APPLICATIONS OF THE COVARIANT FORMALISM FOR THE SUPERSTRING AND SUPERMEMBRANE

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Available superstring formalisms:

Light-cone GS:

- Good for on-shell spectrum
- Bad for scattering amp's because of light-cone op's.
- Consistency conditions for background?

Covariant RNS:

- Good for NS backgrounds but not for R backgrounds.
- Spacetime SUSY not manifest.
- Does not generalize to supermembrane.

Covariant GS:

- Classically OK, but has quantization problems due to second-class constraints.
- Not quadratic in flat background.

Alternative approach:

Light-cone GS

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$SU(4)\times U(1)$ formalism
1992-1994

- Manifest worldsheet $N=2$ superconf. inv.
- Related by field redef. to RNS formalism where $G^+ \to j_{BRST}$ and $G^- \to b$.

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Hybrid formalism
1994-2000

- Useful for compactification on CY

Vafa, Siegel, Witten,
Bershadsky, Hauer, Zhukov, Zwiebach
Vallilo, Gukov

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Pure Spinor formalism
2000-2002

- $SO(9,1)$ super-Poincaré inv. is manifest.
- Worldsheet symmetries described by BRST operator.

Vallilo, Chandía, Howe, Pershin

Tonin, Oda, Sorokin, Matone, Mazzucato; Grassi, van Nieuwenhuizen, Policastro, Porrati

Applications of covariant formalism

Pure spinor formalism:

- Massive superstring states in $D=10$ superspace (NB + O. Chandía)
- Tree amplitudes with manifest $SO(9,1)$ super-Poincaré inv. (NB + B.C. Vallilo)
- Superstring corrections to super-Maxwell and supergravity eqn's in superspace (NB + P. Howe , NB + V. Pershin)
- Generalization to supermembrane and $SO(10,1)$ Covariant M(atrix) theory (work in progress)

$SU(4) \times U(1)$ formalism:

- Actions in plane wave R-R backgrounds with manifest worldsheet $N=2$ superconf. inv. (NB + J. Maldacena)

Pure Spinor Formalism

$$S = \int d^2z \left[\tfrac{1}{2} \partial x^m \bar{\partial} x_m + p_\alpha \bar{\partial} \theta^\alpha + w_\alpha \bar{\partial} \lambda^\alpha \right]$$

$m = 0$ to 9, $\alpha = 1$ to 16

λ^α are ghost variables satisfying the pure spinor constraint $\boxed{\lambda \gamma^m \lambda = 0}$

λ^α has 11 indep. (holomorphic) components

$\Rightarrow$ central charge $= 10 - 2\cdot 16 + 2 \cdot 11 = 0$

Physical states are ghost-number $+1$ states in the cohomology of

$$\boxed{Q = \int dz\, \lambda^\alpha d_\alpha}$$

$d_\alpha = p_\alpha + \partial x_m (\gamma^m \theta)_\alpha$ is the GS supersymmetric Dirac constraint.

$$\boxed{\lambda \gamma^m \lambda = 0 \Rightarrow Q^2 = 0}$$

Why does this work? (unpublished)

- Start with GS superparticle in "semi-light-cone" gauge $(\gamma^+ S)_\alpha = 0$

$$S = \int d\tau [\tfrac{1}{2}\dot{x}^m \dot{x}_m + (S\gamma^- S) P^+ + b\dot{c}]$$

with BRST operator $Q = cP^m P_m$

- Add $[\theta^\alpha, p_\alpha]$ and unconstrained $[\lambda^\alpha, w_\alpha]$

$$S = \int d\tau [\tfrac{1}{2}\dot{x}^m \dot{x}_m + (S\gamma^- S) P^+ + b\dot{c} + p_\alpha \dot{\theta}^\alpha + w_\alpha \dot{\lambda}^\alpha]$$

with $Q = cP^m P_m + \lambda^\alpha p_\alpha$

- Perform unitary transf. so that

$$Q = cP^m P_m + \lambda^\alpha (p_\alpha + (\not{P}\theta)_\alpha + (\not{P} S)_\alpha) + b\lambda\gamma^+\lambda$$

