

**Strings 2016**

Tsinghua University, Beijing

# (Super-)Conformal Bootstrap in 2D

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based on work with Scott Collier, Ying-Hsuan Lin,  
Shu-Heng Shao, David Simmons-Duffin, Yifan Wang

1. Modular bootstrap revisited

Collier, Lin, XY, 1608.?????

2. (4,4) superconformal bootstrap

Lin, Shao, Simmons-Duffin, Wang, XY, 1511.04065

3. (2,2) superconformal bootstrap

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Can we construct such a CFT?

Cannot be rational, because RCFT always contains an extended chiral algebra: extra holomorphic currents that are Virasoro primaries.

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Cannot be free orbifolds, because they also contain extra (higher spin) currents.



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Can we construct such a CFT?

Use exactly marginal deformations?

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Try (2,2) SCFT, gauge away R-current?

Nope, still have conserved higher spin currents of spin 3 and higher...

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Expected to be ubiquitous: e.g. holographic dual to “generic” 3d gravity theory in AdS3.

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Bosonic NLSM on CY? Perturbatively, need infinitely many fine tunings...existence of fixed point questionable...

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Expected to be ubiquitous: e.g. holographic dual to “generic” 3d gravity theory in AdS<sub>3</sub>.

Can we construct such a CFT?

No more tricks in my bag ...

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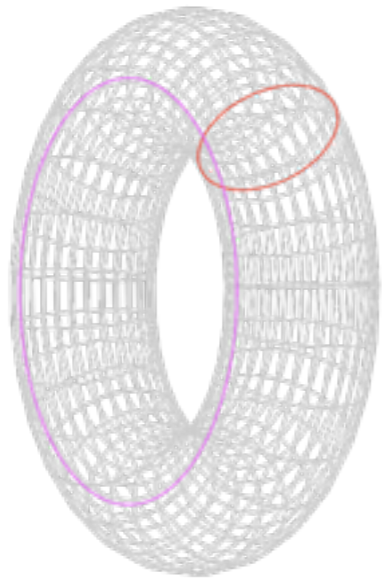
Most likely the latter.

# Conformal Bootstrap in 2D



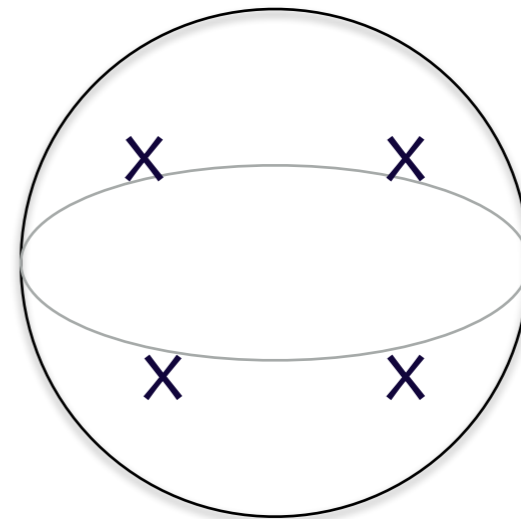
# Conformal Bootstrap in 2D

modular invariance



$$\tau \rightarrow -\frac{1}{\tau}$$

sphere crossing



$$z \rightarrow 1 - z$$

# Conformal Bootstrap in 2D

In principle, modular invariance of torus 1-point functions combined with crossing relation of sphere 4-point functions for **all** Virasoro primaries define a consistent CFT. [BPZ, Friedan-Shenker, Segal, Moore-Seiberg]

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For now, we investigate the consequence of modular invariance of the partition function and the crossing relation of 4-point function separately. **Unitarity** will be assumed throughout this talk.

# Modular Bootstrap

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[For rational CFT, this is textbook stuff]

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[Modern reboot: Hellerman, Friedan-Keller, Qualls-Shapere]



# Modular Bootstrap

[Hellerman, Friedan-Keller, Qualls-Shapere]

Assuming absence of conserved currents and  $c > 1$ , the torus partition function admits character decomposition:

$$Z(\tau, \bar{\tau}) = \chi_0(\tau) \bar{\chi}_0(\bar{\tau}) + \sum_{h, \bar{h} > 0} d(h, \bar{h}) \chi_h(\tau) \bar{\chi}_{\bar{h}}(\bar{\tau}).$$

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$$\chi_0(\tau) = q^{-\frac{c}{24}} \prod_{n=2}^{\infty} \frac{1}{1 - q^n} \quad \text{is the vacuum character}$$

$$\chi_h(\tau) = q^{h - \frac{c}{24}} \prod_{n=1}^{\infty} \frac{1}{1 - q^n} \quad \text{is a non-degenerate Virasoro character}$$

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We simply impose the positivity of  $d(h, \bar{h})$  and modular invariance of  $Z(\tau, \bar{\tau})$ , namely,

$$Z(-1/\tau, -1/\bar{\tau}) = Z(\tau, \bar{\tau})$$

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What sort of constraints are there on the weights and degeneracy of the primaries?

# Modular Bootstrap

Seek linear functionals  $D$  with the property that

$$D [Z(-1/\tau, -1/\bar{\tau}) - Z(\tau, \bar{\tau})] \equiv E_0 + \sum d(h, \bar{h}) E_{h, \bar{h}}$$

(before imposing modular invariance)

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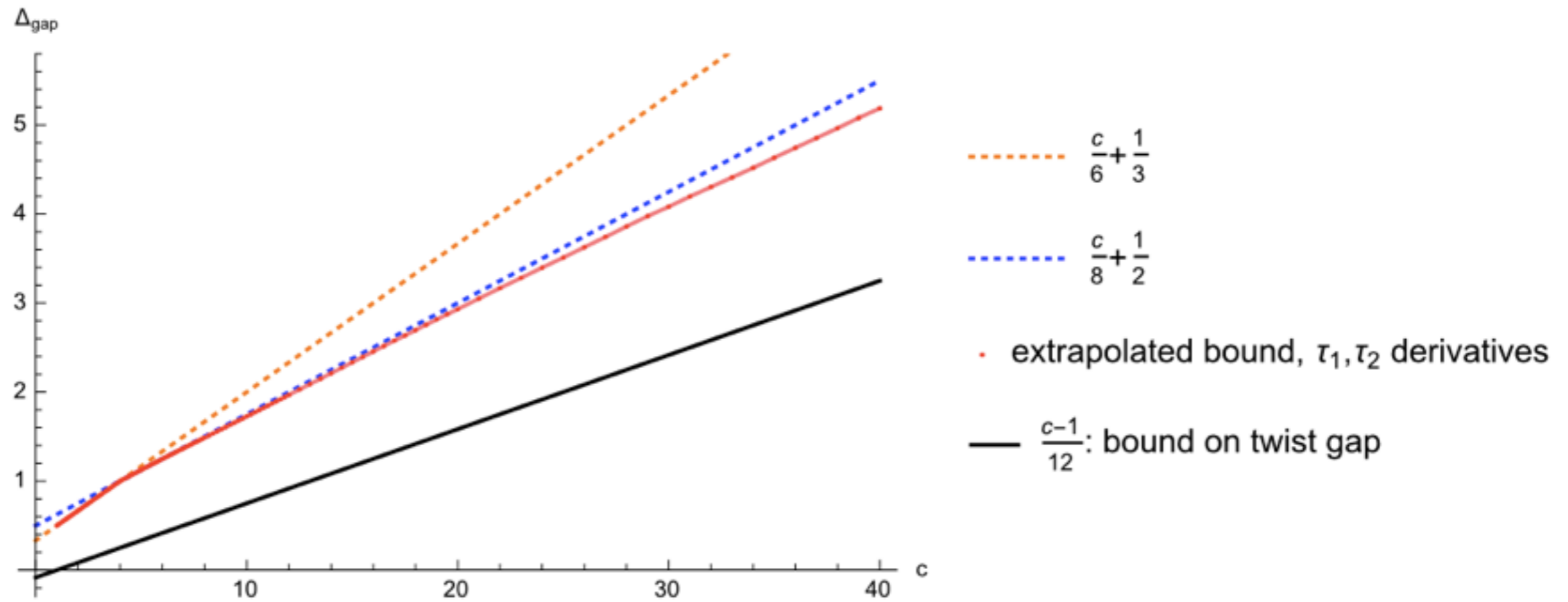
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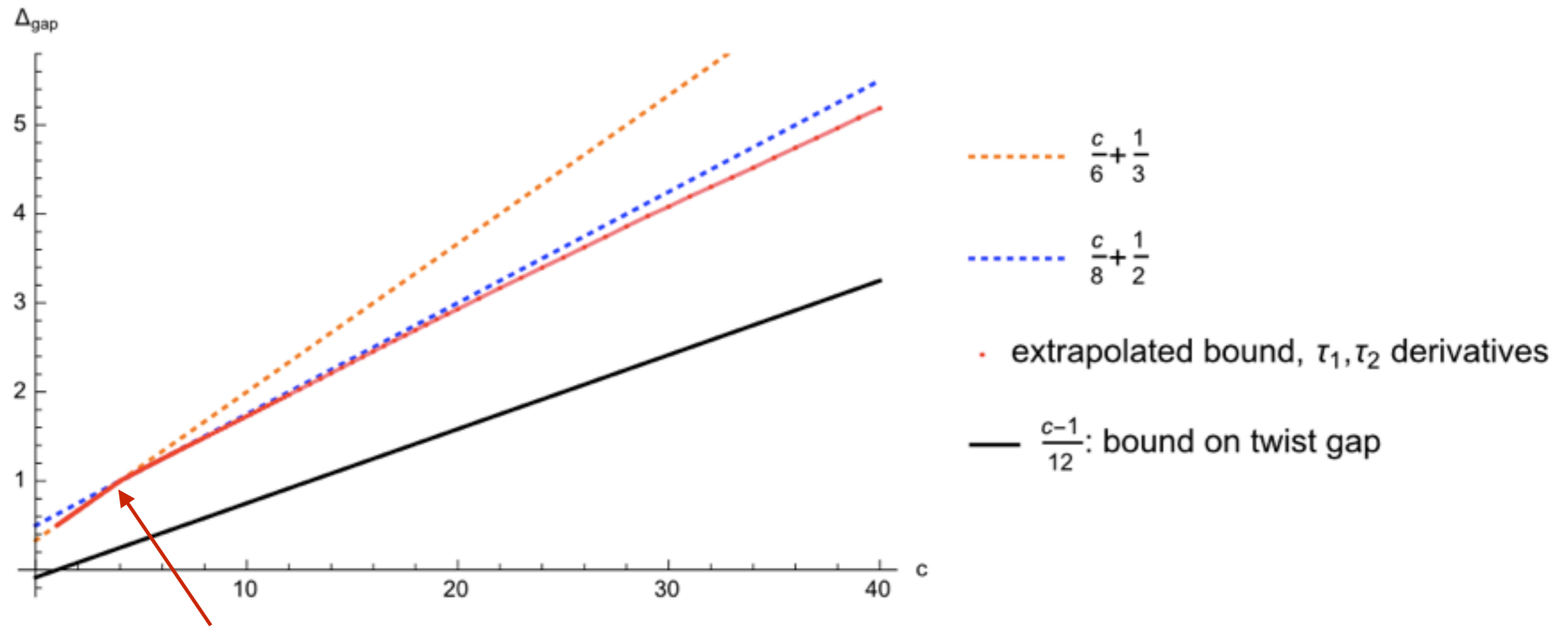
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$$\text{Try } D = \sum_{n+m \leq N} a_{n,m} \partial_z^n \partial_{\bar{z}}^m \Big|_{z=0}, \quad \tau \equiv ie^z, \quad \bar{\tau} = -ie^{\bar{z}}$$

