Resolving Black Holes Using AdS/CFT

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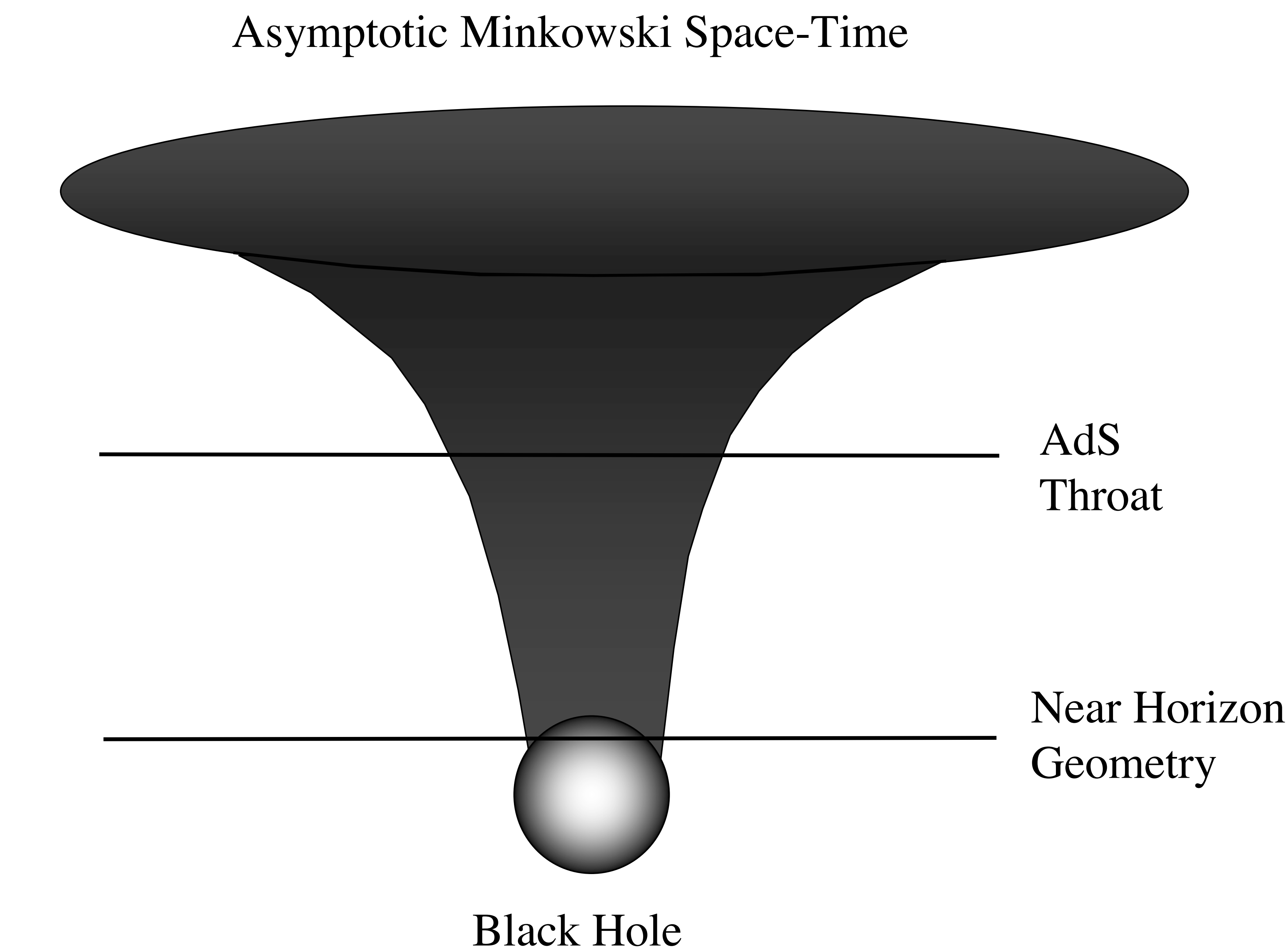
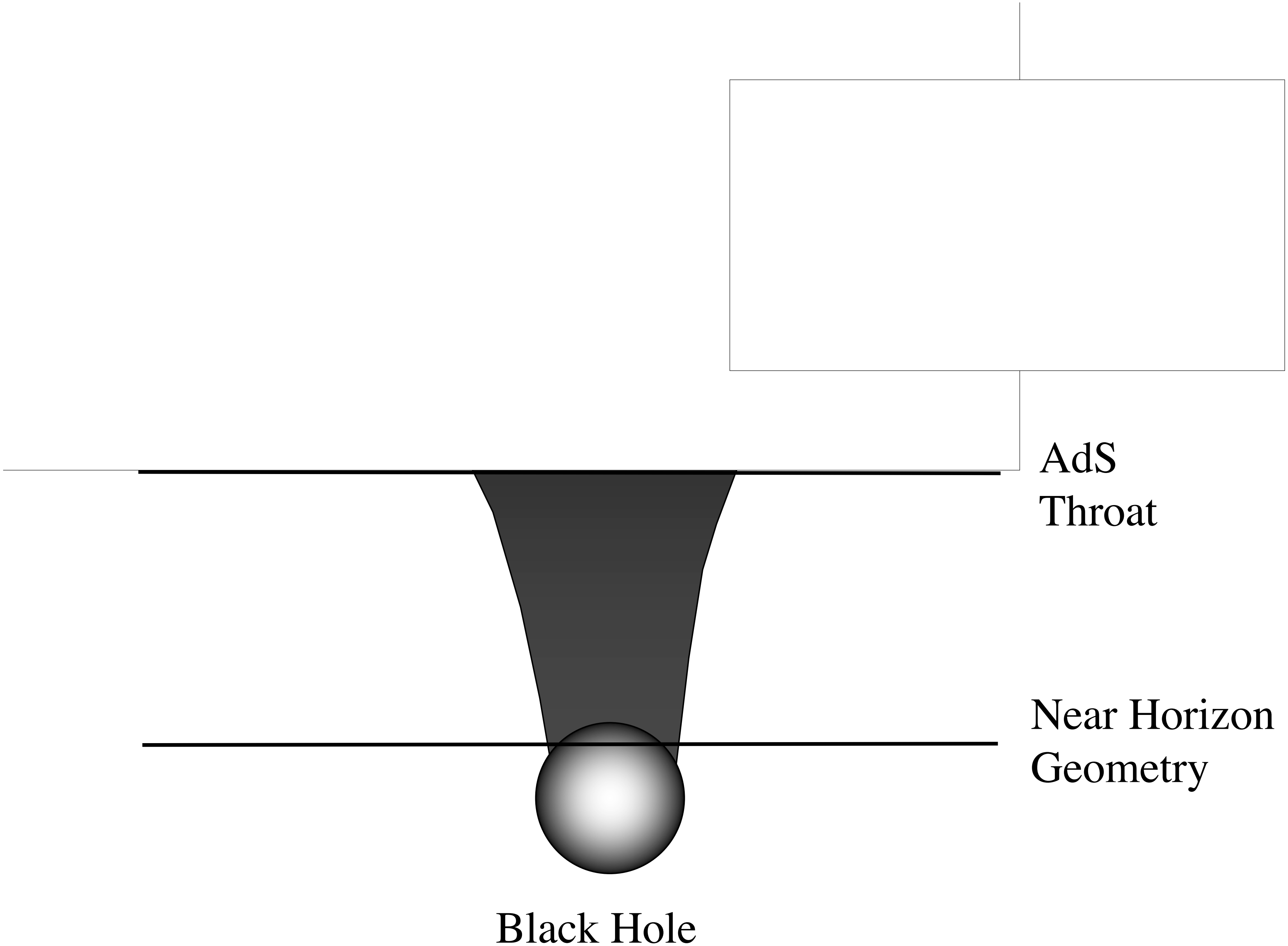
Based mainly on:

