AdS Description of Induced Higher Spin Gauge Theory

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Based on work with S. Giombi, S. Pufu, B. Safdi and G. Tarnopolsky

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Induced Gauge Theory

- A classic example: 3-dimensional QED coupled to N_f massless Dirac fermions and N_b complex scalars. The current 2-point function

$$\langle J^\mu(r) J^\nu(0)\rangle^{\mathbb{R}^3}_{\text{free}} = \frac{N_f + N_b}{8\pi^2} \frac{|r|^2 \delta^{\mu\nu} - 2r^\mu r^\nu}{|r|^6}$$

- Its Fourier transform is the induced parity invariant non-local kinetic operator for the gauge field A_μ (p): Appelquist, Pisarsky

$$K_{\mu\nu}(p) = \frac{N_f + N_b}{16} |p| \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{|p|^2}\right)$$

- The induced term allows us to develop large N expansion for complex scalars and/or fermions. The theory is conformal for $N > N_{crit}$.
- For 3-d CFTs an interesting quantity is the 3-sphere free energy F=- log Z
- Satisfies the F-theorem: for RG flow from UV to IR fixed point $F_{UV} > F_{IR}$
- In QED_3 the leading correction due to the gauge dynamics is IK, Pufu, Sachdev, Safdi

$$F_{\mathrm{QED}} - F_{\mathrm{free}} = \frac{1}{2}\log N + O(N^0)$$

- To understand this result in the dual AdS_4, need to modify boundary conditions for the massless spin 1 field: instead of the standard boundary condition that the magnetic field vanishes, impose the vanishing of electric field. This gauges the U(1) symmetry in the 3-d field theory. Witten; Chang, Minwalla, Sharma, Yin; Giombi, Yin;…
- Similar idea can be applied to spin 2 (induced conformal gravity) Leigh, Petkou; Compere, Marolf;…

$$Z_{\text{3-d gravity}} = \int \frac{[Dg_{\mu\nu}][D\phi^i]}{\text{Vol(Diff)Vol(Weyl)}} e^{-S}$$

- The gravitational dynamics is induced through coupling, e.g., to N conformal scalar fields

$$S = \int d^3x \sqrt{g} \left(g^{\mu\nu} \partial_\mu \phi^i \partial_\nu \phi^i + \frac{1}{8} R (\phi^i)^2 \right)$$

- Generalizations to induced conformal higher-spin theories. Leigh, Petkou; Metsaev; Vasiliev;…
- Such 3-d theories are obtained through modified spin s boundary conditions in Vasiliev's massless higher-spin theory in AdS_4.
- Analogous to the s=0 cases: singlet sector of the critical O(N) model IK, Polyakov or UV fixed point of the Gross-Neveu model Sezgin, Sundell; Leigh, Petkou

Operator Dimensions and Boundary Conditions in AdS/CFT

- A scalar operator $\mathcal{O}(x^\mu)$ in d-dimensional CFT is dual to a field $\Phi(z, x^\mu)$ in AdS_{d+1} which behaves near the boundary as z^Δ
- There are two choices

$$\Delta_\pm = \frac{d}{2} \pm \sqrt{\left(\frac{d}{2}\right)^2 + m^2}$$

- If we insist on unitarity, then Δ_- is allowed only in the Breitenlohner-Freedman range

IK, Witten

$$-(d/2)^2 < m^2 < -(d/2)^2 + 1$$

- Flow from a large N CFT where O has dimension Δ_- to another CFT with dimension Δ_+ by adding a double-trace operator. Witten
- The change in the free energy of the theory on S^d may be calculated using the Hubbard-Stratonovich transformation Gubser, IK
- In d=3 Diaz, Dorn; IK, Pufu, Safdi

$$F_{\rm UV} - F_{\rm IR} = \frac{\pi}{6}\int_{3/2}^{\Delta} (x-\frac{3}{2})(x-1)(x-2)\cot(\pi x) + O(1/N)$$