# Bounds on the gap in scaling dimension

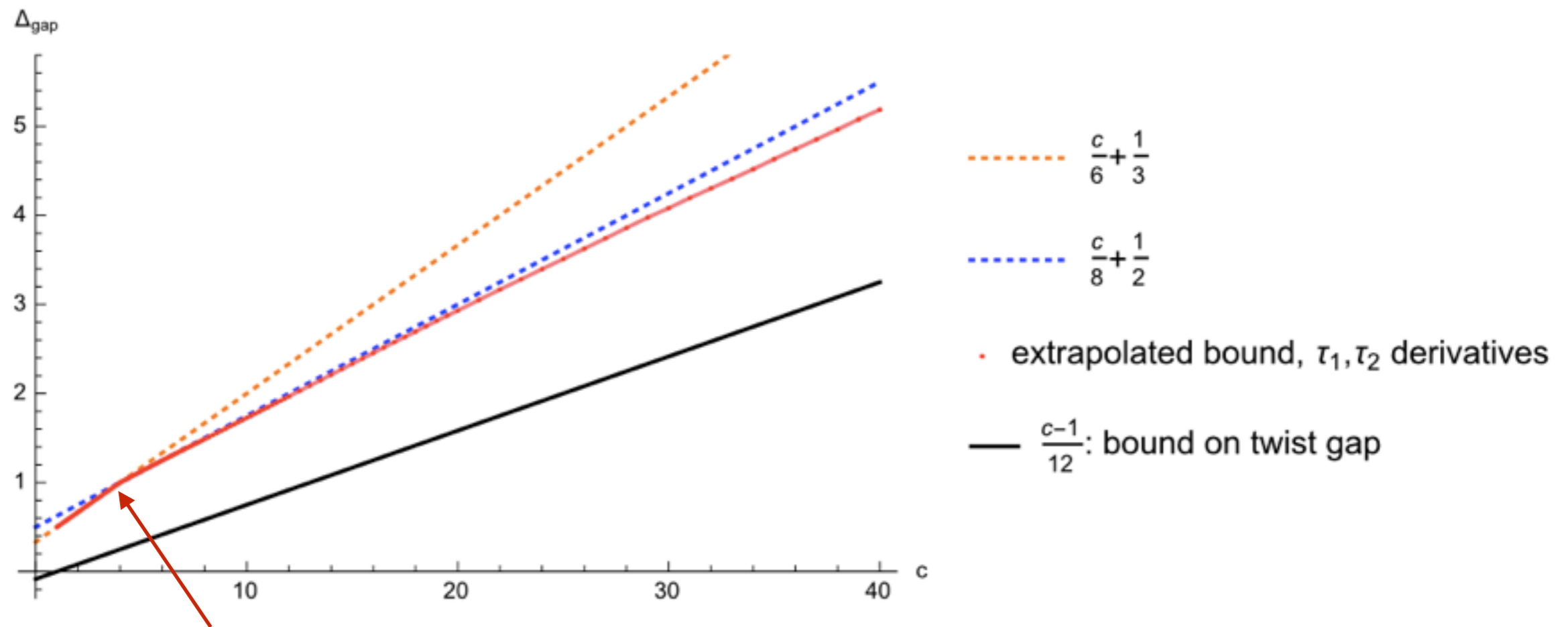


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 $c=4, \text{gap}=1$**

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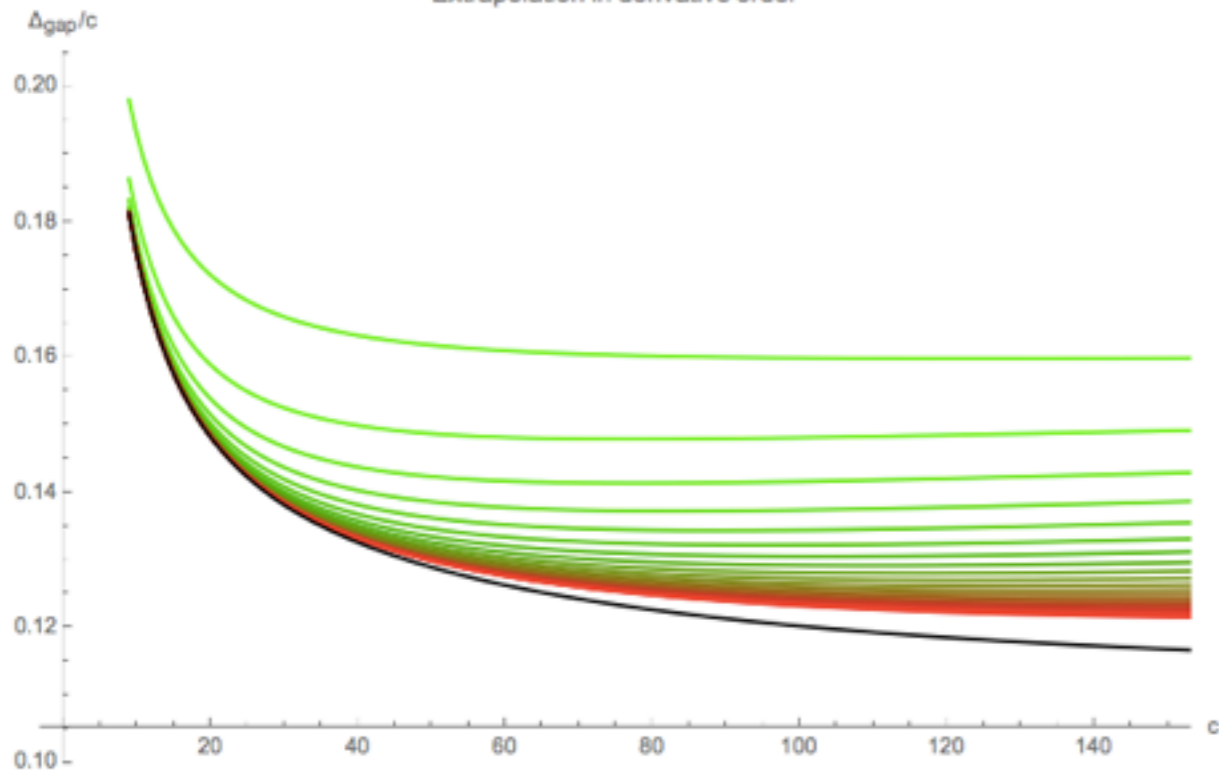
8 free fermions with GSO projection

(despite conserved currents, partition function can be formally decomposed into non-degenerate characters with positive coefficients, due to twist-1 primaries)

