

# M THEORY, $G_2$ -MANIFOLDS AND FOUR DIMENSIONAL PHYSICS.

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M THEORY ON  $X \times M^{3,1}$

WITH  $X$  A 7-MANIFOLD  
COMPACT

WHOSE METRIC HAS HOLONOMY

$G_2$ , IS A NATURAL CONTEXT  
IN WHICH TO OBTAIN A 4D

THEORY OF GRAVITY WITH

$N=1$  SUPERSYMMETRY.

IF  $X$  IS SMOOTH, THE 4D

GAUGE GROUP IS ABELIAN

AND THERE ARE NO

LIGHT CHARGED PARTICLES!

(cf PAPADOPOULOS/TOWNSEND Leh/950618)



I WILL REPORT ON THE RECENT  
PROGRESS IN UNDERSTANDING  
HOW PARTICULAR SINGULARITIES  
IN  $X$  SUPPORT NON-ABELIAN  
GAUGE FIELDS AND CHIRAL  
FERMIONS.

THESE ARE MEMBRANES WRAPPED  
ON SURFACES IN  $X$  WHICH  
HAVE "COLLAPSED" AT THE  
SINGULARITIES.

# NON-ABELIAN GAUGE SYMMETRY

FOR A 4D GAUGE GROUP OF  
A-D-E TYPE,  $X$  CONTAINS A  
3-MANIFOLD  $Q$  OF ORBIFOLD  
SINGULARITIES. NEAR  $Q$ ,  $X$   
LOOKS LIKE  $\frac{\mathbb{R}^4}{\Gamma_{ADE}} \times Q$  WITH

$\Gamma_{ADE}$  A FINITE SUBGROUP OF  
 $SU(2) \subset SO(4)$ .

