



Large $\mathcal{N} = 4$ Holography

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based mainly on

[MRG, R. Gopakumar](#), arXiv:1011.2986, 1205.2472, **1305.4181**



Motivation

One way to get quantitative handle on AdS/CFT correspondence is to consider regime where **gauge theory is weakly coupled**:

$$\left(\frac{R}{l_{\text{Pl}}}\right)^4 = N \quad g_{\text{string}} = g_{\text{YM}}^2 \quad \left(\frac{R}{l_s}\right)^4 = g_{\text{YM}}^2 N = \lambda$$

large small

$l_s \rightarrow \infty$ 'tensionless strings'

[Sundborg '01]
[Witten '01]
[Sezgin, Sundell '01]



Higher spin theory

In this tensionless limit, stringy excitations become massless: **infinite number of massless higher spin fields**, which generate a **very large gauge symmetry**.

maximally unbroken phase of
string theory

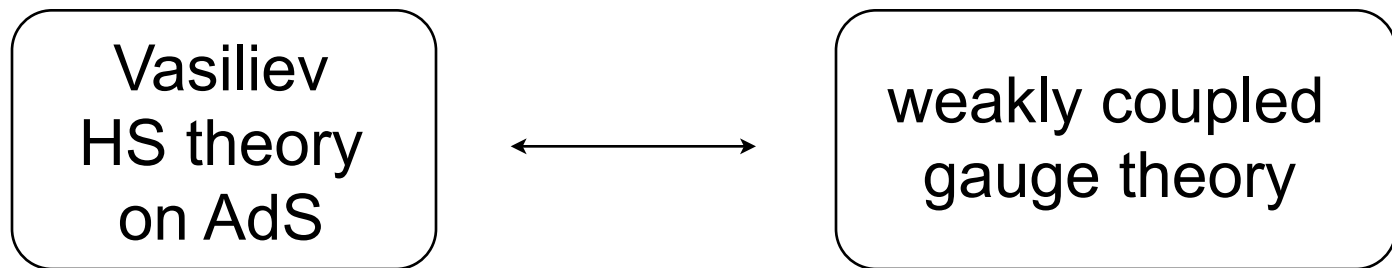
Idea: try to understand AdS/CFT correspondence starting from this perturbative duality!

see also [Sagnotti et.al.], [Jevicki et.al.], [Douglas et.al.], ...



State of the Art

In the past this idea was taken as a **general motivation** to consider dualities relating



However, recently interesting progress about how these dualities fit actually into **stringy AdS/CFT** correspondence has been made....

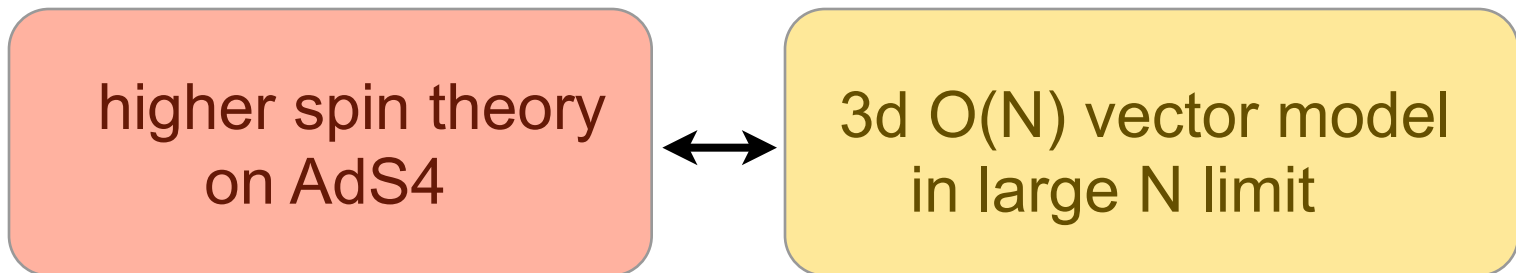
[Chang, Minwalla, Sharma, Yin, '12]
[MRG, Gopakumar, '13]



Higher spin CFT duality

First concrete proposal for HS - CFT duality

[Klebanov-Polyakov, '02]
[Sezgin-Sundell, '02]



Different versions: vector model fields **bosons or fermions**;
free or **interacting** fixed point.

More recently: generalisation to family of parity-violating theories.

[Giombi, Minwalla, Prakash, Trivedi, Wadia, '11]
[Aharony, Gur-Ari, Yacoby, '11]



Checks of the proposal

During the last few years **impressive checks** of the proposal have been performed, in particular

3-point functions of HS fields on AdS₄

have been **matched** to

3-point functions of HS currents in
O(N) model to leading order in $1/N$.

