ROGER PENROSE

ANY SPACE-TIME HAS A PLANE WAVE AS A LIMIT

It is a great pleasure for me to honour André Lichnerowicz on his 60th birthday. And since he has done so much to make relativity respectable as a branch of mathematics, I feel it is appropriate here to point out a simple, yet perhaps surprising, general mathematical property of hyperbolic normal pseudo-Riemannian manifolds which is motivated from knowledge of certain well-known exact solutions in general relativity.

The concept of tangent space at a point p in a manifold M is, of course, basic to differential geometry. Intuitively, one may envisage smaller and smaller neighbourhoods of p in M which are correspondingly scaled up by larger and larger factors. In the limit we obtain the tangent space T_p to M at p. However, it does not appear to have been noted in the literature before that a corresponding procedure can also be applied to any entire (properly embedded) *null geodesic* γ in any space-time or, more generally, in any hyperbolic normal pseudo-Riemannian manifold. But whereas T_p is, in an essential way, a flat space, the corresponding procedure applied to γ yields a curved space W_γ known as a *plane wave*.

I shall give the argument explicitly for a four-dimensional space-time, the generalization to n-dimensions being obvious. It is well-known (cf. [1], for example) that a coordinate system can be set up in some neighbourhood Q of a conjugate point-free portion γ', of γ, the

Any spacetime

$$ds^2 = -2\,du\,dv + dx_i\,dx^i + F(u, x^i)\,du^2$$

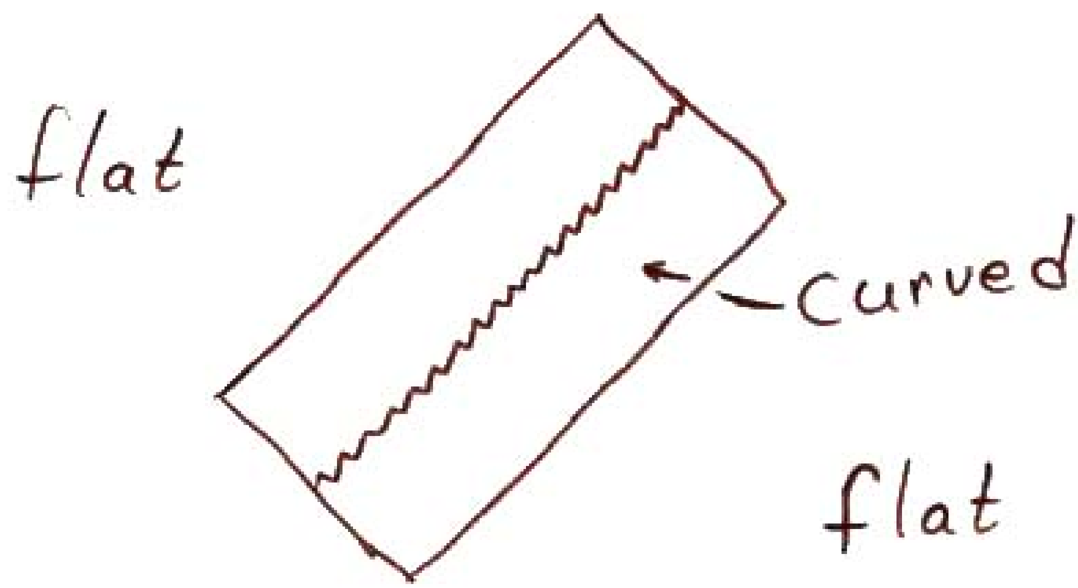
is called a plane-fronted wave with parallel rays or pp-wave

If F is quadratic in x^i, called plane wave (has extra planar symmetry)

Gravitational field of particle boosted to $v \approx c$ is pp-wave with

$$F(u, x^i) \propto \frac{\delta(u)}{|x^i|^{D-4}}$$

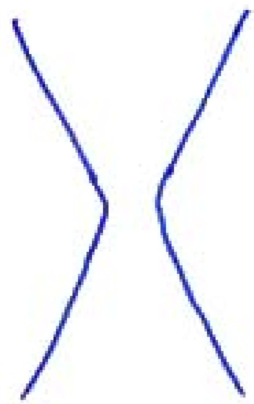
(Aichelburg-Sexl metric)



What is
the nature
of the
Big Bang?

