Progress on AdS Black Holes in String Theory

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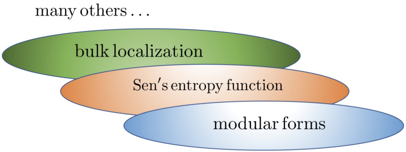
What this talk is about

A major achievement of string theory is the counting of micro-states for a class of asymptotically flat black holes [Vafa-Strominger'96]

- The entropy is obtained by counting states in the corresponding string/D-brane system
- Remarkable precision tests including higher derivatives



No similar results for asymptotically AdS_4 or AdS_5 black holes until very recently.

What this talk is about

In this talk, I review recent progress for AdS_d black holes in diverse dimensions. Using the AdS/CFT correspondence, the entropy is related to a counting of states in the dual CFT.

Disclaimer I: despite holography, the story is still in its infancy.

- computational tools available for BPS black holes
- most of the comparisons are at large N

Disclaimer II: AdS_3 is somehow special and well-studied so we will consider $d \geq 4$.

Generalities on AdS Black Holes

Holographic interpretation

Consider a BPS black hole in $\mathrm{AdS}_{d\geq 4}$. The entropy is a function of the charges Q_I and a set of angular momenta J_i

$$S_{\mathrm{BH}}(Q_I, J_i) = \log n(Q_I, J_i)$$

Holography suggests that the entropy should be recovered by counting states in the dual CFT_{d-1}

$$ds^2 = \frac{dr^2}{r^2} + r^2 ds^2_{M_{d-2}\times\mathbb{R}} + \dots \qquad\qquad r \gg 1$$

set of charged spinful states
of the CFT_{d-1} on $M_{d-2} \times \mathbb{R}$

Q_I become charges under the global symmetries of the CFT_{d-1}

Two interesting string theory classes of BPS black holes, distinguished by supersymmetry algebra and holographic interpretation

- the boundary theory is just the SCFT_{d-1} on $S^{d-2} \times \mathbb{R}$
- the boundary theory on $M^{d-2} \times \mathbb{R}$ is also topologically twisted

characterized by non-zero magnetic fluxes for graviphoton/ R-symmetry: $\int_{\Sigma \subset M} F \in 2\pi\mathbb{Z}$

Most manifest in AdS_4 BH with horizon $\mathrm{AdS}_2 \times S^2$: dichotomy between electrically and magnetically charged BHs first discussed in [Romans 92]

BPS partition function

Counting states with the same susy, charges and angular momenta

$$Z(\Delta_I, \omega_i) = \mathrm{Tr}_{\mathcal{Q}=0}\left(e^{i(Q_I\Delta_I + J_i\omega_I)}\right) = \sum_{Q_I, J_i} n(Q_I, J_i) e^{i(Q_I\Delta_I + J_i\omega_i)}$$

The entropy $S_{\mathrm{BH}}(Q_I, J_i) = \log$ number of states

$$n(Q_I, J_i) = e^{S_{\mathrm{BH}}(Q_I, J_i)} = \int_{\Delta,\omega} Z(\Delta_I, \omega_i) e^{-i(Q_I\Delta_I + J_i\omega_i)}$$

in the limit of large charges, by a saddle point, is a Legendre Transform

$$S_{\mathrm{BH}}(Q_I, J_i) \equiv \mathcal{I}(\Delta, \omega) = \log Z(\Delta_I, \omega_i) - i(Q_I\Delta_I + J_i\omega_i)\,, \qquad \frac{d\mathcal{I}}{d\Delta} = \frac{d\mathcal{I}}{d\omega} = 0$$

PROBLEM: we have efficient tools for counting states preserving four real supercharges. AdS black holes preserve two.

Witten index

It is easier to compute the supersymmetric partition function

$$Z^{\text{susy}}_{M^{d-2}\times S^1}(\Delta_I,\omega_i) = \mathrm{Tr}\left((-1)^F e^{i(Q_I\Delta_I + J_i\omega_I)} e^{-\beta\{\mathcal{Q},\mathcal{Q}^\dagger\}}\right)$$

- superconformal index for SCFT on $S^{d-2}\times S^1$ [Romelsberg 05; Kinney, Maldacena, Minwalla, Raju 05]
- or topologically twisted index for twisted theories [Okuda, Yoshida 12; Nekrasov, Shatashvili 14; Gukov, Pei 15; Benini, AZ 15]

Lower bound on entropy. Index = entropy if there are no large cancellations between bosonic and fermionic ground states. In some cases true at large N.

