# Spacelike Branes

Strings 2002

based on: A. Strominger & M.G. hep-th/0202210
C. Chen, D. Gal'tsov & MG hep-th/0204071

Understanding time dependent string backgrounds is a challenge

- Resolution of Singularities in time dependent solutions (timelike & null orbifolds)

- Cosmology

  Can the universe bounce?
  Big Bang, Inflation, Entropy, Cosm. constant
- holographic construction of a timelike direction (dS/CFT)

- particle & string creation in time dependent backgrounds

- causality, locality, unitarity

- Analog of time evolution: RG-flow

- Exact backgrounds: orbifolds, Nappi-Witten rolling tachyon.

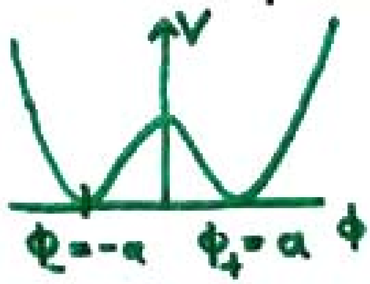
(D) branes have been central in the enhancement of our understanding of string theory (Matrix-theory, AdS-CFT, $D\bar{D}$ & Tachyonic solitons)

Can we generalize to time dependent situations?

## Field Theory examples

Take a real scalar field with double well potential

$$V(\phi) = (\phi^2 - a^2)^2$$




Instead of a spacelike kink construct a timelike kink in order to get an object which is localized in time.

Initial conditions

$$\phi(\vec{x}, t=0) = 0 \qquad \dot{\phi}(\vec{x}, t=0) = v$$

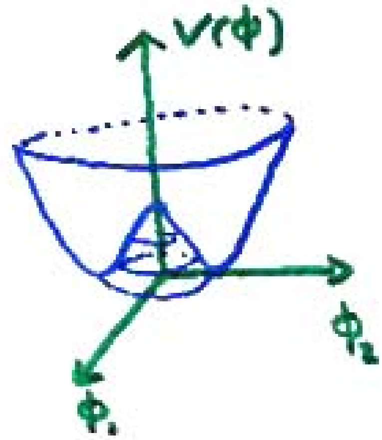
As $t \to \infty$ $\phi$ evolves into $\phi \to \phi_+$ & dissipates energy into radiation

time reversal as $t \to -\infty$ finely tuned radiation pushes the field $\phi$ near $\phi_-$ up to $\phi = 0$

# S1-brane vortex

Generalize to a complex scalar with Mexican hat potential

$$V(\phi) = (\phi^* \phi - a^2)^2$$




Initial conditions at $t=0$

$$\phi(x,y,z,t=0) = a \tanh z$$

$$\dot{\phi}(x,y,z,t=0) = i\, v\, e^{-z^2}$$

As $t \to \infty$ $|\phi| = a$ but at $z = +\infty$ $\phi = a$ and at $z = -\infty$ $\phi = -a$

time reversal implies that finely tuned radiation will make $\phi$ describe a clockwise circle in the $(z,t)$ plane

Charge: $j_\mu = \frac{i}{4\pi a^2} \phi^* \overleftrightarrow{\partial_\mu} \phi$

Coupling a three form fieldstrength $H_3 = dB_2$

$$g \int_{R^4} B \wedge dj$$

On scales large to the thickness of the brane the source can be approximated by a $\delta$-function $t^{\pm} = t \pm z$

$$d * H = g\, \delta(t^+, t^-)\, dt^- \wedge dt^+$$

hence

$$H = dx \wedge dy \wedge d\psi \quad \& \quad 2\partial_+\partial_- \psi = g\, \delta(t^+, t^-)$$

which is solved by

$$\psi(t^+, t^-) = \frac{g}{4}\Big(\underbrace{G_{adv}(t^+, t^-)}_{\text{outgoing radiation}} + \underbrace{G_{ret}(t^+, t^-)}_{\text{incoming radiation}}\Big)$$

This solution carries a 'charge'

$$g = \int_{R^2} d * H = \oint_{S^1} * H$$

which is the analogue of the axionic charge of a vortex.

