Gauge interactions and topological phases of matter

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Based on a work [1604.06184] with Yuji Tachikawa

Classification of QFTs

- Classification of all possible QFTs : too difficult

Instead, let's consider a very crude classification

- Classification of IR limit of QFTs with mass gap and no topological degrees of freedom (i.e., **nothing?**)

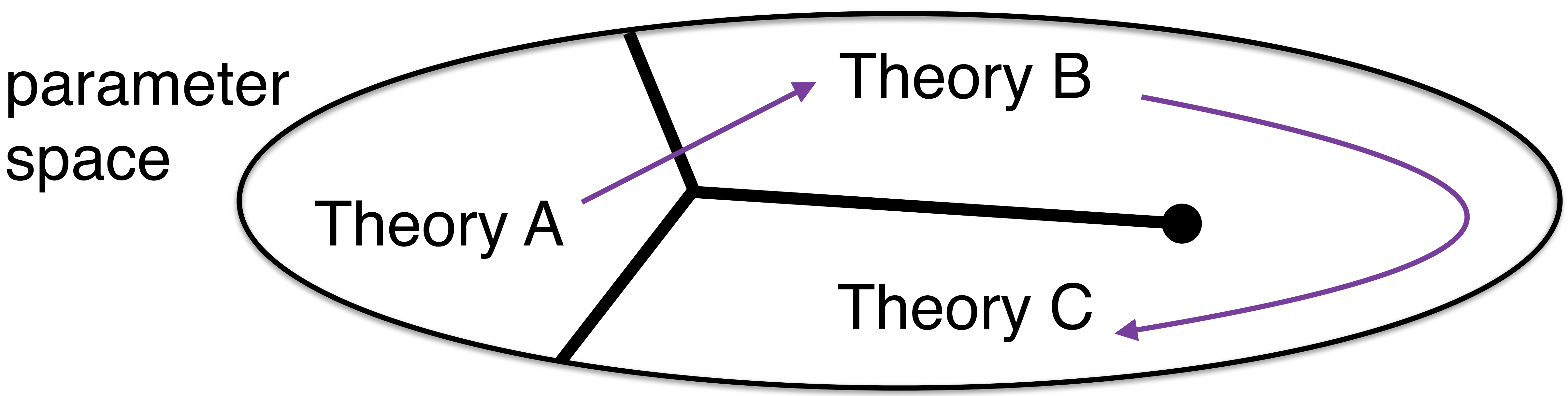
turned out to be very rich!

SPT phases

How can “nothing” be nontrivial?

Symmetry Protected Topological (SPT) phases:

Assume
- mass gap with a unique vacuum
- some global symmetry H



- Theory A $\neq$ Theory B : encounter phase transition
- Theory B $\sim$ Theory C : can avoid phase transition

The IR partition function

The IR limit of gapped QFTs with symmetry H :

Observables:

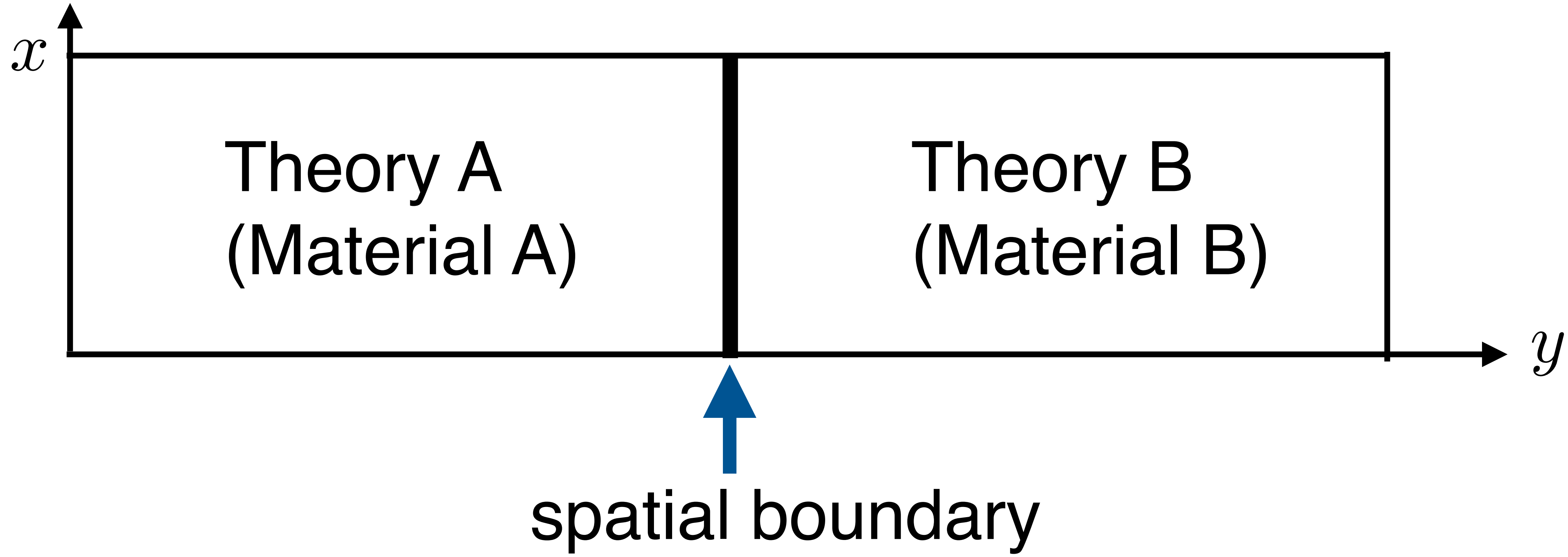
partition functions Z under background H fields

