# On wrapped membranes

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hep-th/0208xxx

## Matrix string theory

Dijkgraaf, Verlinde & Verlinde

Motl

Banks & Seiberg

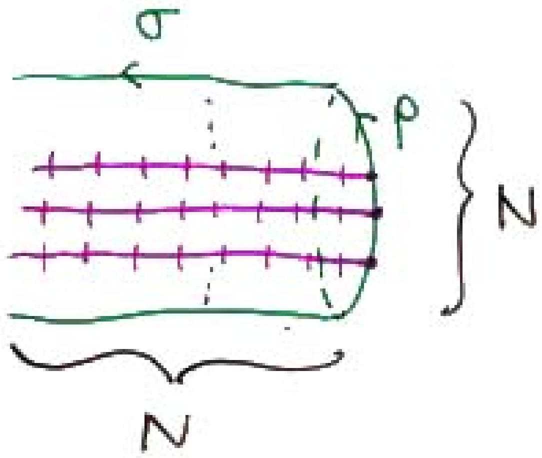
$$
\begin{tikzcd}
\mathrm{D0} \arrow[r, "R_9", "T_9"'] \arrow[d, leftrightarrow, "\text{9-11 flip}"'] & \mathrm{D1} \arrow[d, leftrightarrow, "\text{S duality}"] \\
\text{IIA string} & \text{IIB string} \arrow[l, "T_{11}"]
\end{tikzcd}
$$

wrapped membrane — Duff, Howe, Inami & Stelle

$$
S=\int d^2\sigma\, \mathrm{Tr}\left(\frac{1}{2}(D_\mu X^i)^2+\frac{R_9^3}{4}F_{\mu\nu}^2-\frac{1}{R_9^3}([X^i,X^j])^2 + \theta^T \not{D}\theta+\frac{1}{R_9^{\frac{3}{2}}}\theta^T\gamma_i[X^i,\theta]\right)
$$

$(M_p = 1)$

Sekino & Yoneya proposed a way to discretize wrapped membranes using matrices, which gave a direct relation between wrapped membranes & matrix strings




$\sim X_{kl}(\theta)$

$k, l = 1 \sim N$

I would like to

① show that Sekino-Yoneya regularization is realized by D-branes

② study the Lorentz generators for matrix string theory $(R_9 \neq 0)$

Plan of the talk

§1 Sekino-Yoneya regularization

§2 Rederivation

§3 Lorentz generators for matrix strings

# §1 Sekino-Yoneya Regularization

membrane action (light-cone gauge)

$$S=\int d\tau\int d\sigma\int_0^{2\pi}d\rho\left(\frac{1}{2}(D_0X^i)^2-\frac{1}{4}\{X^i,X^j\}^2+i\theta^T D_0\theta+i\theta^T\gamma_i\{X^i,\theta\}\right)$$

$$\begin{cases} i=1\sim 9 \\ D_0X=\partial_0X-\{A_0,X\} \\ \{A,B\}=\partial_\sigma A\partial_\rho B-\partial_\rho A\partial_\sigma B \end{cases}$$

wrap

$X^9=R_9(\rho+Y)$

$$S=\int d\tau d\sigma d\rho\left(\frac{1}{2}(D_0X^i)^2-\frac{R_9^2}{2}(D_\sigma X^i)^2+\frac{R_9^2}{2}(F_{0\sigma})^2-\frac{1}{4}\{X^i,X^j\}^2+\cdots\right)$$

$$\begin{cases} D_\sigma X=\partial_\sigma X-\{Y,X\} \\ F_{0\sigma}=\partial_0Y-\partial_\sigma A_0-\{A_0,Y\} \end{cases}$$