- Use gauge inv's of Q to gauge-fix

$$\lambda\gamma^+\gamma^j\gamma^- S = \lambda\gamma^m\lambda = c = 0$$

$$\Rightarrow S = \int d\tau [\tfrac{1}{2}\dot{x}^m \dot{x}_m + p_\alpha \dot{\theta}^\alpha + w_\alpha \dot{\lambda}^\alpha]$$

with $Q = \lambda^\alpha d_\alpha$ and $\lambda\gamma^m\lambda = 0$

Pure spinor "covariantly" chooses $\dfrac{SU(4)\times U(1)}{SO(8)}$

Vertex operators (unintegrated)

Massless: $\Phi = \lambda^\alpha A_\alpha(x,\theta)$

$Q\Phi = 0 \Rightarrow \lambda^\alpha \lambda^\beta D_\alpha A_\beta = 0$ $\qquad D_\alpha = \frac{\partial}{\partial\theta^\alpha} + (\gamma^m\theta)_\alpha \partial_m$

$\Rightarrow \underline{D\gamma^{mnpqr}A = 0}$

$\delta\Phi = Q\Lambda = \lambda^\alpha D_\alpha \Lambda \Rightarrow \underline{\delta A_\alpha = D_\alpha \Lambda}$

$\Rightarrow$ super-Maxwell eqns. of motion and gauge inv.

$A_\alpha(x,\theta) = a_m(x)(\gamma^m\theta)_\alpha + \chi^\beta(x)(\gamma^m\theta)_\alpha(\gamma_m\theta)_\beta + \dots$

First massive: $\Phi = \lambda^\alpha \partial\theta^\beta B_{\alpha\beta}(x,\theta)$

$+ \lambda^\alpha \partial X^m C_{\alpha m}(x,\theta) + \lambda^\alpha p_\beta D_\alpha^\beta(x,\theta) + \lambda^\alpha\lambda^\beta \omega_\gamma E^\gamma_{\alpha\beta}(x,\theta)$

$Q\Phi = 0$
and
$\delta\Phi = Q\Lambda$

$\Rightarrow$ Massive spin 3/2 multiplet superspace eqns. of motion and gauge inv.

$C_{\alpha m}(x,\theta) = \chi_{\alpha m}(x) + (\gamma^n\theta)_\alpha g_{mn}(x) + (\gamma^{npq}\theta)_\alpha \partial_n b_{mpq}(x) + \dots$

(NB + O. Chandía)
hep-th/0204121

Super-Poincaré inv. tree amplitudes

Integrated dim. 1 vertex op's V are related to unintegrated dim. 0 vertex op's Φ by $\boxed{QV = \partial\Phi}$,

e.g. $V_{massless} = \partial\theta^\alpha A_\alpha + \partial X^m A_m + d_\alpha W^\alpha + (\lambda\gamma^{mn} w) F_{mn}$.

N-point tree amplitude prescription:

$$\mathcal{A} = \langle \Phi_1(z_1)\, \Phi_2(z_2)\, \Phi_3(z_3) \int dz_4\, V_4(z_4) \ldots \int dz_N\, V_N(z_N) \rangle$$

where $\langle (\lambda\gamma^m\theta)(\lambda\gamma^n\theta)(\lambda\gamma^p\theta)(\theta\gamma_{mnp}\theta) \rangle \equiv 1$.

Justification:

- Functional integral over 11 λ^α zero modes cancels 11 of 16 θ^α zero modes.
- For massless Φ, $\langle \Phi Q \Phi \rangle$ = super-Maxwell action.
- Amplitude agrees with RNS amplitude for external massless states with ≤ 4 fermions

(NB + B.C. Vallilo, hep-th/0004171)

Open superstring corrections to super-Maxwell equations in superspace

$$S_{open} = S_{flat} + \int d\tau\, V^{open}_{massless}$$

$$= \int d^2z \left[\tfrac{1}{2}\partial x^m \bar\partial x_m + p_\alpha \bar\partial \theta^\alpha + \bar p_\alpha \partial \bar\theta^\alpha + w_\alpha \bar\partial \lambda^\alpha + \bar w_\alpha \partial \bar\lambda^\alpha\right] + \int d\tau \left[\partial\theta^\alpha A_\alpha + \partial X^m A_m + d_\alpha W^\alpha + (\lambda\gamma^{mn} w) F_{mn}\right]$$

Need to choose boundary conditions for worldsheet variables such that

(a) Surface term eqns. of motion vanish

and (b) $\lambda^\alpha d_\alpha = \bar\lambda^\alpha \bar d_\alpha$ on boundary

To lowest order, (a) and (b) imply that $[A_\alpha, A_m, W^\alpha, F_{mn}]$ satisfy supersymmetric Born-Infeld equations in $D=10$ superspace.