(IN THIS CONTEXT FIRST CONSIDERED

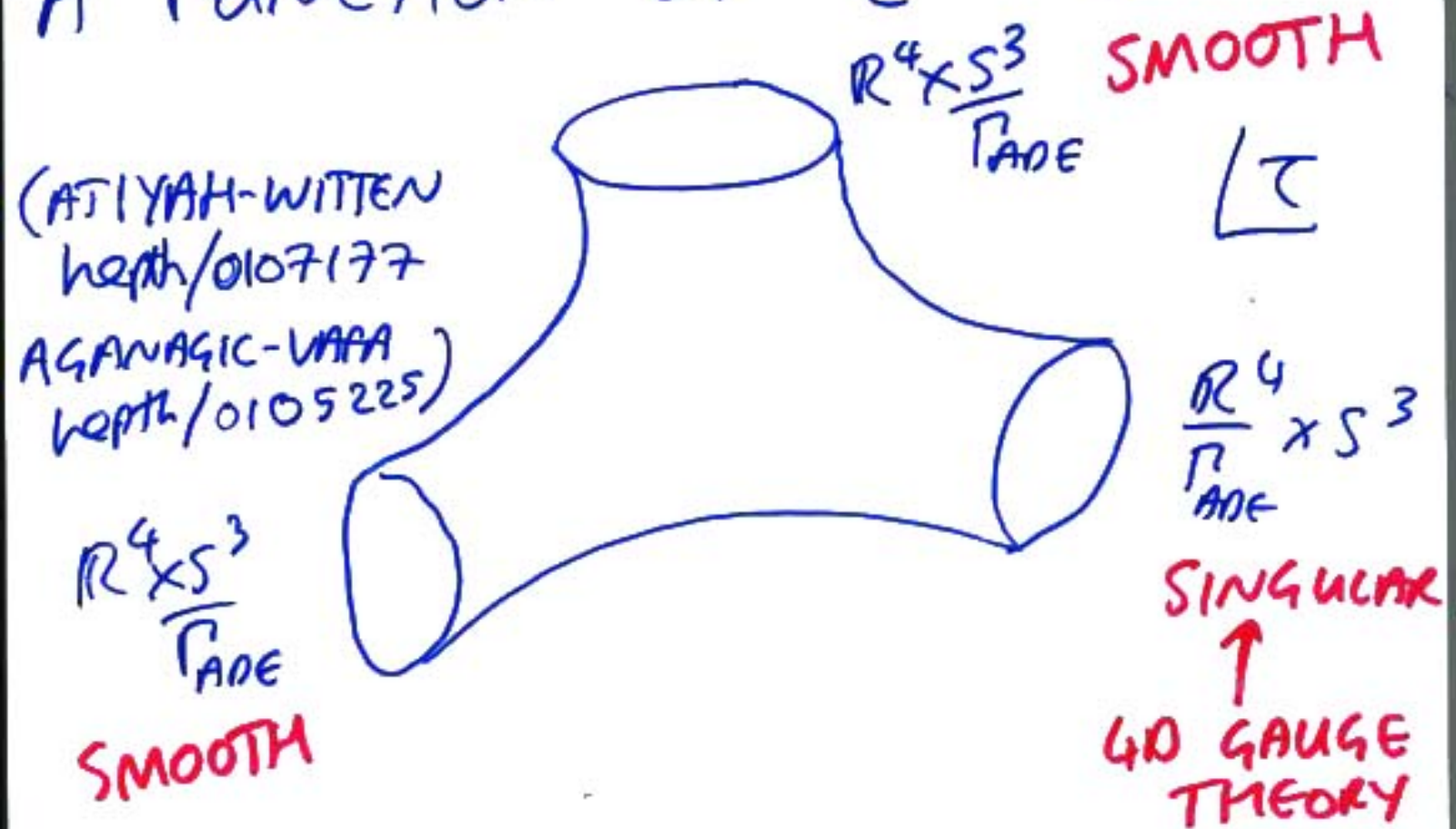
IN BSA hep-th/9812205, 0011089

ATIYAH, MALDACENA, VAFSA  
hep-th/0011256)

NOTE:  $\frac{1}{g_{YM}^2} + i\theta \sim \text{Vol}(Q) + i \int_Q C \equiv \tau$



FOR  $Q = S^3$ , A MORE GLOBAL PICTURE  
 OF THE M THEORY PHYSICS AS  
 A FUNCTION OF  $\tau$  IS



M THEORY PHYSICS SMOOTH  
 AS A FUNCTION OF  $\tau$ .  
 (MEMBRANE INSTANTONS)

$\Rightarrow$  USE SUGRA APPROXIMATION  
 IN SMOOTH, "BIG" REGIONS  
 TO UNDERSTAND DYNAMICS.



# CONFINEMENT, MASS GAP, DOMAIN WALLS.

M2-BRANES WRAPPING 1-CYCLES IN  
THE SMOOTH  $G_2$ -MANIFOLD  $\mathbb{R}^4 \times \frac{S^3}{\Gamma_{ADE}}$   
HAVE PRECISELY THE CHARGES OF  
THE EXPECTED QCD-STRINGS (BSA  
hep-th/0101206)

THE SUGRA ANALYSIS ALSO REVEALS  
A MASS GAP IN THE 4D  $L^2$ -SPECTRUM  
(AMV-hep-th/0011256)  
(A-W-hep-th/0107177)

ONE CAN ALSO COMPUTE THE  
SPECTRUM OF BPS DOMAIN WALLS  
IN THE THEORY COMPACTIFIED ON A  
FURTHER CIRCLE (BSA-Vafa hep-th/0103011)  
(VERIFIED BY EXPLICIT GAUGE THEORY)  
ANALYSIS BY RITZ-SHIFMAN-VALSARAJAN