hep-th/0508023 - Vijay Balasubramanian, JdB, Vishnu Jejjala, Joan Simon
hep-th/0511246 - Luis F Alday, JdB, Ilies Messamah
hep-th/0607222 - Luis F Alday, JdB, Ilies Messamah
to appear - JdB, Sheer El-Showk, Ilies Messamah


Asymptotic Minkowski Space-Time
AdS
Throat
Near Horizon
Geometry
Black Hole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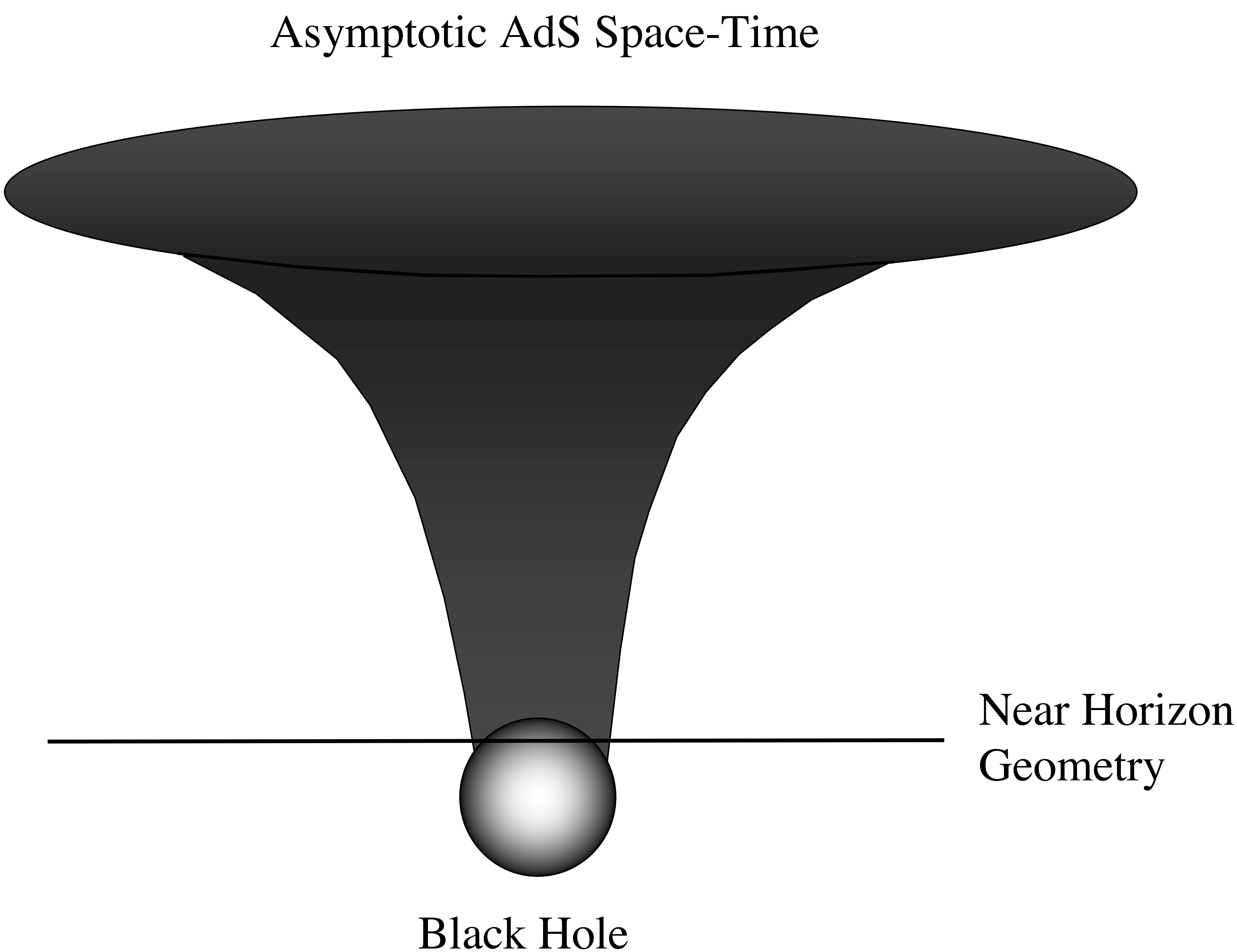

Asymptotic AdS Space-Time
Near Horizon
Geometry
Black Hole

After this decoupling limit, the black hole is embedded in a proper theory of quantum gravity.

Since the black hole describes a normalizable deformation of AdS, it should be dual to a state or density matrix in the dual CFT

BULK		BOUNDARY
$\exp(-S_{\text{bulk}}^{\text{on shell}})$	$\Longleftrightarrow$	$\text{Tr}\,[\rho\mathcal{O}_1\ldots\mathcal{O}_n] = \langle\mathcal{O}_1\ldots\mathcal{O}_n\rangle_\rho$
classical geometries	$\Longleftrightarrow$	semiclassical states; definition = what?
black hole	$\Longleftrightarrow$	$\rho = \sum e^{-\beta E}\|E\rangle\langle E\|$
black object, entropy S	$\Longleftrightarrow$	S= -Tr$(\rho\log\rho)$
bulk has isometry D	$\Longleftrightarrow$	$[\rho, \hat{D}] = 0$
ADM quantum number associated to D	$\Longleftrightarrow$	$\text{Tr}(\rho D) = \langle\hat{D}\rangle = D_{ADM}$

This is a map between quantum states and classical objects (geometries).