# Asymptotics at large $c$ ?

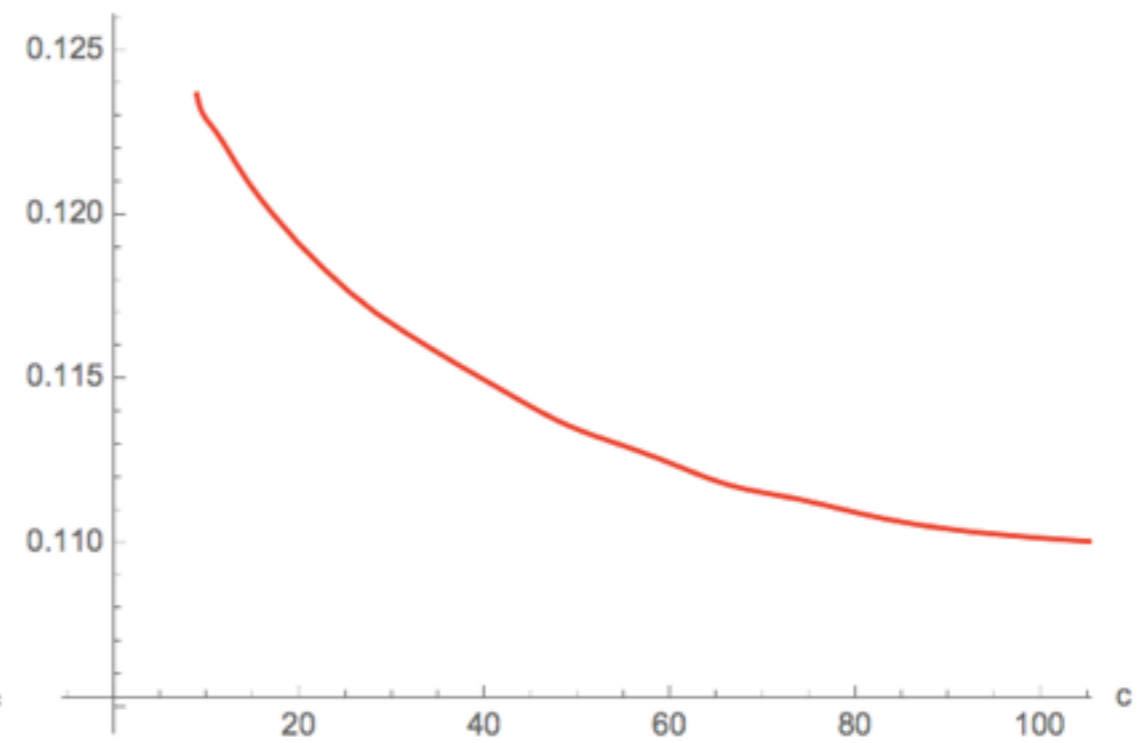
$$\Delta_{gap}/c$$

Extrapolation in derivative order



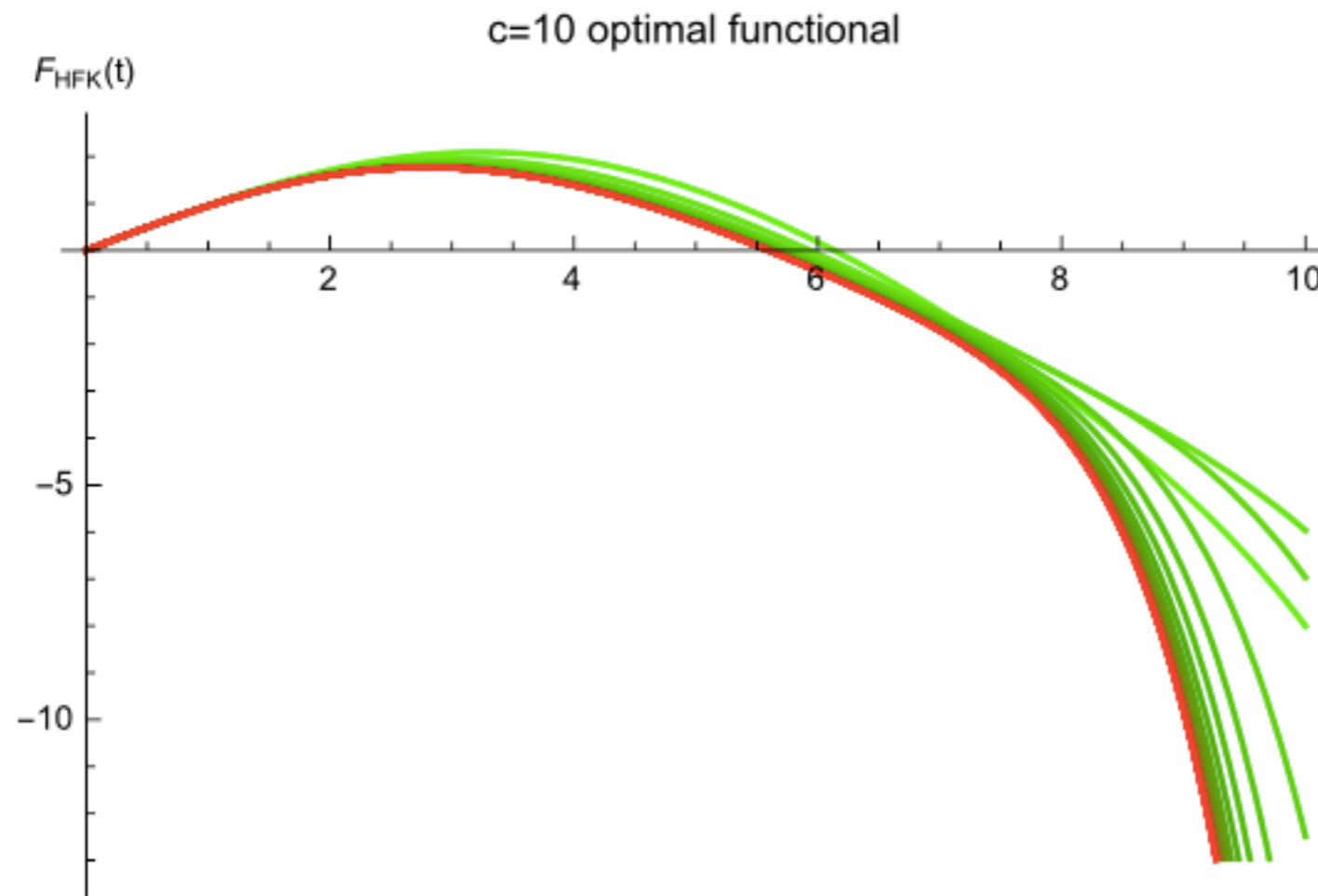
$$\text{slope of } \Delta_{gap}(c)$$

$d\Delta_{gap}/dc$



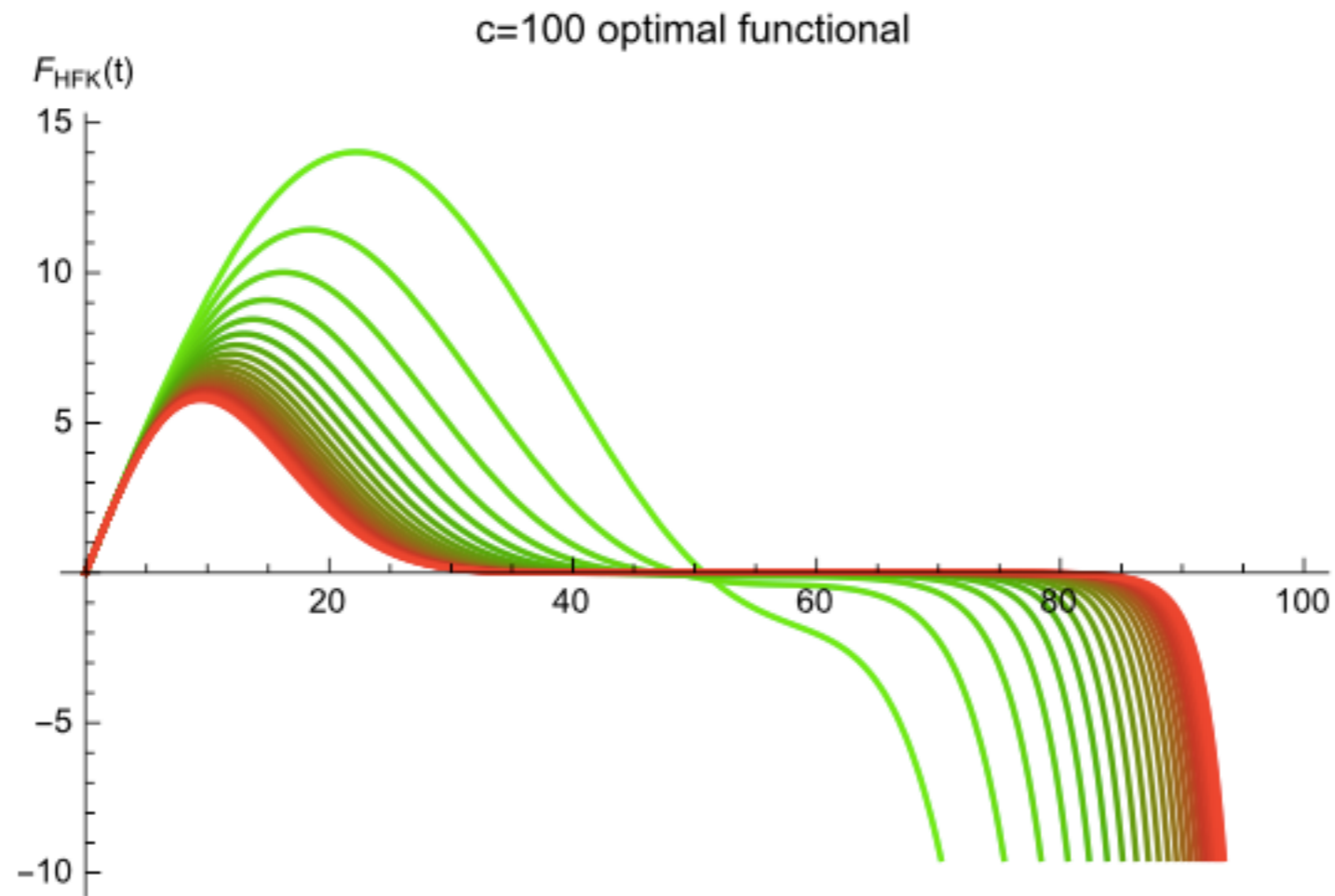
# Why is it hard at large $c$ ?

