FROM BIG BANG TO BIG CRUNCH AND BEYOND

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STRINGS 2002

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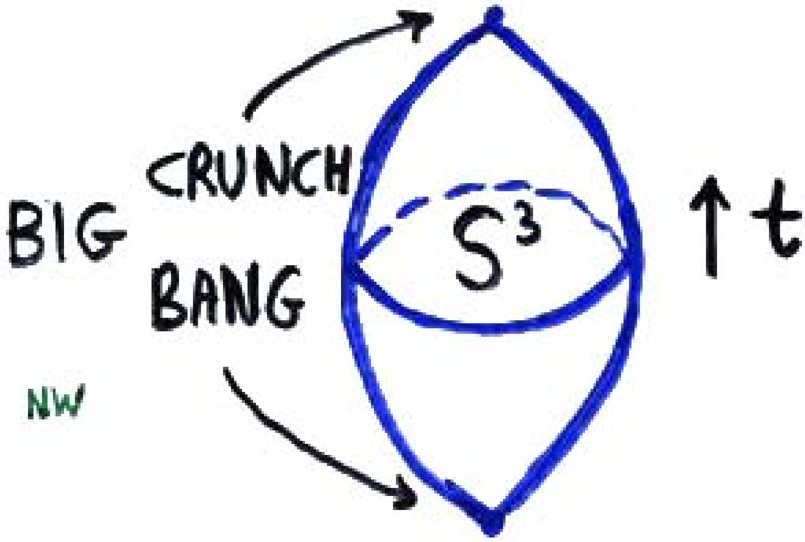
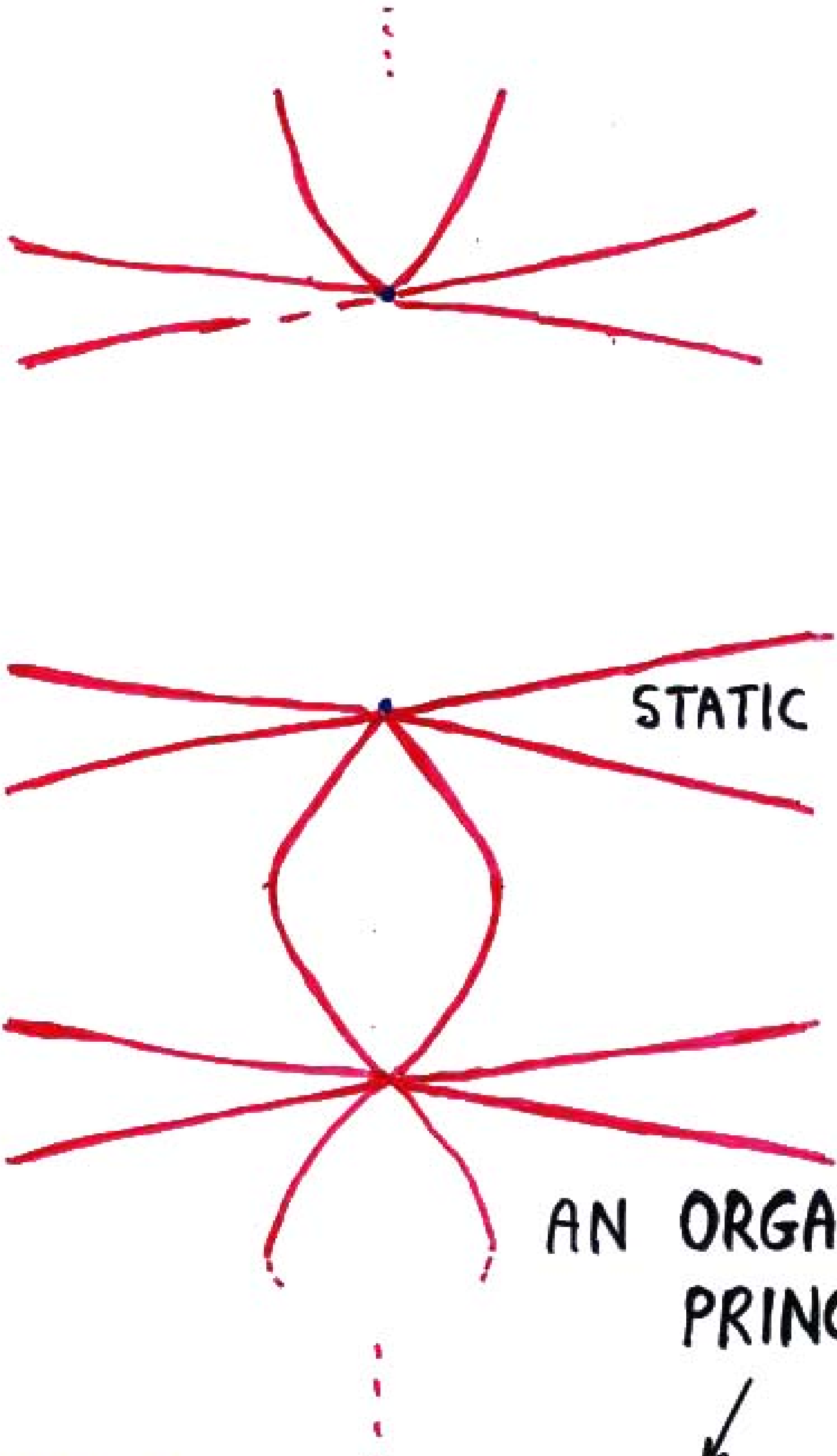


BIG
CRUNCH
BANG
S^3
$\uparrow t$
NW

OUR ATTITUDE:

USE EXACT CFT TO STUDY CONCEPTUAL QUESTIONS IN BACKGROUNDS W/ SINGULARITIES, LIKE:

- NATURE OF OBSERVABLES
- PRE BIG BANG
- MATCHING OF WAVEFUNCTIONS ACROSS THE SINGULARITY
-
-
-



STATIC ~ SPACELIKE LINEAR DILATON

PROVIDES AN ORGANIZING PRINCIPLE

$$\frac{SL(2,\mathbb{R})\times SU(2)}{U(1)\times U(1)}$$

HORAVA

KOUNNAS - LUST

NAPPI - WITTEN

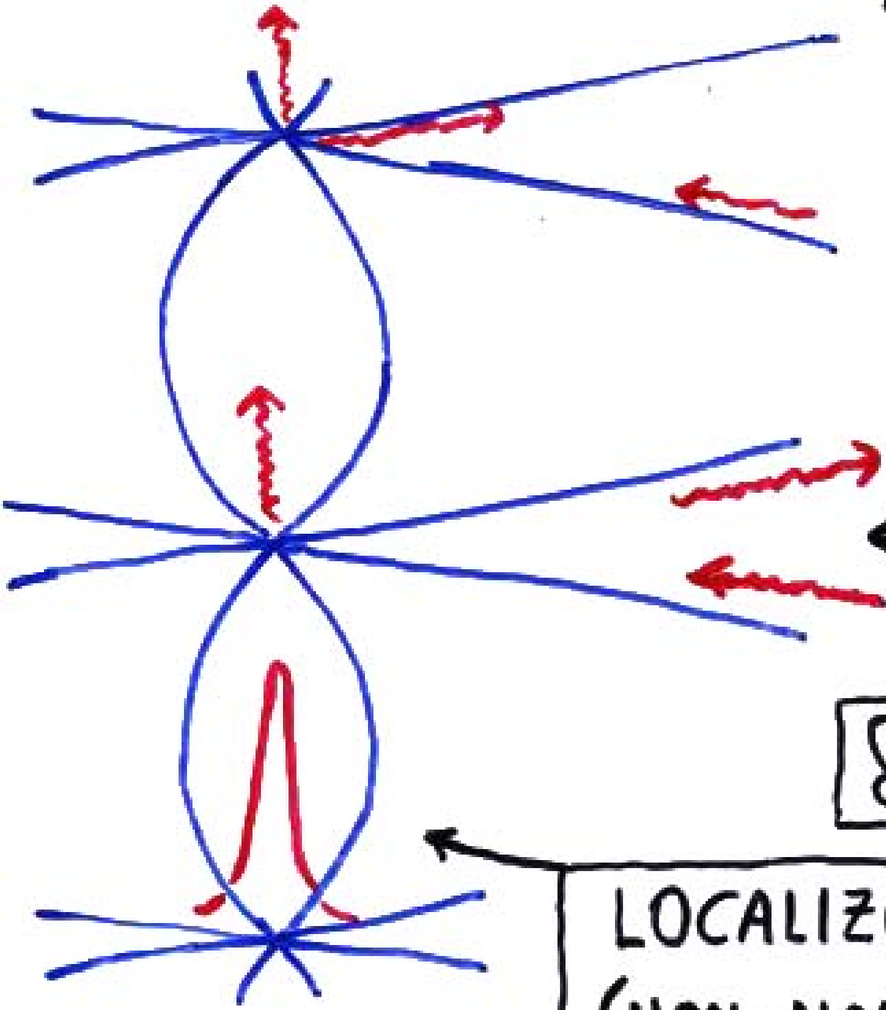
⋮

OBSERVABLES

↕

PERTURBATIVE VERTEX OPERATORS

V



SCATTERING STATES

↕

δ-NOR. W.F.

LOCALIZED W.F. (NON-NOR. V)

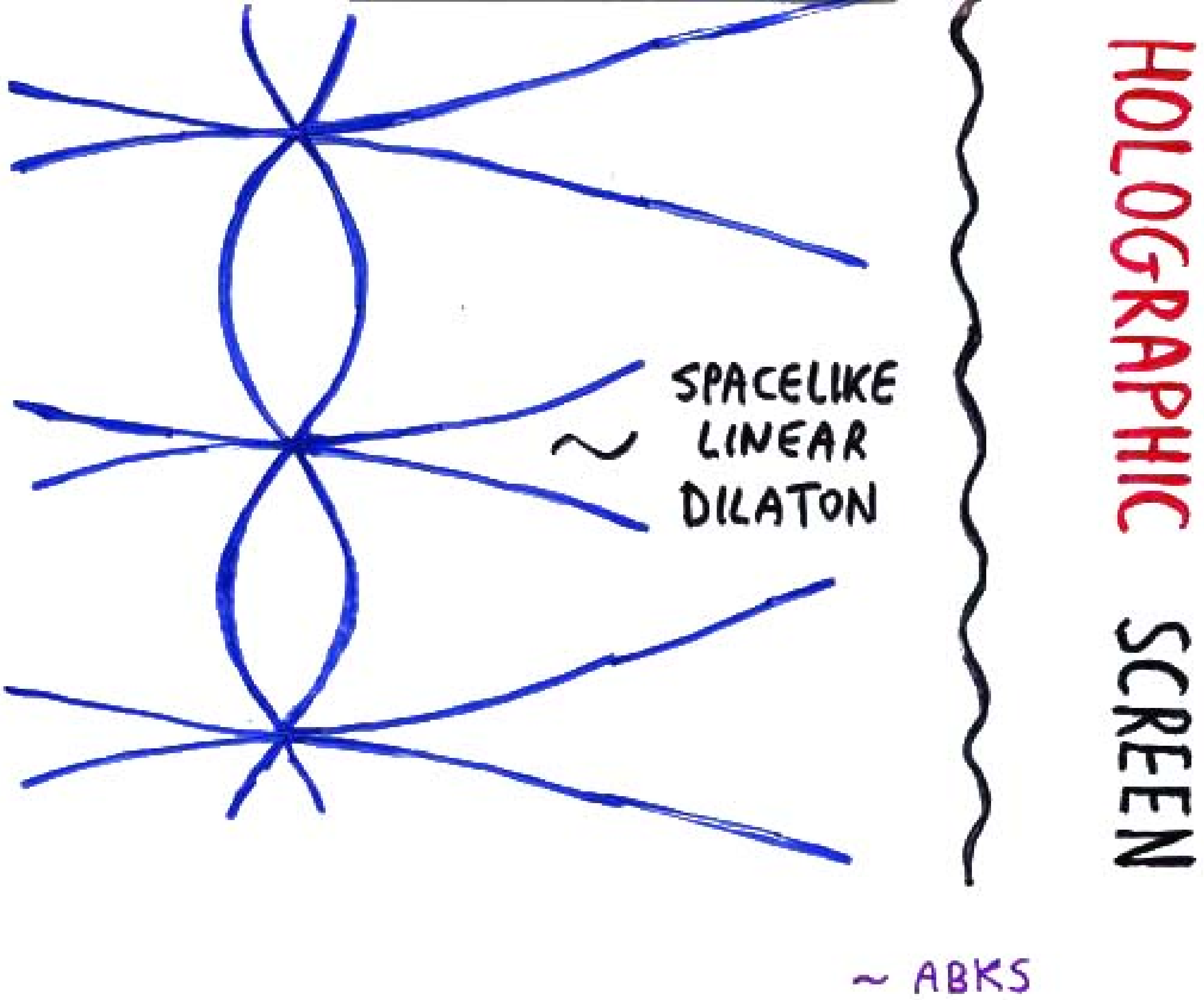
↕

D.O.F. IN THE COMPACT UNIVERSE

INTERACTIONS

2-P.-F. ↔ **R**EFLECTION COEFFICIENT OFF THE SINGULARITY

n-P.-F. ↔ SINGULAR ?



- DEPLETION OF STATES

CONSISTENCY

- INTERACTIONS BETWEEN VARIOUS REGIONS ACROSS THE SINGULARITIES

$$|R| \leq 1$$

- UNITARITY ?
- VIOLATION OF CAUSALITY ?
- EFFECT OF CLOSED TIMELIKE CURVES IN THE WHISKERS ?
- STABILITY ?
- BREAKDOWN OF PERTURBATION THEORY ?

$\uparrow$

BACK REACTION

4-P.-F. ?

$SL(2,\mathbb{R})$

$$g = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$ad - bc = 1 \qquad a,b,c,d \in \mathbb{R}$

NON-COMPACT SPACE-LIKE

$$U(1)_L \times U(1)_R \subset SL(2)_L \times SL(2)_R$$

GENERATED BY σ_3

ACTS ON g AS

$$g \to e^{\alpha \sigma_3} g \, e^{\beta \sigma_3}$$

$$(1)\quad g \to e^{\alpha\sigma_3} g e^{\beta\sigma_3} = \begin{pmatrix} a e^{\alpha+\beta} & b e^{\alpha-\beta} \\ c e^{\beta-\alpha} & d e^{-\alpha-\beta} \end{pmatrix}$$

