Quantum critical transport and AdS/CFT

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hep-th/0701036
arxiv: 0704.1160
arxiv: 0706.3215
arxiv: 0706.3228

Strings-2007, Madrid 28.06.2007

APPLIED STRING THEORY / M-THEORY

- Using AdS/CFT correspondence as a tool to understand various properties of strongly coupled field theories

- Want to apply to real physical systems → focus on 3+1 or 2+1 dim.
  For simplicity → focus on relativistic field theories
  Example: $\mathcal{N}=4$ SYM theory in 3+1 dim. + deformations

However: there is no real physical system whose fundamental d.o.f. are described by $\mathcal{N}=4$ SYM or its cousins.

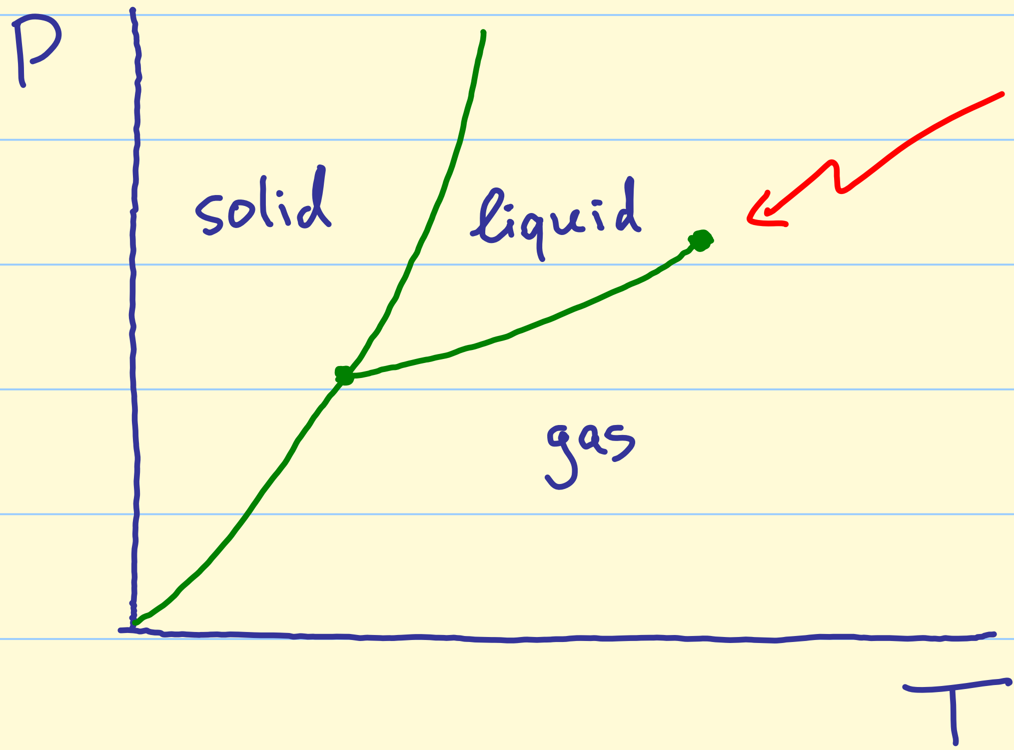
- Construct more realistic models (modified background geometry, probe branes, AdS/QCD ...)

- Look for universal properties of strongly coupled field theories
  Better chance at finite temperature

Analogy: $d=2$ classical Ising model does not describe real ferromagnets, just like $\mathcal{N}=4$ SYM does not describe real QCD. Still, the exact solution of the 2D Ising model proved to be very useful in stat. mech.

Universality is often associated with conformal symmetry.
Let's see where different CFTs show up...

$d=3$ Euclidean CFT



critical point, $\xi(T \to T_c) \to \infty$
3 dim. CFT

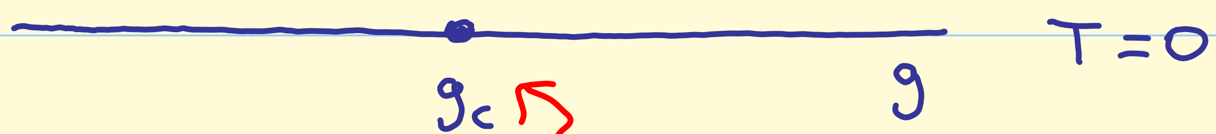
Water: $T_c = 374\,°C$

$P_c = 218$ atm.

Static correlations in the vicinity of liquid-gas critical point are described by a $d=3$ (Euclidean) CFT. For water and many other liquids — Ising universality class.