① discretize $\rho$

$$\rho=\frac{2\pi j}{N}\quad(j=0,1,\cdots,N-1)$$

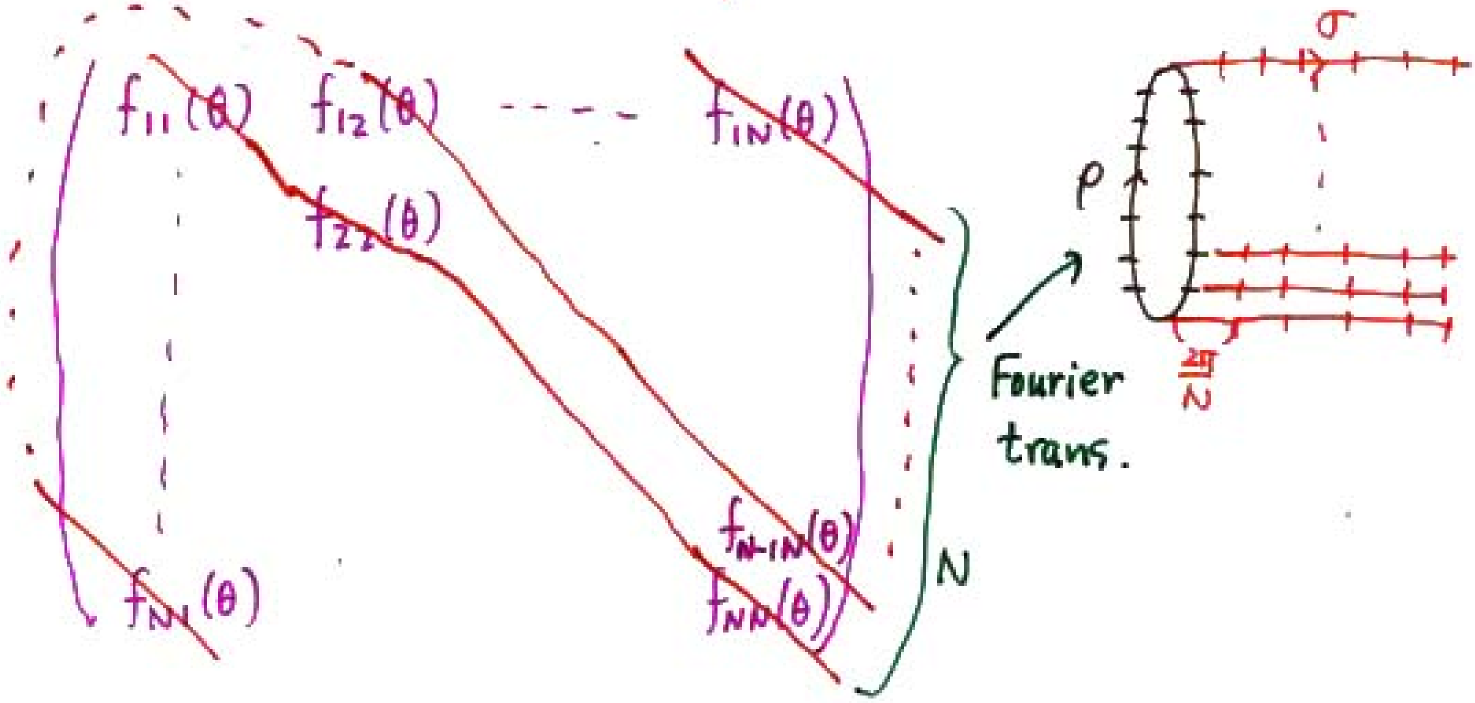
② For any function $f(\sigma,\rho)$

define $f_{kl}(\theta)$

by

$$\begin{cases} 0\le\theta\le 2\pi \\ f_{kl}(2\pi)=f_{k+1\,l+1}(0) \\ k,l=1,\cdots,N \end{cases}$$

$$f_{kl}(\theta)=\frac{1}{N}\sum_{\rho} e^{-i(k-l)\rho}\, f\!\left(\sigma=\frac{k+l-2}{N}\pi+\frac{\theta}{N},\ \rho\right)$$




| function | $\longrightarrow$ matrix |
|---|---|
| $\{f, g\}$ | $\frac{Ni}{2\pi}[f, g]$ |
| $\int d\sigma \frac{d\rho}{2\pi}$ | $\int d\theta \frac{1}{N}\mathrm{Tr}$ |
| $\partial_\sigma$ | $N\partial_\theta$ |

$$S \to \int d\tau\, d\sigma\, \frac{1}{N}\mathrm{Tr}\Big(\frac{1}{2}(D_0X^i)^2-\frac{N^2R_9^2}{2}(D_\theta X^i)^2+\frac{N^2R_9^2}{2}(F_{0\theta})^2 + \frac{N^2}{4}([X^i, X^j])^2+\cdots\Big)$$

After some changes of variables we obtain matrix string action

(but with $X_{kl}(\theta+2\pi)=X_{k+1\,l+1}(\theta)$)

Rem.

- Usually $R_9 \to 0 \Rightarrow$ diagonal elements survive

  $\to$ generalization of IIA string bits

- $X_{kl}(\theta + 2\pi) = X_{k+1\, l+1}(\theta)$

  $\to$ 1 membrane sector of the theory

- only $p$ is discretized

# §2 Rederivation

wrapped D2

$$\begin{tikzcd} D0 \arrow[r, "T_9"'] \arrow[d, leftrightarrow, "9\text{-}11"'] & D1 \arrow[d, leftrightarrow, "S"] \\ \mathrm{IIA} & \mathrm{IIB} \arrow[l, "T_{11}"] \end{tikzcd}$$

wrapped M2

$$\boxed{\begin{array}{ll} \text{① D2 from N D0} & \text{+② compactification} \\ \text{deWit, Hoppe \& Nicolai} & \text{W. Taylor} \\ \text{Townsend} & \\ \text{BFSS --} & \text{+③ field redefinition} \end{array}}$$