EIGENFUNCTIONS OF $U(1)_L \times U(1)_R$

$$D_{m,\bar{m}} \xrightarrow{(1)} e^{2im\alpha + 2i\bar{m}\beta} D_{m,\bar{m}}$$

$$D_{m,\bar{m}} = \left(\frac{a}{d}\right)^{\frac{i}{2}(m+\bar{m})} \left(\frac{b}{c}\right)^{\frac{i}{2}(m-\bar{m})} K(ad)$$

$$\Downarrow$$

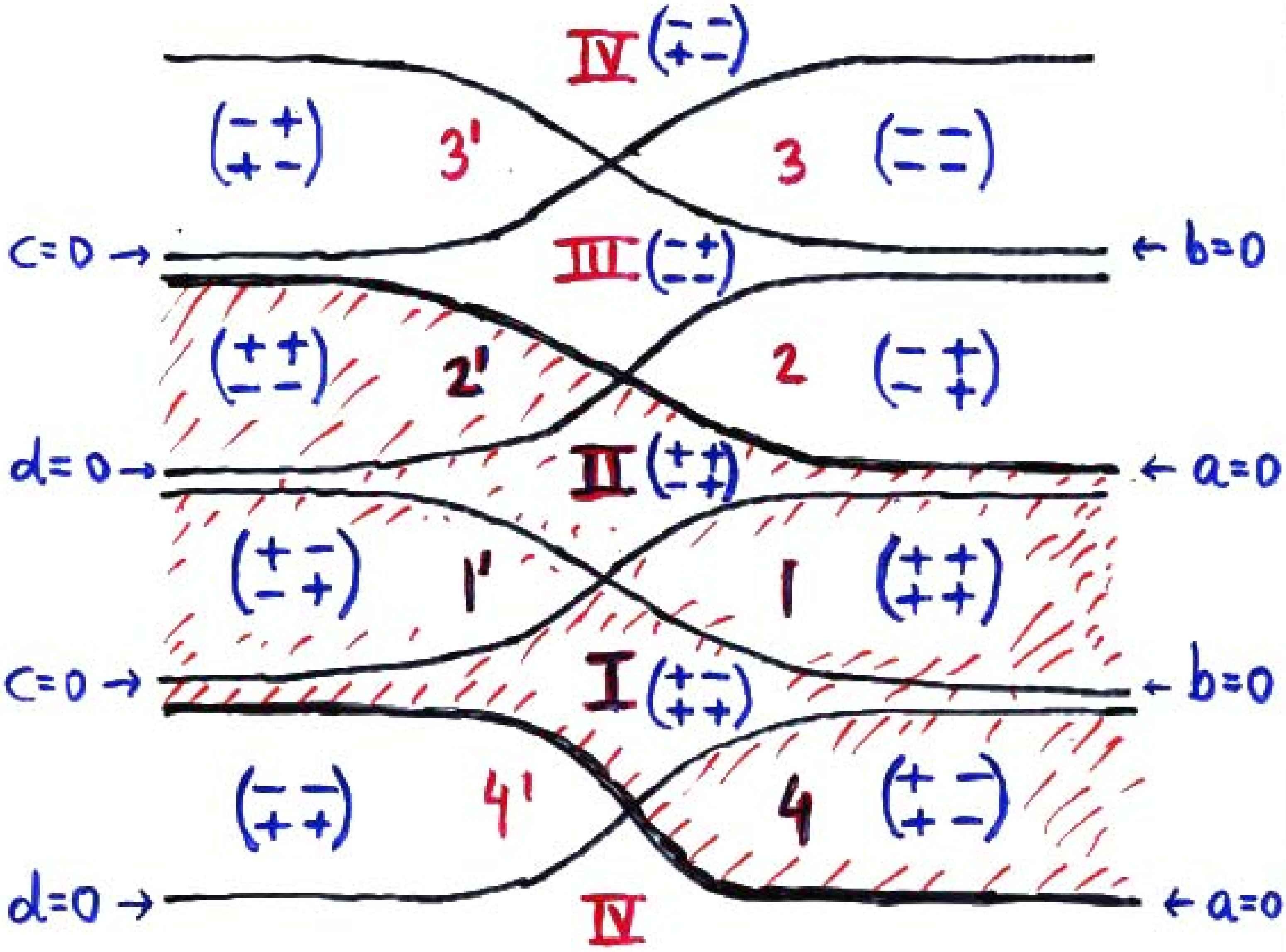
$D_{m,\bar{m}}$ ARE NON-ANALYTIC

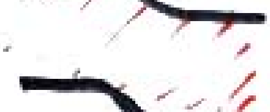
WHEN $a=0$ $\begin{pmatrix} 0 & b \\ -b^{-1} & d \end{pmatrix}$

OR $b=0$ $\begin{pmatrix} * & 0 \\ * & * \end{pmatrix}$

OR $c=0$ $\begin{pmatrix} * & * \\ 0 & * \end{pmatrix}$

OR $d=0$ $\begin{pmatrix} * & * \\ * & 0 \end{pmatrix}$





PSL(2, ℝ) OR POINCARÉ PATCH

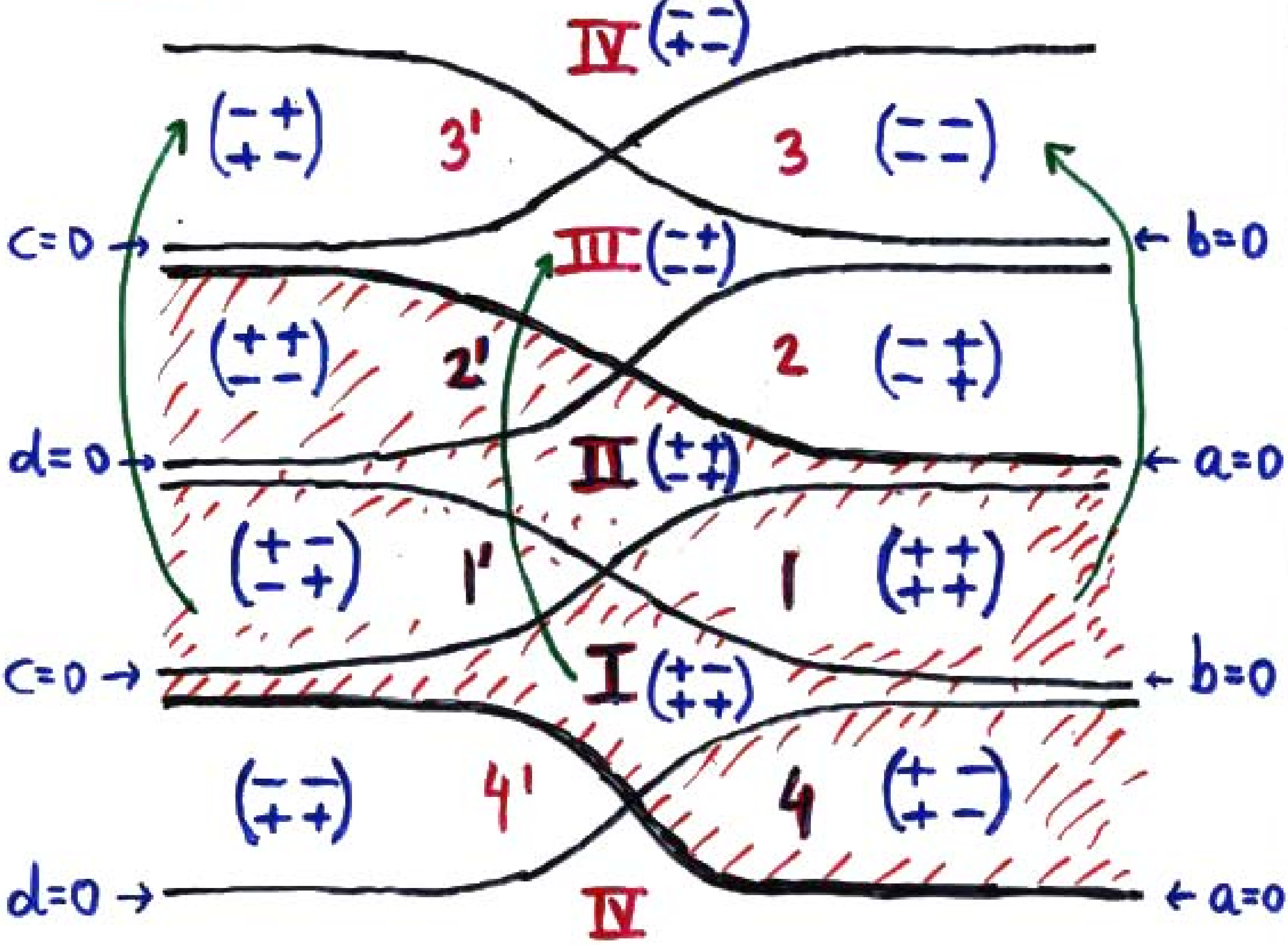
$$W = \mathrm{Tr}(\sigma_3 g \sigma_3 g^{-1}) = 2(2ad-1)$$