Can also compute higher-derivative corrections to these superspace equations.

(NB + V. Pershin, hep-th/0205154)

Closed superstring corrections to supergravity equations in superspace

$$S^{heterotic}_{closed} \simeq S_{flat} + \int d^2z\, V^{closed}_{massless} + \alpha' \int d^2z\, r\, \Phi(x,\theta)$$

$$= \int d^2z \left[(G_{MN} + B_{MN}) \partial Y^M \bar\partial Y^N + E^\alpha_M d_\alpha \bar\partial Y^M \right.$$

$$\left. + A^I_M \partial Y^M \bar J^I + \omega^{ab}_M (\lambda\gamma_{ab} w) \bar\partial Y^M \right] + \alpha' \int d^2z\, r\, \Phi$$

$Y^M = (x^m, \theta^\alpha)$, E^A_M = super-vierbein , ω^{ab}_M = connection

$\bar J^I = E_8 \times E_8$ currents , A^I_M = gauge field

Background superfields must be chosen such that (a) $\bar\partial(\lambda^\alpha d_\alpha) = 0$

(b) $Q^2 = (\int \lambda^\alpha d_\alpha)^2 = 0$

To lowest order, (a) and (b) imply supergravity/super-YM eqns. in N=1 D=10 superspace.

Can compute α' corrections.

Generalizes to IIA/IIB in N=2 D=10 superspace.

(NB + P. Howe, hep-th/0112160)

Can quantize action in $AdS_5 \times S^5$ background but is not solvable (yet).

"Pure" spinors in D = 11

(NB, hep-th/0201151)

D=11 superparticle:

$$S = \int d\tau \left[\tfrac{1}{2} \dot{x}^{\underline{m}} \dot{x}_{\underline{m}} + p_{\underline{\alpha}} \dot{\theta}^{\underline{\alpha}} + w_{\underline{\alpha}} \dot{\lambda}^{\underline{\alpha}} \right]$$

with $\lambda \gamma^{\underline{m}} \lambda = 0$ $\quad \underline{m} = 0$ to 10, $\underline{\alpha} = 1$ to 32

$\lambda^{\underline{\alpha}}$ has 23 indep. (holomorphic) components

Physical states are ghost-number +3 states in cohomology of $\boxed{Q = \lambda^{\underline{\alpha}} d_{\underline{\alpha}}}$

$$d_{\underline{\alpha}} = p_{\underline{\alpha}} + \dot{x}_m (\gamma^m \theta)_{\underline{\alpha}}$$

g.n. = 1 ↔ open string, g.n. = 2 ↔ closed string, g.n. = 3 ↔ membrane

$$\text{g.n.} = 3 \Rightarrow \Phi = \lambda^{\underline{\alpha}} \lambda^{\underline{\beta}} \lambda^{\underline{\gamma}} B_{\underline{\alpha}\underline{\beta}\underline{\gamma}}(x, \theta)$$

$Q\Phi = 0$, $\delta\Phi = Q\Lambda$ $\Rightarrow B_{\underline{\alpha}\underline{\beta}\underline{\gamma}}$ describes linearized D=11 supergravity

$$B_{\underline{\alpha}\underline{\beta}\underline{\gamma}}(x,\theta) = (\gamma^m \theta)_{\underline{\alpha}} (\gamma^n \theta)_{\underline{\beta}} (\gamma^p \theta)_{\underline{\gamma}}\, b_{mnp}(x) + \ldots$$

$\langle \Phi Q \Phi \rangle$ = linearized D=11 supergrav. action

where $\langle \lambda^7 \theta^9 \rangle = 1$.