The optimal linear functional as a function of  $\beta \partial_\beta$   
(evaluated at  $\beta = 1$ )

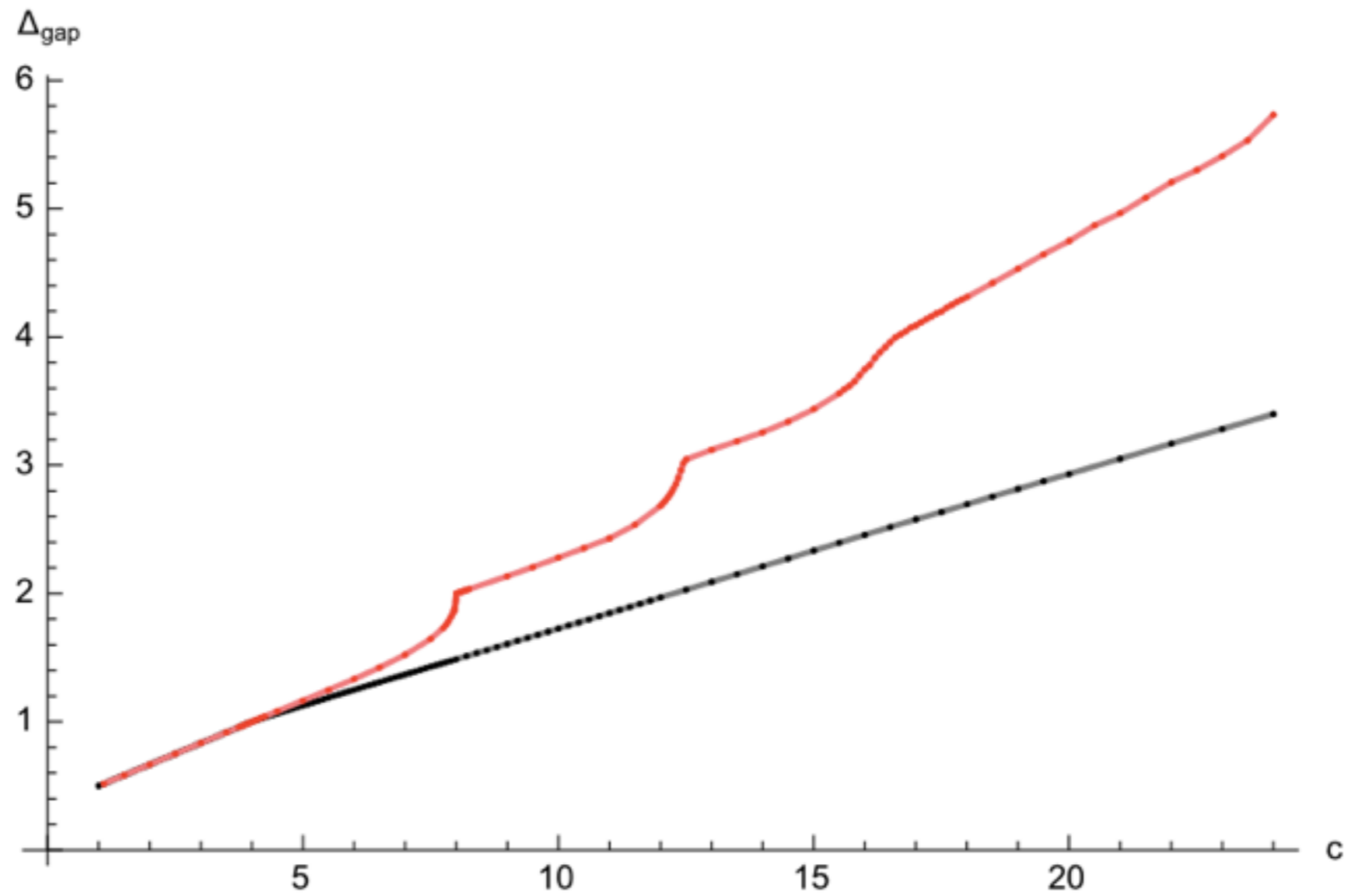


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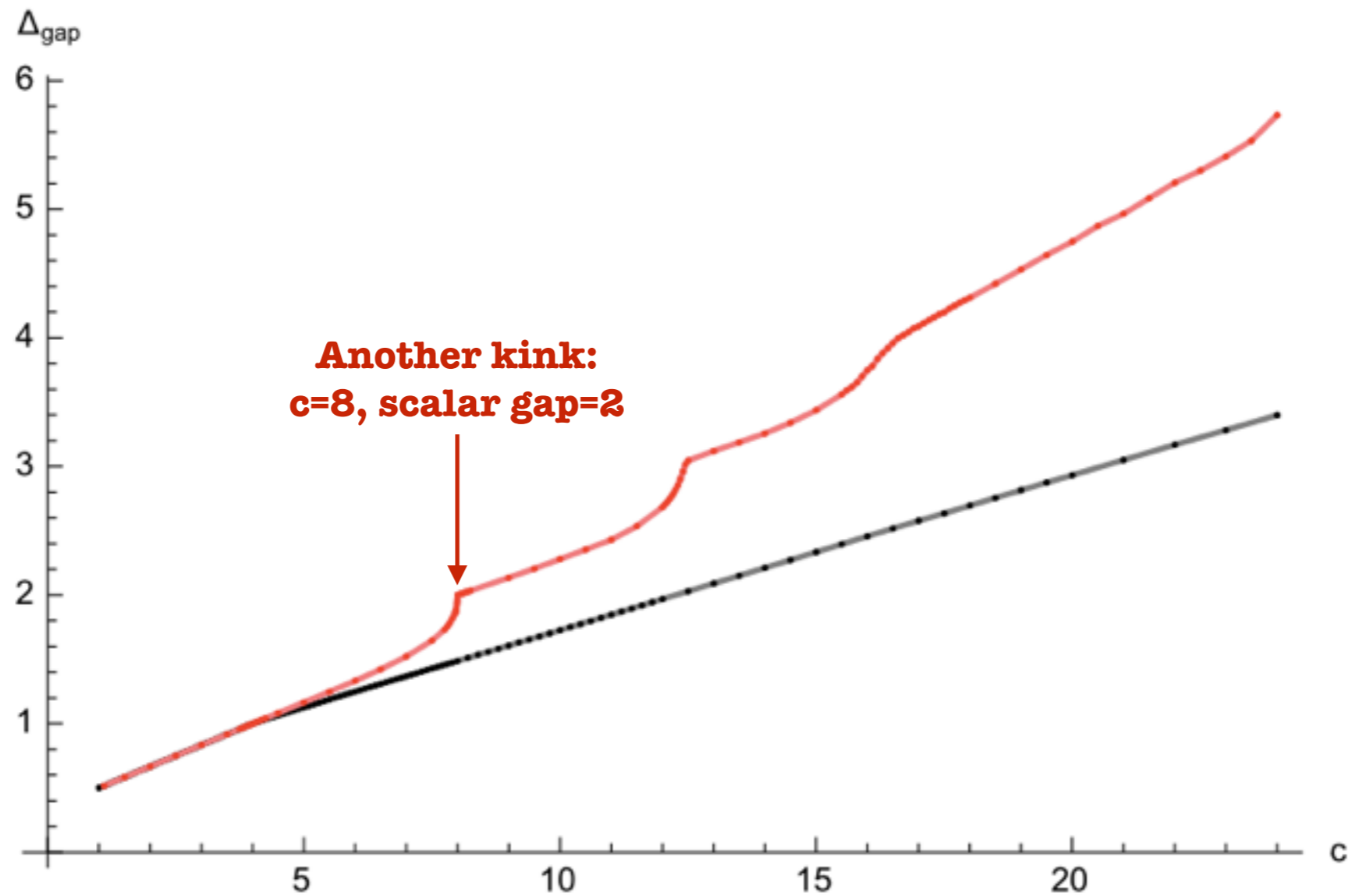