A well-known example of such a map is the map between quantum states and their corresponding classical phase space densities.

It is precisely this map that will allow us to map states into geometries in the ½ BPS sector of AdS_3 and AdS_5.

Phase space distributions (Wigner: arXiv:3303.0010)

For given ρ, $w_\rho(p,q)$ is defined by requiring that for all operators A:

$$\int dp\, dq\, w_\rho(p,q) A(p,q) = \mathrm{Tr}(\rho A(\hat{p},\hat{q}))$$

operator ordering

-Weyl ordering : Wigner distribution $w(p,q) \sim \int dy\, \langle q-y|\rho|q+y\rangle e^{2ipy}$

-Reverse normal ordering: Husimi distribution

Semi-classical state: $w_\rho(p,q)$ is independent of the choice of ordering prescription (as $N \to \infty, \hbar \to 0$).

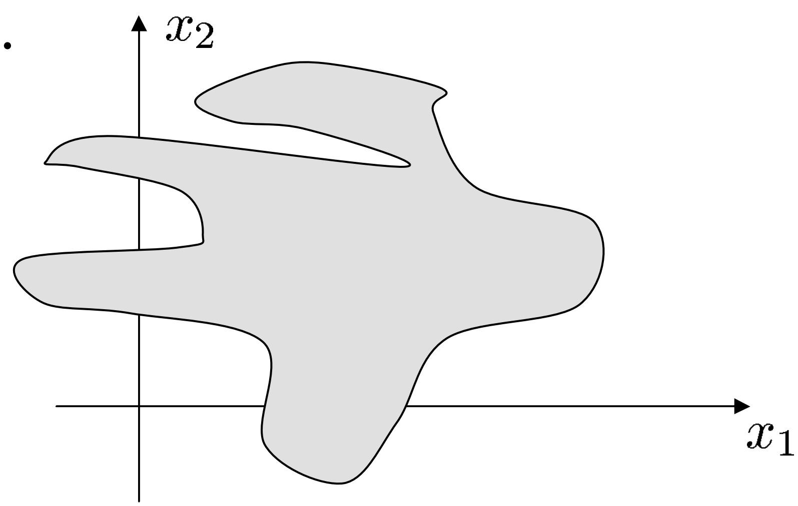
½-BPS states in AdS_5 – geometries classified by Lin, Lunin and Maldacena

$$ds^2 = -h^{-2}(dt + V_i dx^i)^2 + h^2(d\eta^2 + dx^i dx^i) + \eta e^G d\Omega_3^2 + \eta e^{-G} d\tilde{\Omega}_3^2$$

$$h^{-2} = 2\eta \cosh G, \quad \eta \partial_\eta V_i = \eta_{ij} \partial_j V, \quad \eta(\partial_i V_j - \partial_j V_i) = \epsilon_{ij} \partial_\eta z$$

$$z = \tfrac{1}{2} \tanh G, \quad z(\eta, x_1, x_2) = \frac{\eta^2}{\pi} \int dy_1 dy_2 \frac{\frac{1}{2} - u(y_1, y_2)}{[(x-y)^2 + \eta^2]}$$

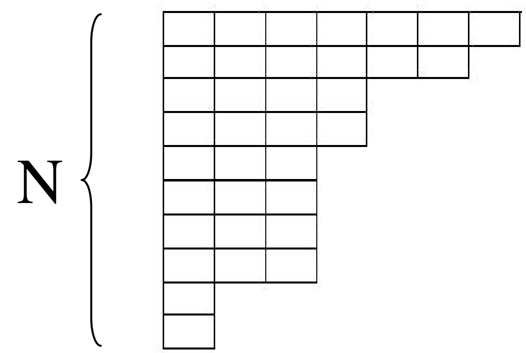
Smooth geometries: $u \in \{0, 1\}$. Function u defines a droplet in the x_1, x_2-plane.



½-BPS states in N=4 U(N) SYM theory: given by Hilbert space of N free fermions in a harmonic oscillator potential.

Corley, Jevicki, Ramgoolam
Berenstein

These can be conveniently enumerated in terms of Young diagrams with N rows.



By quantizing the phase space of smooth gravitational solutions we recover the fermionic description. This confirms that the x_1, x_2-plane is the same as the phase space for the harmonic oscillator.

Mandal
Grant, Maoz, Marsano, Papadodimas, Rychkov
Takayama, Tsuchiya

Balasubramanian, JdB, Jejjala, Simon

Proposal:

State ρ → One-particle phase space density $u(x_1, x_2)$

↓

Geometry

•Most states yield ambiguous`quantum foam' geometries with string scale curvature

•Semiclassical states yield well-defined but still mildly singular space-times

Milanesi, O'Loughlin

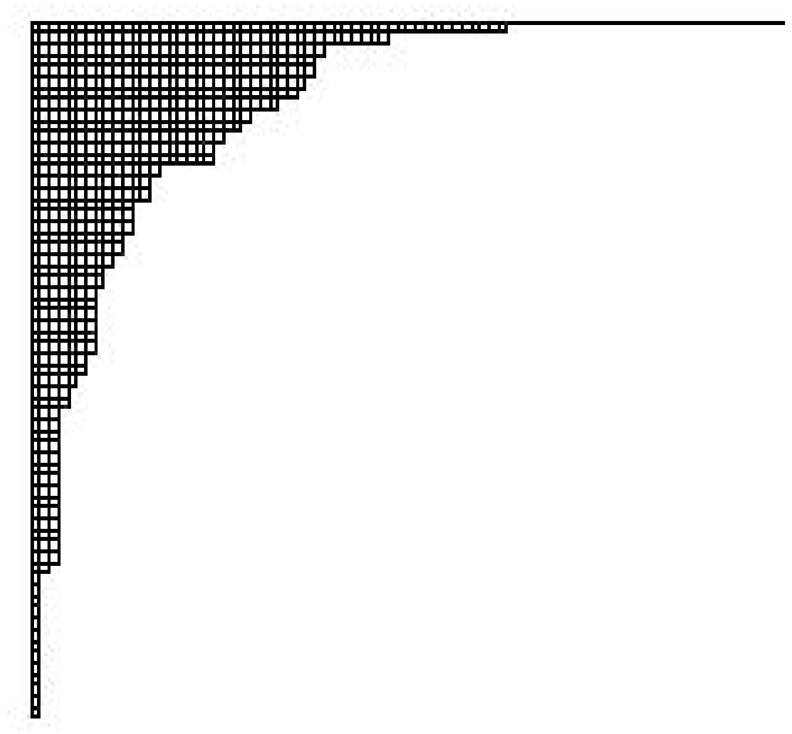
•Can add geometries. Summing (coarse graining) over all states yields the `1/2-BPS' black hole

For large N, all states will start to look the same and are difficult to distinguish from the ensemble average.