Theory A $\neq$ Theory B if $Z(\text{Theory A}) \neq Z(\text{Theory B})$

SPT phases are believed to be classified by the IR limit of partition functions.

e.g. $\log Z = \int \frac{ik}{4\pi} B dB$ (B : background H field)

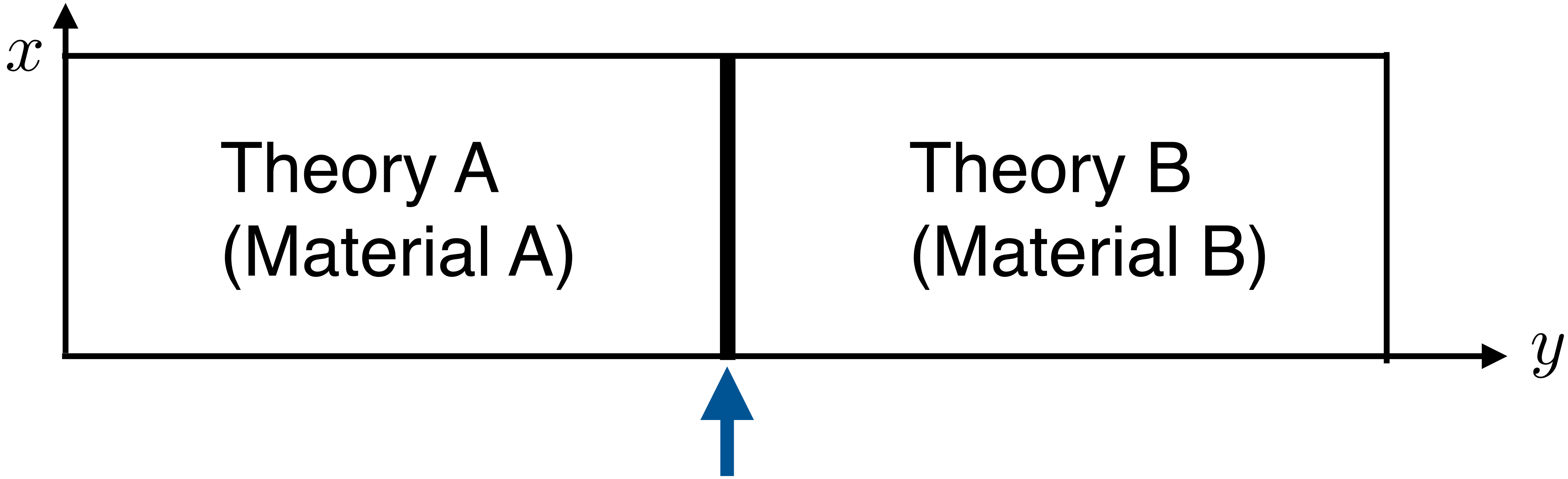
Anomaly inflow



"Anomaly inflow" from both sides of the boundary and some massless or topological degrees of freedom appear on the boundary.

e.g. $\log Z = \int \frac{ik}{4\pi} BdB$ → boundary chiral fermions

Anomaly inflow



In general, what can we get?

The boundary theory needs to match anomalies, but sometimes a variety of theories can be realized.

