Large $\mathcal{N} = 4$ Holography

Matthias Gaberdiel
ETH Zürich

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based mainly on

MRG, R. Gopakumar, arXiv:1011.2986, 1205.2472, **1305.4181**

Motivation

One way to get quantitative handle on AdS/CFT correspondence is to consider regime where gauge theory is weakly coupled:

$$\left(\frac{R}{l_{\mathrm{Pl}}}\right)^4 = N \qquad g_{\mathrm{string}} = g^2_{\mathrm{YM}} \qquad \left(\frac{R}{l_{\mathrm{s}}}\right)^4 = g^2_{\mathrm{YM}} N = \lambda$$

large

small

[Sundborg '01]
[Witten '01]
[Sezgin,Sundell '01]

$l_{\mathrm{s}} \to \infty$ \`tensionless strings'

Higher spin theory

In this tensionless limit, stringy excitations become massless: infinite number of massless higher spin fields, which generate a very large gauge symmetry.

maximally unbroken phase of string theory

Idea: try to understand AdS/CFT correspondence starting from this perturbative duality!

see also [Sagnotti et.al.], [Jevicki et.al.], [Douglas et.al.], ...

State of the Art

In the past this idea was taken as a general motivation to consider dualities relating

Vasiliev HS theory on AdS	$\longleftrightarrow$	weakly coupled gauge theory

However, recently interesting progress about how these dualities fit actually into stringy AdS/CFT correspondence has been made....

[Chang, Minwalla, Sharma, Yin, '12]
[MRG, Gopakumar, '13]

Higher spin CFT duality

First concrete proposal for HS - CFT duality

[Klebanov-Polyakov, ’02]
[Sezgin-Sundell, ’02]

higher spin theory on AdS4 $\longleftrightarrow$ 3d O(N) vector model in large N limit

Different versions: vector model fields bosons or fermions;
free or interacting fixed point.

More recently: generalisation to family of parity-violating theories.

[Giombi, Minwalla, Prakash, Trivedi, Wadia, ’11]
[Aharony, Gur-Ari, Yacoby, ’11]

Checks of the proposal

During the last few years impressive checks of the proposal have been performed, in particular

3-point functions of HS fields on AdS4

have been matched to

3-point functions of HS currents in O(N) model to leading order in 1/N.

[Giombi, Yin, ’09-’10]

Furthermore, the symmetries have been identified.

[Giombi, Prakash, Yin, ’11], [Giombi, Yin, ’11]
[Maldacena, Zhiboedov, ’11-’12]

3d proposal

More recently also lower dimensional version was found:

[MRG,Gopakumar, '10]

AdS3: higher spin theory with a complex scalar of mass M	$\longleftrightarrow$	**2d CFT:** $\mathcal{W}_{N,k}$ minimal models in large N 't Hooft limit with coupling λ

where $\lambda = \dfrac{N}{N+k}$ and $M^2 = -(1-\lambda^2)$

3d proposal

Because 3d HS theories and 2d CFTs are well understood, this duality can be tested in quite some detail: in particular, the quantum symmetries and the spectrum were shown to match precisely.

[Henneaux, Rey, '10] [Campoleoni et.al. '10]
[MRG, Hartman, '11] [MRG,Gopakumar, '13]

[MRG, Hartman, Gopakumar, Raju, '11]
[Castro, Gopakumar, Gutperle, Raeymaekers, '11]
[MRG,Gopakumar, '12]
[Perlmutter, Prochazka, Raeymaekers, '12]

In addition, correlation functions have also been compared successfully.

[Chang, Yin, '11] [Ammon, Kraus, Perlmutter, '11]
[Ahn, '11] [Moradi, '12] [Creutzig, Hikida, Ronne, '12]
[Hijano, Kraus, Perlmutter, '13]

BHs and Generalisations

Furthermore, black hole solutions have been constructed, and their properties matched to the dual CFT.

[Ammon, Gutperle, Kraus, Perlmutter, ’11-’13]
[MRG, Hartman, Jin, ’12] [Perez, Tempo, Troncoso, ’12]
[de Boer, Jottar, ’13] [Kraus, Ugajin, ’13] [Datta, David, ’13]
[Song, Compere, ’13] [MRG, Jin, Perlmutter, to appear]

Finally, various generalisations to other groups and/or supersymmetry have been considered.