This is in perfect agreement with the idea that collapsed heavy pure states are difficult to distinguish from each other and all look like a black hole.

Relevant theorem: all Young diagrams approach with probability one a fixed limit shape (cf crystal melting)

Vershik



½-BPS configurations for $AdS_3 \times S^3 \times M_4$

½-BPS states: States in the Fock space of b_1+b_3 fermions and $b_0+b_2+b_4$ bosons with $L_0=N$. ($b_i = \dim H^i(M_4)$).

½-BPS geometries (Lunin-Mathur):

$$ds^2 = \frac{1}{\sqrt{f_1 f_5}}[-(dt+A)^2 + (dx_1+B)^2] + \sqrt{f_1 f_5}(dx_2^2 + \ldots + dx_5^2) + \sqrt{\frac{f_1}{f_5}} ds_M^2$$

$$f_5 = 1 + \frac{Q_5}{L}\int_0^L \frac{ds}{|\vec{x}-\vec{F}(s)|^2}, \quad f_1 = 1 + \frac{Q_5}{L}\int_0^L \frac{ds|\vec{F}(s)|^2}{|\vec{x}-\vec{F}(s)|^2}$$

$$A_i = \frac{Q_5}{L}\int_0^L \frac{F_i(s)ds}{|\vec{x}-\vec{F}(s)|^2}, \quad dB = *_4 dA, \quad Q_1 = \frac{Q_5}{L}\int_0^L |\vec{F}(s)|^2 ds$$

$\vec{F}(s)$ describes a curve in the $\mathbb{R}^4$ spanned by $x_2, \ldots, x_5$

Quantization of the phase space of classical smooth solutions of supergravity shows that the modes c^i_k of $\vec{F}(s)$

Donos, Jevicki
Rychkov

$$\vec{F}(s) = \mu \sum_{k=1}^{\infty} \frac{1}{\sqrt{2k}} \left(c^i_k e^{\frac{2\pi i k}{L} s} + c.c. \right)$$

become the creation and annihilation modes of the four bosons associated to $H^{(0,0)}(M), H^{(2,0)}(M), H^{(0,2)}(M), H^{(2,2)}(M)$

State ρ ⇒ phase space density $\mu(\vec{F}(s))$

Alday, JdB, Messamah

⇓

Geometry

$$f_5 = 1 + \frac{Q_5}{L} \int \mathcal{D}\vec{F}(s) \mu(\vec{F}(s)) \int_0^L \frac{ds}{|\vec{x} - \vec{F}(s)|^2} \qquad \text{etc.}$$

•Many results from AdS_5 carry over to this case; in particular, almost all states look identical.

•The original classical geometries correspond to point in phase space and are therefore given by coherent states.

•Coarse graining over all states with $L_0 = N$, or over all states weighted with $e^{-\beta L_0}$, yields the M=0 BTZ black hole.

•For M=0 BTZ find $f_1 = Q_1 \frac{1-e^{-\frac{3\beta}{\pi^2\mu^2}x^2}}{x^2}$

•The entropy of the associated stretched horizon scales like $N^{3/4}$ this is larger than the entropy $N^{1/2}$ of M=0 BTZ and disagrees with the result of Lunin-Mathur. What is the right notion of stretched horizon?

Richer set of possibilities: rotations in the $\mathbb{R}^4$ plane.

Denote by a_k^+, a_k^- the modes of two bosons with quantum numbers ±1 under some $U(1) \subset SO(4)$.

<u>Small rotating black hole</u>

$$\boxed{\rho \sim e^{-\beta L_0 - \mu J}}$$

$$J = \sum_{k>0} (a_k^-)^\dagger a_k^- - \sum_{k>0} (a_k^+)^\dagger a_k^+$$

Typical state: $\underbrace{(\text{random})}_{N-J}(a_{-1}^+)^J|0\rangle$

$(a_{-1}^+)^J$ → Bose-Einstein condensate

$N - J$ ↓

Entropy: $S \sim \sqrt{N-J}$

Small black ring

$$\rho \sim e^{-\beta L_0 - \mu J - \nu D}$$

$$D = \sum_{k>0} \frac{1}{k} (a_k^+)^\dagger a_k^+ + \ldots$$

Typical state: $\underbrace{(\text{random})}_{N - DJ} (a^+_{-D})^J |0\rangle$ → Bose-Einstein condensate

↓

Entropy: $S \sim \sqrt{N - DJ}$

Bena Kraus
Iizuka Shigemori
Balasubramanian Kraus Shigemori

•D: `dipole operator'. Its presence in the density matrix is supported by an analysis of the first law of thermodynamics.

Emparan; Copsey, Horowitz

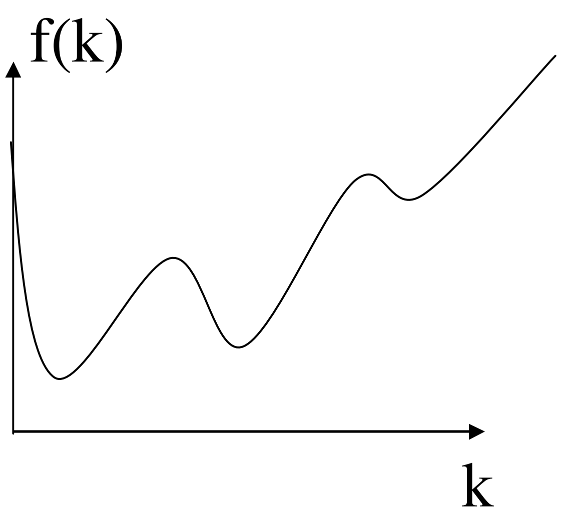
•Not a conserved charge, not clear how to extend definition to interacting theory (cf giant graviton number).

•Reminiscent of non-local conserved charge.