Example : H = Time reversal

Example: topological superconductor

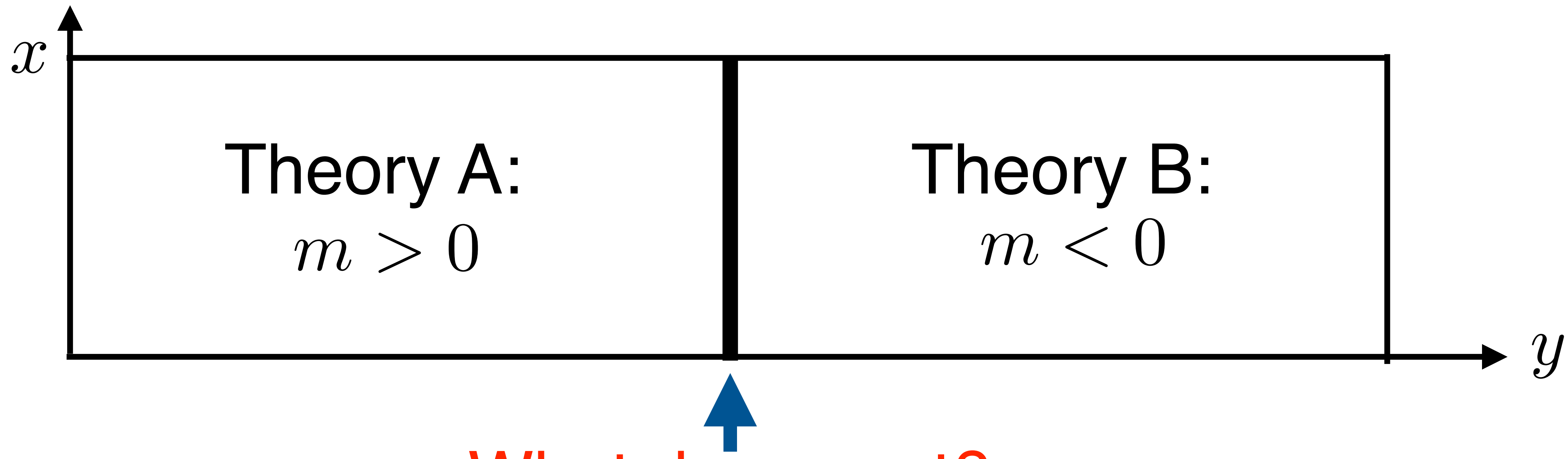
$\nu \in \mathbb{Z}$ copies of massive majorana fermion in 3+1 dim.

$$\mathcal{L} = -\frac{1}{2}\Psi^T C(\gamma^\mu \partial_\mu + m)\Psi$$

H = time reversal symmetry

⟶ m is real: either positive or negative.

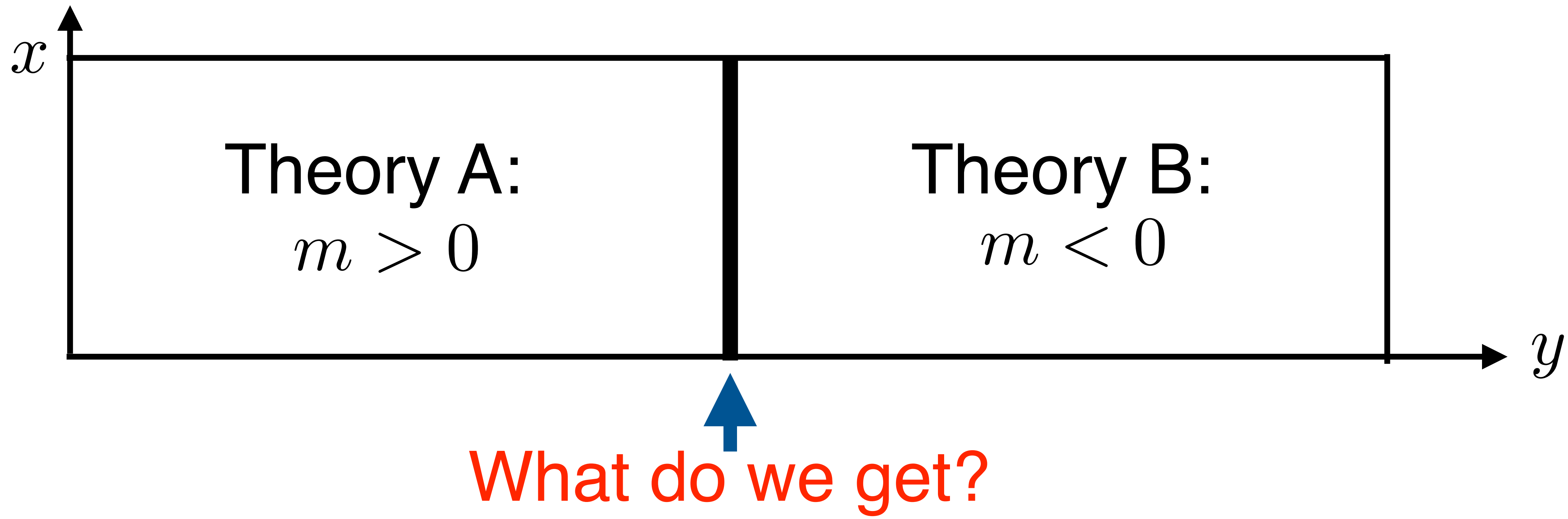
Example : H = Time reversal



What do we get?

- At the free level, we get $\nu \in \mathbb{Z}$ boundary fermions if bulk has $\nu \in \mathbb{Z}$ massive fermions.

Example : H = Time reversal



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- However, classification of partition function is done by $\mathbb{Z}_{16}$ In other words, $\nu = 16$ must be trivial.