Arguments for some asymptotically flat BH [Sen 09]

Magnetically charged black holes

Black holes in $\mathrm{AdS}_4 \times S^7$

Black holes in M theory on $\mathrm{AdS}_4 \times S^7$: [Cacciatori, Klemm 08; Dall'Agata, Gnecchi; Hristov, Vandoren 10; Katmadas; Halmagyi 14; Hristov, Katmadas, Toldo 18]

- preserves two real supercharges (1/16 BPS)
- four electric $\mathfrak{q}_a$ and magnetic $\mathfrak{p}_a$ charges under $U(1)^4 \subset SO(8)$, one angular momentum J in AdS_4; only seven independent parameters
- entropy scales as $O(N^{3/2})$

We focus on $J = 0$: six-dimensional family of dyonic static black holes with horizon $\mathrm{AdS}_2 \times S^2$ (or $\mathrm{AdS}_2 \times \Sigma_g$)

Static black holes in $AdS_4 \times S^7$

Entropy is a complicated function

$$S_{\rm BH}(\mathfrak{p}_a, \mathfrak{q}_a) \sim \sqrt{I_4(\Gamma,\Gamma,G,G) \pm \sqrt{I_4(\Gamma,\Gamma,G,G)^2 - 64 I_4(\Gamma) I_4(G)}}$$

I_4 symplectic quartic invariant

$\Gamma = (\mathfrak{p}_1, \mathfrak{p}_2, \mathfrak{p}_3, \mathfrak{p}_4, \mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \mathfrak{q}_4)$ [Halmagyi 13]

$G = (0, 0, 0, 0, g, g, g, g)$

but it can be written as a Legendre transform

$$S_{\rm BH}(\mathfrak{p}_a, \mathfrak{q}_a) = \log Z(\Delta_a, \mathfrak{p}_a) - \sum_a i \Delta_a \mathfrak{q}_a \Big|_{crit} = \sum\nolimits_a i \mathfrak{p}_a \frac{\partial \mathcal{W}}{\partial \Delta_a} - i \Delta_a \mathfrak{q}_a \Big|_{crit}$$

gauged supergravity prepotential $\mathcal{W} \sim \sqrt{\Delta_1 \Delta_2 \Delta_3 \Delta_4}$

$\sum \Delta_a = 2\pi$ scalar fields at the horizon

- Attractor mechanism: [Ferrara, Kallosh, Strominger 96; Dall'Agata, Gnecchi 10]
- Example of entropy function. See also [Sen 05]

Dual perspective

Dual to ABJM on $\Sigma_{\mathfrak{g}} \times \mathbb{R}$ with a twist on $\Sigma_{\mathfrak{g}}$ parameterized by $\mathfrak{p}_a$

$$U(1)^4 \subset SO(8) \qquad\qquad \frac{1}{2\pi}\int_{\Sigma_{\mathfrak{g}}} F^a = \mathfrak{p}_a \in \mathbb{Z}$$

- Magnetic background for global symmetries: Landau levels on $\Sigma_{\mathfrak{g}}$
- Twisting condition $\sum_{a=1}^{4} \mathfrak{p}_a = 2 - 2\mathfrak{g}$

$$\delta\psi_\mu = \underbrace{\nabla_\mu \epsilon - i \sum_{a=1}^{4} A_\mu^a \epsilon}_{\text{cancel spin connection}} = 0 \qquad\qquad \epsilon = \text{constant} \quad \text{on} \quad \Sigma_{\mathfrak{g}}.$$

The relevant index

Topologically twisted index

$$Z_{\Sigma_{\mathfrak{g}} \times S^1}(\Delta_I, \mathfrak{p}_a) = \underbrace{\mathrm{Tr}_{\mathcal{H}} \left((-1)^F e^{i \sum_{a=1}^{4} Q_a \Delta_a} e^{-\beta H_{\mathfrak{p}}} \right)}_{\sum_{a=1}^{4} \Delta_a \in 2\pi\mathbb{Z}}$$