$-2 < W < 2$: I, II, III, IV

$W > 2$: 1, 1', 3, 3'

$-2 > W$: 2, 2', 4, 4'

$-I_{2\times 2}$



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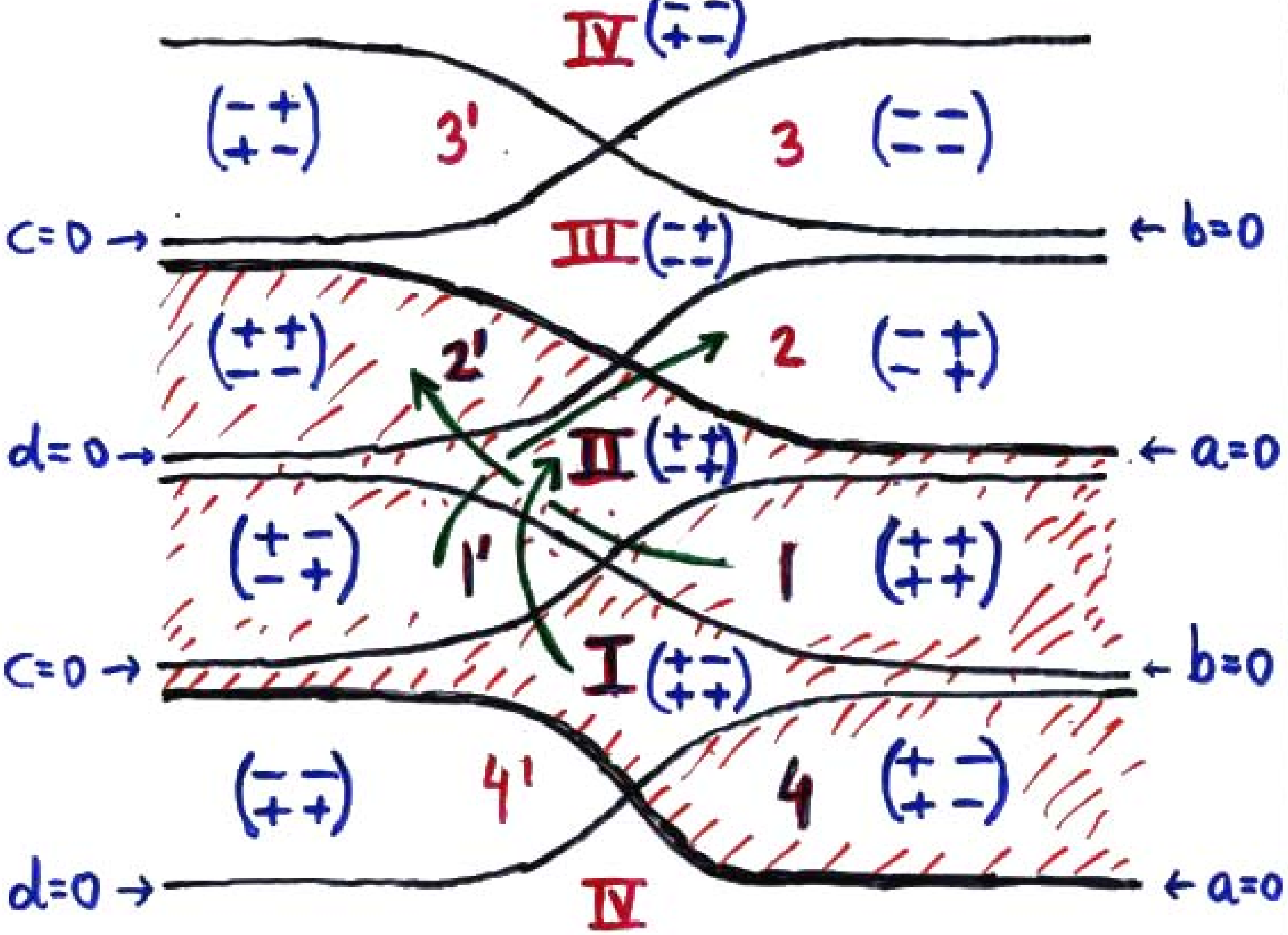
$i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$



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1.3''

$$g_{\varepsilon_{1,2},\delta}(\alpha,\beta,\theta) = e^{\alpha\sigma_3}(-1)^{\varepsilon_1}(i\sigma_2)^{\varepsilon_2}\, g_\delta(\theta)\, e^{\beta\sigma_3}$$

$\varepsilon_{1,2} = 0,1 \qquad \delta = I, 1, 1'$

$$g_I = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad 0 \leq \theta \leq \frac{\pi}{2}$$

$$g_1 = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix} \quad 0 \leq \theta < \infty$$

$$g_{1'} = g_1^{-1}$$

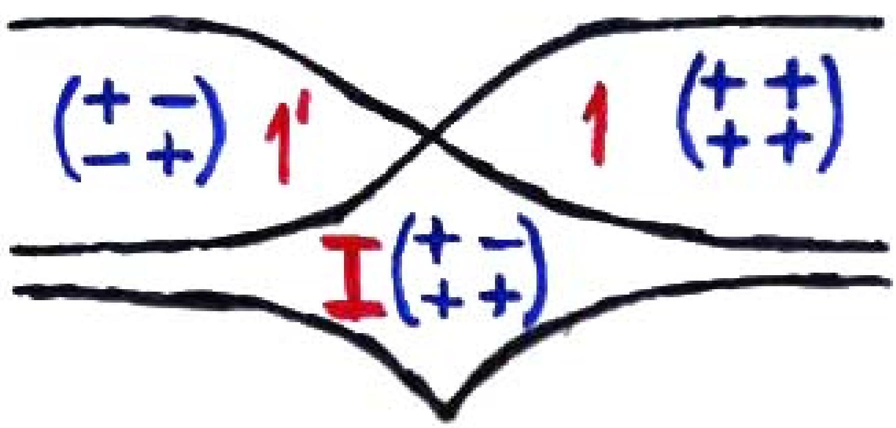
I:
$$g = e^{\alpha\sigma_3}\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}e^{\beta\sigma_3}$$

$$0 \leqslant \theta \leqslant \frac{\pi}{2}$$

1:
$$g = e^{\alpha\sigma_3}\begin{pmatrix}\mathrm{ch}\,\theta & \mathrm{sh}\,\theta \\ \mathrm{sh}\,\theta & \mathrm{ch}\,\theta\end{pmatrix}e^{\beta\sigma_3}$$

$$0 \leqslant \theta < \infty$$

1':
$$g = e^{\alpha\sigma_3}\begin{pmatrix}\mathrm{ch}\,\theta & -\mathrm{sh}\,\theta \\ -\mathrm{sh}\,\theta & \mathrm{ch}\,\theta\end{pmatrix}e^{\beta\sigma_3}$$



WAVEFUNCTIONS

REPS. (UNITARY)

j, ε

$-j(j+1) \in \mathbb{R}$ $\qquad e^{i\pi\varepsilon}$ [REP. OF THE CENTER]

- CONTINUOUS SERIES REPS.

$$j = -\frac{1}{2} + is \qquad s \in \mathbb{R}$$

PSL(2): $\varepsilon = 0$

SL(2): $\varepsilon = 0, 1$

$\widetilde{SL}(2)$: $\varepsilon \in [0, 2)$

- DISCRETE REPS.