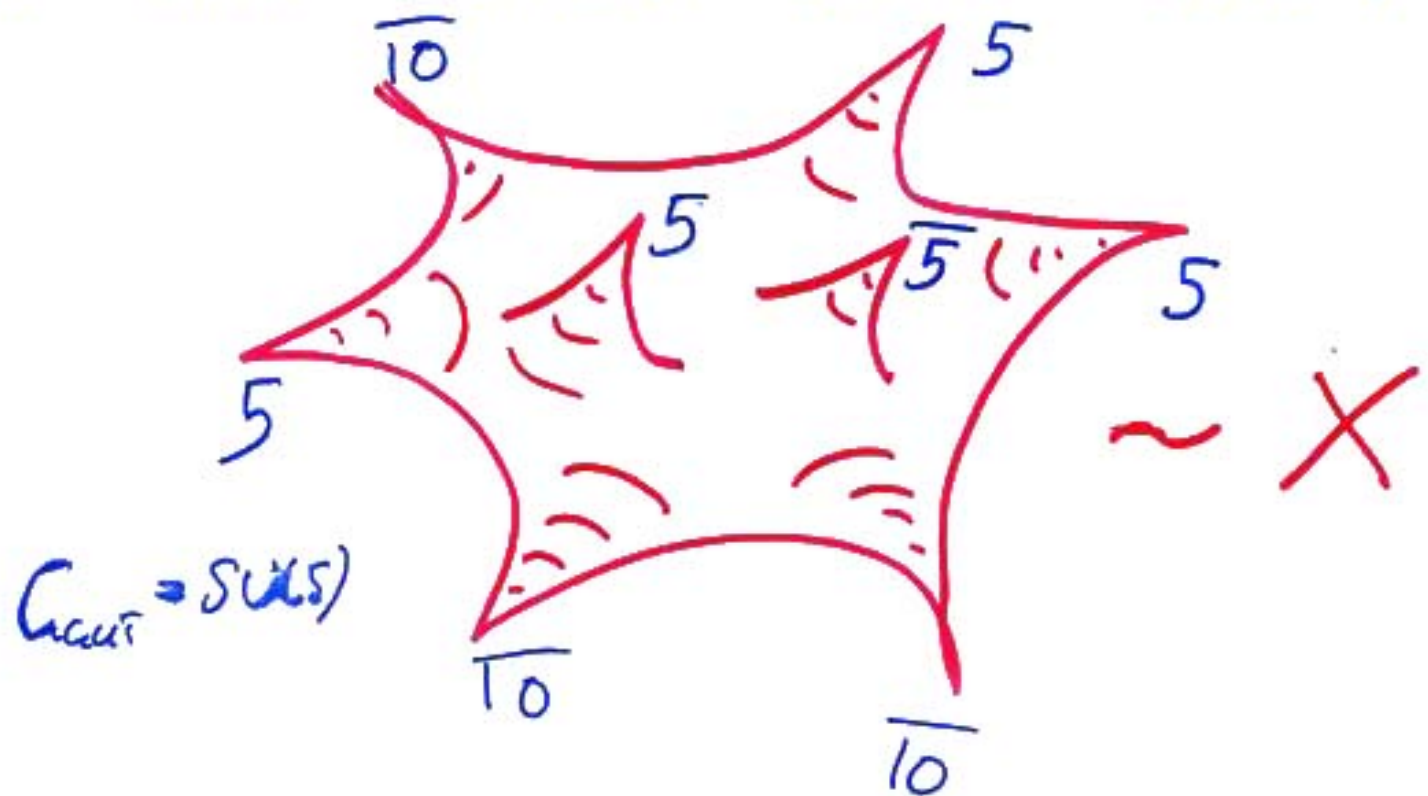
# CHIRAL FERMIONS

CHIRAL FERMIONS CHARGED UNDER THE A-D-E GAUGE GROUP ARISE FROM ADDITIONAL POINTLIKE, CONICAL SINGULARITIES OF  $X$  THROUGH WHICH  $\mathbb{Q}$  PASSES. BY UTILISING A DUALITY WITH TYPE IIA INTERSECTING BRANES, ATIYAH+WITTEN IDENTIFIED SINGULARITIES OF  $X$  WHICH SUPPORT " $\tilde{N}$ 's OF  $SU(N)$ ", CHIRAL " $\tilde{10}$ 's OF  $SU(5)$ ", " $\tilde{16}$ 's OF  $SO(10)$ " AND " $\tilde{27}$ 's OF  $E_6$ " WERE DESCRIBED IN (BSA+E-WITTEN, hep-th/0109152) BY UTILISING A DUALITY WITH HETEROTIC STRINGS THEORY ON CALABI-YAU'S.

(DUAL TYPE IIA ORIENTIFOLDS WERE FIRST DESCRIBED BY CUETIC, SHIU, URANGA hep-th/0107143, 166)