Possible answers

1) Bounce



Veneziano

Steinhardt & Turok

2) Special initial state

Hartle & Hawking

3) Other

Time dependent orbifolds with cosmological-like singularities:

M_2/boost

spacelike singularity

$$ds^2 = -dt^2 + t^2 dx^2$$

$M_3/\text{boost} + \text{rot.}$

null singul.

$$ds^2 = -2dy^+dy^- + (y^+)^2 dy^2$$

Both have S^1 shrink to zero size in finite time.

Claim: These orbifolds are highly unstable.

Consider $(\mathrm{Orb}) \times \mathbb{R}^n$

Adding single particle (localized in $\mathbb{R}^n$) produces curvature singularity everywhere!

J. Polchinski & G.H.
hep-th/0206228

Related work:
Lawrence; Fabinger & McGreevy;
Liu, Moore, Seiberg

GR argument

Single particle on orbifold $\equiv$ ∞ sequence of particles in Minkowski spac

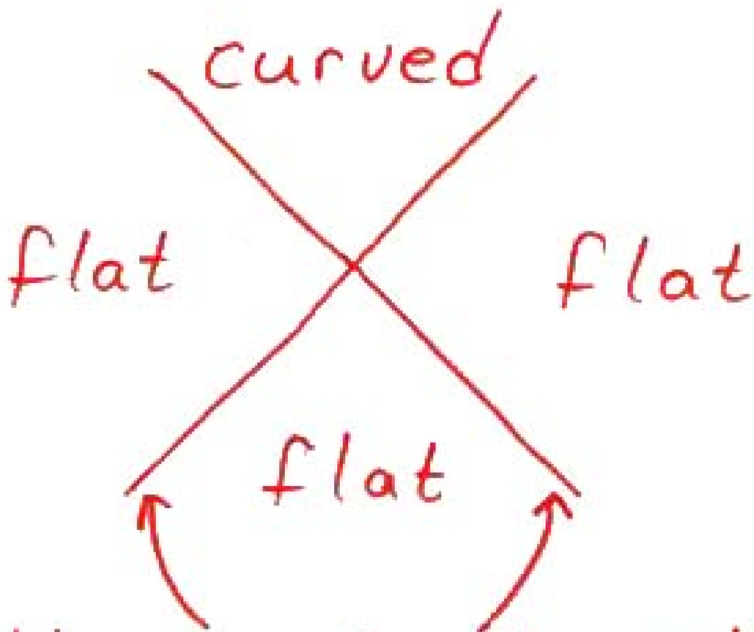
For two particles in D dim

with CM energy E_{cm} satisfying

$$G E_{cm} > b^{D-3}$$

one expects a black hole to form.

In center of mass frame



2 Aichelberg - Sexl metrics

Find a trapped surface when two shock waves collide.
(Eardley & Giddings)

Singularity theorems $\Rightarrow$ there must be a singularity to future.

For a particle and its $n^{\underline{th}}$ boosted image one finds (for both orbifolds)

E_{cm} grows with n

b is independent of n

So always form a BH for large n, and size of BH $\to \infty$ as $n \to \infty$. All spacetime is inside the BH

$\Rightarrow$ really a Big Crunch

Alternative argument:

In rest frame of a particle, gravitational field at large r is linearized Schwarzschild. For a particle on orbifold, field at large r is sum of perturbations of ∞ images. This diverges near orbifold singularity.

Adding shift in $\perp$ direction to orbifold

This removes singularity. Is solution still unstable?

Repeat previous argument and find that for a particle and its n^{th} image:

E_{cm} unchanged

b grows linearly with n

Boost orbifold + shift is unstable as before

$$E_{cm} \propto e^{n}$$

$$\Rightarrow G E_{cm} \gg b^{D-3} \qquad \text{(large } n\text{)}$$

Null orbifold + shift is stable for low energy, and forms finite size BH for high energy (D>5)

$$E_{cm} \propto n^2 p^+ , \quad b \propto n$$

$$\Rightarrow G E_{cm} < b^{D-3} \qquad \text{(large } n\text{)}$$

Implications for string theory

Classical GR corresponds to $\hbar \to 0$ limit of string theory with GE^{D-2} fixed.

$\Rightarrow$ Includes contributions from all loops

Perturbative winding states unlikely to resolve this singularity

Conclusion

Singular, time dependent orbifolds are no simpler than Schwarzschild.

Have to understand strings in highly curved backgrounds.

Promising approach:
Holography

Dual state has well defined evolution, but near big bang it doesn't have a classical bulk spacetime interpretation

What happens when Kaluza-Klein "bubbles of nothing" collide?

They form curvature singularities.

Singularities are spacelike and inside event horizons.

K. Maeda & G.H.
(to appear)