- For flow from free to critical O(N) model, or from interacting to free Gross-Neveu model, Δ_+=2 and

$$F_{\rm UV} - F_{\rm IR} = \frac{\zeta(3)}{8\pi^2} + O(1/N)$$

Higher Spin

- Can consider double-trace perturbations

$$S = S_0 + \frac{\lambda_0}{2} \int d^3x \sqrt{g} J_{\mu_1\mu_2\ldots\mu_s}(x) J^{\mu_1\mu_2\ldots\mu_s}(x)$$

- J is a spin-s single trace operator of dimension Δ. Unitarity bound: $\Delta^{(s)} \geq s+1$
- Generalized Hubbard-Stratonovich transformation

$$S = S_0 - \int d^3x \sqrt{g(x)} \left[h_{\mu_1\ldots\mu_s}(x) J^{\mu_1\ldots\mu_s}(x) + \frac{1}{2\lambda_0} h_{\mu_1\ldots\mu_s} h^{\mu_1\ldots\mu_s} \right]$$

Induced Effective Action

$$Z = Z_0 \int Dh_{\mu_1 \ldots \mu_s} e^{-S_{\rm eff}[h_{\mu_1 \ldots \mu_s}]}$$

$$S_{\rm eff} = -\frac{1}{2} \int d^3x \, d^3y \, \sqrt{g(x)} \sqrt{g(y)} \, h_{\mu_1 \ldots \mu_s}(x) h_{\nu_1 \ldots \nu_s}(y) \langle J^{\mu_1 \ldots \mu_s}(x) J^{\nu_1 \ldots \nu_s}(y) \rangle_0^{\rm conn} + \ldots \, .$$

For large N, find a UV fixed point where

$$\Delta_- = 3 - \Delta + O(1/N)$$

Change in the 3-sphere free energy:

$$\delta F_\Delta^{(s)} = -\log \left| \frac{Z}{Z_0} \right| = \frac{1}{2} \, {\rm tr} \log K + \mathcal{O}(1/N)$$

$$K_{\mu_1 \ldots \mu_s ; \nu_1 \ldots \nu_s}(x, y) = -\langle J_{\mu_1 \ldots \mu_s}(x) J_{\nu_1 \ldots \nu_s}(y) \rangle_0^{\rm conn}$$

- Rank s symmetric traceless tensors on 3-sphere have $SU(2)_L \times SU(2)_R$ decomposition

$$\bigoplus_{n=s+1}^{\infty} \bigoplus_{s'=-s}^{s} (\mathbf{n}+\mathbf{s}', \mathbf{n}-\mathbf{s}')$$

$$\delta F_{\Delta}^{(s)} = \frac{1}{2} \sum_{n=s+1}^{\infty} \sum_{s'=-s}^{s} (n^2 - s'^2) \log k_{n,s'}$$

- We did explicit calculations using tensor spherical harmonics up to s=3, leading us to the general conjecture

$$\delta F_{\Delta}^{(s)} \equiv F_{\rm UV}^{(s)} - F_{\rm IR}^{(s)} = \frac{(2\,s+1)\pi}{6} \int_{3/2}^{\Delta} (x - \frac{3}{2})(x+s-1)(x-s-2)\cot(\pi x)$$

Dual Analysis in AdS_{d+1}

- Fierz-Pauli equations for massive spin s fields

$$\left(\nabla^2 + 2 - (s-2)(d+s-3) - m^2\right) h_{\mu_1\cdots\mu_s} = 0$$
$$\nabla^\mu h_{\mu\mu_2\cdots\mu_s} = 0\,, \qquad g^{\mu\nu} h_{\mu\nu\mu_3\cdots\mu_s} = 0\,.$$

- Follow from minimal actions like the Proca action

$$S = \int d^{d+1}x\sqrt{g}\left(\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{m^2}{2}A_\mu A^\mu\right)$$