- magnetic charges $\mathfrak{p}_a$ enter in the Hamiltonian $H_{\mathfrak{g}}$, electric charges $\mathfrak{q}_a$ introduced through chemical potentials Δ_a
- number of fugacities equal to the number of conserved charges

The relevant index

Topologically twisted index = QM Witten index

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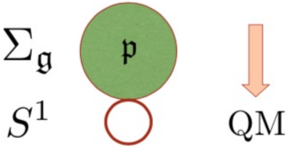


$\rightarrow$ AdS$_2$: reduction to horizon quantum mechanics

Localization formula I

Topologically twisted index $\Longrightarrow$ computable in the UV

$$Z_{\Sigma_{\mathfrak{g}} \times S^1}(\Delta_I, \mathfrak{p}_a) \underset{y_a = e^{i\Delta_a}}{=} \frac{1}{|W|} \sum_{\mathfrak{m} \in \Gamma_{\mathfrak{h}}} \oint_{\mathcal{C}} \frac{dx}{2\pi i x} Z_{\text{int}}(\mathfrak{m}, x; \mathfrak{p}_a, y_a)$$

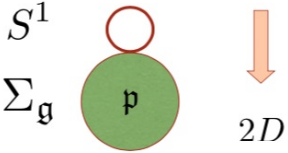
classical piece $Z_{\text{cl}} = x^{km}$

for a chiral multiplet $Z_{1-\text{loop}} = \prod_\rho \left(\frac{\sqrt{x^\rho y_a}}{1 - x^\rho y_a}\right)^{\rho(\mathfrak{m}) - \mathfrak{p}_a + 1 - \mathfrak{g}}$

for a vector multiplet $Z_{1-\text{loop}} = \prod_\alpha \left(1 - x^\alpha\right)^{1-\mathfrak{g}}$

[Localization formula: Benini, AZ 15; Closset, Kim, Willet 16]

Digression: TQFT and Bethe vacua



reduction to two-dimensional theory
(with all KK modes on S^1)

Massive theory with a set of discrete vacua (Bethe vacua)

[Witten 92; Nekrasov, Shatashvili 09]

$$\exp\left(\mathcal{W}'(x^\star)\right) = 1 \qquad\qquad \mathcal{W} = \sum_\rho Li_2(x^\rho y_a) + \dots$$

Many 3d and 4d supersymmetric partition functions can be written as a sum over Bethe vacua [Closset, Kim, Willet 17]

Localization formula II - topological point of view

Sum over Bethe vacua

$$Z_{\Sigma_{\mathfrak{g}} \times S^1}(\Delta_I, \mathfrak{p}_a) = \sum_{x^\star} Z_{\text{cl+1-loop}}(\mathfrak{m} = 0, x^\star; \mathfrak{p}_a, y_a) \left(\det_{ij} \partial_i \partial_j \mathcal{W}(x^\star) \right)^{g-1}$$

$$\exp\left(\mathcal{W}'(x^\star)\right) = 1$$

[Okuda, Yoshida 12; Nekrasov, Shatashvili 14; Gukov, Pei 15; Benini, AZ 15; Closset-Kim-Willet 17]

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For ABJM:

$$\mathcal{W} \underset{x=e^{iu}}{=} \sum_{i=1}^{N} \frac{k}{2}\left(\tilde{u}_i^2 - u_i^2\right) + \sum_{i,j=1}^{N} \left[\sum_{a=3,4} \text{Li}_2\left(e^{i(\tilde{u}_j - u_i + \Delta_a)}\right) - \sum_{a=1,2} \text{Li}_2\left(e^{i(\tilde{u}_j - u_i - \Delta_a)}\right) \right]$$

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Expectation: one Bethe vacuum dominates in the large N limit.