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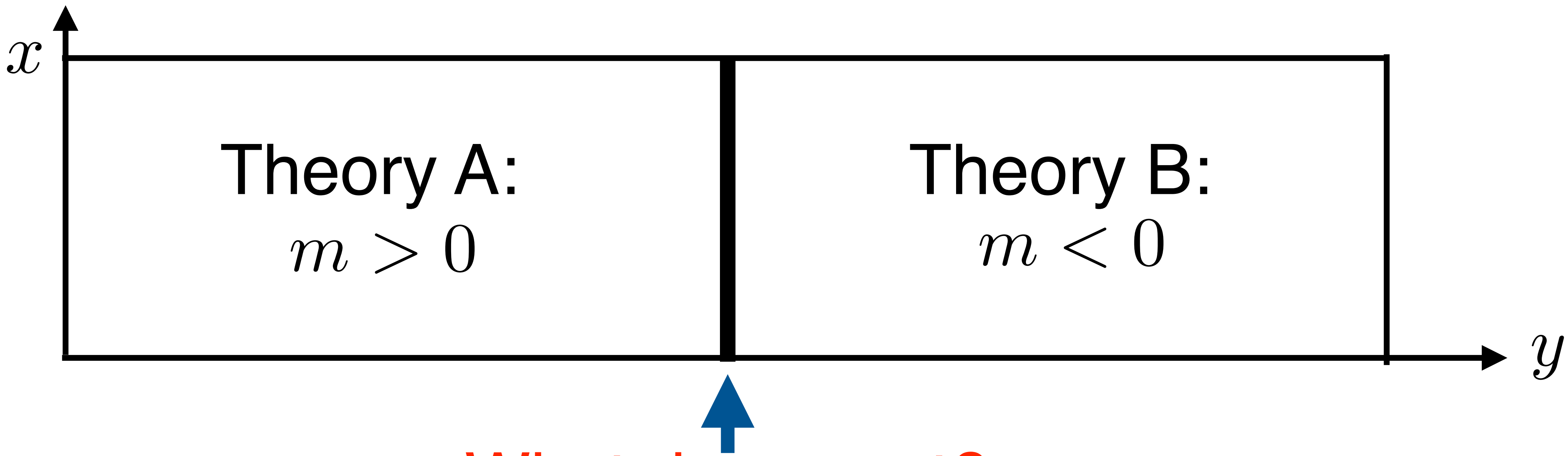


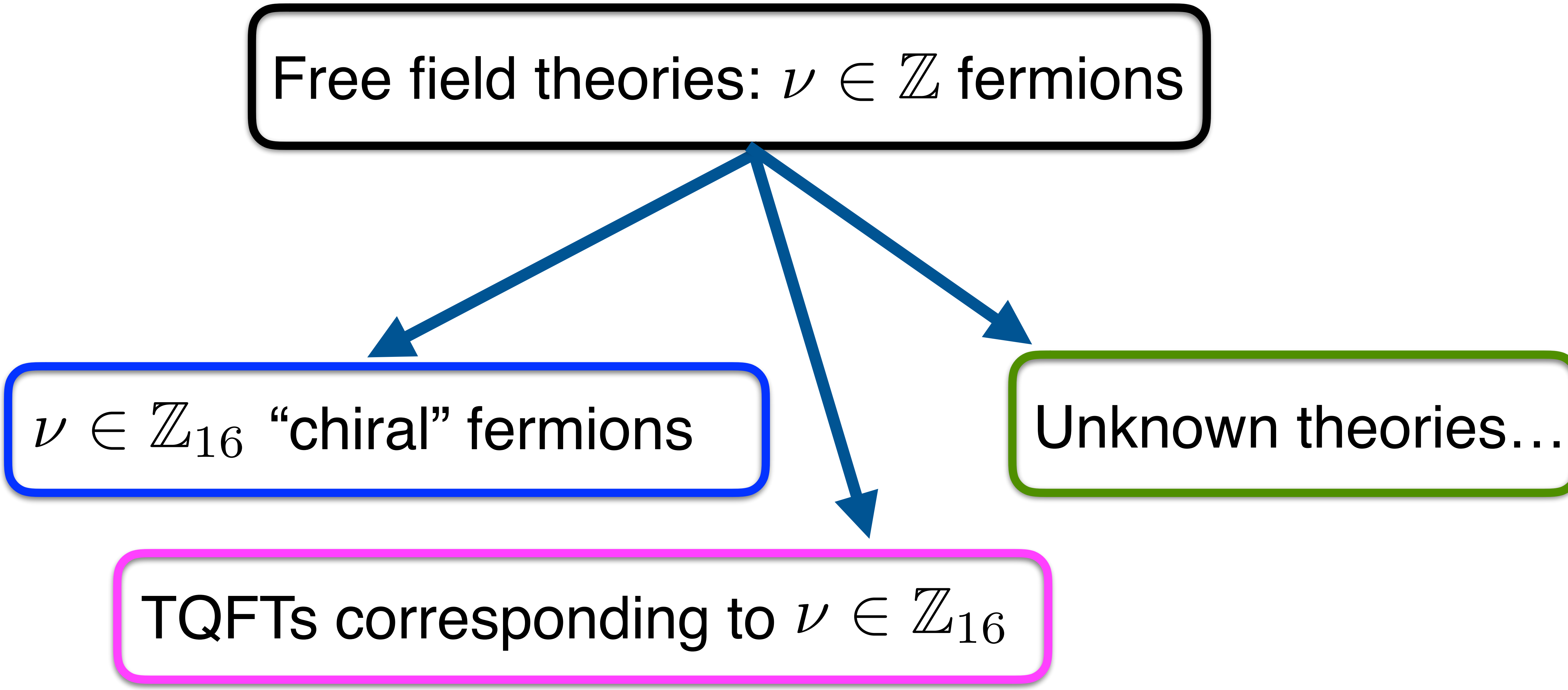
What do we get?