$$j \in \mathbb{Z} + \frac{\varepsilon}{2}$$

BASIS OF EIGENVECTORS OF THE NON-COMPACT SPACE-LIKE U(1)

$g = e^{\alpha\sigma_3}$:

E.V. = $e^{2im\alpha}$ $\qquad m \in \mathbb{R}$

DISCRETE : $\qquad m$

CONTINUOUS: $\qquad m, \pm$

2 VECTORS W/ THE SAME E.V. m

SUMMARY: UNITARY REPS.

$$|j, \varepsilon; m, \pm\rangle \qquad \mathcal{C}_{j,\varepsilon;m,\pm}$$

$j = -\frac{1}{2} + is \qquad m \in \mathbb{R}$

$$j \in \mathbb{Z} + \tfrac{1}{2}\varepsilon \qquad \mathcal{D}_{j;m}$$

$m \in \mathbb{R}$

MATRIX ELEMENTS ($C_{j,\varepsilon;m,\pm}$ FIRST)

$$\langle j,\varepsilon;m,\pm|g|j,\varepsilon;\bar{m},\pm\rangle \equiv$$

$$e^{2im\alpha+2i\bar{m}\beta}e^{i\pi\varepsilon_1\varepsilon}K_{\pm\pm}(\underset{-im-j}{\underset{\parallel}{\lambda}},\underset{-i\bar{m}-j}{\underset{\parallel}{\mu}};j,\varepsilon;g(\theta))$$

REGION 1:

$$K_{++}(\lambda,\mu;j,\varepsilon;g_1) =$$

$$\frac{1}{2\pi i}B(\lambda,-\lambda-2j)\frac{(1-y)^{j+\frac{\lambda+\mu}{2}}}{(-y)^{\frac{\lambda+\mu}{2}}}F(\lambda,\mu;-2j;\tfrac{1}{y})$$

$$y \equiv -\sinh^2\theta$$

$$K_{--}(m,\bar{m};j,\varepsilon;g_1) = K_{++}(\bar{m},m;-(j+1),\varepsilon;g_1)$$

(2 INDEPENDENT K's)

CAN FIND $K_{\pm\pm}$ IN ALL REGIONS

A PARTICULARLY INTERESTING WAVEFUN.:

$$U(\lambda,\mu;\delta,\varepsilon;g) \sim K_{++} - \frac{\sin(\pi\mu)}{\sin(\pi\lambda)} K_{--}$$

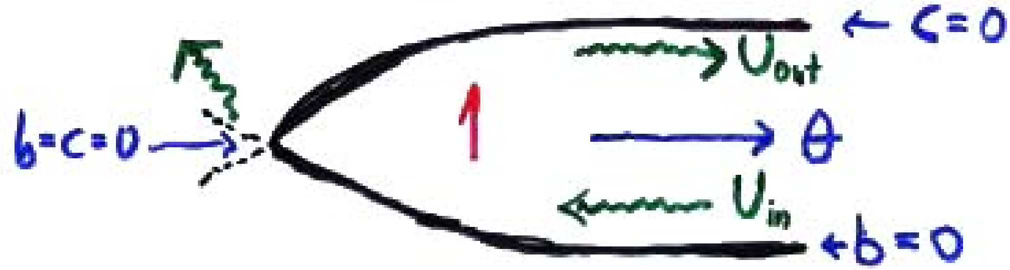
DVV

∞'LY BLUE SHIFTED @ BDRY OF REGION 1:

$$U(b\to 0) \sim b^{i\omega}$$

HENCE WAVE PACKETS VANISH @ $b=0$

ASYMPTOTIC BEHAVIOR:

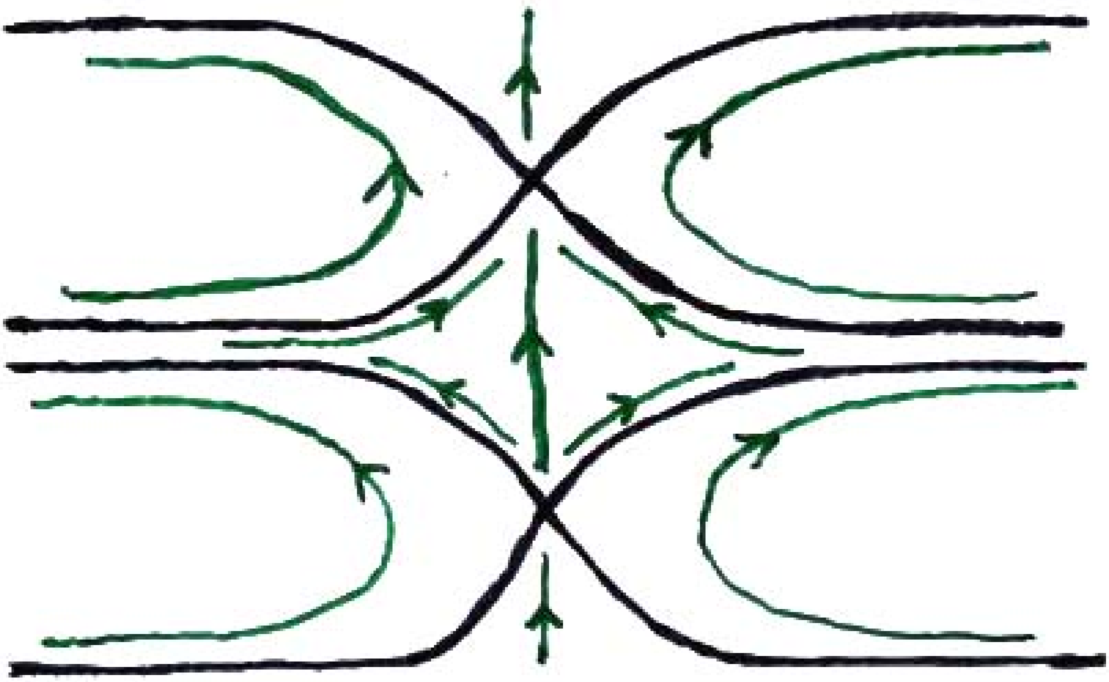


$$U(\gamma,\omega;\delta;\theta\to\infty) \sim e^{2i\gamma\phi} e^{-\theta} \times$$

$$\times \left[e^{2i(\omega t + s\theta)} + R(s;m,\bar{m})\, e^{2i(\omega t - s\theta)} \right]$$

$t = \alpha - \beta \qquad 2\omega = m - \bar{m}$

$\phi = \alpha + \beta \qquad 2\gamma = m + \bar{m}$



DIRECTION OF **time** = ⟵

$$U \overset{\theta\to\infty}{\sim} e^{2i(\omega t+s\theta)} + R(\overset{-\frac{1}{2}+is}{\overset{\shortparallel}{j}};m,\bar{m})\, e^{2i(\omega t-s\theta)}$$