$$u_i = i\sqrt{N} t_i + v_i \qquad \tilde{u}_i = i\sqrt{N} t_i + \tilde{v}_i$$

$\mathcal{I}$-extremization for static black holes in $AdS_4 \times S^7$

In the large N limit [Benini-Hristov-AZ 15]

$$\mathcal{W}_{\text{on-shell}} = \frac{2}{3} i N^{3/2} \sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}$$

$$S(\mathfrak{p}_a, \mathfrak{q}_a) = \log Z(\Delta_a, \mathfrak{p}_a) - \sum_a i\Delta_a \mathfrak{q}_a \Big|_{crit} = \sum\nolimits_a i\mathfrak{p}_a \frac{\partial \mathcal{W}}{\partial \Delta_a} - i\Delta_a \mathfrak{q}_a \Big|_{crit}$$

$$\textstyle\sum_{a=1}^4 \Delta_a = 2\pi \qquad \mathrm{Re}\Delta_a \in [0, 2\pi]$$

- The on-shell superpotential $\mathcal{W}$ coincides with the prepotential of the $\mathcal{N}=2$ gauged supergravity obtained by reducing on S^7. The formula above is the attractor mechanism

Generalisations

- Generalized to other black holes in M theory or massive type IIA. [Hosseini, Hristov, Passias; Benini, Khachatryan, Milan; Azzurli, Bobev, Crichigno, Min, AZ 17; Bobev, Min, Pilch 18; Gauntlett, Martelli, Sparks; Hosseini, AZ 19]

  general formula $\log Z(\Delta_a, \mathfrak{p}_a) = \sum_a \mathfrak{p}_a \frac{\partial F_{S^3}(\Delta)}{\partial \Delta_a}$

  [Hosseini, AZ; Hosseini, Mekareeya '16]

- Including subleading corrections in N [Liu, PandoZayas, Rathee, Zhao; Jeon, Lal 17; Liu, PandoZayas, Zhou 18; Gang, Kim, PandoZayas 19]

- Localization in supergravity [Hristov, Lodato, Reys 17]

- Black holes and black strings in higher dimensions [Hosseini, Nedelin, AZ 16; Hong, Liu 16; Hosseini, Yaakov, AZ 18; Crichigno,Jain,Willet 18; Hosseini, Hristov, Passias, AZ 18; Suh 18]

- Black hole thermodynamics: $\log Z =$ gravity on-shell action [Azzurli, Bobev, Crichigno, Min, AZ 17; Halmagyi, Lal; CaboBizet, Kol, PandoZayas, Papadimitriou, Rathee 17]

- Case with angular momentum still to be worked out.

Electrically charged and rotating black holes

Rotating black holes in $AdS_5 \times S^5$

Most famous BPS examples are asymptotic to $AdS_5 \times S^5$

two angular momenta J_1, J_2 in AdS_5 $\qquad U(1)^2 \subset SO(4) \subset SO(2,4)$

three electric charges Q_I in S^5 $\qquad U(1)^3 \subset SO(6)$

with a constraint $F(J_i, Q_I) = 0$. They must rotate and preserves two supercharges.

$$S_{\rm BH} = 2\pi\sqrt{Q_1Q_2 + Q_2Q_3 + Q_1Q_3 - 2c(J_1 + J_2)} \qquad c = \frac{N^2-1}{4}$$

[Gutowski-Reall 04; Chong, Cvetic, Lu, Pope 05; Kunduri, Lucietti, Reall; Kim, Lee, 06]

The boundary metric is $S^3 \times \mathbb{R}$, no twist. The microstates correspond to states of given angular momentum and electric charge in $\mathcal{N} = 4$ SYM.

Recent examples of hairy black holes with more parameters [Markeviciute, Santos 18]

Entropy function for AdS_5 black holes

- BPS entropy function [Hosseini,Hristov,AZ 17]

$$\mathcal{S}_{\rm BH}(Q_I, J_i) = -i\pi(N^2-1)\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} - 2\pi i\left(\sum_{I=1}^{3} Q_I\Delta_I + \sum_{i=1}^{2} J_i\omega_i\right)\Big|_{\bar{\Delta}_I,\bar{\omega}_i}$$

with $\Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = \pm 1$

- From BH thermodynamics: chemical potentials $\bar{\Delta}_I, \bar{\omega}_i$ can be obtained in a suitable zero-temperature limit for a family of supersymmetric Euclidean black holes [Cabo-Bizet, Cassani, Martelli, Murthy 18]

$$-i\pi(N^2-1)\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} = \text{on-shell action}$$

The critical values $\bar{\Delta}_I, \bar{\omega}_i$ are complex but, quite remarkably, the extremum is a *real* function of the black hole charges.