[Ahn, ’11] [MRG, Vollenweider, ’11]
[Creutzig, Hikida, Ronne, ’12-’13] [Candu, MRG, ’12]
[Candu, MRG, Kelm, Vollenweider, ’12]
[Beccaria, Candu, MRG, Groher, ’13]

Large $\mathcal{N} = 4$ Supersymmetry

In the following I want to explain generalisation to the situation with large $\mathcal{N} = 4$ supersymmetry which is possibly related to string theory on

[MRG, Gopakumar, '13]

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

In order to understand this construction, need to review how symmetries of the two descriptions are related.

The HS theory on AdS3

The AdS3 HS theory can be described very simply.

Recall that pure gravity in AdS3: Chern-Simons theory based on

$$\mathfrak{sl}(2,\mathbb{R})$$

[Achucarro, Townsend, '86]
[Witten, '88]

Higher spin description: replace

[Prokushkin, Vasiliev, '98]

[one spin field for each spin $s = 2, 3, \ldots$]

$$\mathfrak{sl}(2,\mathbb{R}) \;\to\; \mathrm{hs}[\lambda] \cong \mathfrak{sl}(\lambda)$$

where $\mathrm{hs}[\lambda] \oplus \mathbb{C} \cong \dfrac{U(\mathfrak{sl}(2))}{\langle C_2 - \frac{1}{4}(\lambda^2 - 1)\mathbf{1}\rangle}$

[Bordemann et.al., '98]
[Bergshoeff et.al., '90]
[Pope, Romans, Shen, '90]

Higher spin algebra

Generators of $\mathrm{hs}[\lambda]$:

$$V_n^s \quad \text{with} \ |n| < s \ , \ \ s = 2, 3, \ldots$$

\`wedge algebra'

For these higher spin theories asymptotic symmetry algebra can be determined following Brown & Henneaux, leading to classical

$$\mathcal{W}_\infty[\lambda] \ \ \text{algebra}$$

[Henneaux & Rey, '10]
[Campoleoni et al, '10]
[MRG, Hartman, '11]

Asymptotic symmetry algebra

Asymptotic symmetry algebra extends hs algebra
\`beyond the wedge':

pure gravity: $L_0, L_{\pm 1} \;\longrightarrow\; L_n \;,\; n \in \mathbb{Z}$

$\mathfrak{sl}(2,\mathbb{R})$ (Virasoro)

higher spin: $\mathrm{hs}[\lambda] \;\longrightarrow\; \mathcal{W}_\infty[\lambda]$

[Figueroa-O'Farrill et.al., '92]

generated by V_n^s

$s = 2, \ldots, \infty, \;\; n \in \mathbb{Z}$

Large $\mathcal{N} = 4$ Supersymmetry

In the following I want to explain generalisation to the situation with large N=4 supersymmetry which is possibly related to string theory on

[MRG, Gopakumar, ’13]

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

Let us start from this most supersymmetric string background, and `reverse engineer’ the corresponding hs theory....

Towards String Theory

Dual CFT of string background is expected to have

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

Large $\mathcal{N} = 4$

$$\mathrm{Vir} \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2) \oplus \mathfrak{u}(1)$$

with 4 supercharges

[Boonstra, Peeters, Skenderis, ’98; Elitzur, Feinerman, Giveon, Tsabar, ’99;
de Boer, Pasquinucci, Skenderis, ’99; Gukov, Martinec, Moore, Strominger,’04;...]

