A Resolution of the Cosmological Singularity with Orientifolds

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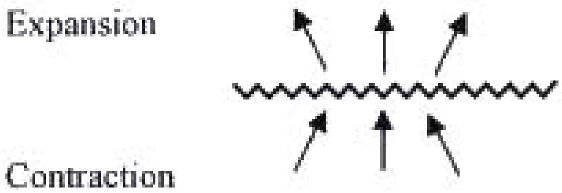
Introduction

- **Cosmological Singularity Problem**



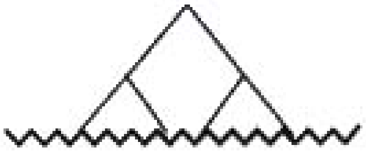
Space-like singularity
(string effects)

[Veneziano]

- **Singularity Theorems :** basic assumptions are global structure and reasonable matter $\rho + 3p > 0$
  Cannot then reverse from contraction to expansion.

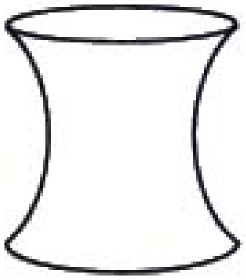
[Hawking & Penrose]

- **Space-like singularity** ⟶ **Horizon Problem**



- **DeSitter Space evades these problems with a positive cosmological constant** Λ

$$p = -\rho$$



Space-Time Global Structure

- Effective (d+1) – dimensional gravitational action

$$\int d^{d+1}x\sqrt{g}\left[R-\frac{\beta}{2}(\nabla\psi)^2-V(\psi)\right]$$

- General solution with a cosmological horizon at $t = 0$ and $SO(1,d)$ symmetry:

Region I – Open Cosmology

$$ds^2_{d+1}=-dt^2+a_I{}^2(t)ds^2(H_d)$$

$$\psi=\psi_I(t)$$

Region II – 'Static' Region

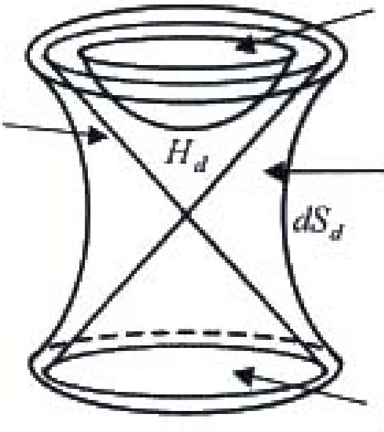
$$ds^2_{d+1}=dx^2+a_{II}{}^2(x)ds^2(dS_d)$$

$$\psi=\psi_{II}(x)$$

- Gluing conditions on the horizon and analytic continuation

$$a_I(t)=t+o(t^3) \qquad \psi_I(t)=\psi_0+o(t^2)$$

$$a_{II}(x)=-ia_I(ix) \qquad \psi_{II}(x)=\psi_I(ix)$$



Expanding

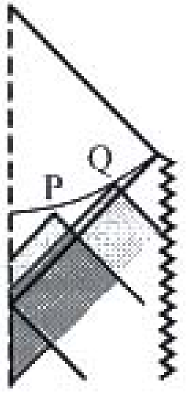
Gluing on horizon

Intermediate :

A singularity will have a *brane-like interpretation*

Contracting

- No Horizon Problem



- Structure of the singularity

$$E^2 ds^2 \equiv \Lambda^{-\frac{1}{2}}\left[\mu\, ds^2(dS_d)\right] + \Lambda^{\frac{1}{2}}\left[d\Lambda^2 + ds^2(\mathbf{T}^{\tilde{d}})\right]$$

$$e^{\phi} = g_s \Lambda^{\frac{4-d}{4}} \qquad F = \frac{1}{g_s E^d}\frac{1}{\Lambda^2} d\Lambda \wedge \sqrt{\mu^d}\,\varepsilon(dS_d)$$

Similar to D(d-1) – brane metric *delocalized along the* $\mathbf{T}^{\tilde{d}}$. Harmonic function

$$H(\Lambda, \mathbf{T}^{\tilde{d}}) = \Lambda$$

But

$$\text{Tension} \propto -\nabla^2 H < 0$$

Negative tension O(d-1)-plane, smeared over the compact directions, with a deSitter world-volume Γ .