Long standing puzzle

Entropy scales like $O(N^2)$ for $Q_I, J_i \sim N^2$.

- difficult to enumerate all 1/16 BPS states. Not enough of them? [Grant, Grassi, Kim, Minwalla 08; Chang, Yin 13; Yokoyama 14]

- the superconformal index

$$Z(\omega_i, \Delta_I) = \mathrm{Tr}(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{2\pi i(\Delta_I Q_I + \omega_i J_i)}$$

number of fugacities equal to the number of conserved charges:

$$p = e^{2\pi i\omega_1}, q = e^{2\pi i\omega_2}, y_I = e^{2\pi i\Delta_I} \qquad \prod_{I=1}^{3} y_I = pq$$

[Romelsberg 05; Kinney, Maldacena, Minwalla, Raju 05]

For real fugacities: $\log Z = O(1)$. Large cancellations between bosons and fermions. [Kinney, Maldacena, Minwalla, Raju 05]

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$$Z(\omega_i, \Delta_I) \sim \oint \frac{dz_i}{2\pi i z_i} \prod_{1 \le i < j \le N} \frac{\prod_{k=1}^{3} \Gamma_e(y_k (z_i/z_j)^{\pm 1}; p, q)}{\Gamma_e((z_i/z_j)^{\pm 1}; p, q)}$$

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The importance of being complex

However, the critical values $\bar{\Delta}_I, \bar{\omega}_i$ of the BPS entropy function are complex. Recent computations for the index

$$Z(\omega_i, \Delta_I) = \mathrm{Tr}(-1)^F e^{-\beta\{Q,Q^\dagger\}} e^{2\pi i(\Delta_I Q_I + \omega_i J_i)}$$

and related/modified quantities with complex fugacities suggest that

- phases may obstruct cancellations in the index
- Stokes phenomena in the complex plane

[Cardy limit: Choi, Kim, Kim, Nahmgoong] [See Kim's talk]

[Modified index/partition function: Cabo-Bizet, Cassani, Martelli, Murthy]

[Large N: Benini, Milan 18]

The importance of being complex

In various limits, index with complex fugacities consistent with:

$$\log Z(\Delta, \omega) \sim -i\pi(N^2 - 1)\frac{\Delta_1 \Delta_2 \Delta_3}{\omega_1 \omega_2} \qquad \Delta_1 + \Delta_2 + \Delta_3 - \omega_1 - \omega_2 = \pm 1$$

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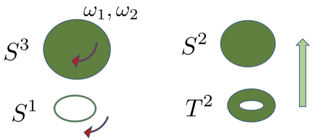
- Large N limit (equal angular momenta)



Reduction on T^2

Two-dimensional Bethe vacua

[Benini, Milan 18]

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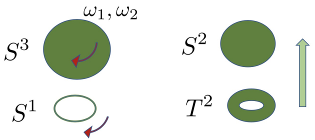

Reduction on T^2
Two-dimensional Bethe vacua
[Benini, Milan 18]

- Cardy limit: $\omega_1,\omega_2 \to 0$ with fixed Δ_I. Large black holes:

$$Q_I \sim \frac{1}{\epsilon^2} \qquad J_i \sim \frac{1}{\epsilon^3} \qquad \omega_1,\omega_2 \sim \epsilon \to 0$$

[Choi, Kim, Kim, Nahmgoong 18; Honda; Arabi Ardehali 19] [See Kim's talk]

The importance of being complex

In various limits, index with complex fugacities consistent with:

$$\log Z(\Delta,\omega) \sim -i\pi(N^2-1)\frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2} \qquad \Delta_1+\Delta_2+\Delta_3-\omega_1-\omega_2=\pm 1$$

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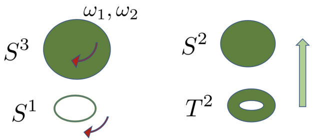

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[Choi, Kim, Kim, Nahmgoong 18; Honda; Arabi Ardehali 19] [See Kim's talk]