# Bounds on the gap of scalar primaries

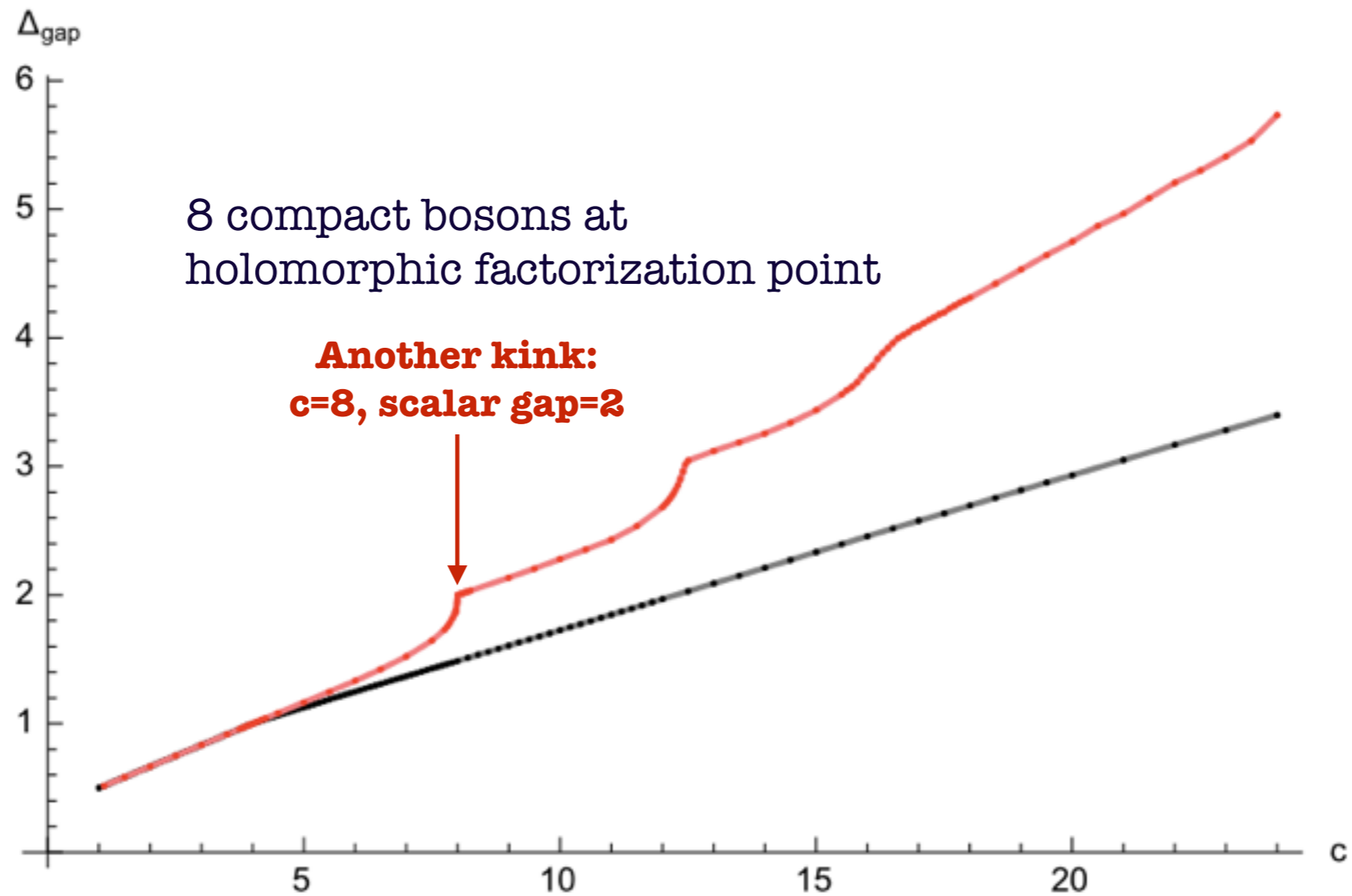




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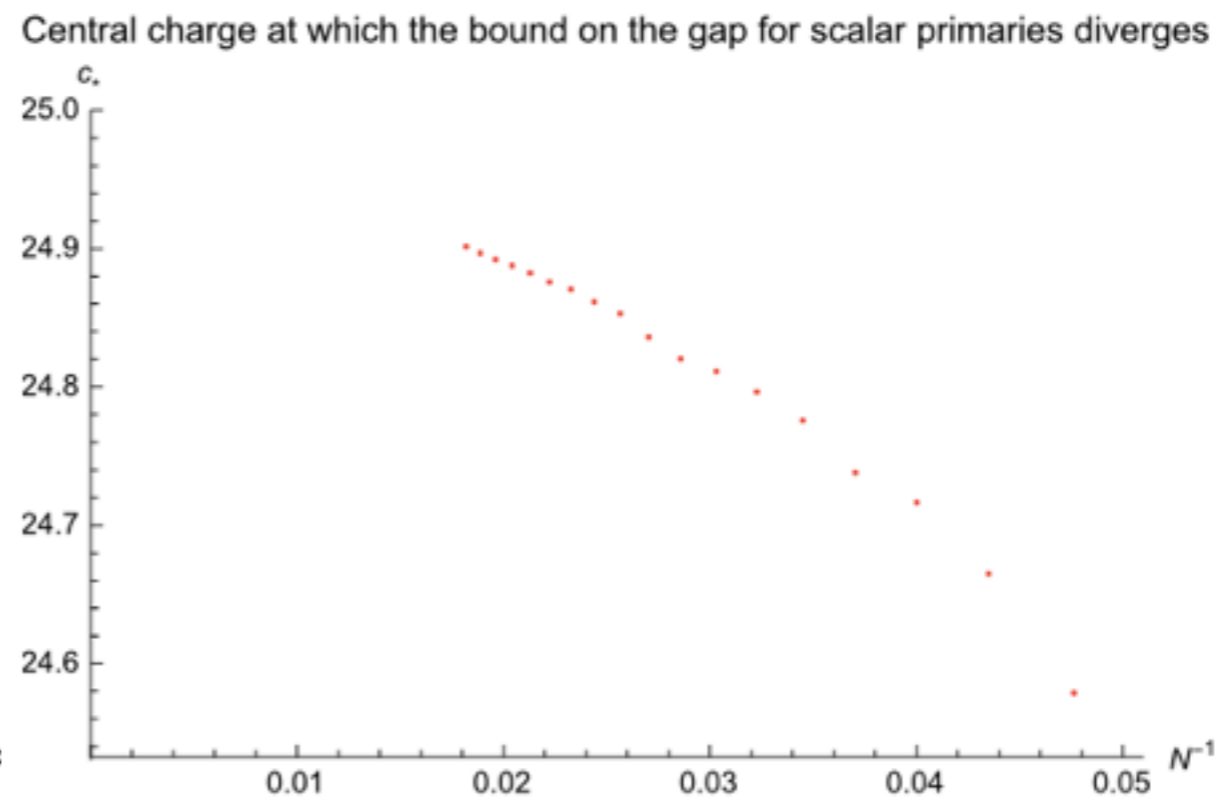
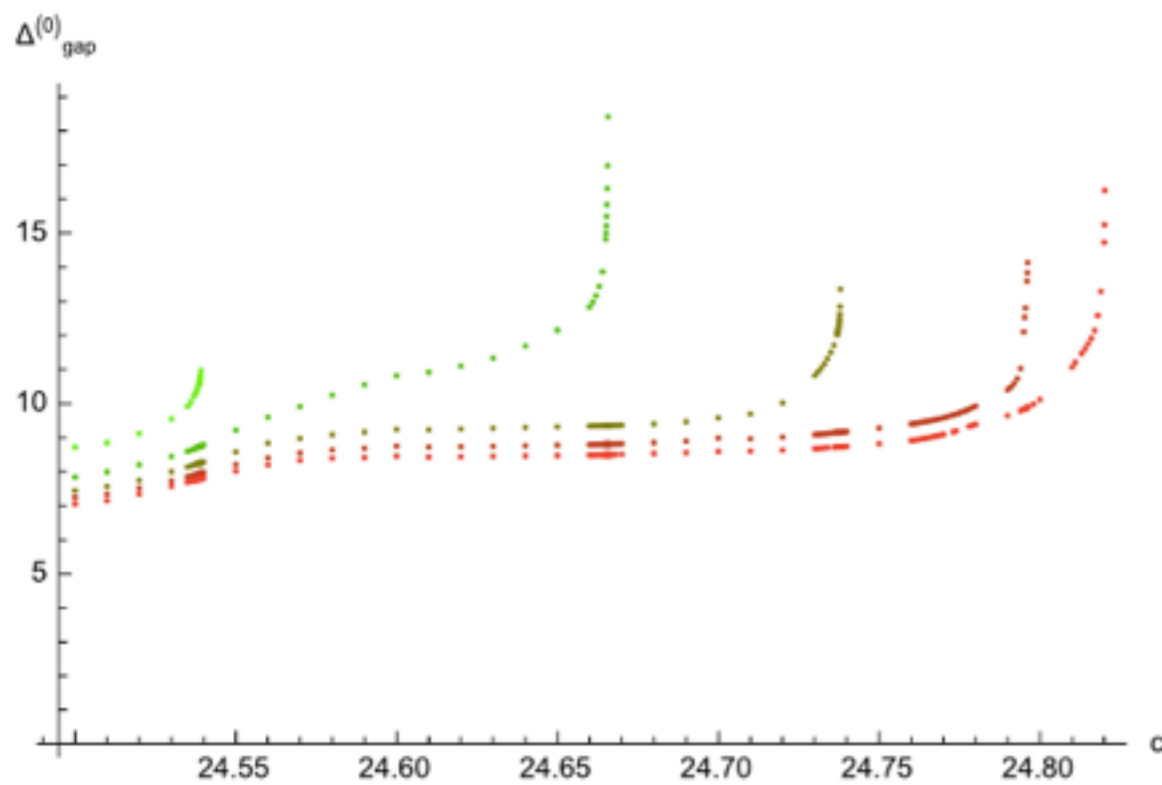