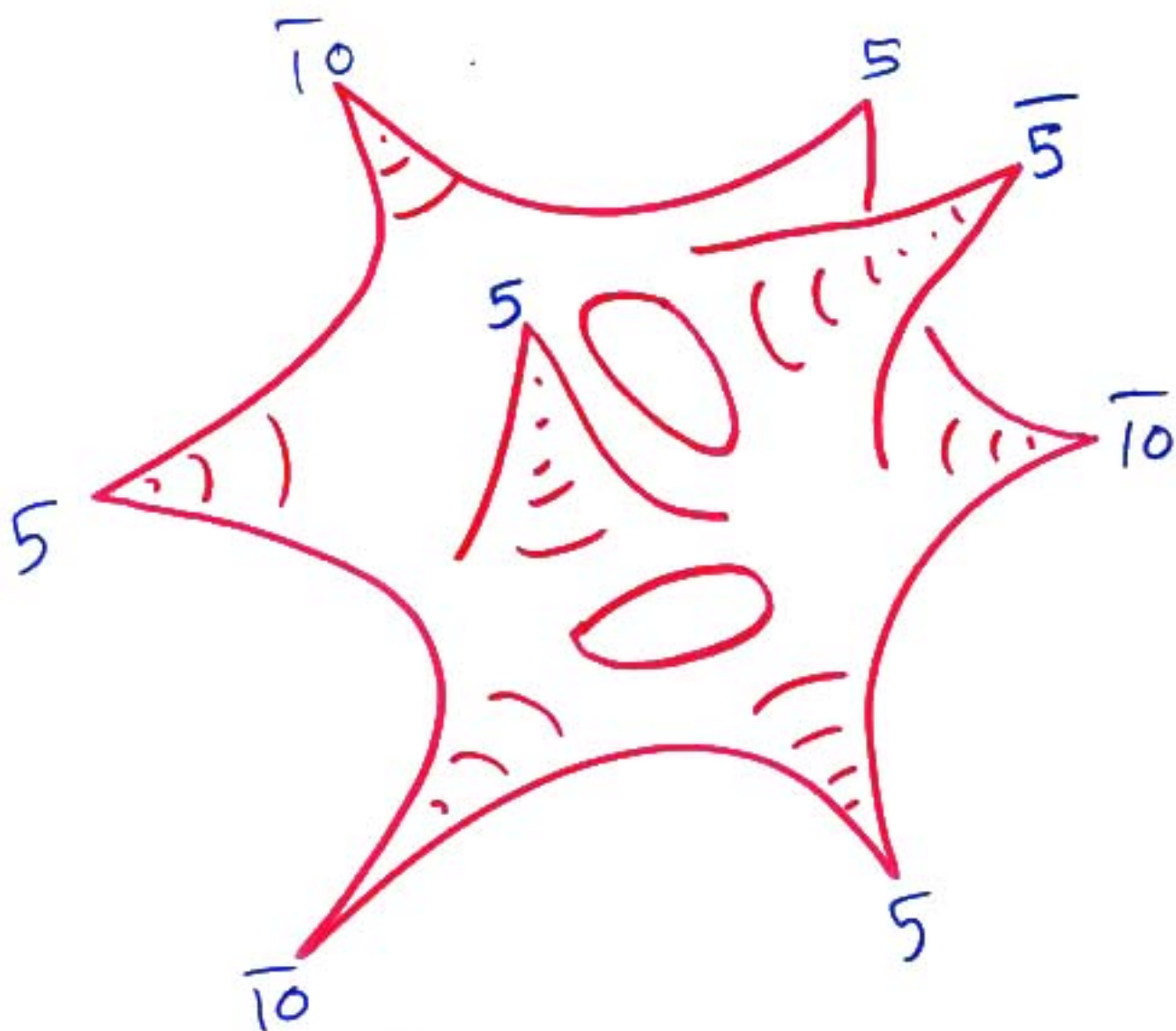
THIS ALL LEADS TO A PICTURE  
 (ACTUALLY AN ARTISTS' IMPRESSION)  
 OF THE "EXTRA" DIMENSIONS:



AT EACH OF THE CONICAL  
 SINGULARITIES A 2-SPHERE FROM  
 THE "BULK" OF  $X$  (TRANSVERSE TO  $Q$ )  
 HAS COLLAPSED TO A POINT.

SINCE THE QUARKS + LEPTONS  
~~RESIDE~~ RESIDE AT DIFFERENT POINTS,  
 THEIR INTERACTIONS ARE NON-LOCAL



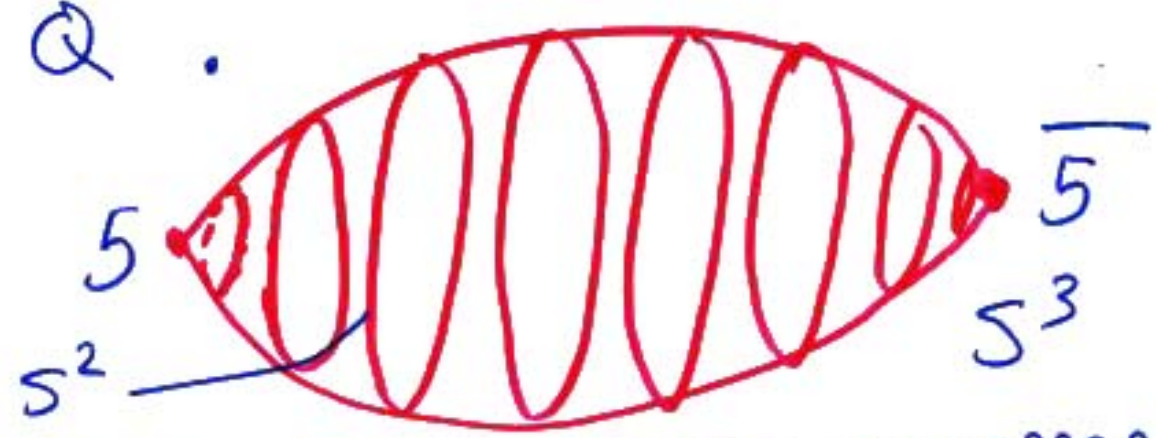


$$G_{\text{GUT}} = \text{SU}(5)$$

X CONTAINS 3-DIMENSIONAL HOLES WHICH ARE "SWEEP OUT" BY THE 2-SPHERES WHICH CONNECT THE CHIRAL SINGULARITIES ALONG LINES

IN Q .