Other saddles at smaller charges/other black holes? [See Kim's talk]

Generalisations

- With a modified index, Cardy limit generalizes to other 4d theories. Finite N corrections. For equal charges:

$$\log Z \underset{3\Delta-\omega_1-\omega_2=\pm 1}{\sim} 2\pi i \frac{\Delta^3}{\omega_1\omega_2}(3c-5a) + 2\pi i \frac{\Delta}{\omega_1\omega_2}(a-c) + O(1)$$

  [Generalize DiPietro-Komargodski 14][Kim, Kim, Song; Cabo-Bizet, Cassani, Martelli, Murthy; Amariti, Garozzo, LoMonaco 19]

- Entropy functions for electrically charged and rotating BH also in AdS_4, AdS_6 and AdS_7 [Hosseini, Hristov, AZ; Choi, Hwang, Kim, Nahmgong 18; Cassani, Papini 19]

- Some index computations in higher dimensions [Choi, Kim, Kim, Nahmgoong 18; Choi, Kim; Kantor, Papageorgakis, Richmond 19]

- Near BPS entropy functions [Larsen, Nian, Zeng 19]

Some general comments

Entropy controlled by anomalies?

- Asymptotically flat black holes in string theory [Strominger, Vafa 96]

$$\log Z \sim \frac{\pi^2}{6\beta} c_{\rm CFT} \qquad \underset{\rm Legendre}{\Longrightarrow} \qquad S_{\rm BH} = 2\pi \sqrt{\frac{n c_{\rm CFT}}{6}}$$

- AdS_5 black holes also controlled by anomalies both at large N and in Cardy limit. For $\mathcal{N} = 4$ SYM

$$\log Z = \underbrace{-4\pi i \frac{(\omega_1 + \omega_2 \pm 1)^3}{27\omega_1\omega_2} a_{\rm CFT}}_{\rm equal\ charges}$$

- same quantity appears as supersymmetric Casimir energy. Hidden modularity? [Hosseini, Hristov, AZ 17; Cabo-Bizet, Cassani, Martelli, Murthy 18]

Some more universality?

We can embed BPS black holes in all maximally supersymmetric $\mathrm{AdS}_{d\geq 4}$ backgrounds

• M theory on $\mathrm{AdS}_4 \times S^7$		ABJM theory
• type IIB on $\mathrm{AdS}_5 \times S^5$	$\Longrightarrow$	$\mathcal{N} = 4$ SYM
• massive IIA on $\mathrm{AdS}_6 \times_W S^4$		5d UV fixed point
• M theory on $\mathrm{AdS}_7 \times S^4$		$(2,0)$ theory

Entropy controlled by anomalies in even dimensions and sphere partition functions in odd dimensions

electrically charged and rotating BH

[BH solutions: Chow; Chong, Gibbons, Cvetic, Lu, Pope; Hristov, Katmadas, Toldo to appear]

$AdS_4 \times S^7$	$\mathcal{F}(\Delta_a) = \sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}$ $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 = 2$	$\log \mathcal{Z}(\Delta_a, \omega_i) = \frac{4\sqrt{2}N^{3/2}}{3} \frac{\sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}}{\omega_1}$ $\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \omega_1 = 2\pi$
$AdS_5 \times S^5$	$\mathcal{F}(\Delta_a) = \Delta_1\Delta_2\Delta_3$ $\Delta_1 + \Delta_2 + \Delta_3 = 2$	$\log \mathcal{Z}(\Delta_a, \omega_i) = -i \frac{N^2}{2} \frac{\Delta_1\Delta_2\Delta_3}{\omega_1\omega_2}$ $\Delta_1 + \Delta_2 + \Delta_3 + \omega_1 + \omega_2 = 2\pi$
$AdS_6 \times_W S^4$	$\mathcal{F}(\Delta_a) = (\Delta_1\Delta_2)^{3/2}$ $\Delta_1 + \Delta_2 = 2$	$\log \mathcal{Z}(\Delta_a, \omega_i) \sim N^{5/2} \frac{(\Delta_1\Delta_2)^{3/2}}{\omega_1\omega_2}$ $\Delta_1 + \Delta_2 + \omega_1 + \omega_2 = 2\pi$
$AdS_7 \times S^4$	$\mathcal{F}(\Delta_a) = (\Delta_1\Delta_2)^2$ $\Delta_1 + \Delta_2 = 2$	$\log \mathcal{Z}(\Delta_a, \omega_i) = -i \frac{N^3}{24} \frac{(\Delta_1\Delta_2)^2}{\omega_1\omega_2\omega_3}$ $\Delta_1 + \Delta_2 + \omega_1 + \omega_2 + \omega_3 = 2\pi$

