$N = 1$ dualities and the dynamics of brane anti-brane systems

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Introduction

One of the most striking results in the modern study of $\mathcal{N} = 1$ supersymmetric gauge theory was Seiberg's discovery of an IR duality between two QCD-like theories, both with N_f flavors of quarks (fundamental chiral superfields), but with different gauge groups.

This discovery connects two quantum theories which at the classical level have no resemblance to each other.

The two theories are

- $SU(N_c)$ supersymmetric gauge theory with N_f flavours $Q_i, \tilde{Q}_i$, and no superpotential.

- $SU(N_F - N_c)$ supersymmetric gauge theory with N_f flavours $q_i, \tilde{q}_i$, meson fields M_{ij} which are singlets of the gauge group, and superpotential $M_{ij} q_i \tilde{q}_j$

A popular way to study $\mathcal{N} = 1$ supersymmetric gauge theories is to realize them geometrically in string theory, as suspended brane constructions, D-branes wrapping cycles in Calabi-Yau manifolds, orbifolds, and otherwise.

- Hanany-Witten
- S. Elitzur, A. Giveon and D. Kutasov
- B. Feng, A. Hanany and Y. H. He (toric duality ?)
- F. Cachazo, B. Fiol, K. A. Intriligator, S. Katz and C. Vafa

They do not explain in a satisfying manner the superpotential data that is necessary to make the dualities work.

The objective of this talk is to show how to overcome this obstacle so that one can derive the (classical) superpotential of the dual theory from first principles.

Outline

- Field theories realized by IIb holomorphic branes on CY.
- Central charge and gauge couplings.
- Duality as change of basis.
- Brane categories and exact sequences.
- Seiberg's original example.
- A self-consistency check.
- Tachyon dynamics and the dual superpotential.
- Applications.
- Outlook.

Field theories realized by IIb holomorphic branes on CY.

- Given a collection of stacks of 'linearly independent' mutually supersymmetric holomorphic D-branes on a CY three-fold $\mathcal{M}$, B_i, we can construct theories by wrapping branes on these cycles as $B \sim \sum N_i B_i$.

- Each D-brane B_i is associated to a coherent sheaf on $\mathcal{M}$.

- The massless spectrum of states between the D-branes is given by calculating $Ext^q(B_i, B_j) = H^{0,q}(X, B_1^* \otimes B_2)$. These are just holomorphic q-forms, and q is the ghost charge. We assume $Ext^0(B_i, B_j) = 0$ for $i \neq j$.

- The category of sheaves has the Serre duality functor $\mathcal{E}xt^q(A, B) = \mathcal{E}xt^{d-q}(B, A)^*$., so that we can just keep $q < d/2 = 3/2$.

- When taking into account the GSO projection Ext^0 become vector multiplets, and Ext^1 become chiral multiplets.

- The theories will have a gauge group $\prod U(N_i)$.

Note:

All of the holomorphic data of the field theory is contained in the twisted B-model (open) topological string theory of the system of branes.

Topological open strings are the massless (field theory) degrees of freedom.

The topological string theory correlation functions determine the superpotential of the theory. This is given by the holomorphic Chern-Simons action of the configuration.

The rest of the physics is determined by Kahler data (e.g. if supersymmetry is spontaneously broken by a configuration or not.)

Central charge and gauge couplings

Let us consider a BPS IIb brane wrapped around some cycle in the CY and the transverse directions to the CY. Take a IIa brane wrapping the same cycle in the CY, but which is a point in the transverse direction to the CY.

This second brane is a BPS particle in a compactification with $\mathcal{N} = 2$ SUSY. The tension is equal to the norm of the central charge

$$T = |Z(B)|$$

In the gauge theory one has

$$\frac{1}{g_B^2} = |Z(B)| \sim vol(B); \qquad \zeta_B \sim \varphi(B) = \frac{1}{\pi} \arg Z(B).$$

for small $\varphi(B)$, and it is determined by the Kahler data.

ζ_B is the FI D-term of the gauge group for brane B.

Two branes A, B are mutually supersymmetric if $\varphi(B) = \varphi(A)$.

