

SOME THOUGHTS

ON THE WAVEFUNCTION

OF THE UNIVERSE

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STRINGS 2002
CAMBRIDGE 2002

STRING / M THEORY

IS A THEORY OF QUANTUM
GRAVITY

⇒ CONTAINS A WAVE FUNCTION

$|\Psi\rangle \equiv$ WAVEFUNCTION OF THE
UNIVERSE

WHAT DO WE KNOW ABOUT IT?

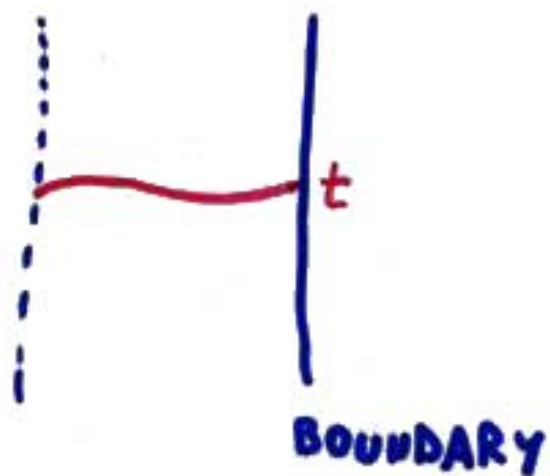
WAVEFUNCTION OF THE UNIVERSE IN AdS/CFT

SEMICLASSICAL GRAVITY
WHEELER deWITT

$$\Psi [g^s, \text{OTHER FIELDS}]$$

↑
SPATIAL METRIC

← WAVE FUNCTION OF QUANTUM GRAVITY



$$\hookrightarrow = |\Psi\rangle_{\text{CFT}}$$

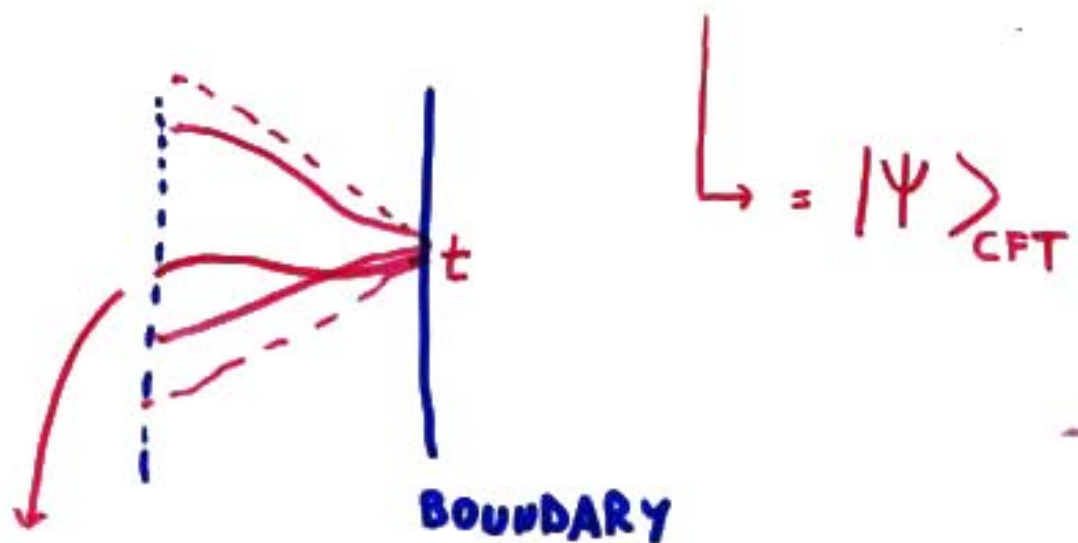
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$$\Psi[g^s, \text{OTHER FIELDS}]$$

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SPATIAL METRIC

← WAVE FUNCTION OF QUANTUM GRAVITY



. ALL THESE SPATIAL SECTIONS ARE CONTAINED IN $\Psi[g]$

. ALL ARE GAUGE EQUIVALENT

$$\mathcal{L}_{\text{DUAL}} \Psi[g, \dots] = 0$$

→ SEMICLASSICAL EXPANSION

$$\Psi \sim e^{\frac{i a}{G_N}} \tilde{\Psi}[\hat{g}, a]$$

↑
TIME..

. HILBERT SPACE \rightarrow CFT ONE

WHAT IS IT IF WE HAVE SEVERAL BOUNDARIES?