Even more general:

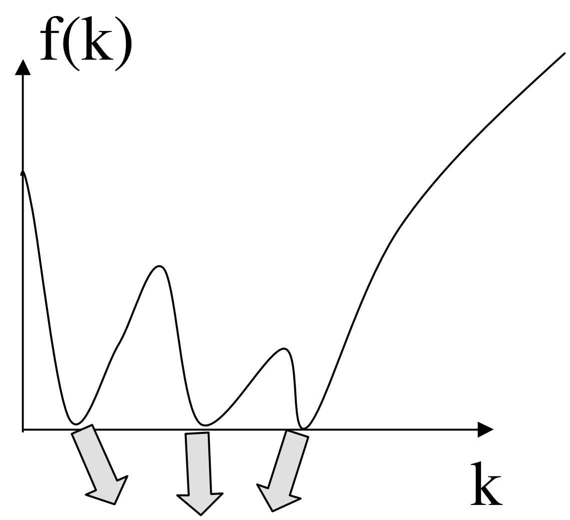
$$\rho = \exp(-\sum_{k>0} f(k)(a_k^+)^\dagger a_k^+ + \ldots)$$

Generic:



Metric depends only on N,J,D
Example of no hair

Fine-tuned:



Multiple Bose-Einstein condensation
Concentric black rings?

So far, no examples with macroscopic horizon. A lot of work has been done in order to find microstates in cases with less supersymmetry.

Giusto, Mathur, Saxena, Srivastava, Potvin, Peet, Ford, Elvang, Emparan, Mateos, Reall, Bena, Kraus, Warner, Wang, Lunin, Balasubramanian, Berglund, Gimon, Levi, Cheng…..

Not many of these are asymptotically AdS.

Denef, Gaiotto, Strominger, van den Bleeken, Yin

A large class of multi-center black holes/rings in $AdS_3 x S^2 x CY$, including many smooth solutions, can be obtained by taking a suitable decoupling limit of the 5D uplift of 4D multi-center BPS black hole solutions.

Denef
Bates, Denef

Gaiotto, Strominger, Yin

The 5D solutions are described in terms of harmonic functions

$$H = h + \sum_p \frac{\Gamma_p}{|\vec{r}-\vec{r}_p|} \in H^{\text{even}}(\text{CY},\mathbb{R})$$

where Γ_p labels the (D0,D2,D4,D6) charges of each of the centers.

Denef, Moore
→ Moore's talk

If the total D6-brane charge vanishes, we can take a decoupling limit where the 11d Planck length is sent to zero, while keeping the size of the 11^{th} dimension, the masses of stretched membranes, and the size of the CY in 11d Planck units fixed.

JdB, El-Showk, Messamah

In this limit, h→0, except for $h \in H^6(CY,\mathbb{R})$ which remains fixed. This constant is crucial in order to have a space which is asymptotically AdS_3.

Write charges as $\Gamma_p = (\Gamma_p^0, \Gamma_p^A, \Gamma_{p,A}, \Gamma_{p,0}) \in (H^0, H^2, H^4, H^6)$

Define a pairing $\langle \Gamma, \Theta \rangle = \int_{CY} (\Gamma_0 \Theta^0 - \Gamma_A \Theta^A + \Gamma^A \Theta_A - \Gamma^0 \Theta_0)$

Then asymptotic AdS_3 has mass and SU(2) angular momentum:

$$M = \tfrac{1}{2}\Gamma_0 - \tfrac{1}{4}\Gamma_A (d_{ABC}\Gamma^C)^{-1}\Gamma_B$$

$$\vec{J} = \tfrac{1}{2} \sum_{p \neq q} \langle \Gamma_p, \Gamma_q \rangle \frac{\vec{r}_p - \vec{r}_q}{|\vec{r}_p - \vec{r}_q|}$$

These define the quantum numbers in the dual N=(0,4) CFT.

Maldacena, Strominger, Witten

Consistency condition:

$$\langle h, \Gamma_p \rangle + \sum_{q \neq p} \frac{\langle \Gamma_q, \Gamma_p \rangle}{|\vec{r}_q - \vec{r}_p|} = 0$$

Because $h_0 \neq 0$ centers that carry non-trivial D6-brane charge cannot be moved all the way to the boundary: honest bound states, part of the Higgs branch.

Cf: giant gravitons in AdS_3 can be moved all the way to the boundary.

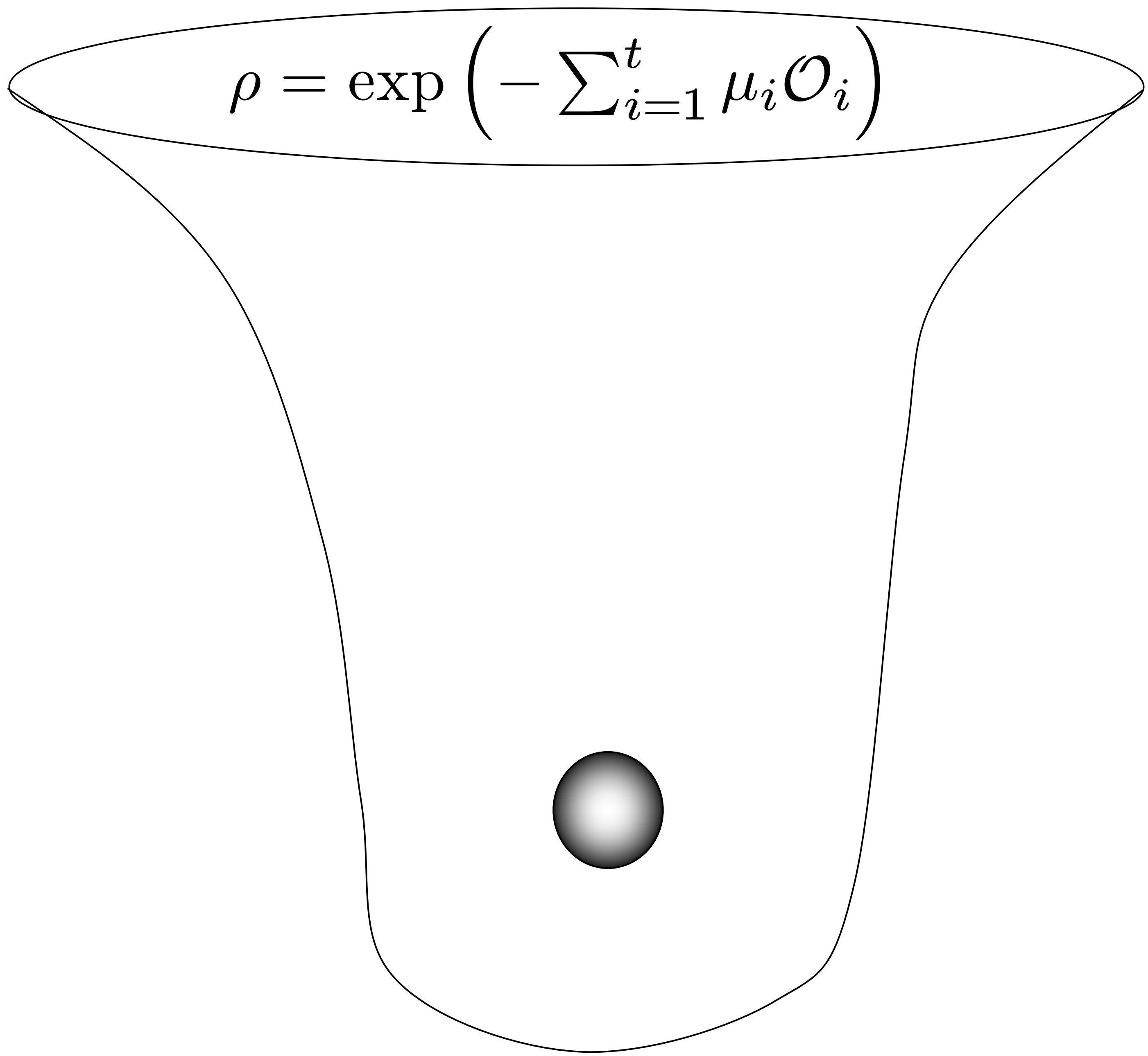
Simplest example: two-centered solution, one with D6-brane charge +1, the other with D6-brane charge -1. These are distinguished by their angular momentum.