①

$$V = \begin{pmatrix} 1 & & & \\ & \omega & & \\ & & \ddots & \\ & & & \omega^{N-1} \end{pmatrix} \qquad U = \begin{pmatrix} 0 & \cdots & \cdots & 0 & 1 \\ 1 & \ddots & & & 0 \\ 0 & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 1 & 0 \end{pmatrix} \equiv e^{iP}$$

$$\omega = e^{\frac{2\pi i}{N}}$$

N D0 $\quad X^i = \sum x^i_{\vec{m}} J_{\vec{m}}$

$$J_{\vec{m}} = \omega^{-\frac{1}{2}m_1 m_2} V^{m_1} U^{m_2}$$

$$\updownarrow$$

$$\begin{pmatrix} m_1 : \text{even} \\ \sigma \sim \sigma + \pi \end{pmatrix}$$

D2 $\quad X^i(\sigma, P) = \sum x^i_{\vec{m}} e^{i(m_1\sigma + m_2 P)}$

for wrapped D2

$$X^9(\sigma, P) = R_9(P + Y) \longleftrightarrow X^9 = R_9(P + Y)$$

② Try

$$X^9 \longrightarrow \begin{pmatrix} \ddots & \vdots & \vdots & \\ \cdots & \boxed{\begin{matrix} R_9(P+Y) \\ +2\pi R_9 \end{matrix}} & \boxed{\phantom{R_9(P+Y)}} & \cdots \\ \cdots & \boxed{\phantom{R_9(P+Y)}} & \boxed{R_9(P+Y)} & \cdots \\ & \vdots & \vdots & \ddots \end{pmatrix} \qquad \begin{array}{cc} & N \\ N & \square \end{array}$$

$x$ : coordinate on the dual torus

$$x \sim x + \underbrace{2\pi R_9'}_{= \frac{1}{R_9}}$$

$$\begin{cases} X^9 \longrightarrow i\partial_x + \underbrace{A_x(x)}_{= R_9(P+Y(x))} \\ X^i \longrightarrow X^i(x) \end{cases}$$

$$\begin{cases} A_x(x+2\pi R_9') = A_x(x) \\ Y(x+2\pi R_9') = Y(x) \end{cases}$$

$\rightarrow$ matrix string theory with a background $A_x(x) = R_9(P+Y(x))$

③ Let us eliminate the background from the action by a field redef.

$$\begin{cases} \tilde{X}^i(x) = e^{-i\frac{x}{2\pi R_9'}P} X^i(x)\, e^{i\frac{x}{2\pi R_9'}P} \\ i\partial_x + \tilde{A}_x(x) = e^{-i\frac{x}{2\pi R_9'}P}\big(i\partial_x + \underbrace{A_x(x)}_{= R_9(P+Y(x))}\big)\, e^{i\frac{x}{2\pi R_9'}P} \end{cases}$$

This does not change the form of the action but changes the boundary cond.

$$\tilde{X}^i(x+2\pi R_9') = U^{-1}\tilde{X}^i(x)U$$

$$\downarrow$$

$$\tilde{X}^i_{kl}(\theta+2\pi) = \tilde{X}^i_{k+1\,l+1}(\theta)$$

$$\theta = \frac{x}{R_9'}$$

- Sekino - Yoneya rule

$$X^i = \sum x^i_{\vec{m}} J_{\vec{m}} \to \tilde{X}^i_{kl} = \sum x^i_{\vec{m}} \left(e^{-i\frac{\theta}{2\pi}P} J_{\vec{m}}\, e^{i\frac{\theta}{2\pi}P}\right)$$

$$= \sum x^i_{\vec{m}}\, \delta_{k-l,m_2} e^{im_1\left(\frac{k+l-2}{N}\pi + \frac{\theta}{N}\right)}$$