eg



INSTANTONIC MEMBRANES WRAPPED ON SUCH 3-CYCLES GENERATE MODULI DEPENDENT INTERACTIONS BETWEEN CHIRAL MATTER MULTIPLICETS

eg

~~$$I \sim e^{-\text{Vol}(S^3)} H \Psi \tilde{\Psi}$$~~

$$I \sim e^{-\text{Vol}(S^3)} H \Psi \tilde{\Psi}$$



BASED UPON THIS PICTURE, E. WITTEN  
(hepph/0201018) ASSESSED THE BASIC  
PARTICLE PHYSICS FEATURES OF SUCH  
COMPACTIFICATIONS OF M THEORY.

ONE BREAKS  $G_{\text{GUT}} = SU(5)$  (SAY) TO  
 $SU(3) \times SU(2) \times U(1)$  BY WILSON LINES.

$\times$  CAN HAVE DISCRETE SYMMETRIES WHICH  
(F)

ALLOW THIS.

THE CHIRAL SINGULARITIES ~~FORM~~ RESIDE  
ON F-ORBITS ALONG WHICH THE  
WILSON LINES MAY OR MAY NOT  
BE TRIVIAL: SO ~~THE~~ DIFFERENT  
MATTER MULTIPLETS WILL CARRY  
DIFFERENT F-QUANTUM NUMBERS,  
DEPENDING ON WHERE THEY ARE  
FROM.



THE DISCRETE QUANTUM NUMBERS  
ALLOW FOR A NATURAL SOLUTION  
TO ~~THE~~ DOUBLET-TRIPLET SPLITTING  
PROBLEM  
i.e. THE MASS SPLITTING BETWEEN  
HIGGS DOUBLETS AND COLOUR TRIPLETS  
(PROTON DECAY SUPPRESSED)

WITTEN ALSO REMARKED ON INTERE-  
STING FEATURES OF THE INTERACTIONS  
WHICH YOU CAN READ ABOUT IN HIS  
PAPER. ~~THESE INTERACTIONS~~

SINCE MASSES ARE MEMBRANE  
INSTANTON EFFECTS THEY VARY  
EXPONENTIALLY WITH THE  
DISTANCE BETWEEN SINGULARITIES.  
NATURAL TO OBTAIN LIGHT QUARKS  
AND LEPTONS.



BUT WHAT OF THE IMPORTANT  
ISSUES OF SUSY BREAKING,  
MODULI FIXING AND THE  
COSMOLOGICAL CONSTANT  
PROBLEM(S)?

FOR ~~THE~~ DETAILED PREDICTIONS  
ON SUSY BREAKING WE NEED  
TO BE ABLE TO CALCULATE  
(AT LEAST) THE SUPERPOTENTIAL  
INTERACTIONS.

ANY PROGRESS IN THIS DIRECTION  
WOULD BE IMPORTANT.

AS FOR THE COSMOLOGICAL  
CONSTANT PROBLEM, IF  
WE ASSUME THAT THE  
INTERACTIONS BREAK  
SUPERSYMMETRY, WE CAN  
EXPECT (BARRING MIRACLES)



AN UNACCEPTABLY LARGE  
COSMOLOGICAL CONSTANT.

THIS WILL <sup>(LOCALISED IN 4D)</sup> INEVITABLY

BACK REACT ON THE "BULK"  
VIA EINSTEIN'S EQUATIONS.

SO, ONE POSSIBILITY IS

THAT THE 4D VACUUM ENERGY  
OF PARTICLE PHYSICS IS CANCELLED

BY ~~THE~~ A RE-ADJUSTMENT OF

THE BULK METRIC AND FLUXES.

~~THE~~ THIS POSSIBILITY RAISES MANY

QUESTIONS (CF ARKANI-HAMED et al hep-th/0001197  
and KACHRU, SCHULZ, SILVERSTEIN  
hep-th/0001206)

... DIFFICULT!