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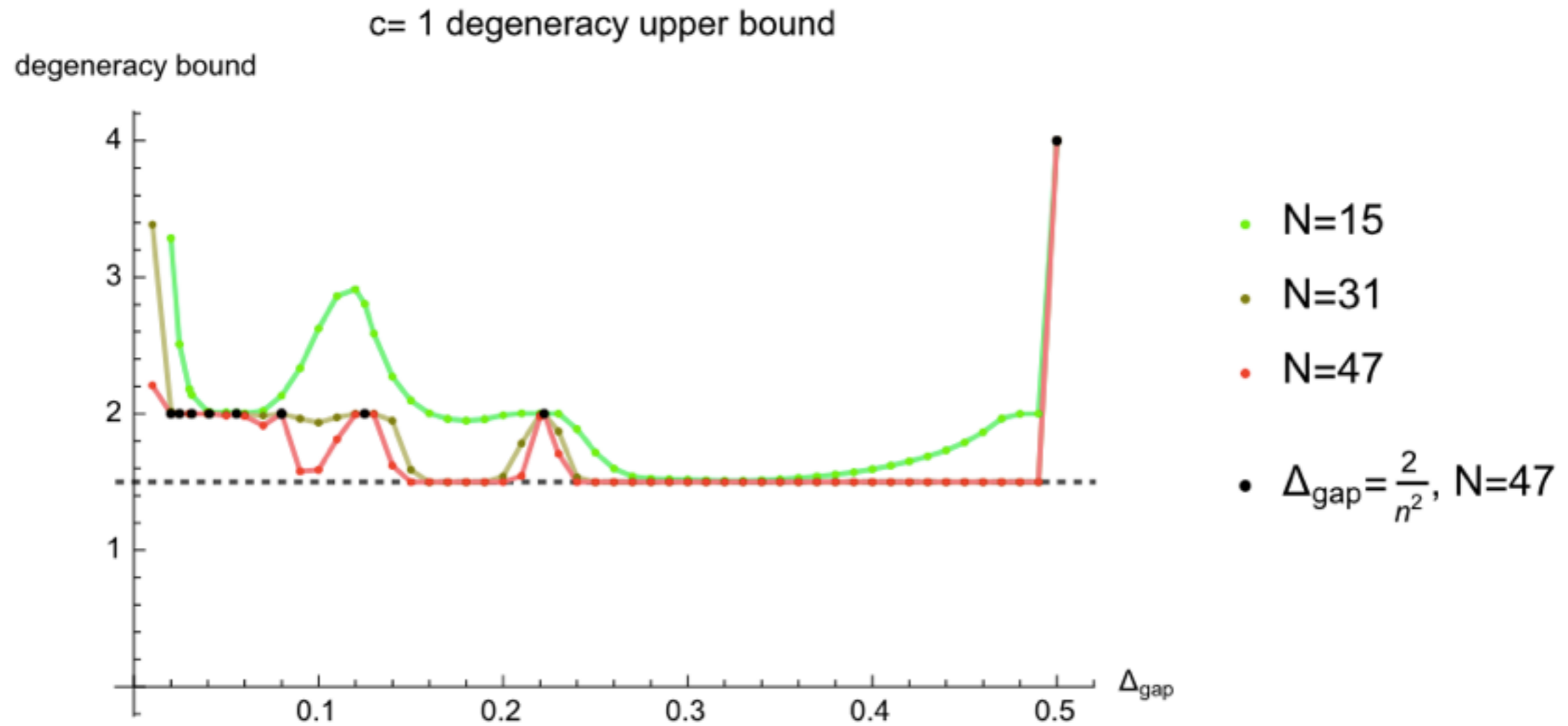


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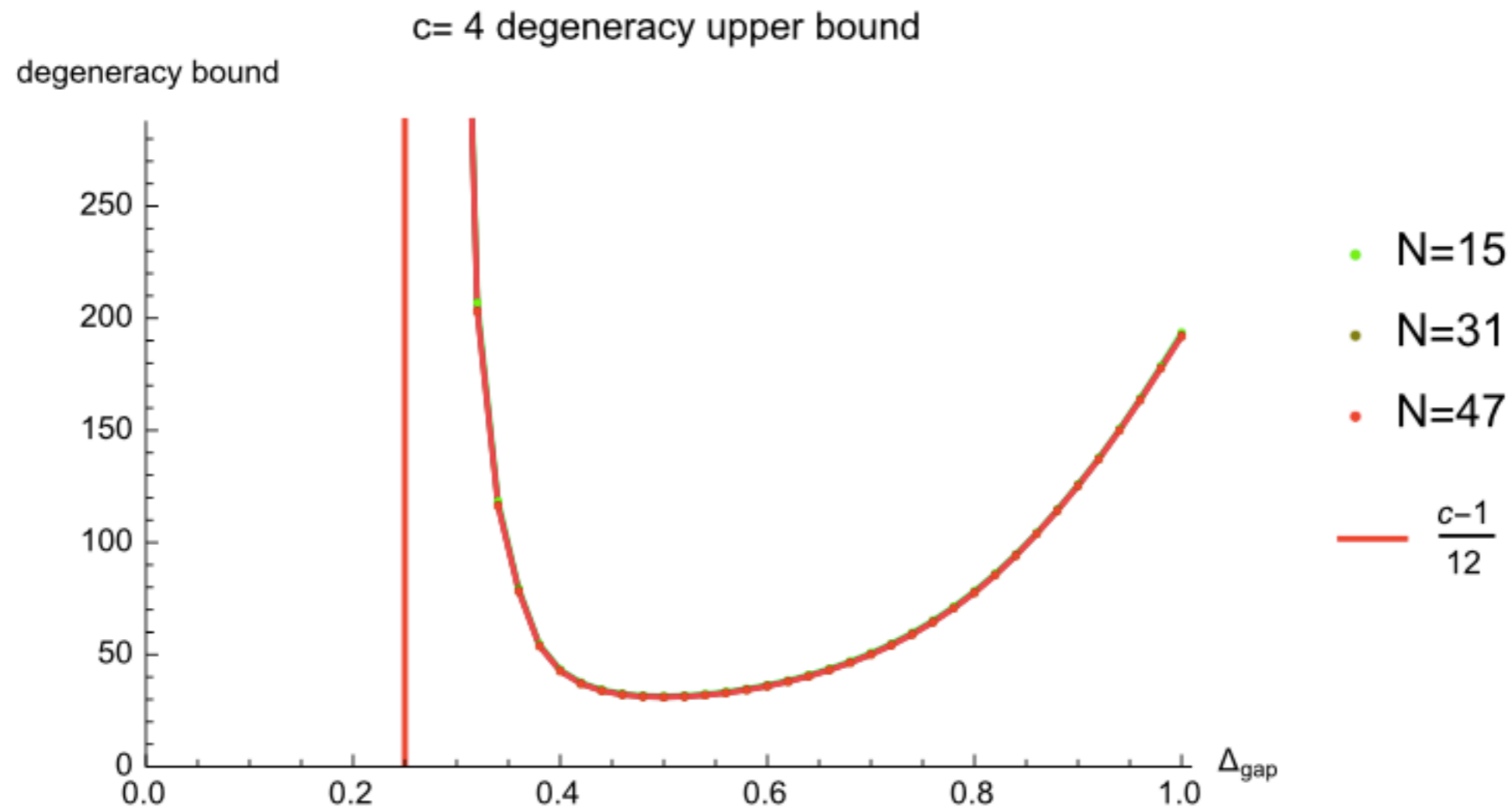
... that disappear at  $c=25$  !?