Lorenzian CFT

Associated with phase transitions driven by quantum (rather than thermal) fluctuations



critical point, $\xi(g \to g_c) \to \infty$
Lorenzian CFT

Lorenzian CFT

Associated with phase transitions driven by quantum (rather than thermal) fluctuations

Quantum critical region: $T \gtrsim 1/\xi(T=0)$

Lorenzian CFT at finite T

$T=0$

critical point, $\xi(g \to g_c) \to \infty$

Lorenzian CFT

Experiment: cold atoms on optical lattices

M. Greiner et al, Nature 415, 39 (2002)

I. Spielman, W. Phillips, J. Porto, PRL 98, 080404 (2007)

Engineer Bose Hubbard model at the phase transition —

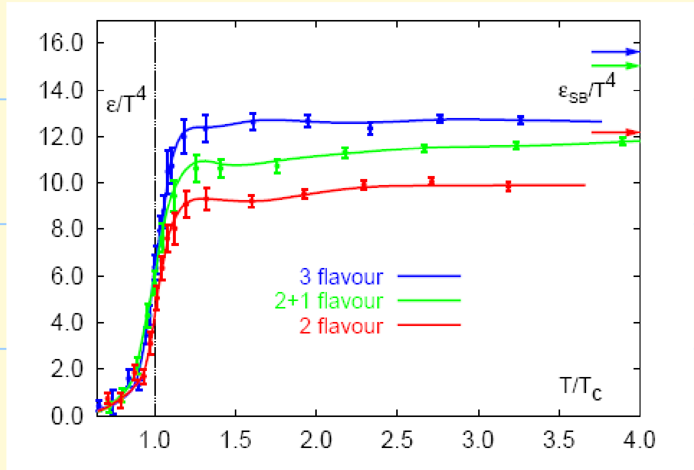
critical point is described by a strongly interacting 2+1 dim CFT

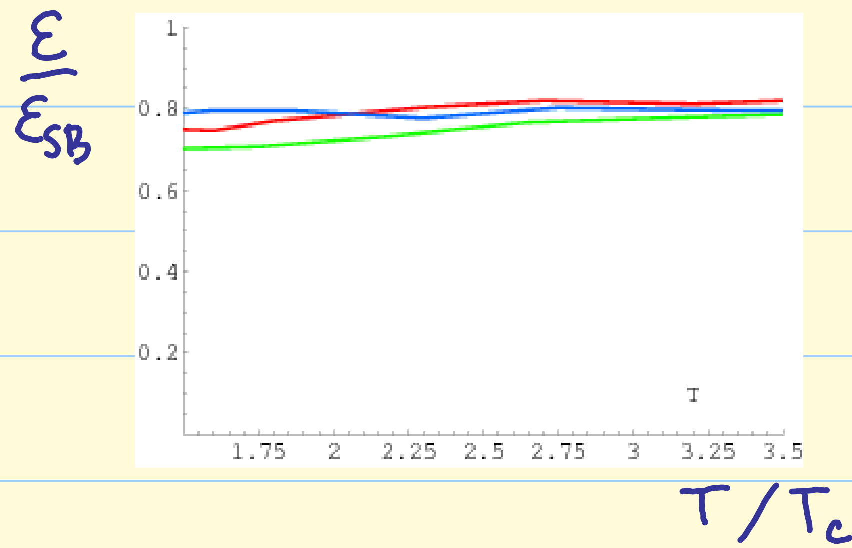
at finite temperature

See also KITP conference "Strongly correlated phases in degenerate atomic gases", April 2007

Are strongly interacting Lorentzian CFTs relevant for particle physics?

QCD thermodynamics:





(F. Karsch, hep-lat/0106019)

Are strongly interacting Lorenzian CFTs relevant for particle physics?

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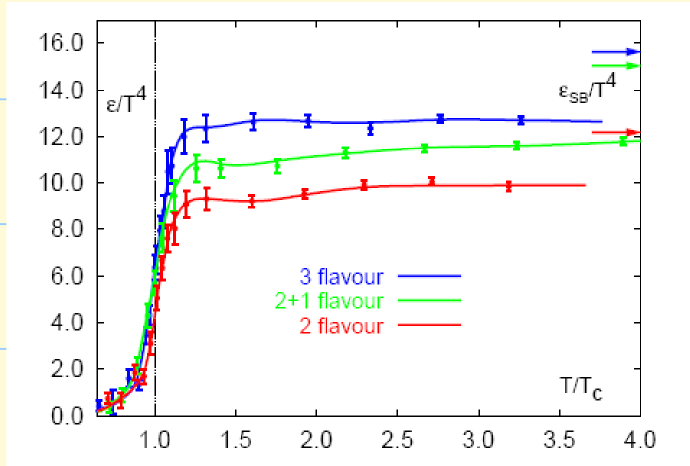
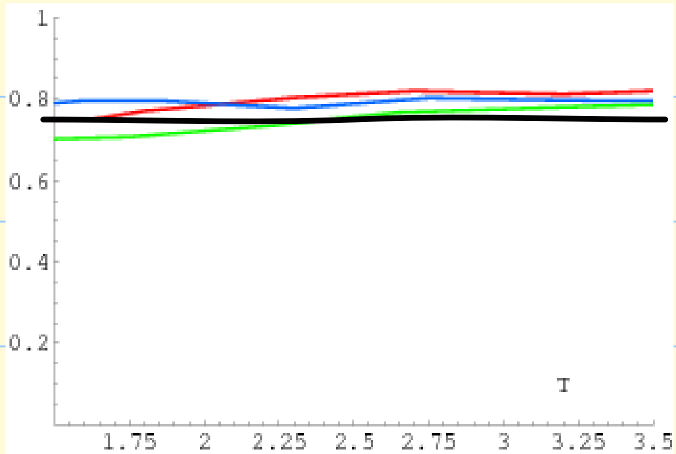