Seiberg duality is related to the physics of the strong coupling region $1/g_E^2 \to 0$. At the classical level this corresponds to going through zero volume or

$$Z(E) = 0$$

Since the central charge is complex, we can avoid this region, going around a point $Z(E) = 0$ on a path with $|Z(E)| >> 1$ in Kahler moduli.

The path will take $Z(E) \to -Z(E)$. If we had a collection of branes B_i that are mutually supersymmetric with E, then in the end, it is the antibrane of E, $\bar{E}$ which is mutually supersymmetric with the B_i.

The field theory $\sum N_i B_i + N_c E$ will not be described by supersymmetric field theory in terms of the branes B_i, E after we take $Z(E) \to -Z(E)$

We can think of the collection B_i, E as a choice of basis of fractional branes.

Duality as change of basis

Given a collection of branes, B_i, E, which are not mutually supersymmetric, they can form bound states which are supersymmetric. For example $\hat{B}_i = B_i + c_i E$.

If a set of branes B_i, E is almost supersymmetric, then a brane configuration can be described as a supersymmetric field theory with small D-terms, and one can do a field theory computation which determines the existence of $\hat{B}_i$ as a BPS brane.

When we take $Z(E) \to -Z(E)$ the branes $\hat{B}_i$ are lighter than the branes B_i, so they are better candidates for fractional branes.

The idea now is that the field theory duality is a change of basis

$$B_i, E \to \hat{B}_i, \bar{E}$$

determined by the change in Kahler data.

The charge of a given brane B does not change, but it's expression in terms of a basis does, (as well as it's stability). So

$$B = \sum N_i B_i + N_c E = \sum N_i B_i + (\sum N_i c_i - N_c)\bar{E}$$

Here we see the familiar shift $N_c \to S - N_c$ that characterizes $N = 1$ dualities.

Brane categories and exact sequences

To understand duality we want to treat branes and anti-branes on the same footing. The data we are interested in is determined by a topological string theory which allows us to treat branes and anti-branes on the same footing.

This is done by replacing coherent sheaves with complexes of coherent sheaves and going to the "derived category". (Douglas)

The standard notation for such a complex is

$$E_{-m} \xrightarrow{d_{-m}} E_{-m+1} \xrightarrow{d_{-m+1}} \dots \xrightarrow{d_{-1}} \underline{E}_0 \xrightarrow{d_0} E_1 \xrightarrow{d_1} \dots \xrightarrow{d_{n-1}} E_n.$$

On a first approximation the cohomology of d is interpreted as the physical brane represented by the complex. The most elementary example of this is

$$B \xrightarrow{1} B$$

which represents brane-anti brane anihilation by tachyon condensation.

One can also represent bound states of branes B_1, B_2 as the middle term in an exact sequence

$$0 \to B_1 \to B \to B_2 \to 0$$

The massless spectrum of strings between B and M can be found from the long exact sequence

$$\begin{aligned} \ldots 0 \to\ & \mathcal{E}xt^0(M, B_2) \to \mathcal{E}xt^0(M, B) \to \mathcal{E}xt^0(M, B_1) & (1) \\ \to\ & \mathcal{E}xt^1(M, B_2) \to \mathcal{E}xt^1(M, B) \to \mathcal{E}xt^1(M, B_1) \quad \ldots. & (2) \end{aligned}$$

or it's dual.

One can generalize the Ext^q to deal with complexes. Now the ghost charge q gets an extra contribution from the grading of the branes.

$$\mathrm{Ext}^q(B_1, B_2) \sim \oplus_{n,m} H(\mathrm{Ext}^{q+n-m}(E_n^1, E_m^2))$$

The idea behind this shift is that when we break supersymmetry by changing $\varphi(A) - \varphi(B)$ the bosonic partners of the fermions will get an additional contribution to their mass

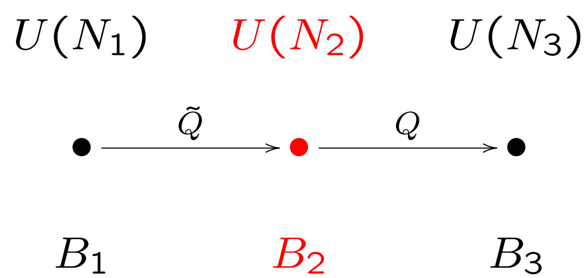
$$m^2 \sim q - 1 + \varphi(A) - \varphi(B)$$

so that they can become tachyonic.