# Upper bound on the degeneracy at the gap

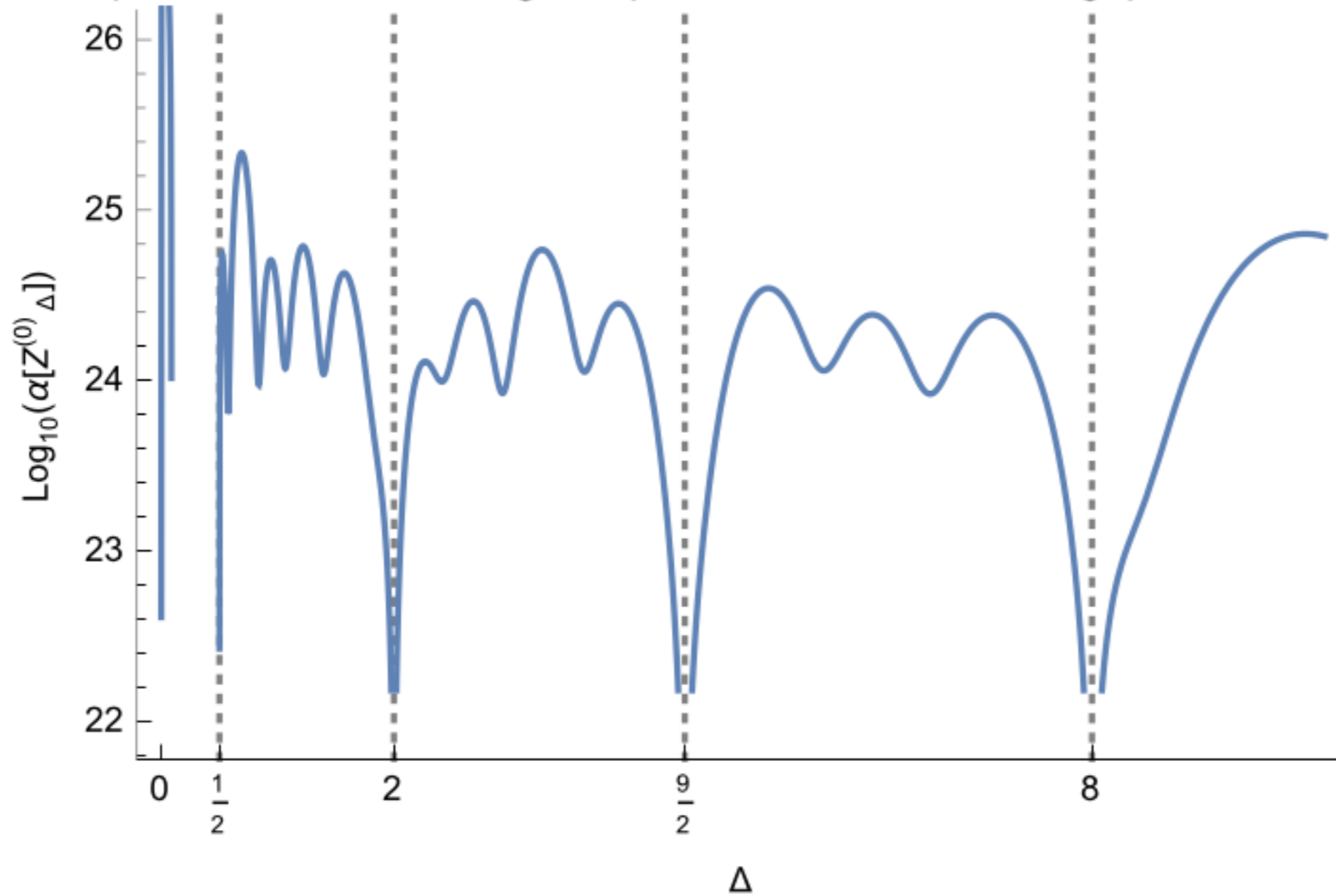


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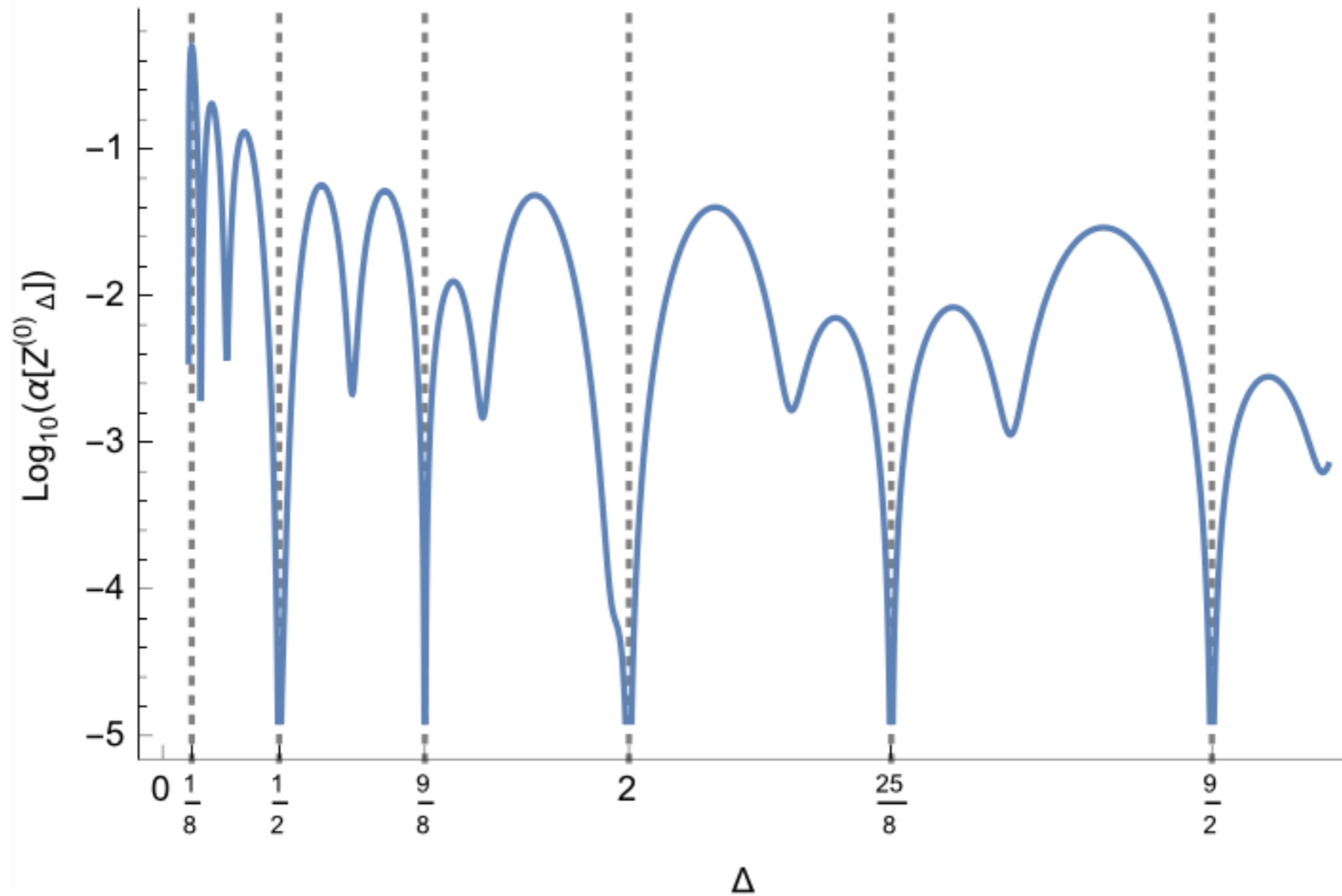
# Extremal spectrum

Optimal functional acting on spin-0 characters,  $c = 1$ , gap =  $1/2$ ,  $N = 47$



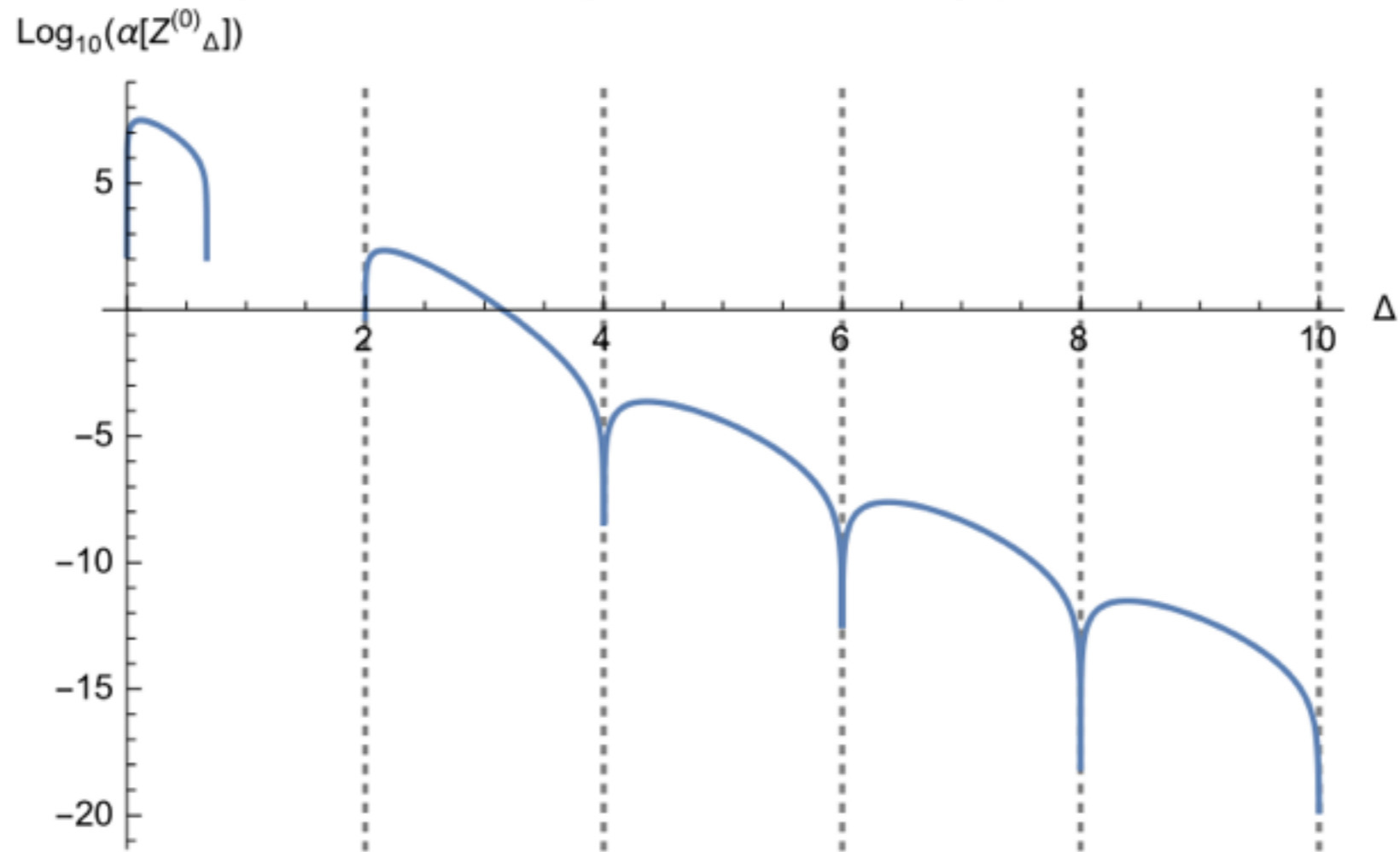
# Extremal spectrum

Optimal functional acting on spin-0 characters,  $c = 1$ , gap =  $1/8$ ,  $N = 47$