$\frac{\varepsilon}{\varepsilon_{SB}}$

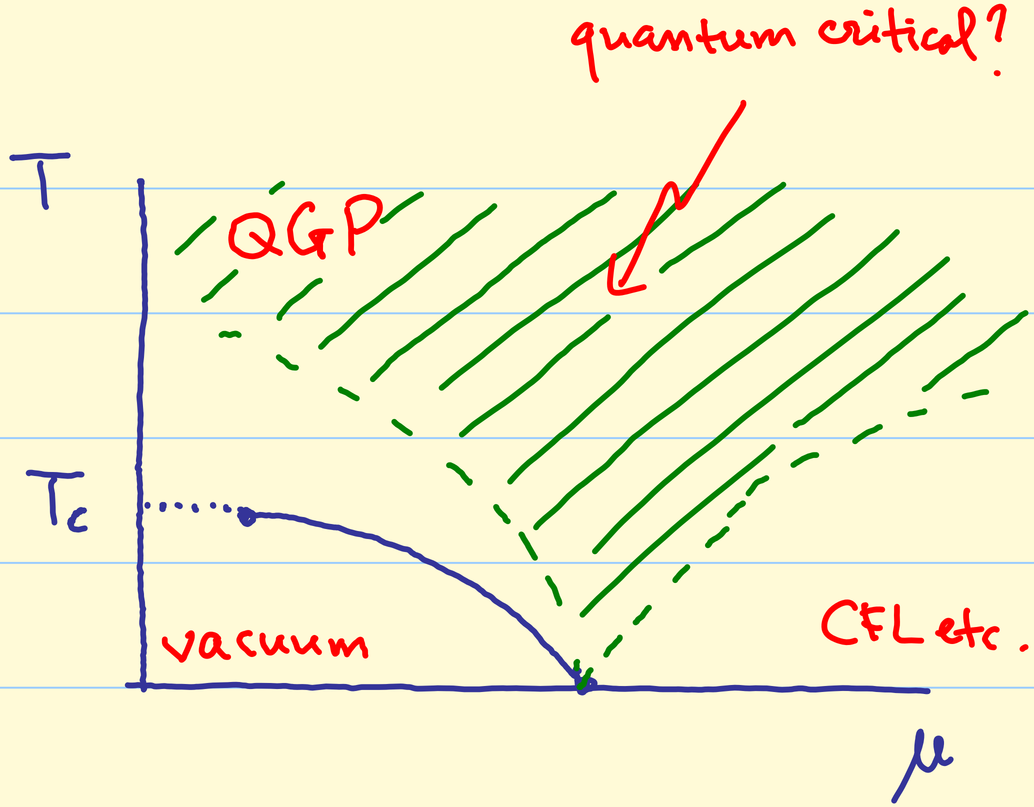


T/T_c

(F. Karsch, hep-lat/0106019)

∴ Thermodynamics of QCD is reasonably consistent with CFT at $T \sim 2-5\,T_c$.

$T_c \lesssim 200$ MeV $\Rightarrow$ strong coupling



Speculation:

Conformal symmetry in QCD arises because of the proximity to the quantum critical point at $T=0$.

See S. Sachdev's talk at the KITP cold atoms conference, April 2007

Phase diagram: Stephanov, hep-lat/0701002
Hands, physics/0105022

Bottomline:

- Strongly interacting CFTs in 2+1 dim and 3+1 dim at finite temperature are not so far removed from the real world

- It makes sense to look for universal properties of real-time response $\Longrightarrow$ study transport

- Momentum transport – controled by viscosities η, ζ

  Universality: $\frac{\eta}{s} = \frac{1}{4\pi}$ P.K., Son, Starinets, hep-th/0405231
  Buchel, hep-th/0408095

  See talk by A. Starinets at Strings 2005

- Charge transport – controled by conductivity σ
- Heat transport – controled by thermal conductivity κ

  However, σ and κ are <u>not</u> universal in AdS/CFT

Are there other universalities in real-time response besides $\frac{\eta}{s} = \frac{1}{4\pi}$?