- At the free level, we get $\nu \in \mathbb{Z}$ boundary fermions if bulk has $\nu \in \mathbb{Z}$ massive fermions.

- However, classification of partition function is done by $\mathbb{Z}_{16}$ In other words, $\nu = 16$ must be trivial.

- More generally, the boundary can be TQFT without any fermions for arbitrary $\nu \in \mathbb{Z}_{16}$

A variety of boundary theories



To get the variety of boundary theories, we need interacting theories

Interactions

What interaction?

- Large 4-fermi interaction (in lattice for d>2).
- Gauge interaction

Where?

- Boundary
- Bulk

Interactions

What interaction?

- Large 4-fermi interaction (in lattice for d>2).
- Gauge interaction ←

Where?

- Boundary
- Bulk ←

Our work considers strongly coupled bulk field theories.

For other cases, see many references including

[Fidkowski-Kitaev,2009],[Wang-Wen,2013],[Seiberg-Witten,2016],[Witten,2016],……..

(many apologies for this very incomplete list)

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Set up

We consider strongly coupled bulk systems with

- mass gap with a unique vacuum
- some global symmetry H (=time reversal)

Explicitly, a gauge group G is coupled to some massive bulk fermions (and scalars).

$$-\frac{1}{4g^2}(F_{\mu\nu})^2 - \Psi D \Psi - m\Psi\Psi - |D_\mu H| - \mu^2 |H|^2$$

The scalars can be infinitely heavy and decoupled:

$$\mu^2 \to +\infty$$

Set up

Does the gauge theory give the same IR SPT phase as the free fermion system?

Free v.s. Strongly coupled

$$-\frac{1}{4g^2}(F_{\mu\nu})^2 - \Psi D \Psi - m\Psi\Psi - |D_\mu H| - V(H)$$

$$V(H) = +M^2|H|^2 + |H|^4$$

Confining phase: strongly coupled

$$V(H) = -M^2|H|^2 + |H|^4$$

Higgs phase: almost free fermions

They can be smoothly connected without phase transition if some conditions are satisfied.

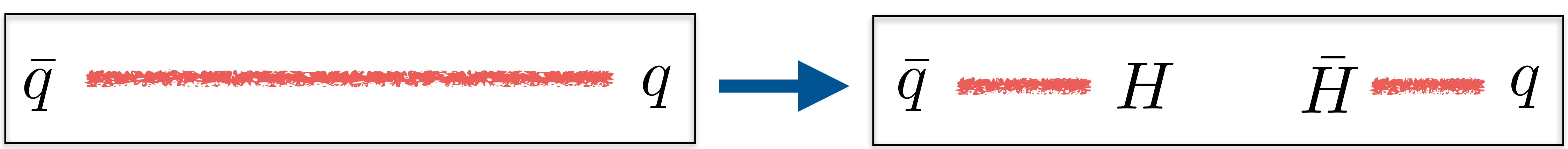
Condition on gauge field 1

Condition 1: topology of gauge group

We need matter field H in the fundamental representation.

[Banks,Rabinovici,1979]
[Fradkin,Shenker,1979]

Color flux tubes of gauge fields are screened by the pair creation of fundamental matter H .



For the fundamental matter to exist, the gauge group must be simply connected.

$$\pi_1(G) = 0$$

Condition on gauge field 1

Condition 1: topology of gauge group

Actually, if $\pi_1(G) \neq 0$, there is nontrivial TQFT in bulk.

⟶ Contradict the assumptions of SPT phases

[Aharony,Seiberg,Tachikawa,2013]
[Tachikawa,2014]

Example:

$G = SU(2)/\mathbb{Z}_2 = SO(3)$ would give bulk topological $\mathbb{Z}_2$ gauge theory.

To avoid topological degrees of freedom, we also impose

$$\pi_0(G) = 0$$

Condition on gauge field 2

Condition 2: theta angle

$$-\frac{1}{4g^2}(F_{\mu\nu})^2 - \bar{\Psi} D \Psi - m\bar{\Psi}\Psi$$

↓ Integrate out massive fermions

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4g_{\text{eff}}^2}(F_{\mu\nu})^2 + \frac{\theta_{\text{eff}}}{64\pi^2} F_{\mu\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$

$$\theta_{\text{eff}} = \begin{cases} 0 & m > 0 \\ \pi t & m < 0 \end{cases}$$

t : Dynkin index of the representation of fermions.

Condition on gauge field 2

Condition 2: theta angle

For the IR limit to have a unique vacuum, we must impose

$$\theta_{\rm eff} = 0 \mod 2\pi$$

If $\theta_{\rm eff} = \pi$, the time-reversal symmetry is spontaneously broken.