The large $\mathcal{N} = 4$ algebra

$$
\begin{aligned}
[U_m, U_n] &= \tfrac{k^+ + k^-}{2}\, m\, \delta_{m,-n} \\
[A^{\pm,i}_m, Q^a_r] &= \mathrm{i}\, \alpha^{\pm\, i}_{ab}\, Q^b_{m+r} \\
\{Q^a_r, Q^b_s\} &= \tfrac{k^+ + k^-}{2}\; \delta^{ab}\, \delta_{r,-s} \\
[A^{\pm,i}_m, A^{\pm,j}_n] &= \tfrac{k^\pm}{2}\, m\, \delta^{ij}\, \delta_{m,-n} + \mathrm{i}\, \epsilon^{ijl}\, A^{\pm,l}_{m+n} \\
[U_m, G^a_r] &= m\, Q^a_{m+r} \\
[A^{\pm,i}_m, G^a_r] &= \mathrm{i}\, \alpha^{\pm\, i}_{ab}\, G^b_{m+r} \mp \tfrac{2k^\pm}{k^+ + k^-}\, m\, \alpha^{\pm\, i}_{ab}\, Q^b_{m+r} \\
\{Q^a_r, G^b_s\} &= 2\, \alpha^{+\, i}_{ab}\, A^{+,i}_{r+s} - 2\, \alpha^{-\, i}_{ab}\, A^{-,i}_{r+s} + \delta^{ab}\, U_{r+s} \\
\{G^a_r, G^b_s\} &= \tfrac{c}{3}\, \delta^{ab}\, (r^2 - \tfrac{1}{4})\delta_{r,-s} + 2\, \delta^{ab}\, L_{r+s} \\
&\qquad + 4\, (r-s)\, \Big(\gamma\, \mathrm{i}\, \alpha^{+\, i}_{ab}\, A^{+,i}_{r+s} + (1-\gamma)\, \mathrm{i}\, \alpha^{-\, i}_{ab}\, A^{-,i}_{r+s}\Big) \ .
\end{aligned}
$$

The large $\mathcal{N} = 4$ algebra

U(1) current

$$[U_m, U_n] = \frac{k^+ + k^-}{2}\, m\, \delta_{m,-n}$$

$$[A_m^{\pm,i}, Q_r^a] = \mathrm{i}\, \alpha_{ab}^{\pm\, i}\, Q_{m+r}^b$$

$$\{Q_r^a, Q_s^b\} = \frac{k^+ + k^-}{2}\, \delta^{ab}\, \delta_{r,-s}$$

$$[A_m^{\pm,i}, A_n^{\pm,j}] = \frac{k^\pm}{2}\, m\, \delta^{ij}\, \delta_{m,-n} + \mathrm{i}\, \epsilon^{ijl}\, A_{m+n}^{\pm,l}$$

$$[U_m, G_r^a] = m\, Q_{m+r}^a$$

$$[A_m^{\pm,i}, G_r^a] = \mathrm{i}\, \alpha_{ab}^{\pm\, i}\, G_{m+r}^b \mp \frac{2k^\pm}{k^+ + k^-}\, m\, \alpha_{ab}^{\pm\, i}\, Q_{m+r}^b$$

$$\{Q_r^a, G_s^b\} = 2\, \alpha_{ab}^{+\, i}\, A_{r+s}^{+,i} - 2\, \alpha_{ab}^{-\, i}\, A_{r+s}^{-,i} + \delta^{ab}\, U_{r+s}$$

$$\{G_r^a, G_s^b\} = \frac{c}{3}\, \delta^{ab}\, (r^2 - \tfrac{1}{4})\delta_{r,-s} + 2\, \delta^{ab}\, L_{r+s} + 4\,(r-s)\, \Big(\gamma\, \mathrm{i}\, \alpha_{ab}^{+\, i}\, A_{r+s}^{+,i} + (1-\gamma)\, \mathrm{i}\, \alpha_{ab}^{-\, i}\, A_{r+s}^{-,i}\Big)\ .$$

The large $\mathcal{N} = 4$ algebra

4 free fermions

$$[U_m, U_n] = \frac{k^+ + k^-}{2}\, m\, \delta_{m,-n}$$

$$[A_m^{\pm,i}, Q_r^a] = \mathrm{i}\, \alpha_{ab}^{\pm\, i}\, Q_{m+r}^b$$

$$\{Q_r^a, Q_s^b\} = \frac{k^+ + k^-}{2}\, \delta^{ab}\, \delta_{r,-s}$$

$$[A_m^{\pm,i}, A_n^{\pm,j}] = \frac{k^\pm}{2}\, m\, \delta^{ij}\, \delta_{m,-n} + \mathrm{i}\, \epsilon^{ijl}\, A_{m+n}^{\pm,l}$$

$$[U_m, G_r^a] = m\, Q_{m+r}^a$$

$$[A_m^{\pm,i}, G_r^a] = \mathrm{i}\, \alpha_{ab}^{\pm\, i}\, G_{m+r}^b \mp \frac{2k^\pm}{k^+ + k^-}\, m\, \alpha_{ab}^{\pm\, i}\, Q_{m+r}^b$$