Look at frequency-dependent conductivity in 2+1 dim

$$C_{\mu\nu}(\omega,\vec{k}) = P^T_{\mu\nu}\,\Pi^T(\omega,k^2) + P^L_{\mu\nu}\,\Pi^L(\omega,k^2)$$

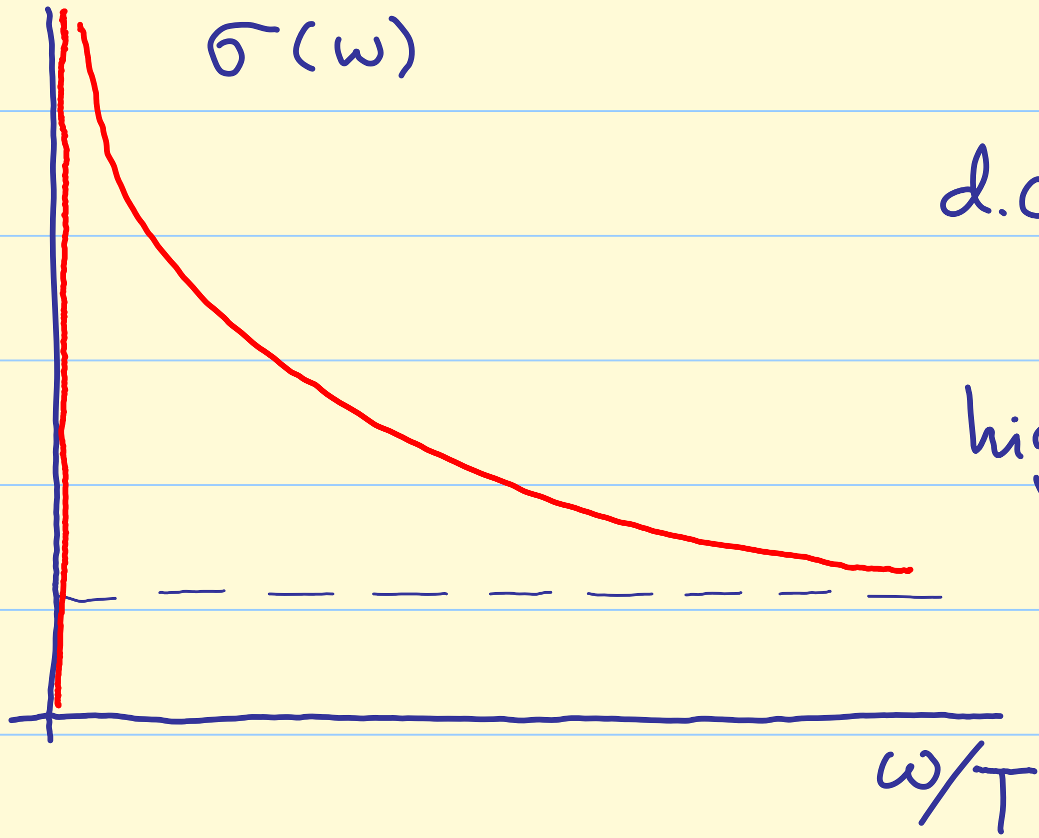
$$P^T_{ij} = \delta_{ij} - \frac{k_i k_j}{\vec{k}^2}\,, \quad P^L_{\mu\nu} = P_{\mu\nu} - P^T_{\mu\nu}$$ ← rotation invariance + current conservation

$$\Pi^T(\omega,k=0) = \Pi^L(\omega,k=0)$$ otherwise Π^T and Π^L are completely unrelated

Kubo formula:

$$\boxed{\sigma(\omega) = -\frac{1}{\omega}\,\mathrm{Im}\,\Pi^T\left(\frac{\omega}{T},\frac{k}{T}=0\right)} = -\frac{1}{\omega}\,\mathrm{Im}\,C_{xx}\left(\frac{\omega}{T},\frac{k}{T}=0\right)$$