- Solution of SUGRA with *negative tension source* on the O-planes

$$|T| \int_{\Gamma} d^d x\, e^{-\phi} \sqrt{-\det G} \pm Q \int_{\Gamma} A$$

- Near singularity orientifold is locally near flat and BPS

- Radius $L \approx \frac{1}{E}$

- Number of O-planes per unit transverse volume $n \approx \frac{l_s E}{g_s}$

Embedding in String Theory

- *Toroidal* Compactification of Type II

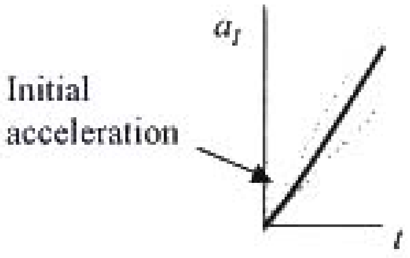
$$E^2 ds^2 = \Lambda^{-\frac{1}{2}\frac{d+1}{d-1}} ds^2_{d+1} + \Lambda^{\frac{1}{2}} ds^2(\mathbf{T}^{\tilde{d}}) \qquad \begin{pmatrix} \tilde{d} = 9 - d \\ \Lambda = e^{2\beta\psi} \end{pmatrix}$$

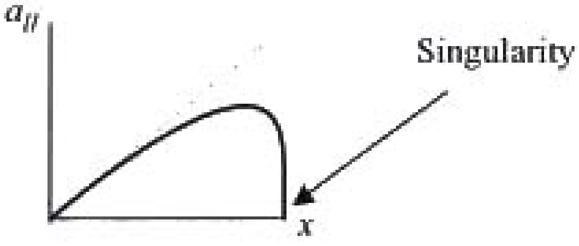
$$e^{\phi} = g_s \Lambda^{\frac{4-d}{4}} \qquad \tilde{F} = \frac{1}{g_s E^{\tilde{d}-1}} \varepsilon(\mathbf{T}^{\tilde{d}})$$

- Effective (d+1) – dimensional scalar potential

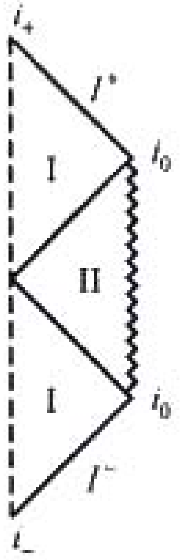
$$V(\psi) = \frac{1}{2} e^{-\psi}$$

- Behavior of scale factor

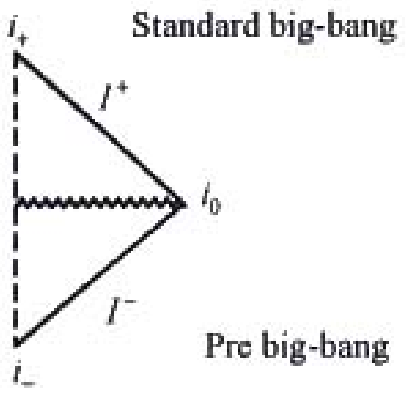




- Carter-Penrose diagram



No big-bang !



Two Dimensional Toy Model

- The case of the O-particles can be obtained as the M-theory compactification

$$\mathbf{M}^3/G \times \mathbf{T}^8$$

where Γ is **Boost & Translation**.