[Giombi, Yin, '09-'10]

Furthermore, the **symmetries** have been identified.

[Giombi, Prakash, Yin, '11], [Giombi, Yin, '11]
[Maldacena, Zhiboedov, '11-'12]



3d proposal

More recently also lower dimensional version was found:

[MRG,Gopakumar, '10]

AdS3:

higher spin theory
with a complex
scalar of mass M



2d CFT:

$\mathcal{W}_{N,k}$ minimal models
in large N 't Hooft limit
with coupling λ

where $\lambda = \frac{N}{N+k}$ and $M^2 = -(1 - \lambda^2)$



3d proposal

Because 3d HS theories and 2d CFTs are well understood, this duality can be tested in quite some detail: in particular, the **quantum symmetries** and the **spectrum** were shown to match precisely.

[Henneaux, Rey, '10] [Campoleoni et.al. '10]
[MRG, Hartman, '11] [MRG, Gopakumar, '13]

[MRG, Hartman, Gopakumar, Raju, '11]
[Castro, Gopakumar, Gutperle, Raeymaekers, '11]
[MRG, Gopakumar, '12]
[Perlmutter, Prochazka, Raeymaekers, '12]

In addition, **correlation functions** have also been compared successfully.

[Chang, Yin, '11] [Ammon, Kraus, Perlmutter, '11]
[Ahn, '11] [Moradi, '12] [Creutzig, Hikida, Ronne, '12]
[Hijano, Kraus, Perlmutter, '13]



BHs and Generalisations

Furthermore, **black hole solutions have been constructed**, and their properties matched to the dual CFT.

[Ammon, Gutperle, Kraus, Perlmutter, '11-'13]
[MRG, Hartman, Jin, '12] [Perez, Tempo, Troncoso, '12]
[de Boer, Jottar, '13] [Kraus, Ugajin, '13] [Datta, David, '13]
[Song, Compere, '13] [MRG, Jin, Perlmutter, to appear]

Finally, various **generalisations** to other groups and/or supersymmetry have been considered.

[Ahn, '11] [MRG, Vollenweider, '11]
[Creutzig, Hikida, Ronne, '12-'13] [Candu, MRG, '12]
[Candu, MRG, Kelm, Vollenweider, '12]
[Beccaria, Candu, MRG, Groher, '13]



Large $\mathcal{N} = 4$ Supersymmetry

In the following I want to explain generalisation to the situation with **large $\mathcal{N} = 4$ supersymmetry** which is possibly related to string theory on

[MRG, Gopakumar, '13]

$$\text{AdS}_3 \times S^3 \times S^3 \times S^1$$

In order to understand this construction, need to review how **symmetries of the two descriptions are related**.



The HS theory on AdS3

The AdS3 HS theory can be described very simply.

Recall that pure gravity in AdS3: **Chern-Simons theory**
based on

$$\mathfrak{sl}(2, \mathbb{R})$$

[Achucarro, Townsend, '86]
[Witten, '88]

Higher spin description: replace

[Prokushkin, Vasiliev, '98]

[one spin field for each
spin $s = 2, 3, \dots$]

$$\mathfrak{sl}(2, \mathbb{R}) \rightarrow \mathfrak{hs}[\lambda] \cong \mathfrak{sl}(\lambda)$$

where $\mathfrak{hs}[\lambda] \oplus \mathbb{C} \cong \frac{U(\mathfrak{sl}(2))}{\langle C_2 - \frac{1}{4}(\lambda^2 - 1)\mathbf{1} \rangle}$

[Bordemann et.al., '98]
[Bergshoeff et.al., '90]
[Pope, Romans, Shen, '90]



Higher spin algebra

Generators of $hs[\lambda]$:

$$V_n^s \text{ with } |n| < s, \quad s = 2, 3, \dots$$

'wedge algebra'

For these higher spin theories **asymptotic symmetry algebra** can be determined following **Brown & Henneaux**, leading to **classical**

$\mathcal{W}_\infty[\lambda]$ algebra

[Henneaux & Rey, '10]
[Campoleoni et al, '10]
[MRG, Hartman, '11]



Asymptotic symmetry algebra

Asymptotic symmetry algebra extends hs algebra
'beyond the wedge':

$$\text{pure gravity: } \begin{array}{ccc} L_0, L_{\pm 1} & \longrightarrow & L_n, n \in \mathbb{Z} \\ \mathfrak{sl}(2, \mathbb{R}) & & \text{(Virasoro)} \end{array}$$

$$\text{higher spin: } \text{hs}[\lambda] \longrightarrow \mathcal{W}_\infty[\lambda]$$

$$\begin{array}{l} \text{generated by } V_n^s \\ s = 2, \dots, \infty, n \in \mathbb{Z} \end{array}$$

[Figueroa-O'Farrill et.al., '92]



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Let us start from this **most supersymmetric** string background, and **'reverse engineer'** the corresponding hs theory....



