String Amplitudes, Topological Strings and the Omega-deformation

Strings @ Princeton
26 - 06 - 2014

Ahmad Zein Assi

CERN

String Amplitudes, Topological Strings and the Omega-deformation

Strings @ Princeton
26 - 06 - 2014

Based on work with

I. Antoniadis
I. Florakis
S. Hohenegger
K. S. Narain

Ahmad Zein Assi

CERN

1302.6993 [hep-th]

1309.6688 [hep-th]

1406.xxxx [hep-th]

Introduction & Motivations

- Topological String: subsector of String Theory
  - Twisted version of type II
  - Free energy Fg = physical coupling $\langle (R_-)^2 \, (T_-)^{2g-2} \rangle$

Witten (88')

Antoniadis, Gava, Narain, Taylor (93')

Bershadsky, Ceccotti, Ooguri, Vafa (93')

Introduction & Motivations

- Topological String: subsector of String Theory
  - Twisted version of type II
  - Free energy Fg = physical coupling $\langle (R_-)^2 (T_-)^{2g-2} \rangle$

Witten (88')

Antoniadis, Gava, Narain, Taylor (93')

Bershadsky, Ceccotti, Ooguri, Vafa (93')

- Geometric engineering of (Ω-deformed) supersymmetric gauge theories

Katz, Klemm, Vafa (96')

$$\sum_{g=0}^{\infty} g_s^{2g-2} F_g \Big|_{\text{field theory}} = \log Z_{\text{Nek}}(\epsilon_+ = 0, \epsilon_- = g_s)$$

Nekrasov et al. (02')

Introduction & Motivations

- Topological String: subsector of String Theory
  - Twisted version of type II
  - Free energy Fg = physical coupling $\langle (R_-)^2 (T_-)^{2g-2} \rangle$

Witten (88')

Antoniadis, Gava, Narain, Taylor (93')

Bershadsky, Ceccotti, Ooguri, Vafa (93')

- Geometric engineering of (Ω-deformed) supersymmetric gauge theories

Katz, Klemm, Vafa (96')

$$\sum_{g=0}^{\infty} g_s^{2g-2} F_g \Big|_{\text{field theory}} = \log Z_{\text{Nek}}(\epsilon_+ = 0, \epsilon_- = g_s)$$

Nekrasov et al. (02')

- Refinement: one-parameter extension of F_g
  - Does it exist?
    Coupling in the string effective action?

Answer from the string effective action

- Ω-background
  - ε_- ↔ $SU(2)_-$ rotation, ε_+ ↔ $SU(2)_+$ rotation
  - T_- ⟶ anti-self-dual graviphoton field strength
  - F_+ ⟶ self-dual gauge field strength
-
-

Answer from the string effective action

- Ω-background
  - $\varepsilon_- \leftrightarrow SU(2)_-$ rotation, $\varepsilon_+ \leftrightarrow SU(2)_+$ rotation
  - $T_- \longrightarrow$ anti-self-dual graviphoton field strength
  - $F_+ \longrightarrow$ self-dual gauge field strength
- Consider $F_{g,n} = \langle (R_-)^2 (T_-)^{2g-2} (F_+)^{2n} \rangle$
  - Heterotic on $K3 \times T^2$: vector partner of T^2 Kähler modulus
  - Contributions start at one-loop
-

Answer from the string effective action

- Ω-background
  - $\varepsilon_- \leftrightarrow SU(2)_-$ rotation, $\varepsilon_+ \leftrightarrow SU(2)_+$ rotation
  - $T_- \longrightarrow$ anti-self-dual graviphoton field strength
  - $F_+ \longrightarrow$ self-dual gauge field strength
- Consider $F_{g,n} = \langle (R_-)^2 (T_-)^{2g-2} (F_+)^{2n} \rangle$
  - Heterotic on $K3 \times T^2$: vector partner of T^2 Kähler modulus
  - Contributions start at one-loop
- Explicit exact evaluation at one-loop in Heterotic

$$\mathcal{F}(\epsilon_-, \epsilon_+) = \sum_{g,n \geq 0} \epsilon_-^{2g} \epsilon_+^{2n} \, \mathcal{F}_{g,n} \xrightarrow{\text{Field Theory}} \int_0^\infty \frac{dt}{t} \frac{-2\cos(2\epsilon_+ t)}{\sin(\epsilon_- - \epsilon_+)\,t \;\sin(\epsilon_- + \epsilon_+)\,t} \, e^{-\mu t}$$