[Dashen,1971] [Baluni,1979] [Witten,1980]

⟶ Contradict the assumptions of SPT phases

So we must require that the fermion representation satisfies

$$\theta_{\rm eff} = \pi t \in 2\pi\mathbb{Z}$$

Summary of conditions

For the gauge interaction to preserve the IR SPT phase, (i.e., a unique vacuum preserving symmetry H)

- $$\pi_0(G) = \pi_1(G) = 0$$
- $$\theta_{\text{eff}} = \pi t = 0 \mod 2\pi$$

One can directly show that the partition function Z of the gauge theory is the same as the free fermion theory up to continuous deformation.

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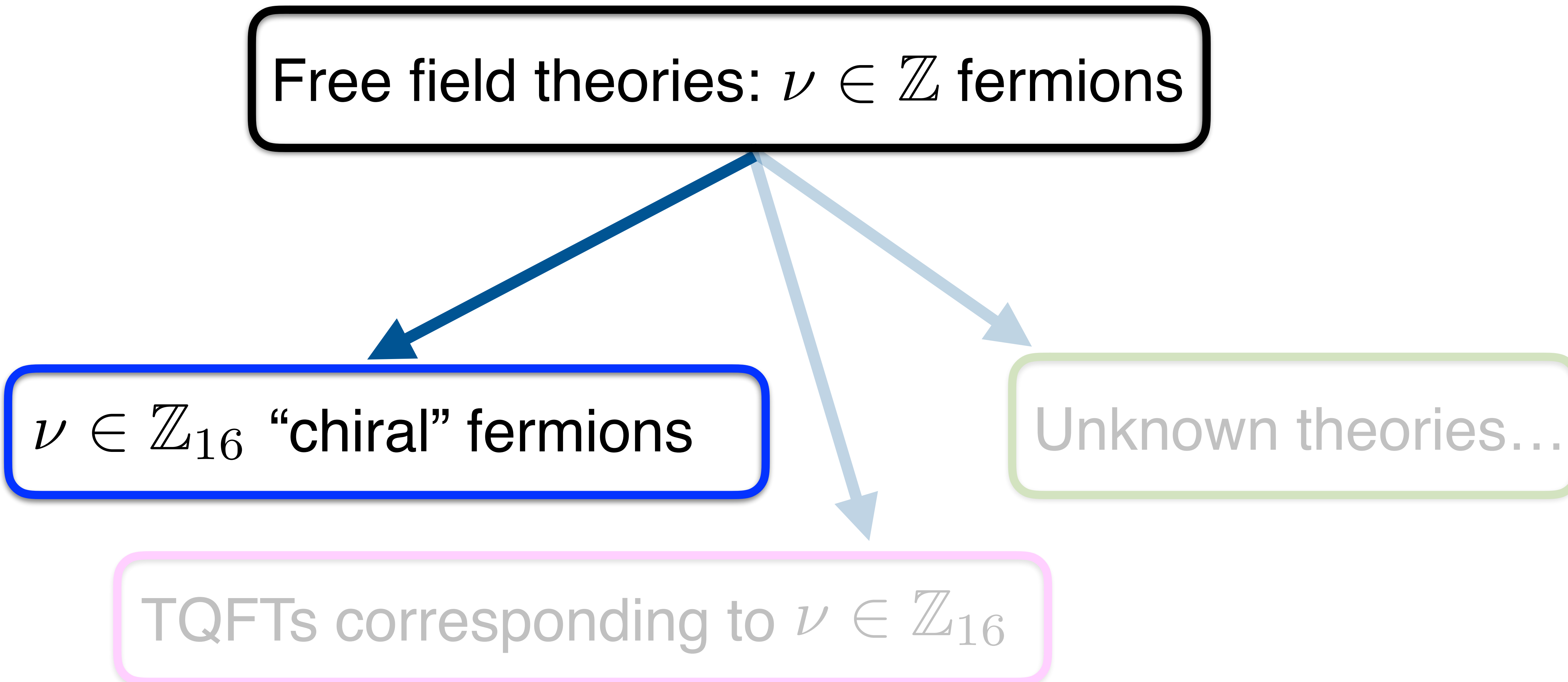
Topological superconductor (1)



We want to demonstrate explicitly that $\nu = 16$ is trivial.

Topological superconductor (1)

$\mathcal{N}=2$ SUSY SU(2) theory with $N_f=4$

- There are 16 fermions contained in quarks.