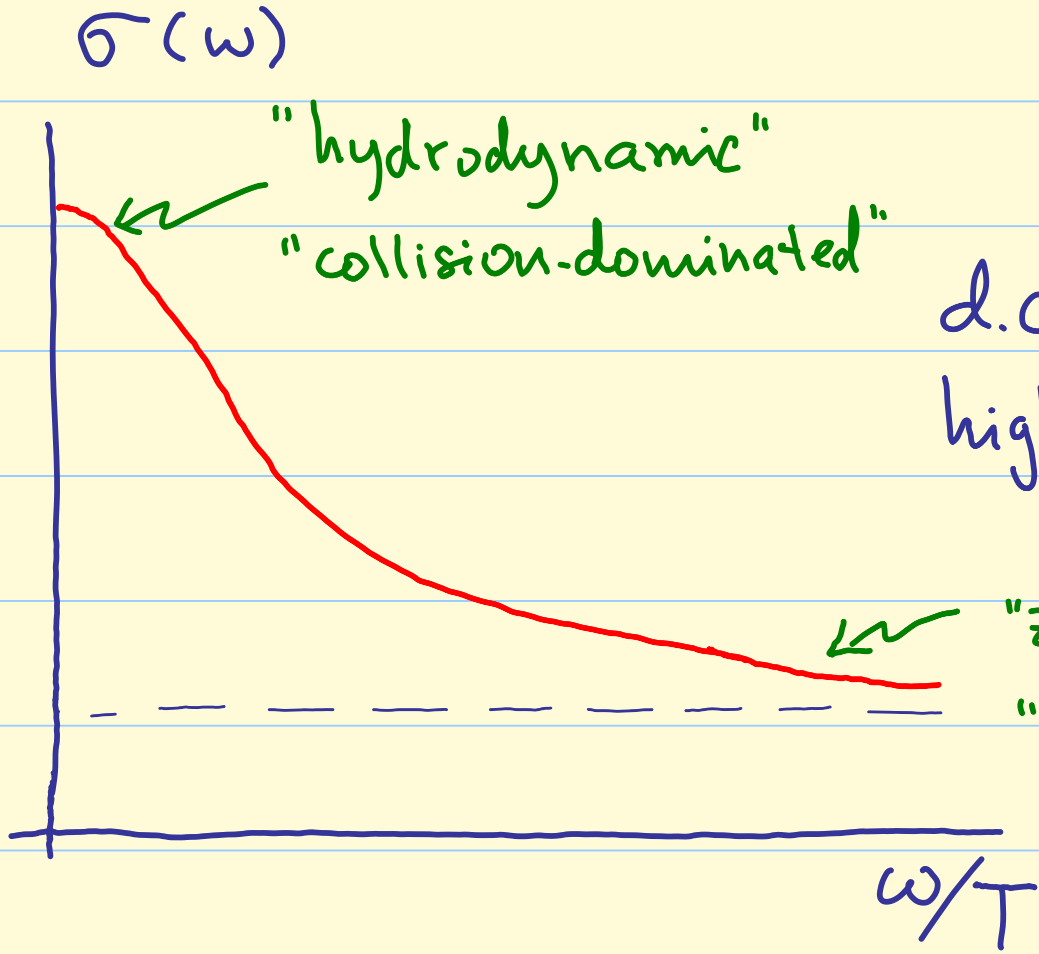
FREE THEORY



d.c. conductivity infinite

high-freq. conductivity constant

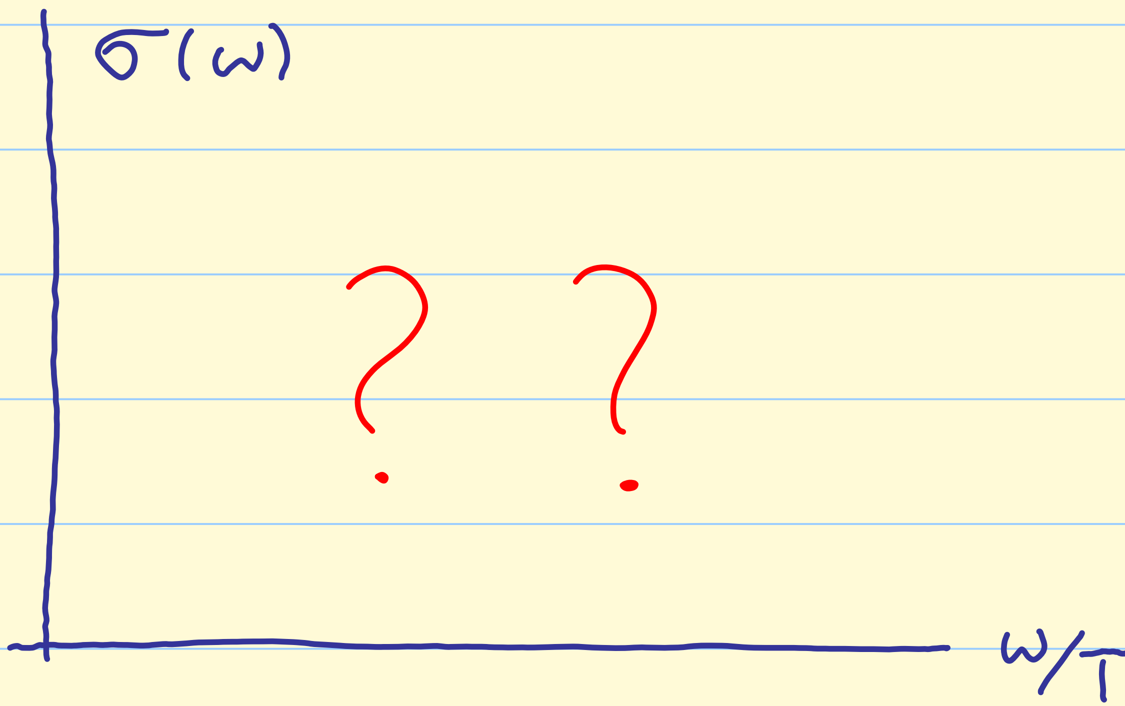
WEAKLY INTERACTING THEORY



d.c. conductivity large $\sim \ell_{mfp}$

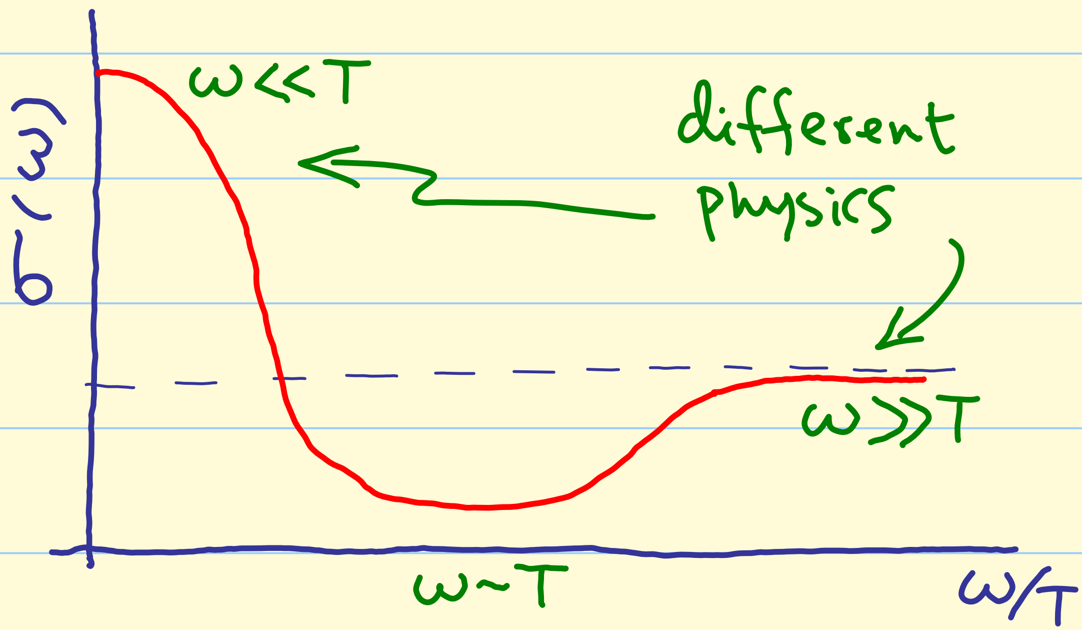
high-freq. conductivity constant

STRONGLY INTERACTING THEORY



AdS/CFT provides toy models where transport can be studied analytically at strong coupling

Note that d.c. limit $\omega \to 0$ does not commute with $T \to 0$



d.c. conductivity at finite T: $\sigma(\frac{\omega}{T} \to 0)$