Towards String Theory

Dual CFT of string background is expected to have

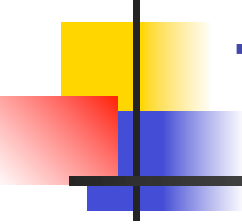
$$\text{AdS}_3 \times S^3 \times S^3 \times S^1$$

Large
 $\mathcal{N} = 4$

$$\text{Vir} \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$$

with 4 supercharges

[Boonstra, Peeters, Skenderis, '98; Elitzur, Feinerman, Giveon, Tsabar, '99; de Boer, Pasquinucci, Skenderis, '99; Gukov, Martinec, Moore, Strominger, '04;...]



The large $\mathcal{N} = 4$ algebra

$$[U_m, U_n] = \frac{k^+ + k^-}{2} m \delta_{m, -n}$$

$$[A_m^{\pm, i}, Q_r^a] = i \alpha_{ab}^{\pm i} Q_{m+r}^b$$

$$\{Q_r^a, Q_s^b\} = \frac{k^+ + k^-}{2} \delta^{ab} \delta_{r, -s}$$

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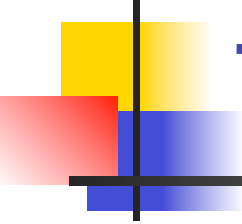
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$$\{G_r^a, G_s^b\} = \frac{c}{3} \delta^{ab} \left(r^2 - \frac{1}{4} \right) \delta_{r, -s} + 2 \delta^{ab} L_{r+s}$$

$$+ 4(r - s) \left(\gamma i \alpha_{ab}^{+i} A_{r+s}^{+, i} + (1 - \gamma) i \alpha_{ab}^{-i} A_{r+s}^{-, i} \right) .$$



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U(1) current

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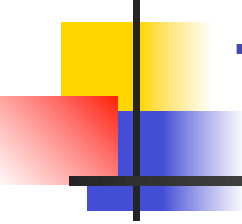
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4 free fermions

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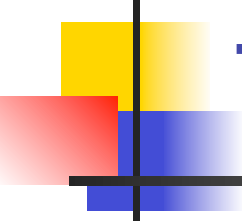
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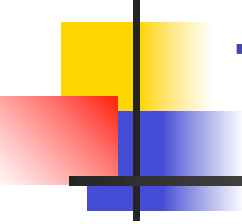
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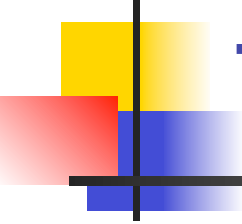
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4 supercharges
free parameter





Large $\mathcal{N} = 4$

Since there are **two current algebras**, the algebra is actually characterised by two parameters: in addition to **central charge**

$$c = \frac{6k^+ k^-}{k^+ + k^-}$$

[Sevrin, Troost, Van Proeyen, Schoutens, Spindel, Theodoridis, '88-'90; Goddard, Schwimmer, '88]

have **parameter**

$$\gamma = \frac{k^-}{k^+ + k^-} \quad \text{or} \quad \alpha = \frac{\gamma}{1 - \gamma} = \frac{k^-}{k^+} \quad \swarrow$$

ratio of the two S3s.

All **other structure constants are determined** in terms of these.



$\mathcal{N} = 4$ coset

One family of $\mathcal{N} = 4$ coset CFTs is known based on Wolf symmetric spaces

$$\frac{\mathfrak{su}(N+2)_\kappa^{(1)}}{\mathfrak{su}(N)_\kappa^{(1)}} \cong \frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2}}$$

[Sevrin, Troost, Van Proeyen, Schoutens, Spindel, Theodoridis, '88-'90]

They contain large $\mathcal{N} = 4$ algebra with

$$\begin{aligned} k^+ &= k + 1 \\ k^- &= N + 1 \end{aligned} \Rightarrow$$

$$\gamma = \frac{N+1}{N+k+2}$$

as well as certain higher spin currents.