$\underline{D=11}$ $\underline{\text{supermembrane}}$:

After double-dimensional reduction, action should reduce to IIA superstring action with pure spinors.

$$\Theta^{\underline{\alpha}} \nearrow \theta^{\alpha} \quad \searrow \bar{\theta}_{\alpha} \qquad \lambda^{\underline{\alpha}} \nearrow \lambda^{\alpha} \quad \searrow \bar{\lambda}_{\alpha}$$

$$Q = \int \lambda^{\underline{\alpha}} d_{\underline{\alpha}} \to Q_L + Q_R = \int \lambda^{\alpha} d_{\alpha} + \int \bar{\lambda}_{\alpha} \bar{d}^{\alpha}$$

In "conformal" gauge ($g^{0j} = 0$, $\det g = 1$),

$$\begin{aligned} S = \int d\tau\, d^2\sigma \Big[&\tfrac{1}{2} \dot{x}^{\underline{m}} \dot{x}_{\underline{m}} + \epsilon^{ij} \epsilon^{kl} \partial_i x^{\underline{m}} \partial_k x_{\underline{m}}\, \partial_j x^{\underline{n}} \partial_l x_{\underline{n}} \\ &+ p_{\underline{\alpha}} \dot{\Theta}^{\underline{\alpha}} + p_{\underline{\alpha}} (\gamma^{\underline{m}} \partial_i \Theta)^{\underline{\alpha}} \partial_j x_{\underline{m}} \epsilon^{ij} + \omega_{\underline{\alpha}} \dot{\lambda}^{\underline{\alpha}} \\ &+ \ldots + \omega_{\underline{\alpha}} (\gamma^{\underline{m}} \partial_i \lambda)^{\underline{\alpha}} \partial_j x_{\underline{m}} \epsilon^{ij} \Big] \end{aligned}$$

$$d_{\underline{\alpha}} = p_{\underline{\alpha}} + (\gamma^{\underline{m}} \Theta)_{\underline{\alpha}} \dot{x}_{\underline{m}} + (\gamma^{\underline{mn}} \Theta)_{\underline{\alpha}}\, \partial_i x_{\underline{m}} \partial_j x_{\underline{n}} \epsilon^{ij}$$

Replacing $\partial_i Y \partial_j Z \epsilon^{ij}$ by $\{Y, Z\}$ suggests action for $SO(10,1)$-covariant M(atrix) theory. (work in progress)

P-P R-R BACKGROUNDS

When is light-cone background consistent?

Bosonic: $S_{LC} = \int d^2z\, (g_{jk}(x)\partial X^j \bar{\partial} X^k + h(x))$

Add ∂X^+ dependence so that action is classically conformally invariant.

$S = \int d^2z\, (\partial X^+ \bar{\partial} X^- + g_{jk}(x)\partial X^j \bar{\partial} X^k + h(x)\partial X^+ \bar{\partial} X^+)$

Check quantum conformal invariance.

RNS: Add ψ^+ and ∂X^+ dependence so that action is classically $N=1$ superconf. inv.

$S = \int d^2z\, d^2\kappa\, (D\mathcal{X}^+ \bar{D}\mathcal{X}^- + g_{jk}(\mathcal{X}) D\mathcal{X}^j \bar{D}\mathcal{X}^k + h(\mathcal{X}) D\mathcal{X}^+ \bar{D}\mathcal{X}^+)$

Check quantum $N=1$ superconf. inv.

Light-cone RNS op. $\partial X^j \psi_j$ comes from $N=1$ supermoduli

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GS: Add (θ^+, θ^-) and (λ^+, λ^-) so that action is $N=2$ superconf. inv. and check quantum inv.

Light-cone GS op. $\partial x^a s^{\bar{a}} + \partial x^{\bar{a}} s^a$ comes from $N=2$ supermoduli (NB, hep-th/9201004)

$SU(4)\times U(1)$ Formalism in Flat Background

Transverse variables: $\boxed{a, \bar{a} = 1 \text{ to } 4}$

$$\mathbb{X}^a = x^a + \kappa^+ s^a + \bar{\kappa}^+ \bar{s}^a + \kappa^+\bar{\kappa}^+ h^a$$

$$\mathbb{X}^{\bar{a}} = x^{\bar{a}} + \kappa^- s^{\bar{a}} + \bar{\kappa}^- \bar{s}^{\bar{a}} + \kappa^-\bar{\kappa}^- \bar{h}^{\bar{a}}$$