In particular A and $\bar{A}$ differ in phase by a shift of ± 1.

Seiberg's original example

We need to start with a set of branes that have the following quiver:



We will call this theory $B = (N_1, N_2, N_3)$.

Anomaly cancellation requires $N_1 = N_3$ and extra matter that connects N_1 and N_3, but we will ignore this for the argument.

We can form the following bound states that break the gauge group to $U(1)$(can be interpreted as 'fractional branes'):

$$B_4 \sim (1,1,0) \quad B_5 \sim (0,1,1) \quad B_6 \sim (1,1,1)$$

Each requires an obvious choice of D-terms to give SUSY vacua.

We are only interested in dualizing B_2, so that we take $\zeta_2 >> 0$. Of the above we can keep B_1, B_5 and $\bar{B}_2$ to make the new basis (B_6 is a bound state of B_1 and B_5).

We can not use B_4 because it is not supersymmetric for $\zeta_2 >> 0$.

In terms of the new basis we have

$$B = N_1 B_1 + (N_3 - N_2)\bar{B}_2 + N_3 B_5$$

This is exactly the dual gauge group predicted by Seiberg.

The matter content

The brane B_5 is a bound state

$$0 \to B_2 \to B_5 \to B_3$$

. Let us calculate the spectrum of open strings between B_5 and B_1.

In the field theory we consider the total brane $B_5 + B_1$. There is one massless quark $\tilde{Q}$, starting at B_1 and ending at B_5. In the dual theory this is invariant under the gauge group of the $\bar{B}_2$ branes, and it has the right quantum numbers to be the meson field of the dual theory. To find the field content associated to brane $\bar{B}_2$ and B_1 we need to shift the brane B_2 by 1. This means we need to shift

$$\mathcal{E}xt^1(B_2, B_1) \to \mathcal{E}xt^2(\bar{B}_2, B_1) \sim \mathcal{E}xt^1(B_1, \bar{B}_2)$$

so we reverse the arrows in the quiver.

Doing the exact sequence for $Ext(B_5, B_2)$ and shifting by one, reverses the arrows between node B_2 and B_3.

This produces the exact spectrum of the dual theory.

$$
\begin{array}{ccccccc}
0 \to \mathcal{E}xt^0(B_2,B_2) & \longrightarrow & \mathcal{E}xt^0(B_5,B_2) & \longrightarrow & 0 & & \\
0 & \longrightarrow & \mathcal{E}xt^1(B_5,B_2) & \longrightarrow & 0 & \longrightarrow & \\
0 & \longrightarrow & \mathcal{E}xt^2(B_5,B_2) & \longrightarrow & \mathcal{E}xt^2(B_3,B_2) & \longrightarrow & \\
\mathcal{E}xt^3(B_2,B_2) & \longrightarrow & \mathcal{E}xt^3(B_5,B_2) & \longrightarrow & 0 & &
\end{array}
$$

The red arrow is a non-trivial connecting homomorphism.

In field theory this is just the Higgs mechanism. ($\mathcal{E}xt^{0,3}$ is the gauge group, and $\mathcal{E}xt^{1,2}$ are matter).

We need to shift $\mathcal{E}xt^0(B_5,B_2) \to \mathcal{E}xt^1(B_5,\bar{B}_2)$

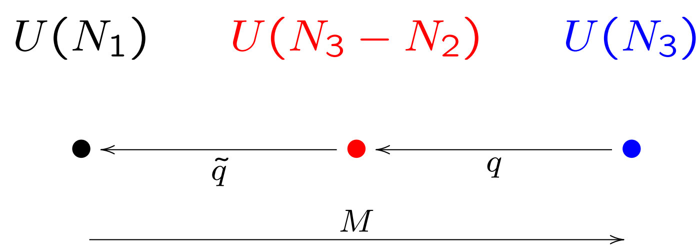
For the antibrane the shift

$$\mathcal{E}xt^{0,3}(B_2,B_2) \to \mathcal{E}xt^{1,4}1(B_2,\bar{B}_2)$$

produces the brane anti-brane tachyons $T,\tilde{T}$.