't Hooft parameter



Wedge algebra

Ansatz: these cosets are dual to $\mathcal{N} = 4$ hs theory in large (N,k) limit!

In order to reverse engineer this hs theory, determine the **wedge algebra (=global subalgebra)** of cosets: it is generated by the `wedge modes`

$$S_n \quad \text{with } |n| < h(S)$$

[Asymptotic symmetry algebra of CS theory based on this wedge algebra should then reproduce the full coset W-algebra.]



From the large $\mathcal{N} = 4$

To begin with consider wedge algebra of the large $\mathcal{N} = 4$ algebra.

$$[U_m, U_n] = \frac{k^+ + k^-}{2} m \delta_{m, -n}$$

free fermions ($h=1/2$) do not contribute
u(1) current: only zero mode --- central

$$[A_m^{\pm, i}, Q_r^a] = i \alpha_{ab}^{\pm i} Q_{m+r}^b$$

$$\{Q_r^a, Q_s^b\} = \frac{k^+ + k^-}{2} \delta^{ab} \delta_{r, -s}$$

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$$+ 4(r-s) \left(\gamma i \alpha_{ab}^{+i} A_{r+s}^{+, i} + (1-\gamma) i \alpha_{ab}^{-i} A_{r+s}^{-, i} \right) .$$

...to the wedge algebra

Surviving wedge generators: L_0 , $L_{\pm 1}$, $G_{\pm \frac{1}{2}}^a$, $A_0^{\pm, i}$

$$[L_m, L_n] = (m - n) L_{m+n}$$

$$[L_m, G_r^a] = \left(\frac{m}{2} - r\right) G_{m+r}^a$$

$$[A_0^{\pm, i}, G_r^a] = i \alpha_{ab}^{\pm i} G_r^b$$

$$[A_0^{\pm, i}, A_0^{\pm, j}] = i \epsilon^{ijkl} A_0^{\pm, l}$$

$$\{G_r^a, G_s^b\} = 2\delta^{ab} L_{r+s}$$

$$+ 4(r - s) \left(\gamma i \alpha_{ab}^{+i} A_{r+s}^{+, i} + (1 - \gamma) i \alpha_{ab}^{-i} A_{r+s}^{-, i} \right) .$$

free parameter
related to

$$\gamma = \frac{\alpha}{1 + \alpha}$$

Isomorphic to exceptional super Lie algebra

$$D(2, 1 | \alpha)$$



$\mathcal{N} = 2$ version

Thus we need to find hs algebra that contains this exceptional superalgebra...

Can be constructed naturally, starting from $\mathcal{N} = 2$ version of hs theory:

[Prokushkin, Vasiliev, '98]

$$sB[\mu] = \frac{U(\mathfrak{osp}(1|2))}{\langle C^{\mathfrak{osp}} - \frac{1}{4}\mu(\mu - 1)\mathbf{1} \rangle}$$

$$= \mathfrak{shs}[\mu] \oplus \mathbb{C}$$

$\mathcal{N} = 2$ \uparrow
hs algebra



$\mathcal{N} = 4$ version

Introduce **Chan-Paton factors**:

$$\begin{aligned} sB_M[\mu] &= sB[\mu] \otimes M_M(\mathbb{C}) \\ &= \text{shs}_M[\mu] \oplus \mathbb{C} \end{aligned}$$

For $M=2$ the resulting Lie algebra contains

[MRG, Gopakumar, '13]

$$\text{shs}_2[\mu] \supset D(2, 1|\alpha) \quad \text{where} \quad \gamma = \frac{\alpha}{1 + \alpha} = \mu$$

(Different strategy from [Henneaux, Licena Gomez, Park, Rey, '12].)



Vasiliev theory

Since just **added Chan-Paton factors**, corresponding hs theory can be constructed by usual methods.