d.c. conductivity at zero T: $\sigma(\frac{\omega}{T} \to \infty)$

From S.Sachdev's QPT book, p.188:

"The distinct physical interpretations of $\sigma(\frac{\omega}{T} \to \infty)$ and $\sigma(\frac{\omega}{T} \to 0)$ make it clear that, in general, there is no reason for them to have equal values (we cannot of course rule out the existence of exotic models or symmetries that may cause these two to be equal."

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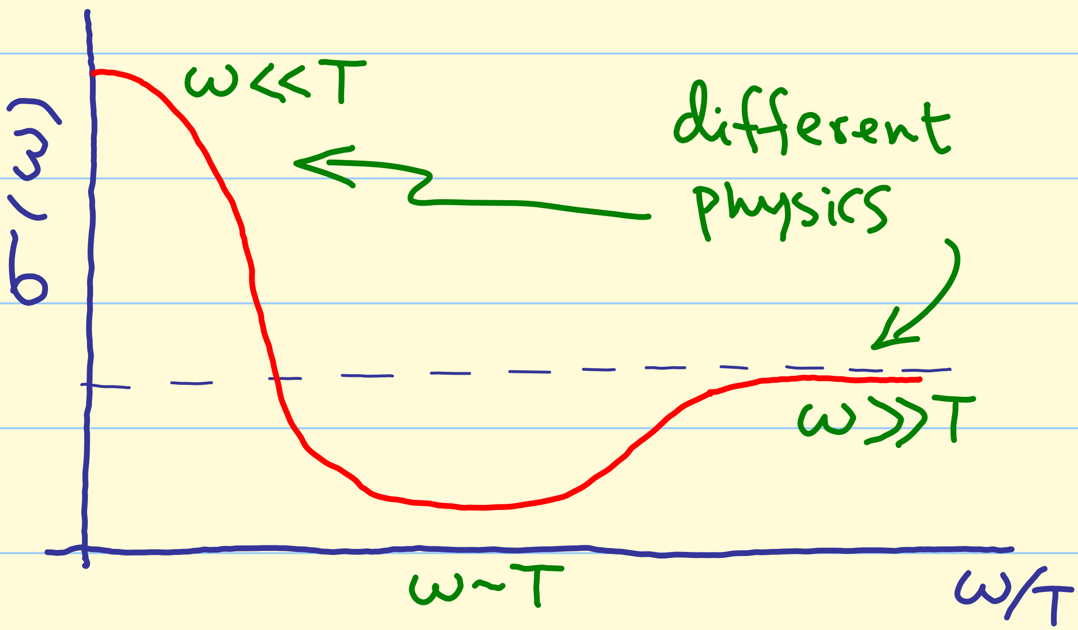