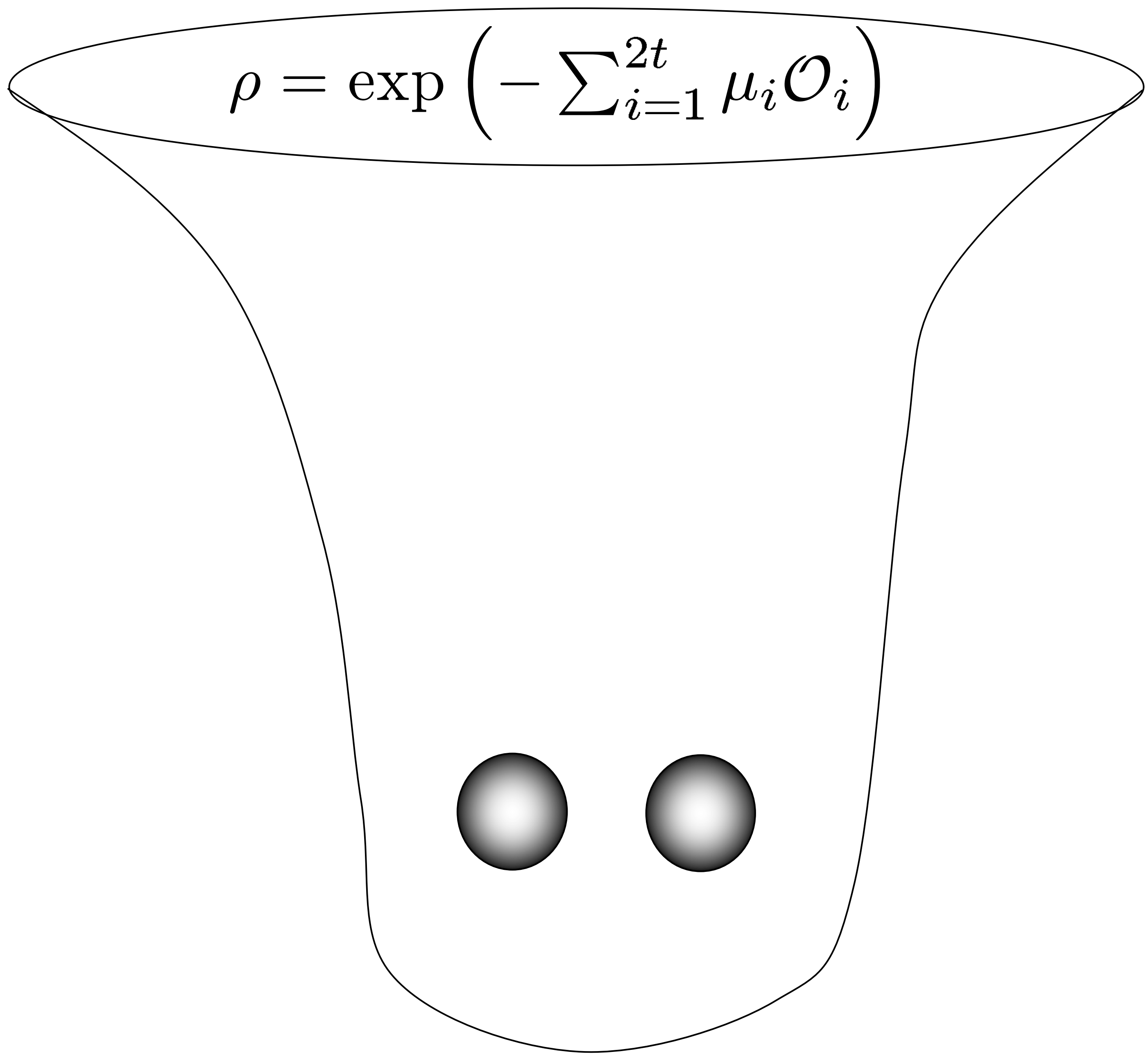
Relevant for `entropy enigma', computation of elliptic genus of the CFT and connection to OSV.