Start with flat metric on the three dimensional space $\mathbf{M}^3$

$$ds^2 = -dX^+dX^- + dY^2$$

Then

$$\Gamma = e^{\kappa} \qquad \kappa = (\Delta \underset{\nearrow}{J} + R \underset{\nwarrow}{P})$$

$$iJ = X^+\partial_+ - X^-\partial_- \qquad\qquad iP = \partial_Y$$

No fixed points

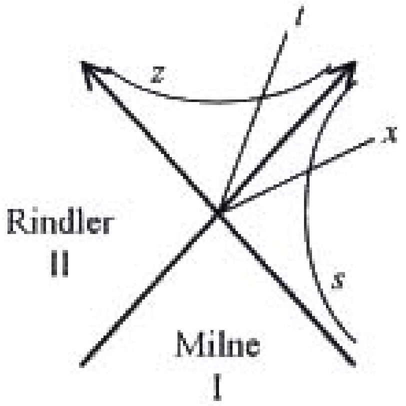
Other models:
[Horowitz, Steif]
[Kounnas, Lust]
[Khoury et al.]
[Balasubramanian et al.]
[Nekrasov]
[Simon]
[Liu et al.]
[Elitzur et al.]
[Craps et al.]
Stability:
[Lawrence]
[Liu et al.]
[Fabinger, McGreevy]
[Horowitz, Polchinski]

- Orientifold singularity where κ becomes null

$$\kappa \cdot \kappa = 0 \qquad \Rightarrow \qquad X^+X^- = -\frac{1}{E^2} \qquad\qquad \left(E \equiv \frac{\Delta}{R}\right)$$

- Compactify to IIA by choosing coordinates where $\kappa = 2\pi R \partial_y$

Natural coordinates in Milne and Rindler wedges : Polar Coordinates



I: Milne

$$EX^{\pm} = te^{\pm(z+y)} \qquad y = Y$$

II: Rindler

$$EX^{\pm} = \pm xe^{\pm(s+y)} \qquad y = Y$$

- Background fields of SUGRA:

Region I

$$E^2 ds^2 = \Lambda^{1/2}\left[-dt^2 + ds^2(\mathbf{T}^8)\right] + \frac{t^2}{\Lambda^{1/2}} dz^2$$

$$e^{\phi} = g_s \Lambda^{\frac{3}{4}} \qquad A = -\frac{1}{g_s E \Lambda} dz$$

$$\Lambda = 1 + t^2$$

Region II

$$E^2 ds^2 = \Lambda^{1/2}\left[dx^2 + ds^2(\mathbf{T}^8)\right] - \frac{x^2}{\Lambda^{1/2}} ds^2$$

$$e^{\phi} = g_s \Lambda^{\frac{3}{4}} \qquad A = -\frac{1}{g_s E \Lambda} ds$$

$$\Lambda = 1 - x^2$$

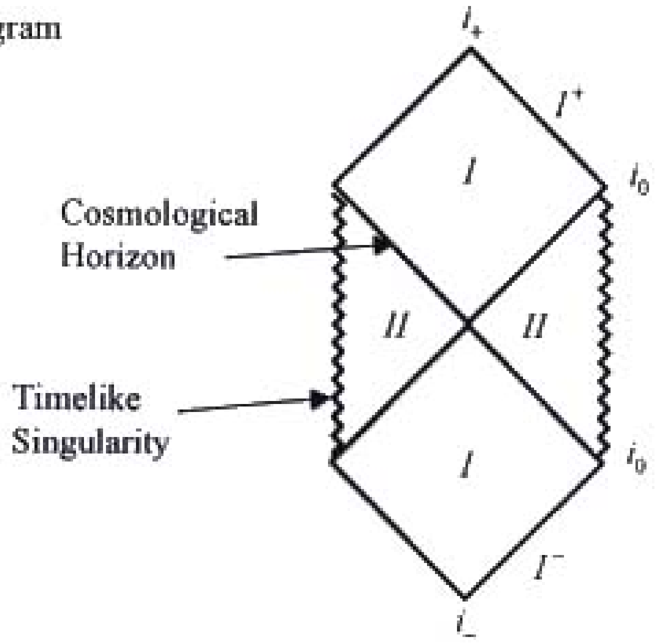
- 2D model with contraction for $t < 0$ and expansion for $t > 0$