$$\nu = 2N_cN_f = 16$$

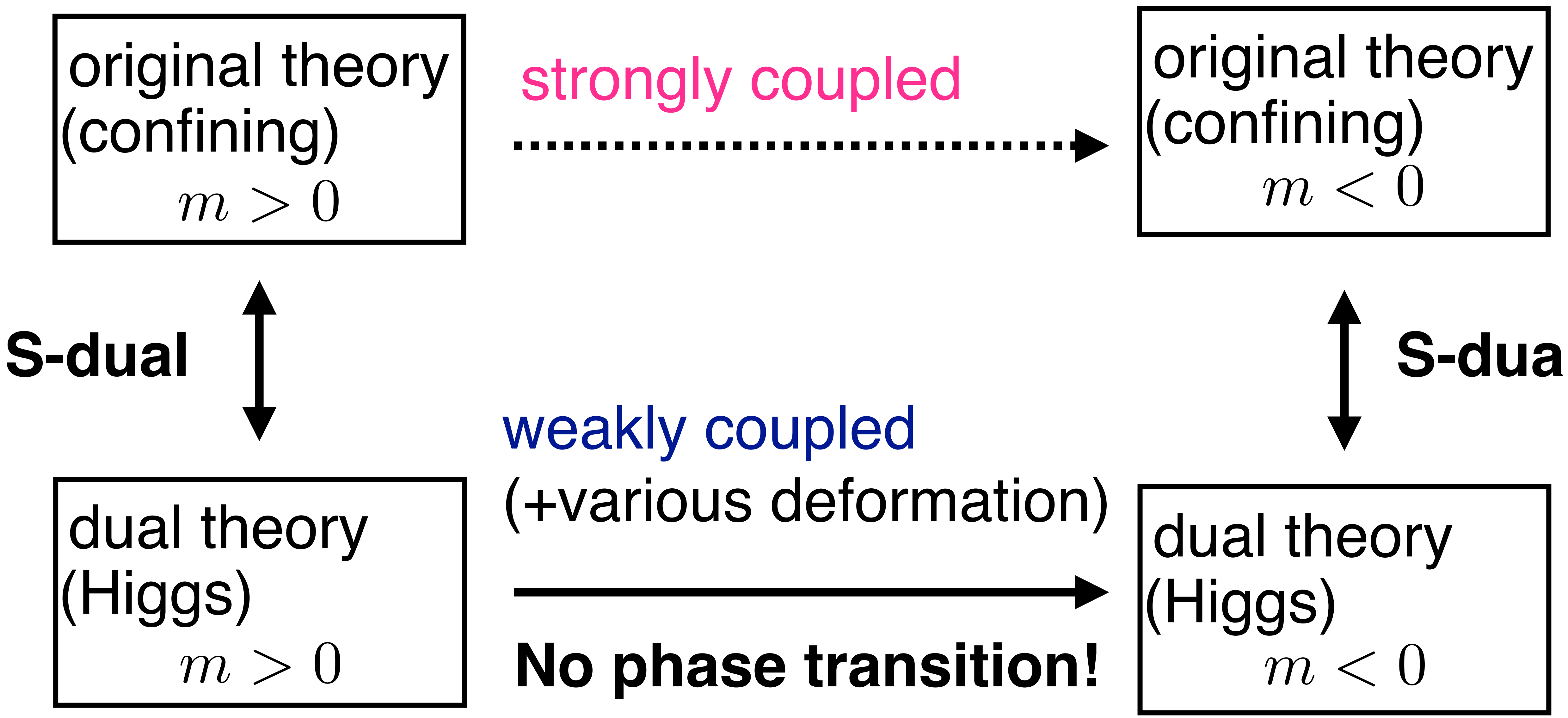
- We change the quark mass m from positive to negative.

Theory A $m>0$ → Theory B $m<0$

No phase transition???

(We also introduce constant SUSY breaking terms.)

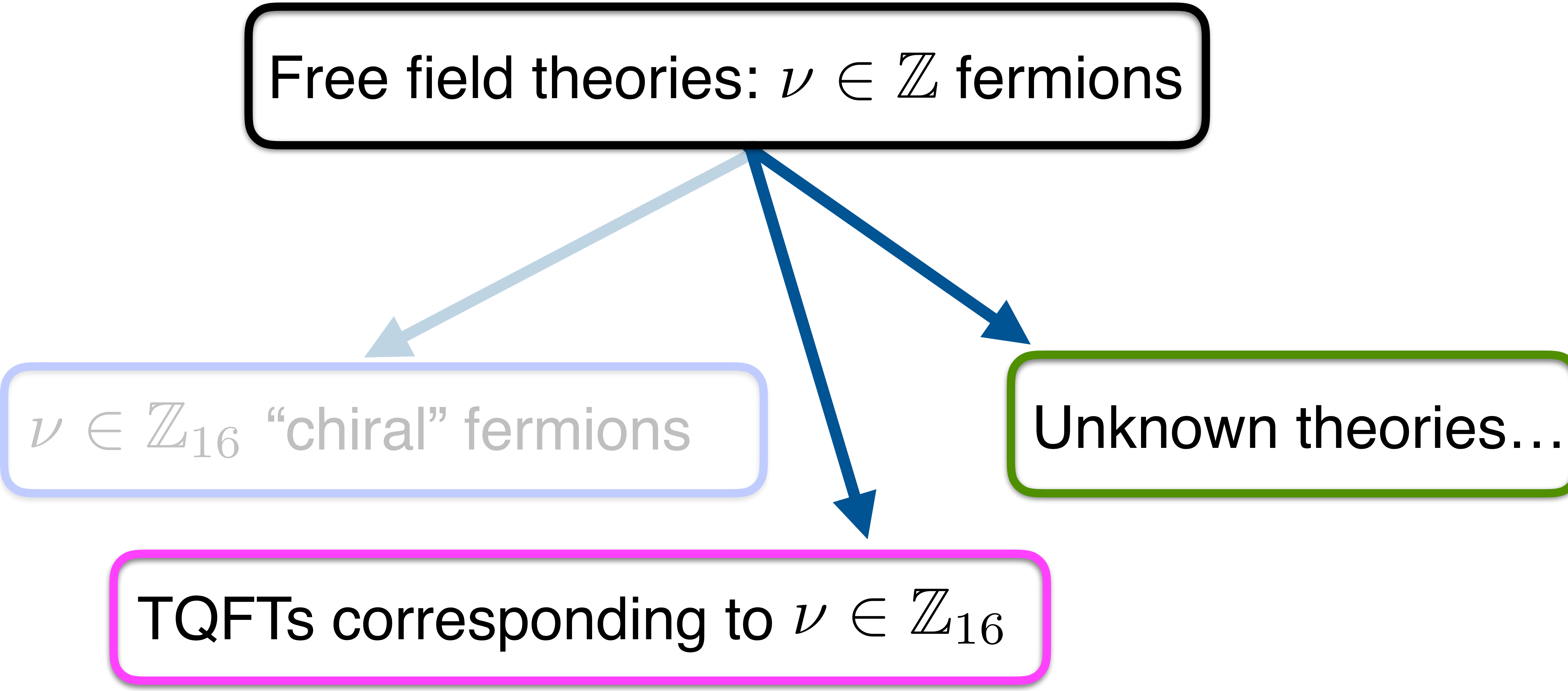
Topological superconductor (1)