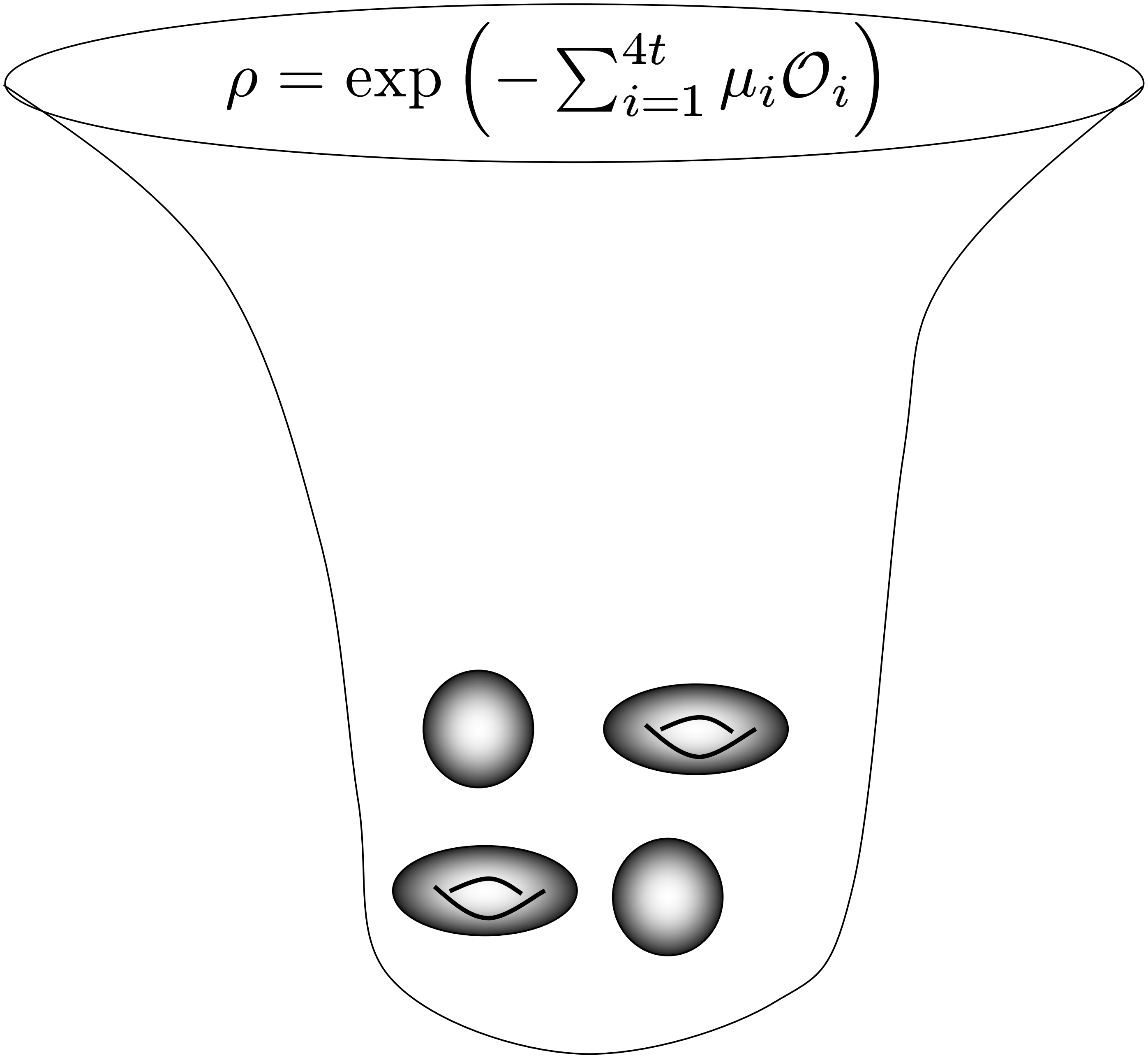
Gaiotto, Strominger, Yin
de Boer, Cheng, Dijkgraaf, Manschot, Verlinde
Denef, Moore

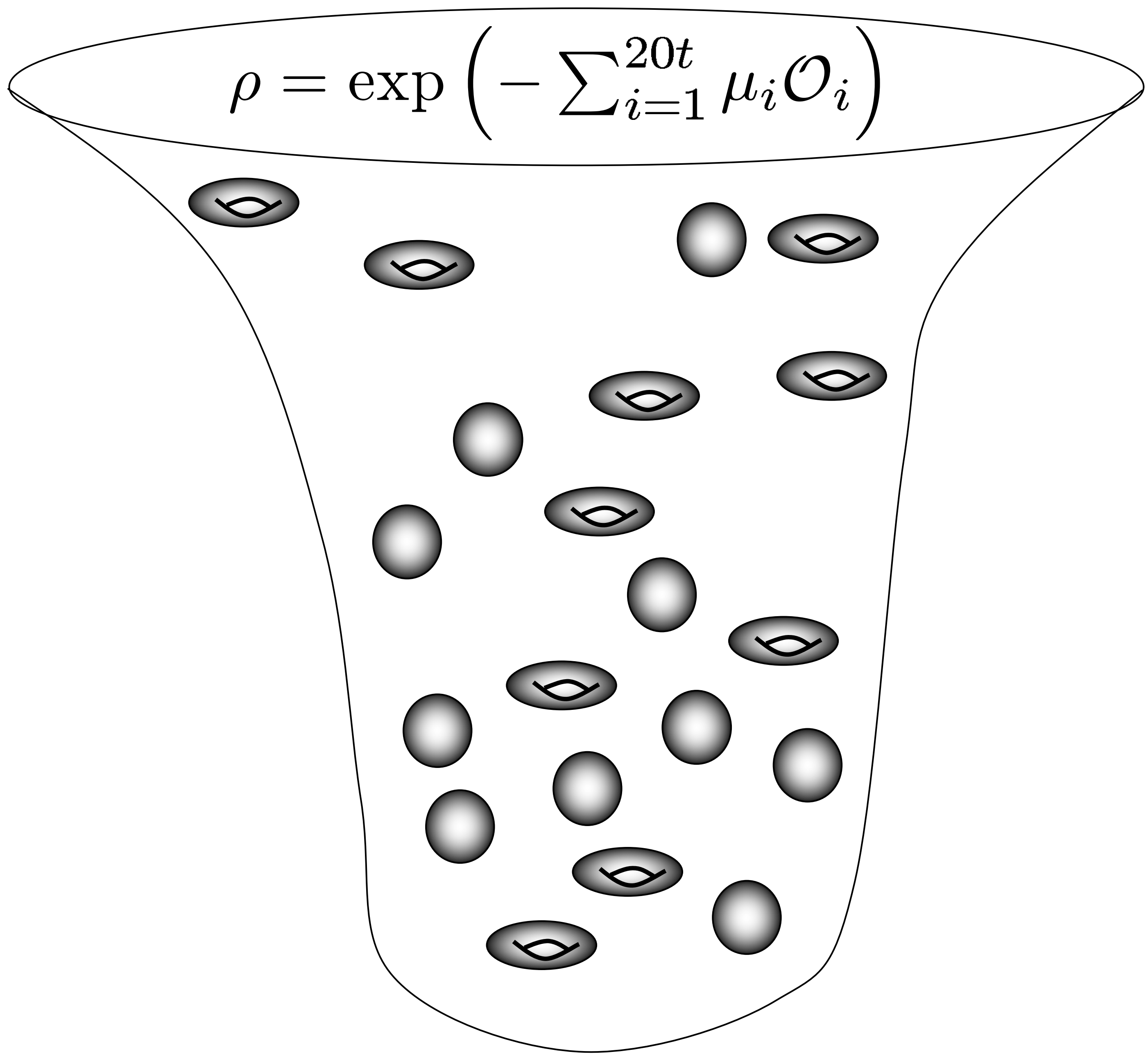
Dual description? Hint: look at thermodynamics. Naively looks like one needs $N(2b_2 + 2)$ potentials for the N-center case.