- Dimensions of single trace spin s operators

$$\Delta_\pm = \frac{d}{2} \pm \nu\,, \qquad \nu = \sqrt{m^2 + \left(\frac{d}{2} + s - 2\right)^2}$$

- Boundary behavior in H^{d+1}

$$ds^2 = \frac{dz^2 + \sum_{i=1}^d dx_i^2}{z^2}$$

$$h_{i_1 \cdots i_s} \sim z^{\Delta - s}$$

- The UV CFT is associated with the more singular boundary behavior $z^{\Delta_- - s}$
- Change in the free energy on S^d Gubser, Mitra

$$\delta F_\Delta^{(s)} = F_{\Delta_-}^{(s)} - F_{\Delta_+}^{(s)} = \frac{1}{2}\left[\mathrm{tr}_-^{(s)} \log(-\nabla^2 + m^2) - \mathrm{tr}_+^{(s)} \log(-\nabla^2 + m^2)\right]$$

$$\partial_\Delta \delta F_\Delta^{(s)} = (2\Delta - d)\frac{\partial \delta F_\Delta^{(s)}}{\partial m^2} = \frac{2\Delta - d}{2} \int \mathrm{vol}_{\mathrm{H}^{d+1}} \left(\mathrm{Tr} G_{\Delta_-}^{(s)}(x,x) - \mathrm{Tr} G_{\Delta_+}^{(s)}(x,x)\right)$$

The Heat Kernel on H^{d+1}

- Known for all spins! Camporesi, Higuchi

$$\left(\frac{\partial}{\partial t} - \nabla^2 + \kappa^2\right) K_{\mu_1\cdots\mu_s}{}^{\nu_1\cdots\nu_s}(x,y,t) = 0\,, \qquad K_{\mu_1\cdots\mu_s}{}^{\nu_1\cdots\nu_s}(x,y,0) = \delta_{(\mu_1\cdots\mu_s)}{}^{(\nu_1\cdots\nu_s)}(x,y)$$

$$\kappa^2 = m^2 - 2 + (s-2)(s+d-3)$$

- The spectral zeta function

$$\zeta^H(z;x) \equiv \frac{1}{\Gamma(z)} \int_0^\infty dt\, t^{z-1} K_{\mu_1\cdots\mu_s}{}^{\mu_1\cdots\mu_s}(x,x,t)$$

$$\zeta^H(z=1) = \int \mathrm{vol}_{H^{d+1}} \mathrm{Tr} G_\Delta^{(s)}(x,x)$$

$$\int \mathrm{vol}_{\mathbb{H}^{d+1}} = \begin{cases} \pi^{d/2}\Gamma\left(-\frac{d}{2}\right)\,, & d \text{ odd}\,, \\ \frac{2(-\pi)^{d/2}}{\Gamma\left(1+\frac{d}{2}\right)} \log R\,, & d \text{ even} \end{cases}$$

- For $\Delta = \Delta_+$, $\zeta^H(z) = \left(\frac{\int \mathrm{vol}_{H^{d+1}}}{\int \mathrm{vol}_{S^d}}\right) \frac{2^{d-1}}{\pi} g(s) \int_0^\infty d\lambda \, \frac{\mu(\lambda)}{\left[\lambda^2 + \left(\Delta_+ - \frac{d}{2}\right)^2\right]^z}$

- The number of propagating degrees of freedom of the massive spin s field is

$$g(s) = \frac{(2s+d-2)(s+d-3)!}{(d-2)!s!}, \qquad d \geq 3$$

- The spectral function (related to eigenvalue density)

$$\mu(\lambda) = \frac{\pi \left[\lambda^2 + \left(s + \frac{d-2}{2}\right)^2\right]}{\left(2^{d-1}\Gamma[(d+1)/2]\right)^2} \left| \frac{\Gamma\left(i\lambda + \frac{d-2}{2}\right)}{\Gamma(i\lambda)} \right|^2$$