[Prokushkin, Vasiliev, '98]

cf. [Chang, Minwalla, Sharma, Yin, '12]

The **spin content of the asymptotic symmetry** algebra is then determined by the full spin content of

$$\text{shs}_2[\mu]$$



$\mathcal{N} = 4$ hs algebra

The hs algebra contains **8 spin fields for each spin $s > 1$** (as well as 7 $s=1$ fields); in terms of **representations of superalgebra**:

$$\text{shs}_2[\mu] = D(2, 1|\alpha) \oplus \bigoplus_{s=1}^{\infty} R^{(s)}$$

where

$$R^{(s)} : \begin{array}{ll} s : & (\mathbf{1}, \mathbf{1}) \\ s + \frac{1}{2} : & (\mathbf{2}, \mathbf{2}) \\ s + 1 : & (\mathbf{3}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{3}) \\ s + \frac{3}{2} : & (\mathbf{2}, \mathbf{2}) \\ s + 2 : & (\mathbf{1}, \mathbf{1}) . \end{array}$$

← reps w.r.t
 $\mathfrak{su}(2) \oplus \mathfrak{su}(2)$



't Hooft limit

The corresponding asymptotic symmetry algebra then **matches precisely the symmetries** of the $\mathcal{N} = 4$ **Wolf space cosets** in the large (N, k) limit, where we take

$$\lambda \equiv \frac{N + 1}{N + k + 2} = \gamma = \mu .$$

In fact this identification is fixed by requiring the **large $\mathcal{N} = 4$ algebra to match**, i.e. by identifying γ .



Higher spin theory

In addition to the spin fields, resulting hs theory has massive BPS representations

$$\phi_+ : (\mathbf{2}, \mathbf{1})_0 \oplus (\mathbf{1}, \mathbf{2})_{1/2}$$

2 complex scalars
with

$$M_+^2 = -1 + (1 - \mu)^2$$

2 Dirac fermions
with

$$m^2 = (\mu - 1/2)^2$$



Higher spin theory

In addition to the spin fields, resulting hs theory has
massive BPS representations

$$\phi_- : (\mathbf{1}, \mathbf{2})_0 \oplus (\mathbf{2}, \mathbf{1})_{1/2}$$

2 complex scalars
with

$$M_-^2 = -1 + \mu^2$$

2 Dirac fermions
with

$$m^2 = (\mu - 1/2)^2$$



't Hooft limit

They correspond in the coset CFT spectrum to the **chiral primaries** (and their conjugates)

$$\phi_+ \cong (\mathfrak{f}; 0) \quad \text{and} \quad \phi_- \cong (0; \bar{\mathfrak{f}}) .$$

In particular, **charges and masses match** precisely those of the hs theory, provided we identify again

$$\lambda \equiv \frac{N + 1}{N + k + 2} \cong \mu$$



Multiparticle states

The CFT also contains **BPS representations** (that appear in the symmetrised/anti-symmetrised fusion products of these minimal reps) with quantum numbers

$$l^+ \in \frac{1}{2}\mathbb{N}_0, \quad l^- \in \frac{1}{2}\mathbb{N}_0$$

These correspond to the **multi-particle states of the hs scalar fields**.

Convincing consistency check!

[CFT also has again light states --- but are expected to disappear in stringy generalisation....]



Stringy Generalisation

In analogy with [Chang, Minwalla, Sharma, Yin, '12]
natural to guess that

- ▶ non-abelian $2M \times 2M$ version of hs theory
- ▶ with $U(M)$ singlet condition

← binds Vasiliev particles in adj
rep of $U(M)$ into strings

may be **dual to suitable string theory** (at a very
symmetrical point in its moduli space).



Consistency Check

This idea passes **one non-trivial consistency check**:
BPS spectrum of resulting hs theory consists of
one (**single-particle**) state for each

$$l^+ \in \frac{1}{2}\mathbb{N}_0, \quad l^- \in \frac{1}{2}\mathbb{N}_0$$

This **matches precisely supergravity spectrum** of

$$\text{AdS}_3 \times S^3 \times S^3 \times S^1$$

[de Boer, Pasquinucci, Skenderis, '99]



Deformation

Furthermore, the (non-abelian) hs theory contains a **double trace BPS operator** with

$$h = \bar{h} = \frac{1}{2}$$

Susy descendant defines modulus that should **break hs symmetry** without destroying the large $\mathcal{N} = 4$ symmetry.

[Note that in string description should really have additional $u(1)$ generator corresponding to S^1 --- should be possible to add this to hs theory.]



Dual CFT

One may also hope that this new perspective point of view will lead to **important clues** as to how to define the **dual CFT of the string background** (which is so far unknown, e.g. symmetric product orbifolds do not even reproduce correct BPS spectrum).

[Gukov, Martinec, Moore, Strominger, '04]

The obvious ideas, however, do not yet seem to work...



Conclusions

- ▶ Explained interesting **large $\mathcal{N} = 4$ generalisation** of bosonic minimal model holography.
- ▶ Promising example to understand **connection between hs theory and string theory** in concrete setting.
- ▶ May shed light on structure of **dual CFT** for strings on **AdS3**.