$\begin{pmatrix}\omega, s>0 \\ \text{w.l.g}\end{pmatrix}$

"INCOMING" PLANE WAVE IN REGION 1

"OUTGOING" IN 1

"SCATTERED" FROM $b=c=0$ w/ "REFLECTION COEFICIENT:"

$$R(j;m,\bar{m}) = \frac{\Gamma(-2j-1)\,\Gamma(j+1+im)\,\Gamma(j+1-i\bar{m})}{\Gamma(2j+1)\,\Gamma(-j+im)\,\Gamma(-j-i\bar{m})}$$

$$\shortparallel \ !$$

2-P-f. $\langle V_{j;m,\bar{m}} V_{j;-m,-\bar{m}} \rangle$ IN

$SL(2)_k$ WZNW CFT

@ $k \to \infty$

MORE PHYSICS IN THE COSET

IN $D_{j;m}$ SERIES

NORMALIZED WAVEFUN. ARE GIVEN BY PARTICULAR LINEAR COMBINATIONS OF $K_{\pm\pm}$

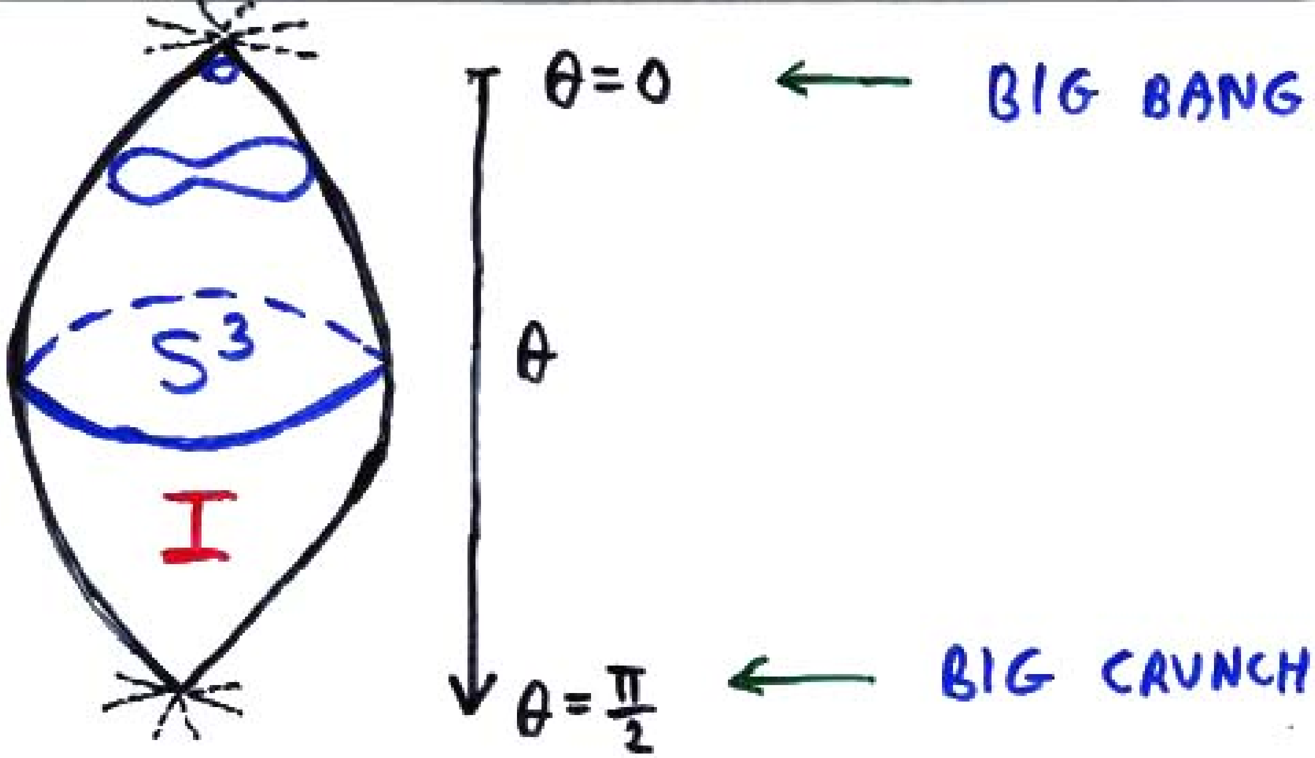
FOR INSTANCE, IN REGION 1:

$$K_{++}(g_1) \quad \text{IF} \quad j < -\tfrac{1}{2}$$

$$K_{--}(g_1) \quad \text{IF} \quad j > -\tfrac{1}{2}$$

$$j \in \mathbb{R}$$

CONTINUATION TO OTHER REGIONS IS NORMALIZABLE IFF $j \in \mathbb{Z} + \frac{1}{2}\varepsilon$



- AT $\theta = 0, \frac{\pi}{2}$: $g_I = \begin{pmatrix} a & 0 \\ 0 & \frac{1}{a} \end{pmatrix}, \begin{pmatrix} 0 & b \\ -\frac{1}{b} & 0 \end{pmatrix}$ HENCE

  $g \to e^{\rho\sigma_3} g e^{\tau\sigma_3}$ CANNOT BE USED FOR A COMPLETE GAUGE FIXING $\Rightarrow$ FIX GAUGE PARTLY IN $SU(2)$

  $\Downarrow$

  LEADS TO "BIG BANG" @ $\theta = 0$

  AND "BIG CRUNCH" @ $\theta = \frac{\pi}{2}$

- UNIVERSE $\equiv$ TIME DEPENDENT $J\bar{J}$ DEFORMATION OF $SU(2)_k$ WZNW

Kiritsis-Kounnas

$$\mathcal{L}_{SU(2)_{WZNW}} + \lambda(R)\, J(R)\, \bar{J}(R)$$

$$E(R) = G(R) + B(R) =$$

$$\begin{array}{l} \theta_1' = \alpha' + \beta' \\ \theta_2' = \alpha' - \beta' \\ \theta' \end{array} \left(\begin{array}{cc} \dfrac{k}{1 + R^2 \cot^2\theta'} \begin{pmatrix} \cot^2\theta' & 1 \\ -1 & R^2 \end{pmatrix} & \\ & k \end{array} \right)$$

$$\Big\downarrow \text{T-duality}$$

$$\frac{\overbrace{k(d\theta'^2 + \tan^2\theta'\, d\tilde{\theta}_1^2)}^{\frac{SU(2)_k}{U(1)}\text{; } \mathbb{Z}_k \text{ Parafermion}} + \overbrace{kR^2 d\tilde{\theta}_2^2}^{S^1_{\sqrt{k}R}}}{\mathbb{Z}_k}$$

A. G.-Kiritsis

$$\tilde{\theta}_{\substack{2\\1}} \sim \theta_1' \pm \theta_2'$$

REGION 1 IN COSMO.: $R \to R(\theta) = \tan\theta$

$$(d\theta_1', d\theta_2', d\theta')\, E\big(R(\theta)\big) \begin{pmatrix} d\theta_1' \\ d\theta_2' \\ d\theta' \end{pmatrix} - d\theta^2$$

2.3'
2.8'

WE PREFER NOW GAUGE (2):

(1) $\boxed{\alpha' = \beta' = 0 \quad \text{IN} \quad SU(2)}$

$\Downarrow$

UNIVERSE RARAMETRIZED BY
$SL(2)$ COOR. α, β, θ
AND $0 < \theta' < \frac{\pi}{2}$

- (1) DOES NOT FIX THE GAUGE COMPLETELY:

$$\alpha' + \beta' \simeq \alpha' + \beta' + 2\pi$$
$$\alpha' - \beta' \simeq \alpha' - \beta' + 2\pi$$

$\Downarrow$

$$\begin{aligned} \rho + \tau &= 2\pi n_1 \\ \rho - \tau &= 2\pi n_2 \end{aligned} \quad \text{LEAVE } g' \text{ UNCHANGED}$$

$\Downarrow$

- RESIDUAL $\mathbb{Z} \times \mathbb{Z}$ GAUGE SYMM. HAS SURVIVED THE GAUGE FIXING

- FIXING THE RESIDUAL $\mathbb{Z}\times\mathbb{Z}$ IN $SL(2)$ LEADS TO THE IDENTIFICATION:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \to \begin{pmatrix} ae^{2\pi n_1} & be^{2\pi n_2} \\ ce^{-2\pi n_2} & de^{-2\pi n_1} \end{pmatrix}$$

- ADDITIONAL FIXED PTS.:

$\begin{pmatrix} 0 & b \\ -b^{-1} & 0 \end{pmatrix}$ IS INVARIANT UNDER $\mathbb{Z}$