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Model: CFT on a stack of N M2 branes

Itzhaki, Maldacena, Sonnenschein, Yankielowicz, hep-th/9802042
Seiberg, hep-th/9705117

$\mathcal{N}=8$ SCFT in 2+1d, large N $\longleftrightarrow$ supergravity on $AdS_4 \times S^7$

thermal state in CFT $\longleftrightarrow$ black hole in AdS_4

SO(8) global symmetry $\longleftrightarrow$ isometry of S^7

conserved currents J^a_μ $\longleftrightarrow$ gauge fields A^a_μ in AdS_4

$$SO(8) \Rightarrow \quad \langle J^a_\mu J^b_\nu \rangle = C_{\mu\nu}\,\delta^{ab}$$

↑ suffices to look at abelian gauge fields in AdS_4

Action: $S = -\frac{1}{4g^2}\int d^4x\,\sqrt{-g}\,F^{\mu\nu}F_{\mu\nu}$, $\quad \frac{1}{g^2} \sim N^{3/2}$

Need to solve Maxwell equations on a background of AdS_4 + b.h.

- CFT currents live in 2+1 dim
- dual AdS_4 gauge fields live in 3+1 dim
- abelian gauge fields in 3+1 dim have electric-magnetic duality

See also Witten hep-th/0307041

EM duality in the 3+1 dim bulk implies

$$\boxed{\Pi^T(\omega,k)\,\Pi^L(\omega,k) = \frac{-1}{g^2}\left(\omega^2 - k^2\right)}$$

Herzog, PK, Sachdev, Son, hep-th/0701036

on the 2+1 dim boundary. Take $k \to 0$: $\sigma(\omega) = 1/g^2$ for all ω.

EM duality of the dual 3+1 dim theory implies ω-independent conductivity in 2+1 dim CFT at $T \neq 0$.

Comments on $\boxed{\Pi^T(\omega,k)\,\Pi^L(\omega,k) = \text{const}\,(\omega^2-\vec{k}^{\,2})}$, $T\neq 0$

- Very non-trivial relation at $T\neq 0$: a priory no reason to expect any connection between Π^T and Π^L

- Besides AdS/CFT, no other examples are known where $\Pi^T\Pi^L \sim p^2$

- Assuming AdS/CFT: there is a large class of exotic 2+1 dim CFTs where $\Pi^T\Pi^L\sim p^2$. These are field theories whose AdS dual contains $S = \frac{-1}{4g^2}\int d^4x\,\sqrt{-g}\,F^{\mu\nu}F_{\mu\nu}$

details of metric not important

The relation $\Pi^T(\omega,k)\,\Pi^L(\omega,k) = \mathrm{const}\,(\omega^2 - k^2)$ should be viewed as another example of universality in AdS/CFT

CHARGE TRANSPORT IN EXTERNAL MAGNETIC FIELD

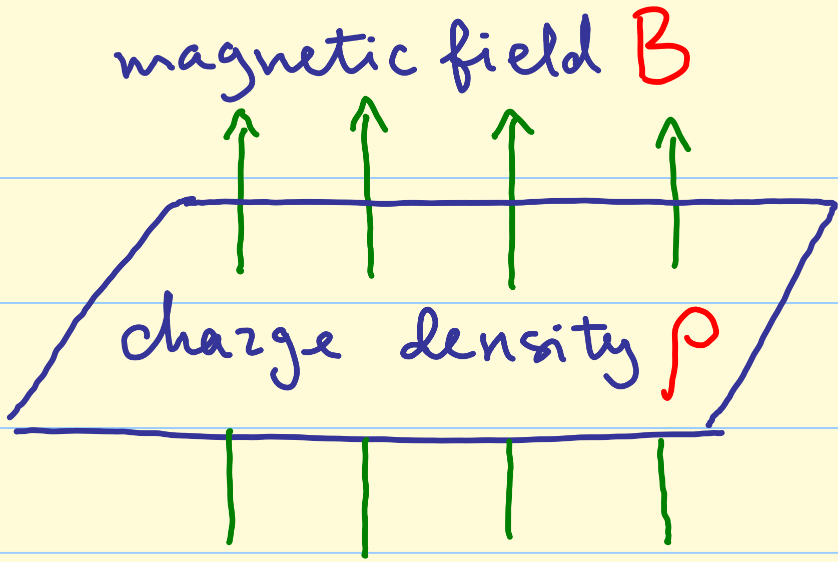
$$\boxed{\sigma_{ij}(\omega) = -\frac{1}{\omega}\,\mathrm{Im}\, C_{ij}(\omega, k=0)}$$