Solution is invariant under Lorentz transformations in transverse space $\Rightarrow$

An Sp brane in D dimensions has an $SO(D-p-2,1)$ 'R-symmetry'

# S-branes in String theory

- Sen constructed lower dimensional D branes as kinks/vortices of the tachyon in $D\bar{D}$ or non BPS D brane systems.

  Hence a construction described above can use the tachyon as the scalar field for example a real tachyon in a non BPS p brane has a coupling

  $$\int dT \wedge C^{RR}_{(p)}$$

  which implies that a timelike kink carries a RR-'charge'

  Note a rolling tachyon solution which has initial conditions $T=0$ $\dot{T}>0$ has the same properties.

- D3 brane has a coupling

  $$\int C \, \mathrm{tr}\, F \wedge F$$

  can consider a time dependent solution which at $t \to \pm\infty$ ends up in different winding number sectors

# Boundary States for S-branes

Boundary states are very useful tools for D-branes in particular they make the coupling to closed strings easy to determine

The simplest guess for a S-brane boundary state is one which satisfies Dirichlet boundary conditions in time

$$(a_n^\mu + \sigma^\mu{}_\nu \bar{a}_{-n}^\nu)|B,\eta\rangle = 0 \qquad \mu = 0,\dots,9$$

$$(\psi_r^\mu - i\eta\sigma^\mu{}_\nu \bar{\psi}_{-r}^\nu)|B,\eta\rangle = 0 \qquad \text{"}$$

$$(q^a - y^a)\ |B,\eta\rangle = 0 \qquad a = 0,1,\dots,8-p$$

$$p^i\ \ |B,\eta\rangle = 0 \qquad i = 9-p,\dots,9$$

Dirichlet b.c for $9-p$ directions, Neuman for the rest

$$\sigma^{\mu\nu} = \mathrm{diag}\,(-\eta^{ab}, \delta^{ij})$$

D brane boundary states are combinations

$$|B\rangle = |B\ \eta=-1\rangle_{NS} - |B,\eta=-1\rangle_{NS} + |B,\eta=+1\rangle_{RR} + |B,\eta=-1\rangle_{RR}$$

the boundary state is source for closed string fields.

$$(L_0 + \bar{L}_0 - a)|\phi\rangle = |B\rangle$$

⇒ a S-Dp brane carries RR-charge.

However because the boundary state is localized in time

$$|B\rangle_0 = \int dk_0\, d^pk\, e^{-ik_0x_0 + i\vec{k}\vec{x}}\, |k_0, \vec{k}\rangle$$

the boundary state contains states of arbitrary $k^2$, hence $|B\rangle$ contains physical states for momenta on the light cone & also for massive string states.

This is to be contrasted with standard D-p brane where only $k^2=0$ state is physical and leads to $\frac{1}{r^{d-p-3}}$ metric corrections via Fischler-Susskind mechanism.

- Large backreaction
- R symmetry, thickness of brane, energy
- Analytic structure of $\langle B, y_1 | \Delta | B\, y_2 \rangle$ Green

# Supergravity Solutions

Standard brane solutions: $ISO(p,1) \times SO(D-p-1)$

S-brane solutions: $ISO(p+1) \times SO(D-p-?,1)$

Not to be confused with E brane solutions in

$\mathrm{II}^*$ theories (Hull)

References: Strominger & MG hep-th/0202210
Chen, Galtsov & MG hep-th/0204071
Kruczenski, Myers & Peet hep-th/0204144

See also relatedly: Lu, Mukherji & Pope hep-th/9612224
Lukas, Ovrut & Waldram hep-th/9610238
Grojean, Quevedo, Tasinato & Zavala hep-th/0106120

<u>Simple example:</u> d = 4 Einstein Maxwell theory

$$S = \int d^4x \sqrt{g}\,(R - F^2)$$

Ansatz has $SO(2,1) \times R$ symmetry

$$ds^2 = -\frac{dt^2}{\lambda(t)^2} + \lambda(t)^2 dz^2 + R(t)^2 dH_2^2$$

$$F_{(2)} = Q\,\mathcal{E}_2$$

equations of motion

$$(R^2\lambda^2)'' = 2$$

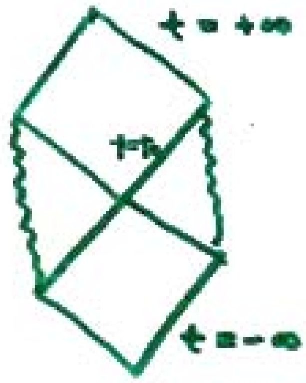
$$\left((t^2 - t_0^2)(\ln \lambda^2)'\right)' = -\frac{2Q^2\lambda(t)^2}{t^2 - t_0^2}$$