$$\updownarrow$$

$$X^i(\sigma,\rho) = \sum x^i_{\vec{m}} e^{im_1\sigma + im_2\rho}$$

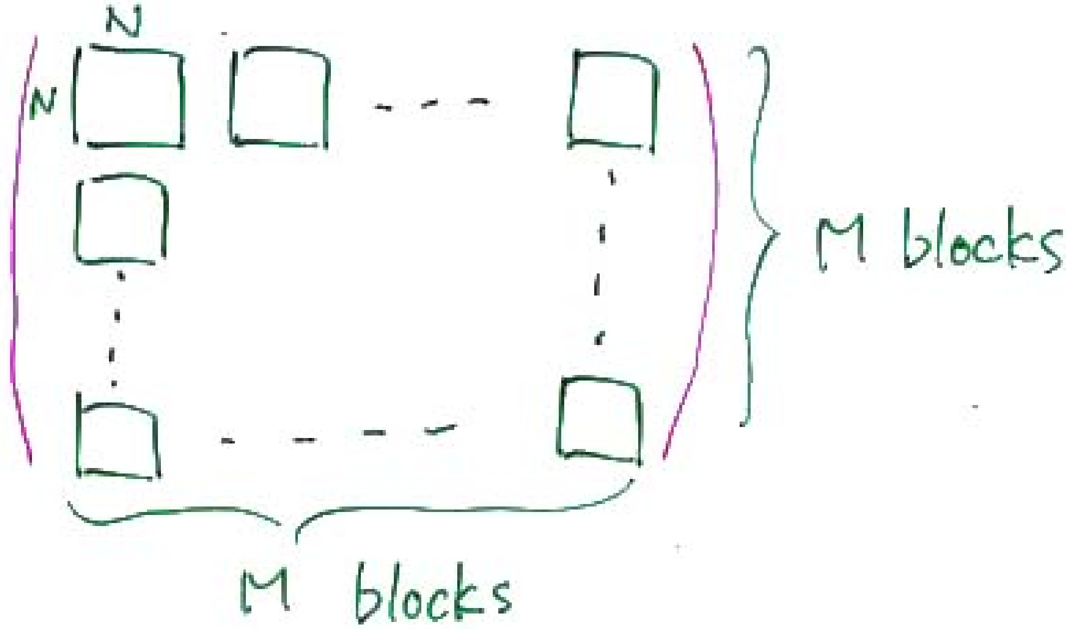
$$\therefore\ \tilde{X}^i_{kl}(\theta) = \frac{1}{N}\sum_{\rho} e^{-i(k-l)\rho} X^i\left(\sigma = \frac{k+l-2}{N}\pi + \frac{\theta}{N}, \rho\right)$$

$\sigma$ is discretized in this set-up

$$\Delta\sigma \sim \frac{2\pi}{N}$$

# More general set-up

$U, V$ : $MN \times MN$ matrices




$\sim$ M copies of N D0 branes

$M \to \infty$ represent $X^9 \sim X^9 + 2\pi R_9$

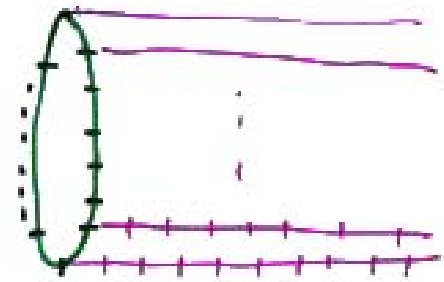
Things become more complicated but one can get the Sekino-Yoneya rule essentially in the same way.

$$\begin{cases} \Delta\sigma \sim \frac{2\pi}{MN} \xrightarrow[M\to\infty]{} 0 \\ \Delta p \sim \frac{2\pi}{N} \end{cases}$$

# §3 Lorentz generators for matrix strings

matrix string theory with $\sim$

$X_{k\ell}(\theta+2\pi) = X_{k+1\,\ell+1}(\theta)$



N strings interacting

(with each other

$g^2 \propto \frac{1}{R_9^3}$

$$S_{membrane} = \int d\tau\, d\sigma\, d\rho \left( \frac{1}{2}(D_0 X^i)^2 - \frac{1}{2}(D_\sigma X^i)^2 + \frac{R_9^3}{4} F_{\mu\nu}^2 - \frac{1}{4R_9^3}(\{X^i, X^j\})^2 + \cdots \right)$$

$$X^9 = R_9(\rho + Y)$$

$$\left.\begin{array}{l} \text{fix APD by } Y=0 \\ \text{solve Gauss law constr.} \end{array}\right\} \longrightarrow \left.\begin{array}{l} \text{action} \\ \text{constraints} \end{array}\right\} \text{coincides with usual strings}$$

$\downarrow$ matrix

$$\left.\begin{array}{l} \text{fix } U(N) \text{ by } A_0 = 0 \\ \text{solve Gauss law constr.} \end{array}\right\}$$

$$M^{i-} = \underbrace{M^{i-(0)}}_{= \sum_{\alpha=1}^{N} M_\alpha^{i-}} + \frac{1}{R_9^3} M^{i-(1)} + \cdots$$

$M_\alpha^{i-}$: $M^{i-}$ for each string

etc.

$M^{i-(0)}$ etc. are guaranteeed to form super Poincaré alg.

useful for M theory with $R_9 \gg 1$