conductivity becomes a tensor, with $\sigma_{xy} \neq 0$: this is Hall effect

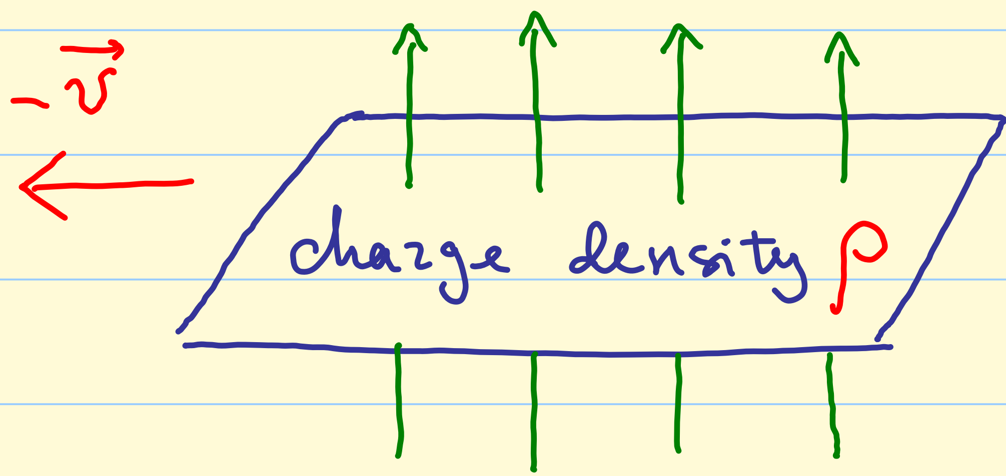
(What we computed earlier was $\sigma = \sigma_{xx} = \sigma_{yy}$)

In a relativistic theory, σ_{xy} is fixed by boost invariance

D. C. conductivity



$$\vec{j} = 0$$
$$\vec{B} = (0,0,B)$$
$$\vec{E} = 0$$



$$\vec{j} = \rho \vec{v}$$
$$B = (0,0,B)$$
$$\vec{E} = -\vec{v} \times \vec{B}$$
$$\Downarrow$$
$$\vec{E} = -\frac{B}{\rho}\vec{j} \times \hat{z}$$

$$\therefore \quad j_i = \sigma_{ij} E_j \,, \quad \boxed{\sigma_{xy} = \frac{\rho}{B} \,, \quad \sigma_{xx} = 0}$$

DOES AdS/CFT KNOW ABOUT THE HALL EFFECT?

- Need background charge density in CFT (= chemical potential) $\} \rightarrow$ electrically charged b.h. in AdS_4

- Need background magnetic field in CFT $\} \rightarrow$ magnetically charged b.h. in AdS_4

Need to solve Maxwell equations in 3+1 dim. bulk, with both electric and magnetic charges.

Linear perturbations of b.h. + Kubo formula $\Rightarrow$ $\sigma_{xy}(\omega=0) = \frac{\rho}{B}$

$\sigma_{xx}(\omega=0) = 0$

P.K., S.Hartnoll, arXiv: 0704.1160

Note that the d.c. limit $\omega \to 0$ does not commute with $B \to 0$.

(At $B=0$, we found $\sigma_{xx}(\omega\to 0) =$ finite, $\sigma_{xy}(\omega \to 0) = 0$)

Nernst effect (electric field induced by thermal gradient)

S. Hartnoll, P.K., M. Müller, S. Sachdev, arxiv: 0706.3215

S. Hartnoll, C. Herzog, arxiv: 0706.3228

Some challenges for applied AdS/CFT

- How close are AdS/CFT models to QCD quantitatively ?

- Fermions at finite density (get rid of scalars)

- Field theory in presense of disorder? Electric and magnetic charges are quantized ⇒ Quantum Hall ?

- So far – linear response only. Non-linear (convective) transport ?

- Flux compactifications ⇒ landscape of AdS_4 vacua ⇝ landscape of universality classes. Find critical Ising model ?

Conclusion

AdS/CFT has been arguably useful in application to strongly interacting QCD matter and RHIC collisions.

Can it be useful in application to condensed matter?

For example: if transport measurements at some quantum critical point find $\Pi^T(\omega, k)\,\Pi^L(\omega, k) = \text{const}\,(\omega^2 - k^2)$, then:

we have found M-theory in the lab!

the end!