$$\{Q_r^a, G_s^b\} = 2\, \alpha_{ab}^{+\, i}\, A_{r+s}^{+,i} - 2\, \alpha_{ab}^{-\, i}\, A_{r+s}^{-,i} + \delta^{ab}\, U_{r+s}$$

$$\{G_r^a, G_s^b\} = \frac{c}{3}\, \delta^{ab}\, (r^2 - \tfrac{1}{4})\delta_{r,-s} + 2\, \delta^{ab}\, L_{r+s} + 4\,(r-s)\, \Big(\gamma\, \mathrm{i}\, \alpha_{ab}^{+\, i}\, A_{r+s}^{+,i} + (1-\gamma)\, \mathrm{i}\, \alpha_{ab}^{-\, i}\, A_{r+s}^{-,i}\Big)\ .$$

The large $\mathcal{N} = 4$ algebra

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\{Q^a_r, Q^b_s\} &= \tfrac{k^+ + k^-}{2}\, \delta^{ab}\, \delta_{r,-s} \\
[A^{\pm,i}_m, A^{\pm,j}_n] &= \tfrac{k^\pm}{2}\, m\, \delta^{ij}\, \delta_{m,-n} + \mathrm{i}\, \epsilon^{ijl}\, A^{\pm,l}_{m+n} \qquad \longleftarrow \qquad 2\ \mathfrak{su}(2)'\mathrm{s} \\
[U_m, G^a_r] &= m\, Q^a_{m+r} \\
[A^{\pm,i}_m, G^a_r] &= \mathrm{i}\, \alpha^{\pm\, i}_{ab}\, G^b_{m+r} \mp \tfrac{2k^\pm}{k^+ + k^-}\, m\, \alpha^{\pm\, i}_{ab}\, Q^b_{m+r} \\
\{Q^a_r, G^b_s\} &= 2\, \alpha^{+\, i}_{ab}\, A^{+,i}_{r+s} - 2\, \alpha^{-\, i}_{ab}\, A^{-,i}_{r+s} + \delta^{ab}\, U_{r+s} \\
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[A_m^{\pm,i}, A_n^{\pm,j}] &= \tfrac{k^\pm}{2}\, m\, \delta^{ij}\, \delta_{m,-n} + \mathrm{i}\, \epsilon^{ijl}\, A_{m+n}^{\pm,l} \\
[U_m, \boxed{G_r^a}] &= m\, Q_{m+r}^a \\
[A_m^{\pm,i}, \boxed{G_r^a}] &= \mathrm{i}\, \alpha_{ab}^{\pm\, i}\, \boxed{G_{m+r}^b} \mp \tfrac{2k^\pm}{k^+ + k^-}\, m\, \alpha_{ab}^{\pm\, i}\, Q_{m+r}^b \\
\{Q_r^a, \boxed{G_s^b}\} &= 2\, \alpha_{ab}^{+\, i}\, A_{r+s}^{+,i} - 2\, \alpha_{ab}^{-\, i}\, A_{r+s}^{-,i} + \delta^{ab}\, U_{r+s} \\
\{\boxed{G_r^a}, \boxed{G_s^b}\} &= \tfrac{c}{3}\, \delta^{ab}\, (r^2 - \tfrac{1}{4})\delta_{r,-s} + 2\, \delta^{ab}\, L_{r+s} \\
&\quad + 4\, (r - s)\, \left(\gamma\, \mathrm{i}\, \alpha_{ab}^{+\, i}\, A_{r+s}^{+,i} + (1-\gamma)\, \mathrm{i}\, \alpha_{ab}^{-\, i}\, A_{r+s}^{-,i}\right)\ .
\end{aligned}
$$