- **$O\overline{O}$ pair delocalized on T^8**

- **T-duality** along all the T^8 directions gives a **$O8$-$\overline{O8}$ pair at a distance L** in massive SUGRA

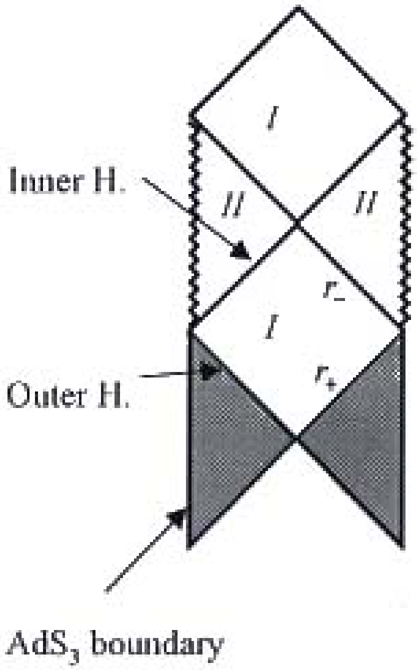
$$L = \frac{1}{E} \qquad 1 = N = \frac{l_s E}{g_s}$$

- Carter-Penrose Diagram



Similar 2D structure:
[Kounnas, Lust]
[Grojean et al.]
[Burgess et al.]

- 2D geometry as a limit of the BTZ black hole



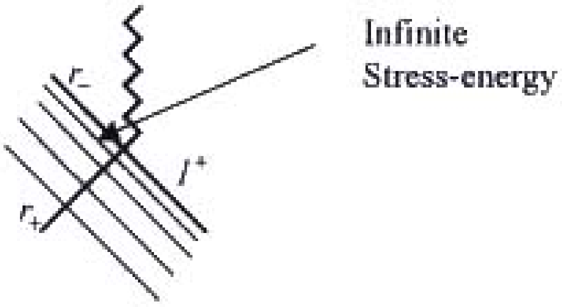
Limit of large AdS_3 radius $L \rightarrow \infty$ with

$$r_- = R \qquad r_+ = \Delta L \rightarrow \infty$$

Region *outside the black hole* is removed in the limit. We have

$$r_+ \rightarrow I^-$$

- Penrose-Simpson instability of inner horizon



These modes do not exist when

$$r_+ \rightarrow I^-$$

- Instability of orbifold models $\nRightarrow$ Instability of orientifold geometry

Excision of region after singularity & boundary condition at singularity

Perturbative String Description

- O8-plane BPS configuration: Type IIA on

$S^1/I\,\Omega \longleftarrow$ World-sheet parity

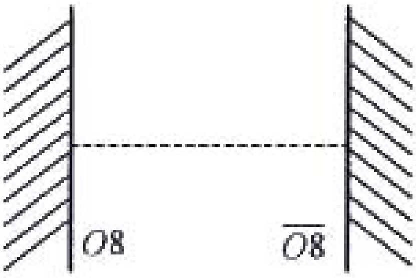
$\nwarrow$ Reflection

- $O8-\overline{O8}$ non-BPS configuration: Type IIA on

[Antoniadis et al.]
[Kachru et al.]

$S^1/G \longleftarrow$ G generated by

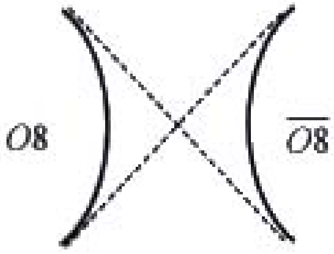
$$g_1 = I\,\Omega$$
$$g_2 = I\,\Omega(-)^F\delta \qquad \delta : x \to x + \pi R$$



g_1 and g_2 break the opposite half of SUSY

- Usually one adds D-branes for **Tadpole cancellation**.