Longtitudinal variables:

$$\Theta^\pm = \theta^\pm + \kappa^\pm \lambda^\pm + \ldots$$

$$\bar{\Theta}^\pm = \bar{\theta}^\pm + \bar{\kappa}^\pm \lambda^\pm + \ldots$$

$$W^\pm = \ldots + \kappa^\pm p^\pm + \kappa^+\kappa^- \omega^\pm$$

$$\bar{W}^\pm = \ldots + \bar{\kappa}^\pm \bar{p}^\pm + \bar{\kappa}^+\bar{\kappa}^- \bar{\omega}^\pm$$

$$S = \int d^2z \int d^4\kappa \left[\mathbb{X}^a \mathbb{X}^{\bar{a}} + \Theta^+ W^- + \Theta^- W^+ + \bar{\Theta}^+ \bar{W}^- + \bar{\Theta}^- \bar{W}^+ \right]$$

$$= \int d^2z \left[\partial x^a \bar{\partial} x^{\bar{a}} + s^a \bar{\partial} s^{\bar{a}} + \bar{s}^a \partial \bar{s}^{\bar{a}} + p_+ \bar{\partial}\theta^+ + p_- \bar{\partial}\theta^- + \omega_+ \bar{\partial}\lambda^+ + \omega_- \bar{\partial}\lambda^- + c.c. \right]$$

$\partial x^+ \equiv \lambda^+\lambda^- - \theta^+\partial\theta^- - \theta^-\partial\theta^+$ is "composite" field

$w^\pm \approx x^- \lambda^\pm$ as in Penrose twistors

In light-cone gauge, $\Theta^\pm = \kappa^\pm$ and $\bar{\Theta}^\pm = \bar{\kappa}^\pm$

$\Rightarrow \theta^\pm = \bar{\theta}^\pm = 0, \; \lambda^\pm = \bar{\lambda}^\pm = 1, \; \partial x^+ = 1.$

SU(4) × U(1) Formalism in R-R Background

Maldacena recently found R-R plane-wave backgrounds preserving 4 susy's which are described by holomorphic function $Y(\mathcal{X}^a)$.

$Y(\mathcal{X}^a) = \mathcal{X}^a \mathcal{X}^a$ describes Penrose limit of $AdS_5 \times S^5$ background.

$$S_{LC} = \int d^2z \left[\int d^4\kappa\, \mathcal{X}^a \mathcal{X}^{\bar{a}} + \int d^2\kappa\, Y(\mathcal{X}^a) + \int d^2\bar{\kappa}\, \bar{Y}(\mathcal{X}^{\bar{a}}) \right]$$

$$\rightarrow S = \int d^2z \int d^4\kappa \left[\mathcal{X}^a \mathcal{X}^{\bar{a}} + Y(\mathcal{X}^a)\Theta^- \bar{\Theta}^- + \bar{Y}(\mathcal{X}^{\bar{a}})\Theta^+ \bar{\Theta}^+ \right]$$

Can also have R-R backgrounds preserving 2 susy's described by real harmonic function

$$S = \int d^2z \int d^4\kappa \left[\mathcal{X}^a \mathcal{X}^{\bar{a}} + (\Theta^+ + \Theta^-)(\bar{\Theta}^+ + \bar{\Theta}^-)\, V(\mathcal{X}^a, \mathcal{X}^{\bar{a}}) \right]$$

These actions are $N=2$ superconf. inv. to all (perturbative) orders in α'.

Light-cone op's come from integration over worldsheet $N=2$ supermoduli $\Rightarrow$ no contact terms

Might be useful for amplitude computations.

(NB + J. Maldacena, to appear)

Conclusions

Applications of covariant formalism:

- Super-Poincaré inv. superstring tree amp's
- Superstring corrections in superspace to low-energy eqns. for background
- Solvable superconf. inv. actions for R-R plane-wave backgrounds

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Future applications ?

- Super-Poincaré inv. superstring loop amp's ?
- $SO(10,1)$ - covariant M(atrix) theory?
- Solvable action for $AdS_5 \times S^5$ background?

Speculation (inspired by comments of Witten and Vafa)

R-R plane wave $SO(4)\times SO(4) \rightarrow AdS_5 \times S^5$ $SO(4,1)\times SO(5)$

$SU(4)\times U(1)$ formalism $\rightarrow SU(5)\times U(1)$ formalism (Wick-rotated)

$\boxed{a = 1 \text{ to } 5}$

$$G^+ = \partial x^a s^{\bar{a}} + b\gamma$$

$$G^- = \partial x^{\bar{a}} s^a + c\partial\beta + 2\beta\partial c + b$$

$$J = s^a s^{\bar{a}} + bc + 2\beta\gamma$$