4 supercharges

The large $\mathcal{N} = 4$ algebra

$$
\begin{aligned}
[U_m, U_n] &= \tfrac{k^+ + k^-}{2}\, m\, \delta_{m,-n} \\
[A_m^{\pm,i}, Q_r^a] &= \mathrm{i}\, \alpha_{ab}^{\pm\, i}\, Q_{m+r}^b \\
\{Q_r^a, Q_s^b\} &= \tfrac{k^+ + k^-}{2}\, \delta^{ab}\, \delta_{r,-s} \\
[A_m^{\pm,i}, A_n^{\pm,j}] &= \tfrac{k^\pm}{2}\, m\, \delta^{ij}\, \delta_{m,-n} + \mathrm{i}\, \epsilon^{ijl}\, A_{m+n}^{\pm,l} \\
[U_m, \boxed{G_r^a}] &= m\, Q_{m+r}^a \\
[A_m^{\pm,i}, \boxed{G_r^a}] &= \mathrm{i}\, \alpha_{ab}^{\pm\, i}\, \boxed{G_{m+r}^b} \mp \tfrac{2k^\pm}{k^+ + k^-}\, m\, \alpha_{ab}^{\pm\, i}\, Q_{m+r}^b \\
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&\quad + 4\,(r-s)\, \Big(\gamma\, \mathrm{i}\, \alpha_{ab}^{+\, i}\, A_{r+s}^{+,i} + (1-\gamma)\, \mathrm{i}\, \alpha_{ab}^{-\, i}\, A_{r+s}^{-,i}\Big)\ .
\end{aligned}
$$

4 supercharges

free parameter

Large $\mathcal{N} = 4$

Since there are two current algebras, the algebra is actually characterised by two parameters: in addition to central charge

$$c = \frac{6k^+k^-}{k^+ + k^-}$$

[Sevrin, Troost, Van Proeyen, Schoutens, Spindel, Theodoridis, '88-'90; Goddard, Schwimmer, '88]

have parameter

$$\gamma = \frac{k^-}{k^+ + k^-} \quad \text{or} \quad \alpha = \frac{\gamma}{1-\gamma} = \frac{k^-}{k^+}$$

ratio of the two S3s.

All other structure constants are determined in terms of these.

$\mathcal{N} = 4$ coset

One family of $\mathcal{N} = 4$ coset CFTs is known based on Wolf symmetric spaces

$$\frac{\mathfrak{su}(N+2)^{(1)}_{\kappa}}{\mathfrak{su}(N)^{(1)}_{\kappa}} \cong \frac{\mathfrak{su}(N+2)_k \oplus \mathfrak{so}(4N+4)_1}{\mathfrak{su}(N)_{k+2}}$$

[Sevrin, Troost, Van Proeyen, Schoutens, Spindel, Theodoridis, '88-'90]

They contain large $\mathcal{N} = 4$ algebra with

$$\begin{aligned} k^+ &= k+1 \\ k^- &= N+1 \end{aligned} \quad \Rightarrow \quad \boxed{\gamma = \frac{N+1}{N+k+2}}$$

't Hooft parameter

as well as certain higher spin currents.

Wedge algebra

Ansatz: these cosets are dual to $\mathcal{N} = 4$ hs theory in large (N,k) limit!

In order to reverse engineer this hs theory, determine the wedge algebra (=global subalgebra) of cosets: it is generated by the `wedge modes'

$$S_n \quad \text{with } |n| < h(S)$$

[Asymptotic symmetry algebra of CS theory based on this wedge algebra should then reproduce the full coset W-algebra.]

From the large $\mathcal{N} = 4$ .....

To begin with consider wedge algebra of the large $\mathcal{N} = 4$ algebra.

free fermions (h=1/2) do not contribute
u(1) current: only zero mode --- central

$$
\begin{aligned}
[U_m, U_n] &= \tfrac{k^+ + k^-}{2}\, m\, \delta_{m,-n} \\
[A_m^{\pm,i}, Q_r^a] &= \mathrm{i}\, \alpha_{ab}^{\pm\, i}\, Q_{m+r}^b \\
\{Q_r^a, Q_s^b\} &= \tfrac{k^+ + k^-}{2}\, \delta^{ab}\, \delta_{r,-s} \\
[A_m^{\pm,i}, A_n^{\pm,j}] &= \tfrac{k^\pm}{2}\, m\, \delta^{ij}\, \delta_{m,-n} + \mathrm{i}\, \epsilon^{ijl}\, A_{m+n}^{\pm,l} \\
[U_m, G_r^a] &= m\, Q_{m+r}^a \\
[A_m^{\pm,i}, G_r^a] &= \mathrm{i}\, \alpha_{ab}^{\pm\, i}\, G_{m+r}^b \mp \tfrac{2k^\pm}{k^+ + k^-}\, m\, \alpha_{ab}^{\pm\, i}\, Q_{m+r}^b \\
\{Q_r^a, G_s^b\} &= 2\, \alpha_{ab}^{+\, i}\, A_{r+s}^{+,i} - 2\, \alpha_{ab}^{-\, i}\, A_{r+s}^{-,i} + \delta^{ab}\, U_{r+s} \\
\{G_r^a, G_s^b\} &= \tfrac{c}{3}\, \delta^{ab}\, (r^2 - \tfrac{1}{4})\delta_{r,-s} + 2\, \delta^{ab}\, L_{r+s} \\
&\quad + 4\,(r-s)\, \Big(\gamma\, \mathrm{i}\, \alpha_{ab}^{+\, i}\, A_{r+s}^{+,i} + (1-\gamma)\, \mathrm{i}\, \alpha_{ab}^{-\, i}\, A_{r+s}^{-,i}\Big)\ .
\end{aligned}
$$