A self consistency check

We have constructed the dual theory (except for the superpotential). It looks as follows



The double dual should be the original theory. The dual theory is supposed to have the superpotenatial

$$\boxed{\mathrm{tr}(qM\tilde{q})}$$

Consider the bound state $0 \to \bar{B}_2 \to B_7 \to B_1 \to 0$, and the change of basis. There should be no double dual 'meson' field.

$$
\begin{array}{ccccccc}
0 & \longrightarrow & 0 & \longrightarrow & \mathcal{E}xt^0(B_5, B_7) & \longrightarrow & 0 \longrightarrow \\
 & & 0 & \longrightarrow & \mathcal{E}xt^1(B_5, B_7) & \longrightarrow & \mathcal{E}xt^1(B_5, \bar{B}_2) \longrightarrow \\
 & & \mathcal{E}xt^2(B_5, B_1) & \longrightarrow & \mathcal{E}xt^2(B_5, B_7) & \longrightarrow & 0 \longrightarrow \\
 & & 0 & \longrightarrow & \mathcal{E}xt^3(B_5, B_7) & \longrightarrow & 0
\end{array}
$$

When we turn on the field q (form B_7), we should have no massless matter between B_7 and B_5. This requires a connecting homomorphism for the arrow in red. This has to come from a superpotential term, which is expected in the dual

$$\boxed{\mathrm{tr}(qM\tilde{q})}$$

We still need to prove this is the superpotential

What connecting homomorphisms mean:

$\mathrm{Ext}^0 \to \mathrm{Ext}^1$ and $\mathrm{Ext}^2 \to \mathrm{Ext}^3$ are dual to each other. These take care of the (super) Higgs mechanism: Gauge fields eat superfields to become massive together.

$\mathrm{Ext}^1 \to \mathrm{Ext}^2$ is the connecting homomorphism that takes care of giving masses due to superpotential terms when we give vevs to some fields.

"Unification of gauge and superpotential dynamics"

Tachyon dynamics and the dual superpotential.

How can we get the superpotential from first principles?

The superpotential is determined by the holomorphic Chern-Simons action.

The brane-antibrane system give rise to a holomorphic graded Chern-Simons theory. $NB_2 + N'\bar{B}_2$ gives a $U(N|N')$ structure. The tachyon is part of the 'generalized connection'.

In particular, tachyon condensation to eliminate cohomology requires terms

$$\mathrm{tr}(T\bar{\tilde{Q}}Q) + \mathrm{tr}(\tilde{T}Q\bar{Q})$$

. This also follows from $U(N|N')$ symmetry.

For the dual theory $M \sim Q$, $q \sim T$ and $\tilde{q} \sim \bar{\tilde{Q}}$. The term in blue is the dual superpotential.

Applications

One can generalize the simple example above to many other situations. One needs to understand what BPS bound states can be form to obtain the new basis.

In general one can show that all the possible configurations between the various basis match.

One can deal with many examples in the literature:

- Kutasov's $SU(N_c)$ with N_F flavours and and adjoint X with superpotential tr(X^{k+1}). This requires integrating out some fields which become massive to get to the dual superpotential.
- More complicated quivers with various arrows between the nodes and fairly generic superpotentials.
- The procedure described is a generalization of the Fourier-Mukai transform.

Outlook

- We have seen how dualities can be described in terms of brane-antibrane systems. This is what the change of basis argument requires.

- Tachyons play a fundamental role in this derivation of duality.

- We have seen how string theory arguments allow one to derive (classical) superpotentials for dual theories from first principles.

- Can one find new dualities with these methods?

- Can one generalize this procedure to include the SO/Sp groups?

- Can one understand the quantum duality between field theories from this perspective (instantons and non-perturbative superpotentials)?