- Gravity solution corresponds to the **backreaction** of closed strings to the O-planes (at the end-point of the annihilation of the D-branes)



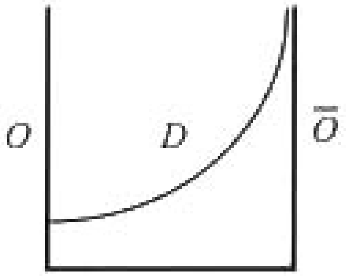
- Question: what is dynamics **behind** such configuration?

Orientifold Repulsion

Naïve point of view : the $O8-\overline{O8}$ should ***attract and annihilate***

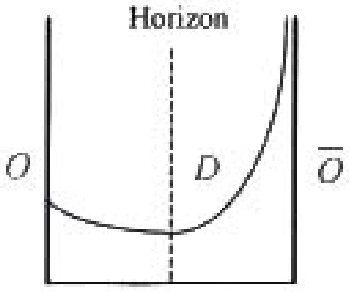
Gravity solution shows ***repulsion***

- Consider a D-brane probe between $O\overline{O}$ pair and calculate static potential.



Naïve potential without backreaction

- OD is SUSY
- $\overline{O}D$ repel



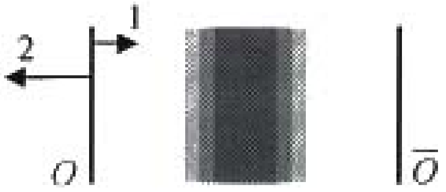
Complete potential

D-brane attracted to core of geometry

⇩

Positive energy density at the core

- Orientifolds are repelled by energy density at the core of geometry.



Energy	-	+	-
Charge	-	0	+

Two competing forces:

1. $O\overline{O}$ attraction
2. Repulsion from energy density at the core

Second force wins !

Cosmological Thermal Radiation

- Consider **massless scalar** in 2D model.

  Laplace equation $\nabla^2\chi = 0$ gives the mode decomposition

  Region I: $e^{ipz} J_{\pm ip}(pt)$

  Region II: $e^{ips} J_{\pm ip}(ipx)$

[Tolley, Turok]

- Orientifold **boundary conditions** at singularity

$$\chi = 0 \qquad \text{or} \qquad \partial_x \chi = 0$$

- Assume **trivial vacuum in the far past**. Expansion of the field is

$$\chi_{\text{in}} = \frac{1}{\sqrt{p\,|t|}} e^{ip(t+z)}$$

In the future one has (using continuity at the horizon + boundary condition) non-trivial Bogolubov coefficients

$$\chi_{\text{out}} = \frac{1}{\sqrt{p\,|t|}} \left[\alpha\, e^{ip(t+z)} + \beta\, e^{ip(-t+z)}\right]$$

Observer in the expanding region detects an average number of particles of momentum p given by

$$\langle N(p) \rangle = |\beta|^2 = \frac{1}{e^{p/\tau(p)} - 1}$$

- For large p the dimensionless temperature is (holds also for d>1)

$$\tau(p) \approx \frac{1}{2\pi}$$

- $\frac{\kappa}{2\pi}$ where $\kappa = 1$ surface gravity of the horizon

- Acceleration of singularity at $E^2 X^+ X^- = -1$
  Accelerating mirror

- At late cosmological time, due to the large red-shift, one obtains an exactly thermal spectrum with *physical* temperature

$$T \equiv \frac{E}{2\pi\, a(t)}$$

Expected for radiation in (d+1) – dimensional FRW geometry

$$\rho \approx T^{d+1}$$

$$\rho \approx \frac{1}{a} \cdot \frac{1}{a^d}$$

Redshift Expansion

Open Problems

1. Gravity solutions
    - Similar causal structure. How generic ?
    - Gravity + negative-tension boundary conditions ⇨ Presence of horizon ?
2. CFT
    - Precise relation with a CFT description (rolling tachyon in $D\overline{D}$ annihilation)
3. Cosmology
    - Make a "realistic" model with a more complex compactification and an exit to standard radiation domination.
    - Revisit standard literature allowing for the possibility of a cosmological horizon replacing the big-bang singularity.