...to the wedge algebra

Surviving wedge generators: $L_0\ , L_{\pm 1}\ ,\quad G^a_{\pm\frac{1}{2}}\ ,\quad A_0^{\pm,i}$

$$
\begin{aligned}
[L_m, L_n] &= (m-n)\, L_{m+n} \\
[L_m, G^a_r] &= (\tfrac{m}{2} - r) G^a_{m+r} \\
[A_0^{\pm,i}, G^a_r] &= \mathrm{i}\, \alpha^{\pm\, i}_{ab} G^b_r \\
[A_0^{\pm,i}, A_0^{\pm,j}] &= \mathrm{i}\, \epsilon^{ijl} A_0^{\pm,l} \\
\{G^a_r, G^b_s\} &= 2\delta^{ab}\, L_{r+s} \\
&\quad + 4\,(r-s)\,\Big(\gamma\, \mathrm{i}\, \alpha^{+\,i}_{ab}\, A^{+,i}_{r+s} + (1-\gamma)\, \mathrm{i}\, \alpha^{-\,i}_{ab}\, A^{-,i}_{r+s}\Big)\ .
\end{aligned}
$$

free parameter
related to

$$\gamma = \frac{\alpha}{1+\alpha}$$

Isomorphic to exceptional super Lie algebra

$$D(2,1|\alpha)$$

$\mathcal{N} = 2$ version

Thus we need to find hs algebra that contains this exceptional superalgebra...

Can be constructed naturally, starting from $\mathcal{N} = 2$ version of hs theory:

[Prokushkin,Vasiliev, '98]

$$sB[\mu] = \frac{U(\mathfrak{osp}(1|2))}{\langle C^{\mathfrak{osp}} - \frac{1}{4}\mu(\mu-1)\mathbf{1}\rangle}$$

$$= \mathrm{shs}[\mu] \oplus \mathbb{C}$$

$\mathcal{N} = 2$ hs algebra

$\mathcal{N} = 4$ version

Introduce Chan-Paton factors:

$$sB_M[\mu] = sB[\mu] \otimes \mathrm{M}_M(\mathbb{C})$$
$$= \mathrm{shs}_M[\mu] \oplus \mathbb{C}$$

For M=2 the resulting Lie algebra contains

[MRG,Gopakumar, '13]

$$\mathrm{shs}_2[\mu] \supset D(2,1|\alpha) \quad \text{where} \quad \gamma = \frac{\alpha}{1+\alpha} = \mu$$

(Different strategy from [Henneaux, Licena Gomez, Park, Rey, '12].)

Vasiliev theory

Since just added Chan-Paton factors, corresponding hs theory can be constructed by usual methods.

[Prokushkin,Vasiliev, ’98]

cf. [Chang, Minwalla, Sharma, Yin, ’12]

