delta_{n+m,0} \left[\frac{1}{\eta + i\frac{w}{n}\eta\mathcal{J}}\right]^{\mu\nu}$$

$$[\tilde{\alpha}_n^\mu, \tilde{\alpha}_m^\nu] = n\delta_{n+m,0} \left[\frac{1}{\eta - i\frac{w}{n}\eta\mathcal{J}}\right]^{\mu\nu}$$

⇒ Unusual exchange algebra in the w-twisted sector

$$\partial X^{\mu_1}(z_1)\partial X^{\mu_2}(z_2) = \partial X^{\mu_2}(z_2)\partial X^{\mu_1}(z_1) + i\frac{w}{z_1 z_2}\left(e^{-iw\log z_1\mathcal{J}}\mathcal{J}e^{iw\log z_2\mathcal{J}}\eta^{-1}\right)^{\mu_1\mu_2}$$

$(|z_1| > |z_2|)$

This nontrivial exchange algebra is very similar to the exchange algebras of chiral vertex operators of RCFT. △

The exchange algebra is also suggestive of non-commuting coordinates and hence of non-commutative geometry. △

This might be related to remarks of Nekrasov re quantum groups.

Physical twisted states

There are sensible solutions of

$$(L_0 - 1)|0; p^+\rangle = (\tilde{L}_0 - 1)|0; p^+\rangle = 0$$

DDF Operators:

$$A_n = \oint \frac{dz}{2\pi} \left[\partial_z x + iw \log z \partial_z x^+ + \frac{w}{znk_0} \right] e^{ink_0 x^+}(z),$$

$n = \pm 1, \pm 2 \cdots$

$\Rightarrow$ usual tower of oscillator states, in the twisted sector.

• This is confirmed by analysis of one-loop partition functions.

• Compare hyperbolic orbifold, which has *no* physical states in the twisted sector (Nekrasov).

$\Rightarrow$ Open problem: Do these twisted states lead to important physical effects?

Related Models

- Other subgroups

$$\Gamma \subset G := \left(\mathrm{Spin}(7) \ltimes \mathbb{R}^8\right) \ltimes \mathbb{R}^{1,9}$$

- (Time-dependent) orbifolds of pp waves

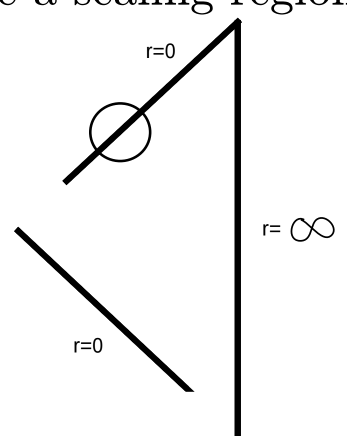
- Melvin models, fluxbranes, generalized fluxbranes, ...

- BTZ black holes:

$M = J = 0$ BTZ $= \widetilde{SL(2,\mathbb{R})}/g_0^{\mathbb{Z}}$

parabolic orbifold $= sl(2,\mathbb{R})/g_0^{\mathbb{Z}}$

Physically: Take a scaling region at the horizon.



Conjecture/Worry

In general, gauging WZW models by noncompact gauge groups might not lead to string backgrounds well-defined in string perturbation theory.

Conclusions & Future Directions

We have described a class of models with

- Weak coupling,
- Supersymmetry,
- All orders of α' under control,

Yet, amazingly (to me), not all is well.

⇒String perturbation theory in time-dependent orbifolds is not necessarily well-defined.

One of our positive results is that "good orbifolds" seem to exist: for example, the null brane $R > 0$.

Open problem: Formulate general consistency conditions for "good orbifolds"

Another result is a new phenomenon of "UV enhanced IR divergences."

Further explanations in talk II tomorrow.