This explicitly shows that $\nu = 16$ is really trivial.

(An approach based on boundary gauge theory; see [Witten,2016])

Topological superconductor (2)

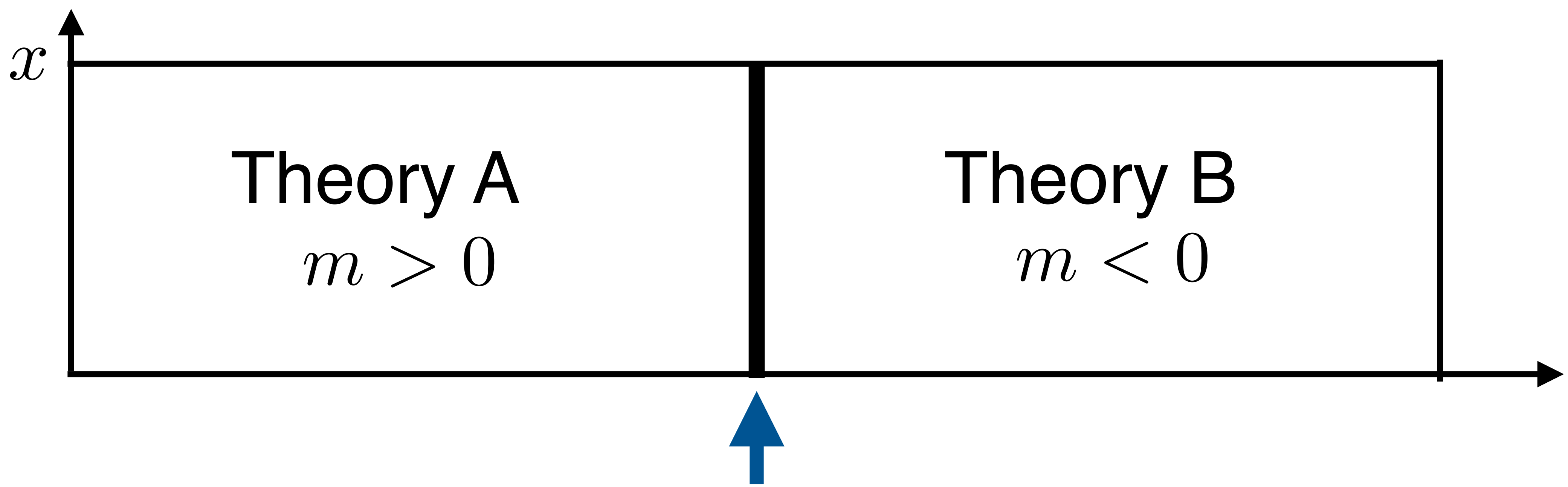


What boundary theory can we get other than massless fermions?

Topological superconductor (2)

$\mathcal{N}=1$ pure Super-Yang-Mills with gaugino mass

Condition: $t_{\mathrm{adj}} = h^{\vee}_{G} \in 2\mathbb{Z}, \quad \pi_0(G) = \pi_1(G) = 0$



Strongly coupled domain wall
$\nu = \dim G$

TQFTs supported on the domain walls. [Acharya-Vafa,2001] [Gaiotto,2013]

More details: work in progress…

Contents

Summary

- Gauge interaction is a powerful way to obtain a variety of boundary theories of bulk SPT phases.

- Bulk gauge interaction preserves SPT phases if

$$\pi_0(G) = \pi_1(G) = 0$$

$$\theta_{\text{eff}} = \pi t \in 2\pi\mathbb{Z}$$