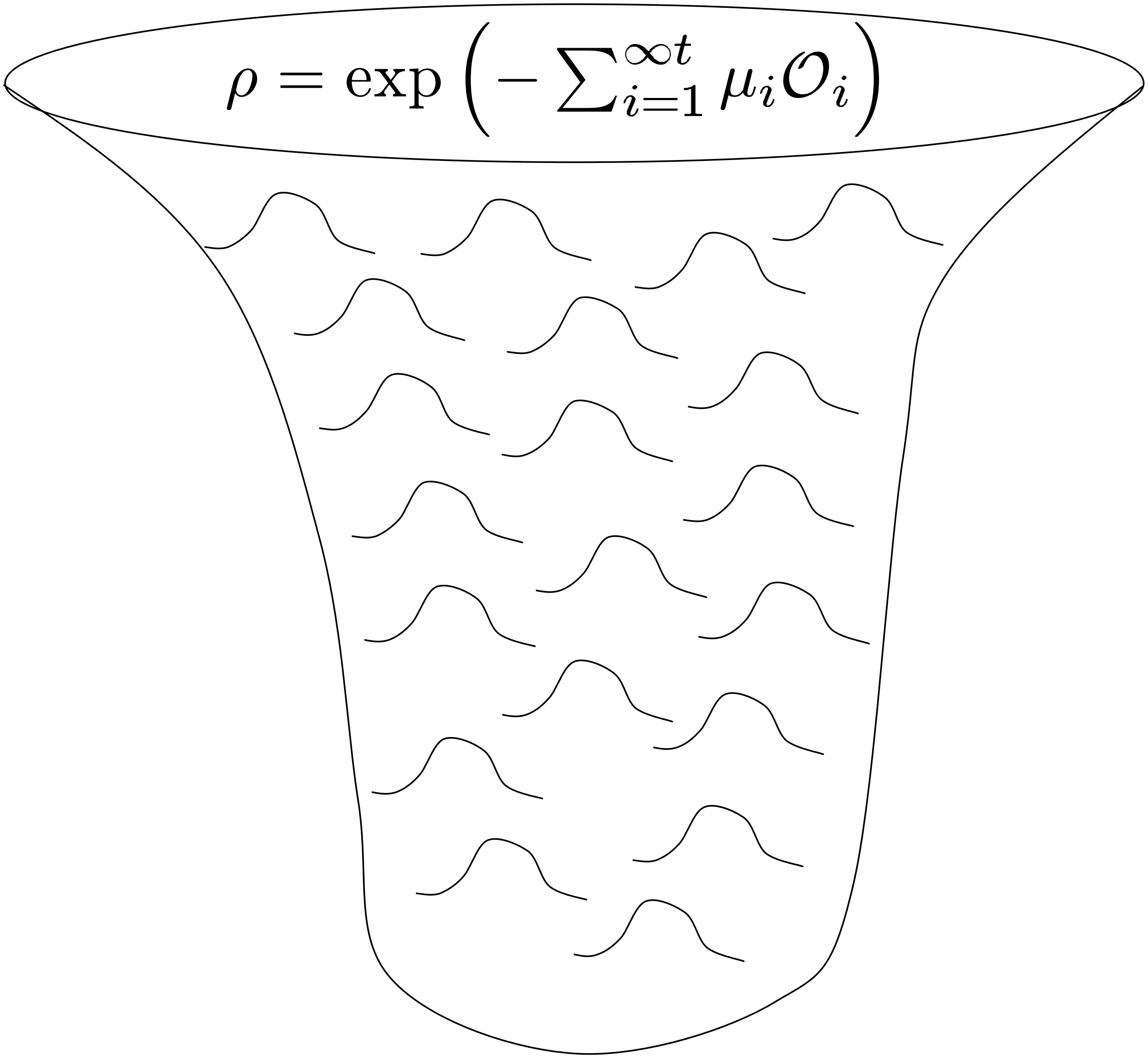
This suggests the following picture:

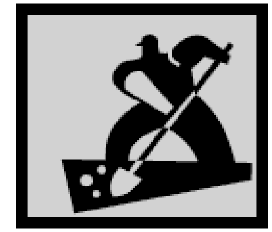

$\rho = \exp\left(-\sum_{i=1}^{t} \mu_i \mathcal{O}_i\right)$


$\rho = \exp\left(-\sum_{i=1}^{2t} \mu_i \mathcal{O}_i\right)$


$\rho = \exp\left(-\sum_{i=1}^{4t} \mu_i \mathcal{O}_i\right)$


$\rho = \exp\left(-\sum_{i=1}^{20t} \mu_i \mathcal{O}_i\right)$


$\rho = \exp\left(-\sum_{i=1}^{\infty t} \mu_i \mathcal{O}_i\right)$



TO DO:

•Explore space of solutions.

•How fine tuned are the multi-center solutions?

•Analyze space of smooth solutions. Phase space large enough to account for (a finite fraction) of the entropy of a macroscopic black hole?? Or are these smooth solutions all highly atypical?

•Connection and corrections to OSV, implications for the partition function of the dual CFT.

•Understand `split RG flows’.

•Understand BMPV black hole in this context.

SUMMARY: RESOLVING THE BLACK HOLE

•Gravity is thermodynamic, a black hole is a thermodynamic description of an underlying large set of microstates.

•Almost all states in the dual ensembles are typical and very difficult to distinguish from each other (no hair)

•By adding more operators and potentials to the ensemble we can describe more elaborate black objects.

•In the limit where we include infinitely many operators the ensemble can become a coherent state, and by that time the dual geometry has become smooth and horizonless.