. ETERNAL BLACK HOLE

(Banco)



$$H = H_{\text{CFT}_1} \times H_{\text{CFT}_2}$$

$$|\Psi(t_1, t_2)\rangle_{\text{CFT}} = \Psi[g, t_1, t_2]$$

. WE WOULD LIKE TO UNDERSTAND HOW THE SEMICLASSICAL GRAVITY APPROXIMATION EMERGES FROM THE CFT

. Q.M. OF CLOSED SYSTEMS

. EMERGENCE OF TIME

⋮

ETC...

CURIOUS FEATURES

- . HILBERT SPACE \rightarrow ASSOCIATED TO THE BOUNDARY
- . DYNAMICAL EVOLUTION IN BOUNDARY TIME \rightarrow DEPENDS ON THE BOUNDARY CONDITIONS
 \rightarrow SET BY IR ASYMPTOTICS IN THE BULK
- . VERY SIMILAR TO CHERN-SIMONS THEORY
WITTEN
- . NEED IR ASYMPTOTICS IN THE BULK TO HAVE MATHEMATICALLY PRECISELY DEFINED QUANTITIES

WAVE FUNCTION IN EUCLIDEAN QUANTUM GRAVITY



GROSS, KLAR
POLYAKOV
WITTE

$$\Psi\left[\frac{1}{\epsilon}\hat{g}, \dots\right] = Z_{\text{CFT}}[\hat{g}, \epsilon] e^{-\frac{1}{\epsilon} S[\hat{g}]}$$

↑
PARTITION FUNCTION
OF THE REGULATED
FIELD THEORY

- OK FOR LENGTH SCALES
 $\ell \gg \epsilon$

• E.D.G. \rightarrow EXPONENTIAL TAILS OF THE
WAVE FUNCTION OF THE UNIVERSE

• WE KNOW A LOT ABOUT VERY
IMPROBABLE CONFIGURATIONS

• WHEELER de WIT \approx R.G. FLOW

de BOER
VERLINDEN

dS/CFT ??

STROMINGER
WITTMAN
...

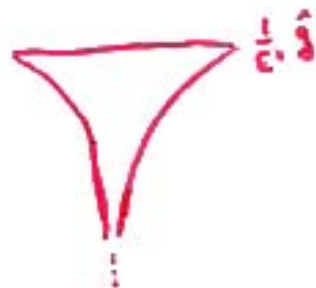
$$\Psi\left[\frac{1}{\epsilon}\hat{g}, \dots\right] = Z_{\text{CFT}}[\hat{g}, \epsilon]_{L_{\text{CUTOFF}}}$$

HOT BIG BANG

————— END OF INFLATION

APPROX. de SITTER

- - - - - BEGINNING



- WHEN WE LOOK AT PRIMORDIAL FLUCTUATION WE ARE MEASURING Ψ (AT THE END OF INFLATION)
- STANDARD INFLATIONARY THEORY TELLS US HOW TO COMPUTE $\Psi\left[\frac{1}{\epsilon}\hat{g}, \dots\right]$ SEMICLASSICALLY
- WE ARE META-OBSERVERS OF THE EARLY UNIVERSE de-SITTER PHASE

DAMBL...

$$\hat{g}_{ij} = \frac{1}{\epsilon^2} \left[\underbrace{(1+\gamma)}_{\text{SCALAR}} \delta_{ij} + \underbrace{h_{ij}^T}_{\text{TENSOR}} \right] \quad \text{FLUCTUATION}$$

$$\langle \gamma(k, t) \gamma(-k, t) \rangle = \frac{1}{\epsilon} \frac{H^2}{M^2} \frac{1}{k^3}$$

$$\langle h_{\mu\nu}^T(k, t) h_{\mu\nu}^T(-k, t) \rangle = \frac{H^2}{M^2} \frac{1}{k^3} \underbrace{\frac{1}{c}}$$

C ~ CENTRAL CHARGE OF "CFT" = ENTROPY OF dS SPACE

ϵ : SLOW ROLL PARAMETER

$$\epsilon = \left(\frac{M_{pl}}{V} \right)^2$$

$$\text{TRACE} \langle T_{\mu\nu} T_{\mu\nu} \rangle_{\text{CFT}} \sim \frac{\partial^2}{\partial \gamma \partial \gamma} \Psi[\gamma, k^T]$$

$$\text{TRACELESS} \langle T_{\mu\nu} T_{\mu\nu} \rangle_{\text{CFT}} \sim \text{SINGULAR}$$

$$\langle \gamma \gamma \rangle = \int d\gamma |\Psi|^2$$

• QUADRATIC APPROX. TO THE ACTION

→ GAUSSIAN WAVE FUNCTION

• LEADING ORDER NON GAUSSIAN FEATURES

- INTERACTIONS IN G.R.