The spin content of the asymptotic symmetry algebra is then determined by the full spin content of

$$\mathrm{shs}_2[\mu]$$

$\mathcal{N} = 4$ hs algebra

The hs algebra contains 8 spin fields for each spin s>1 (as well as 7 s=1 fields); in terms of representations of superalgebra:

$$\mathrm{shs}_2[\mu] = D(2,1|\alpha) \oplus \bigoplus_{s=1}^{\infty} R^{(s)}$$

where

$$R^{(s)}: \quad \begin{array}{ll} s: & (\mathbf{1},\mathbf{1}) \\ s+\frac{1}{2}: & (\mathbf{2},\mathbf{2}) \\ s+1: & (\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3}) \\ s+\frac{3}{2}: & (\mathbf{2},\mathbf{2}) \\ s+2: & (\mathbf{1},\mathbf{1})\ . \end{array}$$

reps w.r.t
$\mathfrak{su}(2) \oplus \mathfrak{su}(2)$

't Hooft limit

The corresponding asymptotic symmetry algebra then matches precisely the symmetries of the $\mathcal{N}=4$ Wolf space cosets in the large (N,k) limit, where we take

$$\lambda \equiv \frac{N+1}{N+k+2} = \gamma = \mu \ .$$

In fact this identification is fixed by requiring the large $\mathcal{N}=4$ algebra to match, i.e. by identifying γ .

Higher spin theory

In addition to the spin fields, resulting hs theory has massive BPS representations

$$\phi_+ : \;\; (\mathbf{2},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{2})_{1/2}$$

2 complex scalars
with
$$M_+^2 = -1 + (1-\mu)^2$$

2 Dirac fermions
with
$$m^2 = (\mu - 1/2)^2$$

Higher spin theory

In addition to the spin fields, resulting hs theory has massive BPS representations

$$\phi_- \; : \; (\mathbf{1},\mathbf{2})_0 \oplus (\mathbf{2},\mathbf{1})_{1/2}$$

2 complex scalars
with
$$M_-^2 = -1 + \mu^2$$

2 Dirac fermions
with
$$m^2 = (\mu - 1/2)^2$$

‘t Hooft limit

They correspond in the coset CFT spectrum to the chiral primaries (and their conjugates)

$$\phi_+ \cong (\mathrm{f};0) \quad \text{and} \quad \phi_- \cong (0;\bar{\mathrm{f}}) \ .$$

In particular, charges and masses match precisely those of the hs theory, provided we identify again

$$\lambda \equiv \frac{N+1}{N+k+2} \cong \mu$$

Multiparticle states

The CFT also contains BPS representations (that appear in the symmetrised/anti-symmetrised fusion products of these minimal reps) with quantum numbers

$$l^{+} \in \frac{1}{2}\mathbb{N}_0 \ , \qquad l^{-} \in \frac{1}{2}\mathbb{N}_0$$

These correspond to the multi-particle states of the hs scalar fields.

Convincing consistency check!

[CFT also has again light states --- but are expected to disappear in stringy generalisation....]

Stringy Generalisation

In analogy with [Chang, Minwalla, Sharma, Yin, ’12] natural to guess that

- non-abelian 2M x 2M version of hs theory
- with U(M) singlet condition

binds Vasiliev particles in adj rep of U(M) into strings

may be dual to suitable string theory (at a very symmetrical point in its moduli space).

Consistency Check

This idea passes **one non-trivial consistency check**:
BPS spectrum of resulting hs theory consists of
one (single-particle) state for each

$$l^+ \in \tfrac{1}{2}\mathbb{N}_0 \ , \qquad l^- \in \tfrac{1}{2}\mathbb{N}_0$$

This matches precisely supergravity spectrum of

$$\mathrm{AdS}_3 \times \mathrm{S}^3 \times \mathrm{S}^3 \times \mathrm{S}^1$$

[de Boer, Pasquinucci, Skenderis, ’99]

Deformation

Furthermore, the (non-abelian) hs theory contains a double trace BPS operator with

$$h = \bar{h} = \frac{1}{2}$$

Susy descendant defines modulus that should break hs symmetry without destroying the large $\mathcal{N} = 4$ symmetry.

[Note that in string description should really have additional u(1) generator corresponding to S1 --- should be possible to add this to hs theory.]

Dual CFT

One may also hope that this new hs point of view will lead to important clues as to how to define the dual CFT of the string background (which is so far unknown, e.g. symmetric product orbifolds do not even reproduce correct BPS spectrum).

[Gukov, Martinec, Moore, Strominger, '04]

The obvious ideas, however, do not yet seem to work...

Conclusions

- Explained interesting large $\mathcal{N} = 4$ generalisation of bosonic minimal model holography.
- Promising example to understand connection between hs theory and string theory in concrete setting.
- May shed light on structure of dual CFT for strings on AdS3.