[Disclaimer: normalizations and signs for sake of exposition]
[See "Generalisations" slides for refs]

magnetically charged BH and black strings

$\mathrm{AdS}_4 \times S^7$	$\mathcal{F}(\Delta_a) = \sqrt{\Delta_1\Delta_2\Delta_3\Delta_4}$ $\Delta_1+\Delta_2+\Delta_3+\Delta_4 = 2$	$\log \mathcal{Z} = -\frac{2\sqrt{2}N^{3/2}}{3}\sum_{a=1}^4 \mathfrak{p}_a \frac{\partial \mathcal{F}(\Delta)}{\partial \Delta_a}$ $\Delta_1+\Delta_2+\Delta_3+\Delta_4 = 2\pi$
$\mathrm{AdS}_5 \times S^5$	$\mathcal{F}(\Delta_a) = \Delta_1\Delta_2\Delta_3$ $\Delta_1+\Delta_2+\Delta_3 = 2$	$\log \mathcal{Z} = -\frac{N^2}{2\beta}\sum_{a=1}^3 \mathfrak{p}_a \frac{\partial \mathcal{F}(\Delta)}{\partial \Delta_a}$ $\Delta_1+\Delta_2+\Delta_3 = 2\pi$
$\mathrm{AdS}_6 \times_W S^4$	$\mathcal{F}(\Delta_a) = (\Delta_1\Delta_2)^{3/2}$ $\Delta_1+\Delta_2 = 2$	$\log \mathcal{Z} \sim N^{5/2}\sum_{a,b=1}^2 \mathfrak{p}_a\tilde{\mathfrak{p}}_b \frac{\partial^2 \mathcal{F}(\Delta)}{\partial \Delta_a \partial \Delta_b}$ $\Delta_1+\Delta_2 = 2\pi$
$\mathrm{AdS}_7 \times S^4$	$\mathcal{F}(\Delta_a) = (\Delta_1\Delta_2)^2$ $\Delta_1+\Delta_2 = 2$	$\log \mathcal{Z} \sim \frac{N^3}{\beta}\sum_{a,b=1}^2 \mathfrak{p}_a\tilde{\mathfrak{p}}_b \frac{\partial^2 \mathcal{F}(\Delta)}{\partial \Delta_a \partial \Delta_b}$ $\Delta_1+\Delta_2 = 2\pi$

[Note: AdS_5 and AdS_7 refer to black strings in Cardy limit]
[See "Generalisations" slides for refs]

Similar results for more complicated CFTs at large N suggest:

- 3d: $\mathcal{F}(\Delta) \sim F_{S^3}(\Delta)$ trial sphere partition function

[Hosseini, AZ; Hosseini, Mekareeya]

- 4d: $\mathcal{F}(\Delta) \sim a(\Delta) \sim \mathrm{Tr} R(\Delta)^3$ trial a-central charge

[Hosseini, Nedelin, AZ 16; Hosseini, Hristov, AZ 18; Kim, Kim, Song; Amariti, Garozzo, LoMonaco 19]

$\mathcal{F}(\Delta)$ can be also related to

- the twisted superpotential on the dominant Bethe vacuum
- the prepotential of the relevant gauged supergravity

and similarly for higher dimensions. [See "Generalisations" slides for refs]

Conclusions

Puzzles remain

- Many different approaches and intricate structure of saddles in AdS_5; other black holes?
- Comparison for general $AdS_4 \times SE_7$ black holes. Large N limit not always known.

and a long way to go

- finite N corrections
- extremal non-supersymmetric and near-BPS black holes?

But the main message of this talk is that there is still a lot of interesting physics in AdS black holes.