Solution:

$$ds^2 = -\frac{t^2}{t^2-t_0^2}dt^2 + \frac{t^2-t_0^2}{t^2}dz^2 + t^2 dH_2^2$$

asymptotic regions $t \to \pm\infty$

horizon $t \to \pm t_0$




**More general solutions:**

Supergravity action:

$$S = \int d^d x \sqrt{g}\left(R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2q!}e^{a\phi}F_q^2\right)$$

ansatz:

$$ds^2 = -e^{2A}dt^2 + e^{2B}(dx_1^2 + \ldots + dx_{p+1}^2) + e^{2C}d\Sigma_{d-p-2}^2$$

$$F_{[d-p-2]} = q\,\mathrm{Vol}(\Sigma_{d-p-2})$$

$\Sigma_{d-p-2}$ hyperbolic space

$$d\Sigma_{d-p-2}^2 = d\psi^2 + \sinh\psi^2 d\Omega_{d-p-3}$$

simplifying ansatz:

$$-A + (p+1)B + (d-p-2)C = 0$$

The equations of motion become

$$(1)\quad -\ddot{A} + \dot{A}^2 - (p+1)\dot{B}^2 - (d-p-2)\dot{C}^2 - \frac{1}{2}\dot{\phi}^2 - b^2\frac{d-p-3}{2(d-2)} e^{a\phi + 2(p+1)B} = 0$$

$$(2)\quad \ddot{B} + b^2\frac{(d-p-3)}{2(d-2)} e^{a\phi + 2(p+1)B} = 0$$

$$(3)\quad \ddot{C} - (d-p-3)e^{2A-2C} - b^2\frac{p+1}{2(d-2)} e^{a\phi + 2(p+1)B} = 0$$

$$(4)\quad \ddot{\phi} + \frac{ab^2}{2} e^{a\phi + 2(p+1)B} = 0$$

Equations can be simplified

$$\phi = a\frac{(d-2)}{d-p-3} B$$

$$A = (d-p-2)g - \frac{p}{d-p-3} f$$

$$B = f$$

$$C = g - \frac{p}{d-p-3} f$$

& become equivalent to the first order system:

$$\dot{f}^2 + \frac{(d-p-3)b^2}{(d-2)\chi} e^{\chi} = \alpha^2$$

$$\dot{g}^2 - e^{2(d-p-3)g} = \beta^2$$

where $\chi = 2(p+1) + \frac{a^2(d-2)}{d-p-3}$, $\beta^2 = \frac{(d-2)\chi\alpha^2}{2(d-p-2)(d-p-3)}$

Solution:

$$f(t) = \frac{2}{\chi} \ln\left(\frac{\alpha}{\cosh\left(\frac{\chi\alpha}{2}(t-t_0)\right)}\right) + \frac{1}{\chi} \ln\left(\frac{(d-2)\,\chi}{(d-p-3)\,b^2}\right)$$

$$g(t) = \frac{1}{d-p-3} \ln\left(\frac{\beta}{\sinh[(d-p-3)\beta(t-t_1)]}\right)$$

asymptotic behavior

$t \to t_1$ metric becomes flat (infinity)

$t \to \infty$, singularity of $R^2$ curvature invariants

Generalizations:

- take $d\Sigma_{d-p-2}$ to be flat or spherical related by analytic continuation to known black brane solutions
- Intersecting branes, multiply charged solution
- Resolution of singularities
- Breaking of $SO(d-p-1,1)$ R symmetry
- Non isotropic solution (breaking $ISO(p+1)$) can be nonsingular & look like S 0-brane

Grojean et al, Kruczenski et al

# Conclusions

- Time dependence is a 'timely' problem
- Spacelike branes might prove useful as simple realizations of time-dependence
- 3 different point of views

  Tachyon 'solitons'

  Boundary state / CFT

  Supergravity solutions

  Are they all consistent?