↳ CORRECTION OF THE VACUUM
+ INTERACTIONS (ORDER H^2/M^2)

$$S = \int \psi \psi + \int \psi \psi \psi$$

FIND

$$\langle \psi \psi \psi \rangle \sim \left(\frac{H}{M}\right)^4$$

$$\frac{\langle \psi \psi \psi \rangle}{\langle \psi \psi \rangle^2} \sim \epsilon$$

• SHIBUICKI
• GAUSS et al.
• J.M. WITTE

TOO SMALL TO BE MEASURED IN
THE CMB $l < 2000$ DUE TO
COSMIC VARIANCE

KOMATSU - SPERGE

IS THE NUMBER OF STATES IN
de SITTER FINITE?

$$N_{\text{STATES}} = e^S$$

$$N_{\text{q-BITS}} \sim S$$

. CONSERVATIVE :

- NUMBER OF STATES AN OBSERVER CAN MEASURE
- S : ENTANGLEMENT ENTROPY BETWEEN THE HILBERT SPACE OF AN OBSERVER AND THE REST

. RADICAL

- N_{ST} IS THE DIMENSION OF THE HILBERT SPACE IN TERMS OF WHICH WE CAN EXPRESS ANY PHYSICAL MEASUREMENT IN dS.

BANKS
⋮

EARLY UNIVERSE

$$S \geq 10^{12} \quad (\epsilon \sim 0.1)$$

. COMPUTE THE NUMBER OF MODES OF δ RELEVANT FOR STRUCTURE FORMATION

$$N_{\text{BITS}} \sim N_{\text{MODES}} \gtrsim N_{\text{GALAXIES}} \geq 10^{12}$$

. THE STANDARD INFLATIONARY COMPUTATION REGARDS THEM AS INDEPENDENT

. HOW BIG IS THE META OBSERVER'S HILBERT SPACE?

. COSMIC VARIANCE IS A BIG LIMITATION
(IT IS HARD TO MEASURE EVEN THE SIMPLEST DEPARTURE FROM GAUSSIANITY)

. EVAPORATING BLACK HOLE \rightarrow WE ONLY MEASURE OF ORDER S QUANTA

ANALOGY BETWEEN dS & BLACK HOLES



ETERNAL
BH



dS



HERE ENTROPY
IS ENTANGLEMENT
ENTROPY



SEEMS SIMILAR HERE



- CONSERVATIVE : ENTANGLED
- RADICAL : SAME HILBERT SPACE

WORMHOLES?

J.M. & L. MAOZ



GIORGIO
STROMBERG
COSTA-BUFFONE
CE



2 BOUNDARIES
CONNECTED THROUGH
THE INTERIOR.

WHAT IS THE AdS/CFT INTERPRETATION
OF THESE?

• WITTEB YAU \rightarrow CAN'T BE DONE IF THE BOUNDARIES
HAVE $R > 0$.

• IN $AdS_3 \leftrightarrow CFT_2 \rightarrow$ NEGATIVE CURVATURE
RIEMANN SURFACES



$$ds^2 = dg^2 + \frac{1}{r^2} ds_{H^2}^2$$

$$\Psi[g_{S_1 + S_2}] \stackrel{?}{=} Z_{CFT}[g_1] Z_{CFT}[g_2]$$

IDEAS:

①

$$\Psi[g_{S_1} + g_{S_2}] = \sum_{\mathcal{R}} Z_{\text{CFT}, \mathcal{R}}[g_1] Z_{\text{CFT}, \mathcal{R}}[g_2]$$

↑
SECTORS OF THE CFT
?

②

$$\sum_{\text{ALL CONNECTING GEOMETRIES}} \text{[Diagram of a connected surface with two holes]} = \text{[Diagram of two separate surfaces, each with one hole]}$$

FACTORIZES

HOW DOES IT
HAPPEN?