(SUBGROUP CORRESPONDING TO $n_2=0$)

$\begin{pmatrix} a & 0 \\ 0 & a^{-1} \end{pmatrix}$ $\mathbb{Z}$ INV. ($n_1=0$ sub $\mathbb{Z}\times\mathbb{Z}$)

$$\Downarrow$$

- ORBIFOLD SINGULARITY @ SURFACES

$a=0=d$ AND $b=0=c$

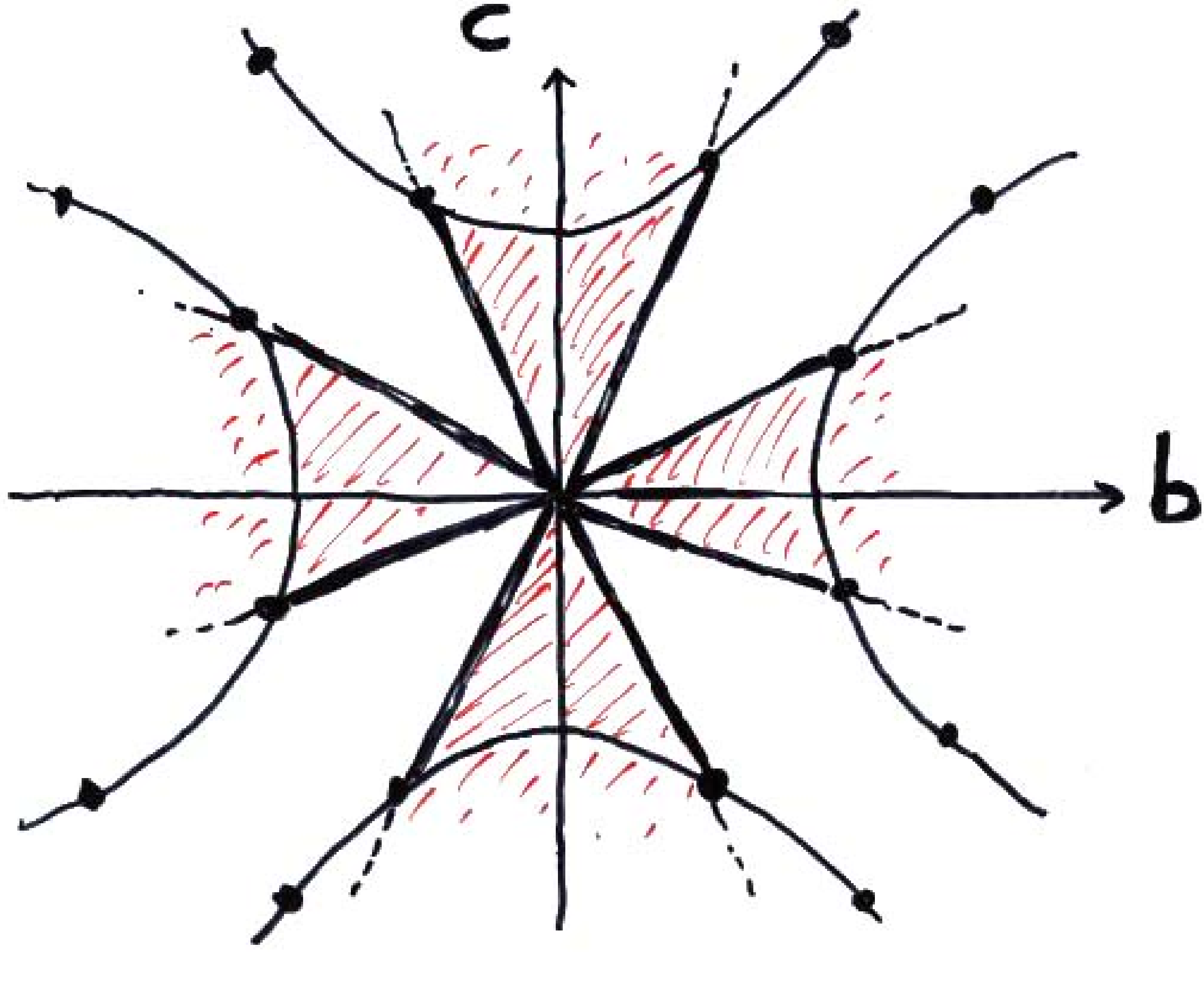
FUNDAMENTAL DOMAIN

$$e^{-4\pi} \leqslant \left|\frac{b}{c}\right|, \left|\frac{a}{d}\right| \leqslant 1$$

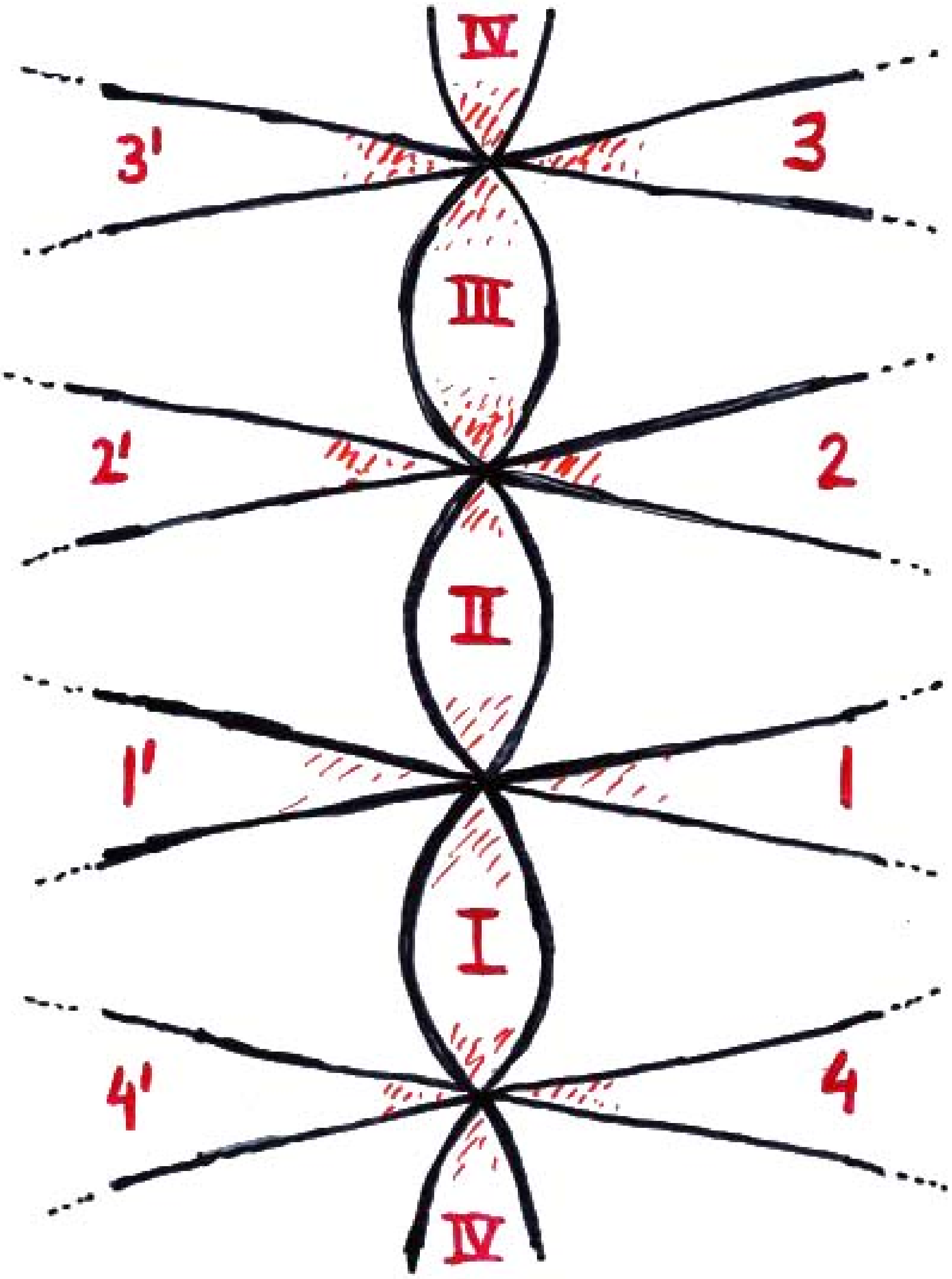
NEAR $b = c = 0$:

$$\sim \left\{\begin{matrix}\text{FINITE INTERVAL} \\ \text{IN } a;\ \theta'\end{matrix}\right\} \times \frac{\mathbb{R}^{1,1}_{bc}}{\text{BOOST}}$$