- For d=3 need the integral

$$I_3(z) = \int_0^\infty d\lambda\, \lambda \left[\lambda^2 + \left(s+\frac{1}{2}\right)^2\right] \frac{\tanh \pi\lambda}{[\lambda^2+\nu^2]^z}\,, \qquad \nu \equiv \Delta_+ - \frac{d}{2}$$

$$I_3(z\approx 1) = \left[\left(s+\frac{1}{2}\right)^2 - \nu^2\right]\frac{1}{2(z-1)} + \left[\nu^2 - \left(s+\frac{1}{2}\right)^2\right]\psi\left(\nu+\frac{1}{2}\right) - \frac{1}{24} - \frac{\nu^2}{2} + O(z-1)\,.$$

- To get the alternate boundary condition, continue $\nu \to -\nu$. This confirms our conjecture!

$$\zeta^H(z) - \zeta^H_-(z)\big|_{z=1} = -\frac{\pi}{3}\left(s+\frac{1}{2}\right)(\Delta_+ - s - 2)(\Delta_+ + s - 1)\cot \pi\Delta_+$$

Conserved Currents

- When $J_{\mu_1\mu_2\ldots\mu_s}$ is conserved then $\Delta = s + 1$
- The auxiliary field becomes a spin s gauge field with gauge transformation

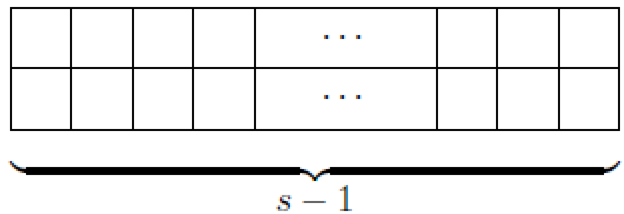
$$\delta h_{\mu_1\mu_2\ldots\mu_s} = \left(\mathcal{O}_g v\right)_{\mu_1\mu_2\ldots\mu_s}$$

$$\left(\mathcal{O}_g v\right)_{\mu_1\mu_2\ldots\mu_s} = \nabla_{(\mu_1} v_{\mu_2\mu_3\ldots\mu_s)} - \frac{s-1}{d+2(s-2)} g_{(\mu_1\mu_2} \nabla^{\nu} v_{\mu_3\mu_4\ldots\mu_s)\nu}$$

- The gauge transformations that act trivially are the conformal Killing tensors of rank s-1.
- They correspond to traceless Killing tensors in AdS_{d+1}

$$\nabla_{(\mu_1}\xi_{\mu_2\ldots\mu_s)} = 0, \qquad \xi^{\mu}{}_{\mu\mu_3\ldots\mu_{s-1}} = 0$$

- They form the [s-1,s-1] representation of the SO(d+1,1) conformal group:



- The dimension of this irrep is

$$n_{s-1} = \frac{(d+2s-4)(d+2s-3)(d+2s-2)(d+s-4)!(d+s-3)!}{s!(s-1)!d!(d-2)!}$$

- Due to the trivial gauge transformations, the QED_3 result is generalized to

$$\delta F = \frac{n_{s-1}}{2}\log N + O(N^0) \qquad n_{s-1}^{d=3} = \frac{s(4s^2-1)}{3} = \frac{(2s+1)!}{3!(2s-2)!}$$

Massless spin s in AdS

- The one-loop partition function is Gibbons, Hawking, Perry; Gaberdiel, Gopakumar, Saha; Gaberdiel, Grumiller, Vassilevich; …

$$Z_{(s)} = \frac{\left[\det_{s-1}^{STT}\left(-\nabla^2 + (s-1)(d+s-2)\right)\right]^{\frac{1}{2}}}{\left[\det_{s}^{STT}\left(-\nabla^2 + (s-2)(d+s-3)-2\right)\right]^{\frac{1}{2}}}$$