③ OTHER

e.g. FOR SOME NOT SO OBVIOUS REASON THE
FIELD THEORY IS NOT WELL DEFINED...

de - SITTER

→ SIMILAR CONNECTION

WITTEN
BALASUBRAMANIAN
& DEBNER
NIVEC

• THE FAR FUTURE DOES NOT CARE ABOUT THE PAST ~~AND~~ ($l \ll R_{dS}$)

• STANDARD RULES OF PERTURBATION THEORY → NEED A DEFINITION OF THE VACUUM



LORENTZIAN

EUCLIDEAN

HARTLE
HAWKING

BUNCH-DAMES

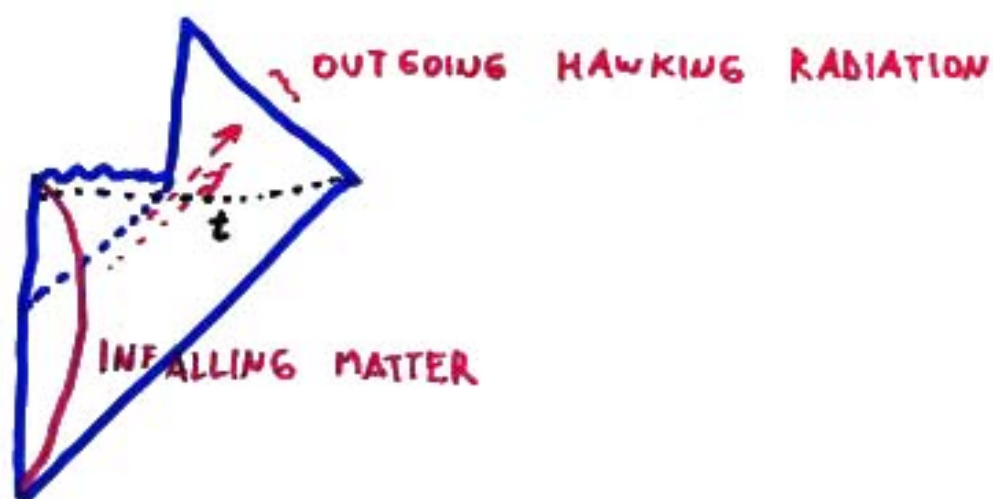
→ LEADS TO H-H WAVE FUNCTION

→ TIME:



BLACK HOLE SINGULARITIES

EVAPORATING BLACK HOLE



STANDARD PICTURE:

HAWKING

AT t

$$|\Psi\rangle \in \underbrace{H_{\text{FALLING}} \times H_{\text{IN}} \times H_{\text{OUT}}}_{H_{\text{INTERIOR}}}$$

• LIGHT FIELDS $\phi \rightarrow$ HAWKING QUANTA

$$|0\rangle \rightarrow \sum_{\epsilon} |E\rangle_{\text{IN}} |E\rangle_{\text{OUT}} = \text{ENTANGLED STATE IN } H_{\text{IN}} \times H_{\text{OUT}}$$

TRACE OVER $H_{\text{IN}} \rightarrow$ DENSITY MATRIX IN H_{OUT} (MICRO CANONICAL ENSEMBLE)

ONE IDEA IS THAT $H_{IN} = H_{OUT}$
(BLACK HOLE COMPLEMENTARITY)

1/2 HOOPY
SUSSEKIND
...

G. HORWITZ, J.M.

• ANOTHER IDEA

• STATE AT THE FUTURE OF $H_{INTERIOR}$
IS FIXED

HARTLE & GELMAN

• CALL IT $|BH\rangle \in H_{FALLING} \times H_{IN}$

• RULE IS THAT STATES EVOLVE
ACCORDING TO THE RULE

$$|\Psi_{FINAL}\rangle = \langle BH | |\Psi\rangle \in H_{OUT}$$

MORE EXPLICITLY

$$|\Psi\rangle = |i_0\rangle \times \sum_{\epsilon} \frac{1}{\sqrt{N}} |-\epsilon\rangle_{IN} \otimes |\epsilon\rangle_{OUT}$$

ENTANGLED STATE
IN THE FIELD THEORY

$$\langle BH | = \sum_{i, \epsilon} \sqrt{N} S_{i, \epsilon} \langle i | \langle \epsilon |$$

$$\langle BH | \Psi \rangle = \sum_E S_{iE} |E\rangle_{\text{OUT}} = |\Psi\rangle_{\text{FINAL}}$$

S_{iE} = UNITARY MATRIX WITH
GENERIC COEFFICIENTS DETERMINED
BY FULL QUANTUM GRAVITY.