MILNE UNIVERSE



SIMILARLY NEAR $a = d = 0$: $\dfrac{\mathbb{R}^{1,1}_{ad}}{\text{boost}} \times \left\{\begin{matrix}\text{finite interval} \\ \text{in } b;\ \theta'\end{matrix}\right\}$



 ~ Milne × 2 intervals

$|W| < 2$ (SAY I)

$$ds^2 = -d\theta^2 + d\theta'^2 + \frac{\cot^2\theta'}{1+\tan^2\theta\cot^2\theta'}\,d\lambda_+^2$$

$$+ \frac{\tan^2\theta}{1+\tan^2\theta\cot^2\theta'}\,d\lambda_-^2$$

$$\lambda_\pm = \alpha \pm \beta \in [0, 2\pi)$$

$$0 \leqslant \theta, \theta' \leqslant \frac{\pi}{2}$$

$$B_{\lambda_+\lambda_-} = \frac{1}{1+\tan^2\theta\cot^2\theta'}$$

$$\Phi = \Phi_0 - \frac{1}{2}\log\left(\cos^2\theta\sin^2\theta' + \sin^2\theta\cos^2\theta'\right)$$

- SINGULARITIES: $\theta = \theta' = 0$ BIG BANG

  $\theta = \theta' = \frac{\pi}{2}$ BIG CRUNCH

- $J\bar{J}$ DEF. OF $SU(2)$ w/ $R(\theta) = \tan\theta$

$|w| > 2$ (SAY 1)

$$ds^2 = d\theta^2 + d\theta'^2 + \frac{\cot^2\theta'}{1-\tanh^2\theta\cot^2\theta'}\,d\lambda_+^2 - \frac{\tanh^2\theta}{1-\tanh^2\theta\cot^2\theta'}\,d\lambda_-^2$$

$$0 \leq \theta < \infty$$
$$0 \leq \theta' \leq \frac{\pi}{2}$$

$$B_{\lambda_+\lambda_-} = \frac{1}{1-\tanh^2\theta\cot^2\theta'}$$

$$\Phi = \Phi_0 - \frac{1}{4}\log(\cosh^2\theta\sin^2\theta' - \sinh^2\theta\cos^2\theta')^2$$

- CLOSED TIMELIKE CURVES: $\lambda_\pm \simeq \lambda_\pm + 2\pi$
- SINGULARITY: $\tan\theta' = \tanh\theta$

WAVEFUNCTIONS

VERTEX OPERATOR IN $SL(2) \times SU(2)$

$$V^{j;j'}_{m,m';\bar{m},\bar{m}'} = K^{j}_{m,\bar{m}}(g)\, D^{j'}_{m',\bar{m}'}(g')$$

$$K^{j}_{m,\bar{m},\epsilon_1,\epsilon_2,\delta}(\theta)\, e^{2i(m\alpha + \bar{m}\beta)}$$

$$D^{j'}_{m',\bar{m}'}(\theta')\, e^{2i(m'\beta' + \bar{m}'\alpha')}$$

V INV. UNDER GAUGE TRANS.

$$\left.\begin{array}{l} \alpha \to \alpha + \rho \\ \alpha' \to \alpha' + \rho \end{array}\right\} \Rightarrow \boxed{\bar{m}' = m}$$

$$\left.\begin{array}{l} \beta \to \beta + \tau \\ \beta' \to \beta' + \tau \end{array}\right\} \Rightarrow \boxed{m' = \bar{m}}$$

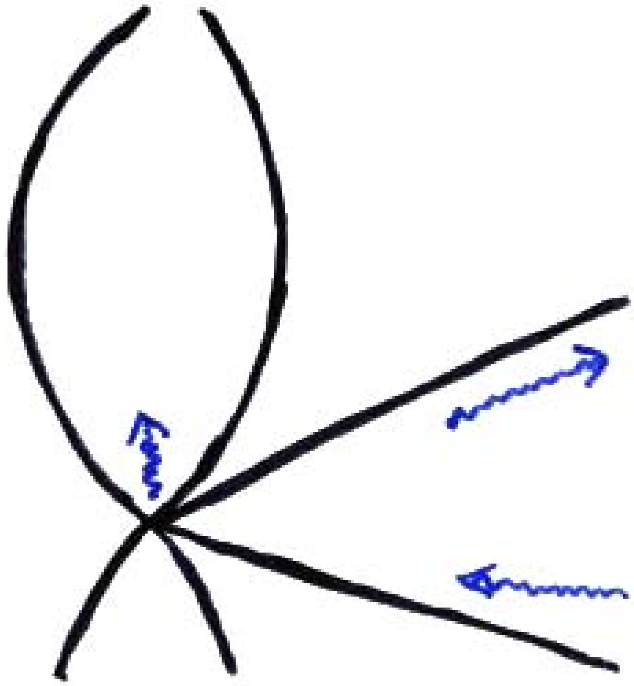
E.V. OF $\bar{J}^3_{SU(2)}$, $J^3_{SU(2)}$

E.V. OF $\left.\begin{array}{l} J^3_{SL(2)} \\ \bar{J}^3_{SL(2)} \end{array}\right\}$ NON-COMPACT, SPACE-LIKE

SCATTERING FROM THE BIG BANG

$$V = U(\lambda, \mu; j, \varepsilon; g)\, D^{j'}_{m', m}(g')$$

$-im-j$ $\quad -im'-j \quad -\frac{1}{2}+is$



REMOVING SINGULARITIES

TURNING ON A_μ:

$$\frac{SL(2)\times SU(2)}{U(1)^2}\times U(1) \rightarrow \frac{SL(2)\times SU(2)\times U(1)}{U(1)^2}$$

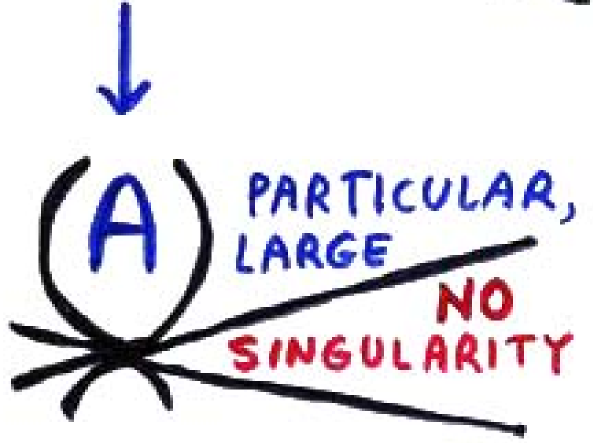
1-PARAMETER FAMILY

NW

3-PARAMETER FAMILY

GP, GMV

3+1 COSMO + GAUGE FIELD



UNCHARGED

FULLY REFLECTED

W.F. EXIST

IFF

BIG BANG/CRUNCH

SINGULARITY EXISTS