- Roughly, the numerator is due to spin s-1 ghosts.
- The ghost boundary conditions are correlated with those of spin s fields $\delta h_{\mu_1\dots\mu_s} = \nabla_{(\mu_1}\xi_{\mu_2\dots\mu_s)}$

$$\xi_{i_1\dots i_{s-1}}(z,x_i) \sim z^{\delta_\pm} c_{i_1\dots i_{s-1}}(x_i), \qquad \delta_+ = d, \qquad \delta_- = 2-2s$$

- The ghost zero modes appear only for the alternate boundary conditions! They are the traceless Killing tensors in AdS_{d+1} which diverge near the boundary.
- For example, for s=2 they are the (d+1)(d+2)/2 Killing vectors.
- This sensitivity to boundary conditions explains, from the dual point of view, why for odd d

$$F^{(s)}_{\Delta_-} - F^{(s)}_{\Delta_+} = \frac{1}{2} n_{s-1} \log N + \mathcal{O}(N^0)$$

Even d and Weyl Anomalies

- In even dimensions d, the sphere free energy may be used to calculate the a-coefficient of the Weyl anomaly, i.e. the coefficient of the Euler density.
- The induced gauge theories have local actions which are classically Weyl invariant.
- In d=4, for spin 1 it is the Maxwell theory; for spin 2 it is the conformal gravity with Weyl-squared action.
- Quadratic actions have been generalized to s>2 by Fradkin and Tseytlin. These conformal higher spin theories have higher derivative actions.

4-d Weyl Anomalies from AdS_5

- The 1-loop partition function in AdS_5 provides a remarkably simple approach to the Weyl anomaly calculations: $a_s = a_s^{\rm phys} - a_{s-1}^{\rm ghost}$

$$a_s = \frac{s^2}{180}(1+s)^2[3+14s(1+s)]$$

- For s=1 this reproduces 31/45 of the Maxwell field; for s=2 it reproduces 87/5 Fradkin, Tseytlin. For higher s these are predictions of AdS/CFT!
- Anomaly-free theory with all positive integer spins:

$$\sum_{s=1}^{\infty} a_s = \frac{1}{90}[10\zeta(-3) + 21\zeta(-5)] = 0$$

2-d Central Charges from AdS_3

- Higher-spin theories in AdS_3 are dual to 2-d CFTs with W symmetries. Henneaux, Rey; Campoleoni et al; Gaberdiel, Gopakumar;…
- Alternate boundary cond. on spin s field corresponds to gauging W_s symmetry.
- 1-loop partition functions in AdS_3 give

$$c_s = c_s^{\text{gauged}} - c_s^{\text{ungauged}} = -2\big[1 + 6\, s\,(s-1)\big] \quad (s \geq 2)$$

- They agree with that of the bc ghost system of weight (s, 1-s) found in W_s gravity. Pope

26 from AdS_3

- In particular, we find Polyakov's classic result c=-26 for the ghost system of 2-d gravity.
- We have found yet another way to see that the critical dimension of bosonic string is 26: this time from 1-loop analysis of gravity in AdS_3 and taking the difference of free energy for the two boundary conditions on the graviton.

Conclusions

- We have presented new evidence for AdS_{d+1}/IGT_d correspondence.
- IGT=Induced Gauge Theory.
- AdS calculations tend to be simpler; they make some new predictions about IGT.
- Our other results: half-integer spins; adding Chern-Simons terms in d=3; etc.

- Interesting applications to the Vasiliev theories which have an infinite number of massless higher spin fields. If we change boundary conditions for all of them, we should get an interacting non-linear conformal higher-spin theory.
- Such theories have been discussed for many years. Segal; Vasiliev; Bekaert, Joung, Mourad; …
- We find evidence for an anomaly-free 4-d conformal higher-spin theory including each positive integer spin once.
- Many further directions to explore…