. IF WE PERFORM A MEASUREMENT
OVER A SMALL NUMBER OF OUTGOING
QUANTA \rightarrow WE NEED TO AVERAGE OVER
A RANGE OF $|E\rangle$'s

$$\sum_{\text{RANGE OF } E\text{'s}} |S_{iE}|^2 \sim \text{CONSTANT}$$

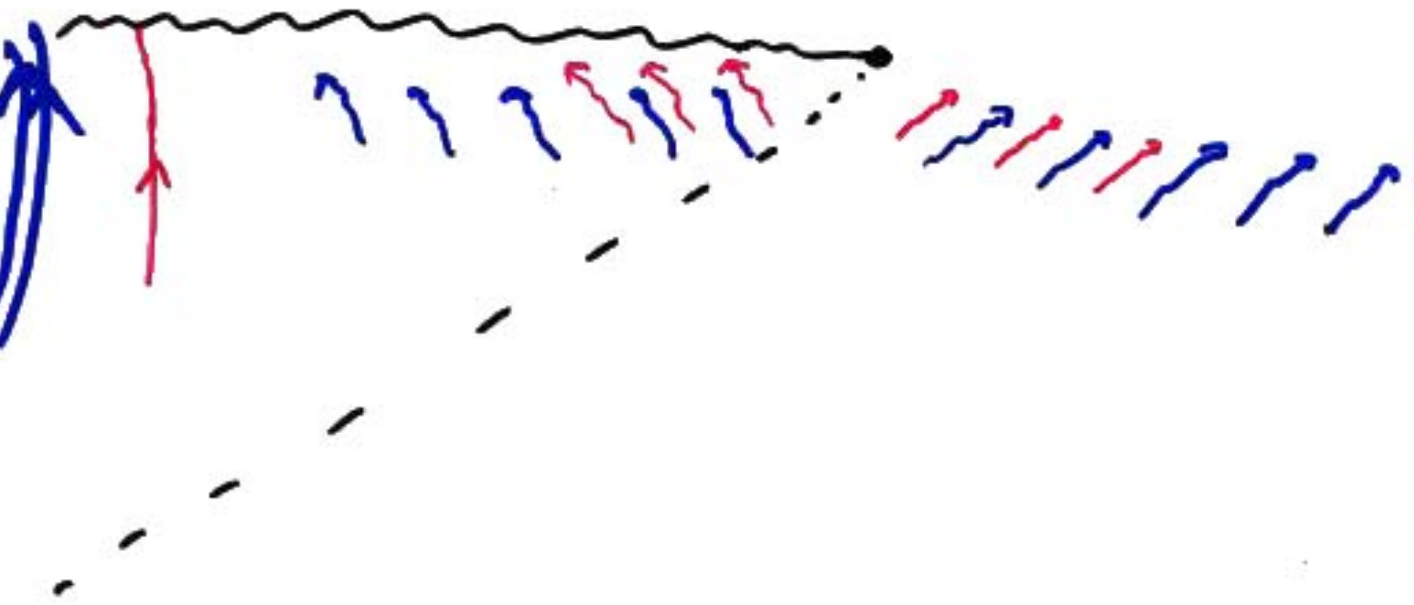
↑
THERMAL ANSWER

ARROW OF TIME?

2 ENERGY (LENGTH) SCALES

- . LOCALIZED INFALLING OBSERVERS
 $l \ll R_s$
- . HAWKING RADIATION
 $l \sim R_s$

. INFALLING OBSERVER \rightarrow DOES NOT MEASURE
HAWKING QUANTA \rightarrow ITS FUTURE IS NOT CONSTRAINED



1

FOCUS ON WHAT IS OBSERVABLE?

e.g. \rightarrow d.S OBSERVER \rightarrow FINITE NUMBER OF STATES

\rightarrow FINITE DIMENSIONAL HILBERT SPACE?

$$|\psi\rangle \in H \sim \mathbb{C}^N$$

\rightarrow CAN'T REPEAT THE MEASUREMENT

\rightarrow SHOULD WE THINK THAT THE
'WAVEFUNCTION' LIVES IN A DISCRETE
SPACE?

BY THE NEPTUNE STRING
CONFERENCE WE SHOULD KNOW
THE ANSWER TO THESE PROBLEMS !

ITP
SANTA BARBARA

WORKSHOP

ON

STRING

COSMOLOGY

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