Answer from the string effective action

- Ω-background
  - $\varepsilon_- \leftrightarrow SU(2)_-$ rotation, $\varepsilon_+ \leftrightarrow SU(2)_+$ rotation
  - $T_- \longrightarrow$ anti-self-dual graviphoton field strength
  - $F_+ \longrightarrow$ self-dual gauge field strength
- Consider $F_{g,n} = \langle (R_-)^2 (T_-)^{2g-2} (F_+)^{2n} \rangle$
  - Heterotic on $K3 \times T^2$: vector partner of T^2 Kähler modulus
  - Contributions start at one-loop
- Explicit exact evaluation at one-loop in Heterotic

$$\mathcal{F}(\epsilon_-, \epsilon_+) = \sum_{g,n\geq 0} \epsilon_-^{2g} \epsilon_+^{2n}\, \mathcal{F}_{g,n} \xrightarrow{\text{Field Theory}} \log Z_{\text{Nek}}^{\text{Pert}}(\epsilon_+, \epsilon_-)$$

Answer from the string effective action

- Non-perturbative corrections
  - Instantons in the Ω-background: deformed ADHM
  - Gauge instantons: Dp-Dp+4 configuration
  - Closed string background: T_- and F_+

-

-

Answer from the string effective action

- Non-perturbative corrections
  - Instantons in the Ω-background: deformed ADHM
  - Gauge instantons: Dp-Dp+4 configuration
  - Closed string background: T_- and F_+
- Compute Ω-deformed ADHM action
  - Take $\alpha' \to 0$ limit
-

Answer from the string effective action

- Non-perturbative corrections
  - Instantons in the Ω-background: deformed ADHM
  - Gauge instantons: Dp-Dp+4 configuration
  - Closed string background: T_- and F_+
- Compute Ω-deformed ADHM action
  - Take $\alpha' \to 0$ limit
- Evaluate the instanton path-integral

$$\sum_{g,n} \epsilon_-^{2g} \epsilon_+^{2n} \, \mathcal{F}_{g,n}^{Inst.} \Big|_{\text{f.t.}} = \log \left\langle e^{-S_{\text{ADHM}}(\epsilon_+,\epsilon_-,A)} \right\rangle = \log Z_{\text{Nek}}^{\text{NP}}(\epsilon_+, \epsilon_-)$$

Answer from the string effective action

- Non-perturbative corrections
  - Instantons in the Ω-background: deformed ADHM
  - Gauge instantons: Dp-Dp+4 configuration
  - Closed string background: T_- and F_+
- Compute Ω-deformed ADHM action
  - Take $\alpha' \to 0$ limit
- Evaluate the instanton path-integral

$$\mathcal{F}_{g,n}\Big|_{\text{field theory}} = \mathcal{F}_{g,n}^{\text{Nek}}(\epsilon_+, \epsilon_-)$$

Holomorphicity Properties of the Refined Couplings

- Explicit breaking of holomorphicity
  - Related to the compactness of the CY
-
-
-

Holomorphicity Properties of the Refined Couplings

- Explicit breaking of holomorphicity
  - Related to the compactness of the CY
- Non-compact CY for
  - R-symmetry current
  - Decoupling of hypers
  - Define generating function refined topological invariants

Huang, Kashani-Poor, Klemm, Vafa, etc.

-
-

Holomorphicity Properties of the Refined Couplings

- Explicit breaking of holomorphicity
  - Related to the compactness of the CY
- Non-compact CY for
  - R-symmetry current
  - Decoupling of hypers
  - Define generating function refined topological invariants

Huang, Kashani-Poor, Klemm, Vafa, etc.

- Use the generic CY compactification + appropriate limit
-

Holomorphicity Properties of the Refined Couplings

- Explicit breaking of holomorphicity
  - Related to the compactness of the CY
- Non-compact CY for
  - R-symmetry current
  - Decoupling of hypers
  - Define generating function refined topological invariants

Huang, Kashani-Poor, Klemm, Vafa, etc.

- Use the generic CY compactification + appropriate limit
- $F_{g,n}$ satisfies a generalised holomorphic anomaly equation (*à la* Klemm, Walcher, etc.)

Accident? Coincidence?

- Background of anti-self-dual graviphotons + self-dual T-vectors = consistent string theory uplift of the Ω-background
  - Perturbative Nerkrasov partition function = one-loop effective action of generalized F-terms (in the field theory limit)
  - Non-perturbative part = tree-level effective action of a Dp-D(p+4) bound state
-
-

Accident? Coincidence?

- Background of anti-self-dual graviphotons + self-dual T-vectors = consistent string theory uplift of the Ω-background
  - Perturbative Nerkrasov partition function = one-loop effective action of generalized F-terms (in the field theory limit)
  - Non-perturbative part = tree-level effective action of a Dp-D(p+4) bound state
- Generalised holomorphic anomaly equations
-

Accident? Coincidence?

- Background of anti-self-dual graviphotons + self-dual T-vectors = consistent string theory uplift of the Ω-background
  - Perturbative Nerkrasov partition function = one-loop effective action of generalized F-terms (in the field theory limit)
  - Non-perturbative part = tree-level effective action of a Dp-D(p+4) bound state
- Generalised holomorphic anomaly equations
- Promising candidate for a worldsheet realization of the refined topological string

String Amplitudes, Topological Strings and the Omega-deformation

Thank You!

Strings @ Princeton
26 - 06 - 2014

Ahmad Zein Assi

CERN