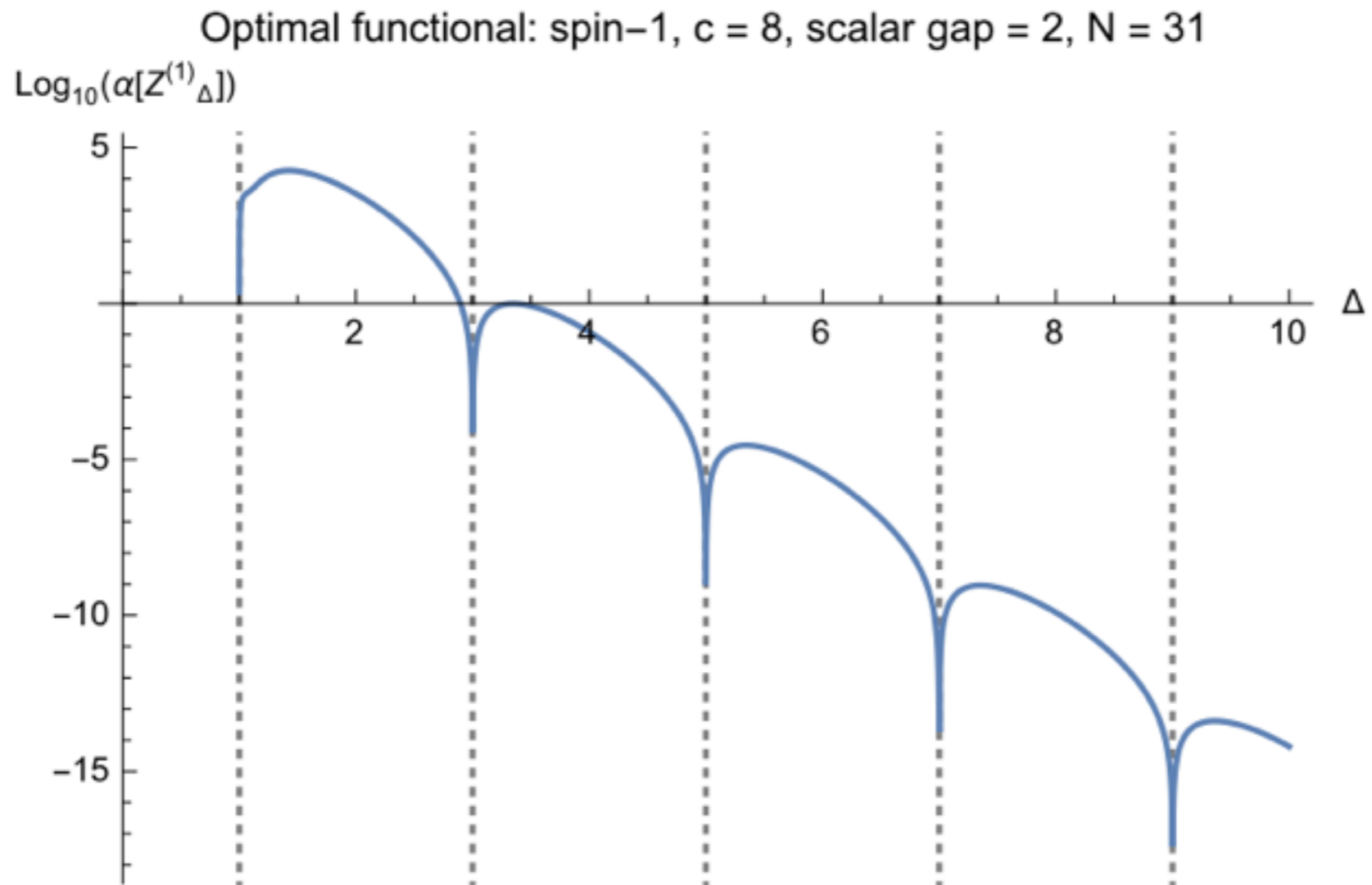
# Extremal spectrum

Optimal functional: spin-0,  $c = 8$ , scalar gap = 2,  $N = 31$

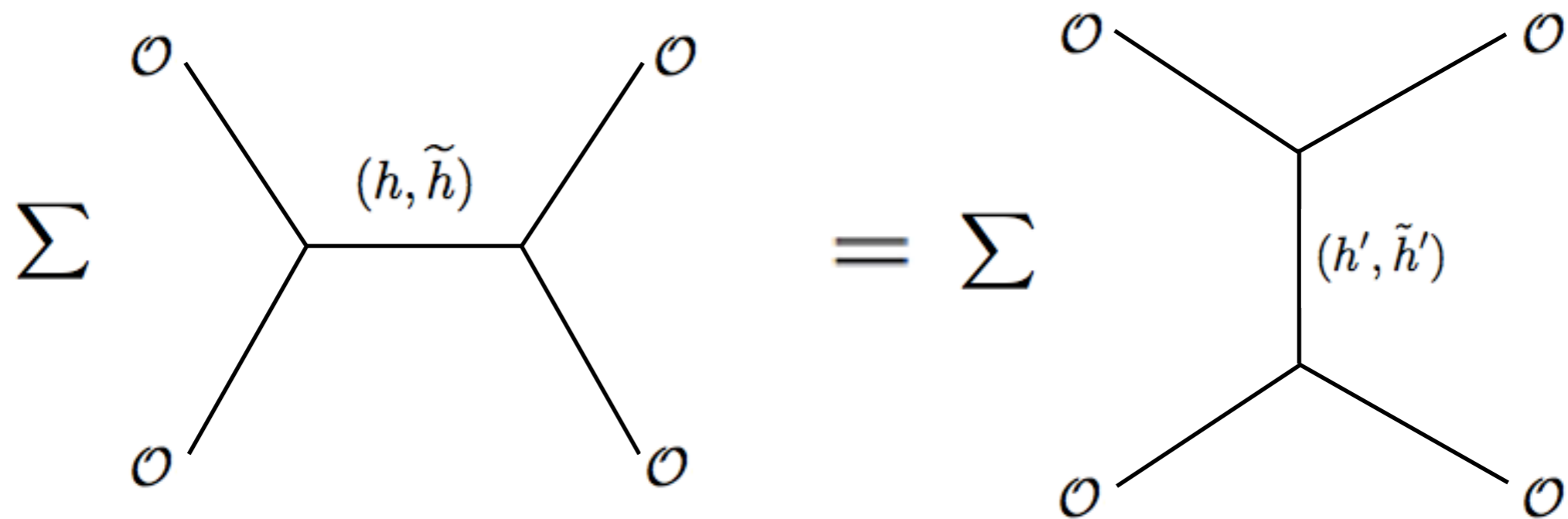




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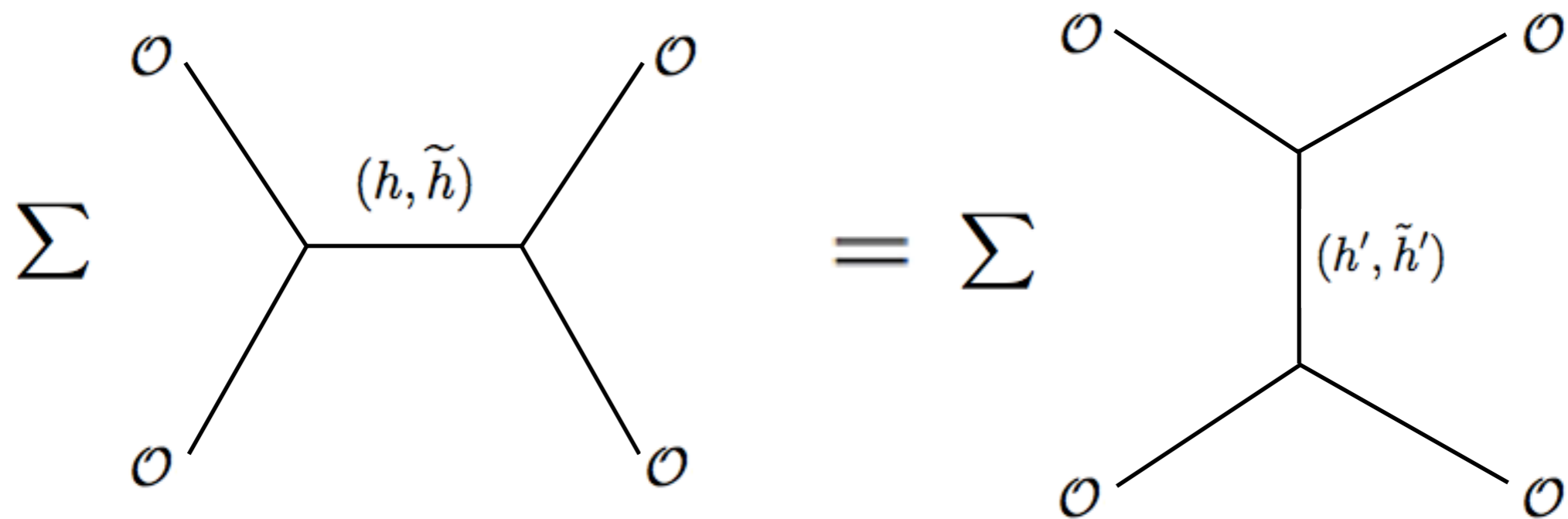


# Conformal bootstrap: the crossing equation



[Belavin, Polyakov, Zamolodchikov, Rattazzi, Rychkov, Vichi, Tonni, Poland, Simmons-Duffin, El-Showk, Paulos, ....]

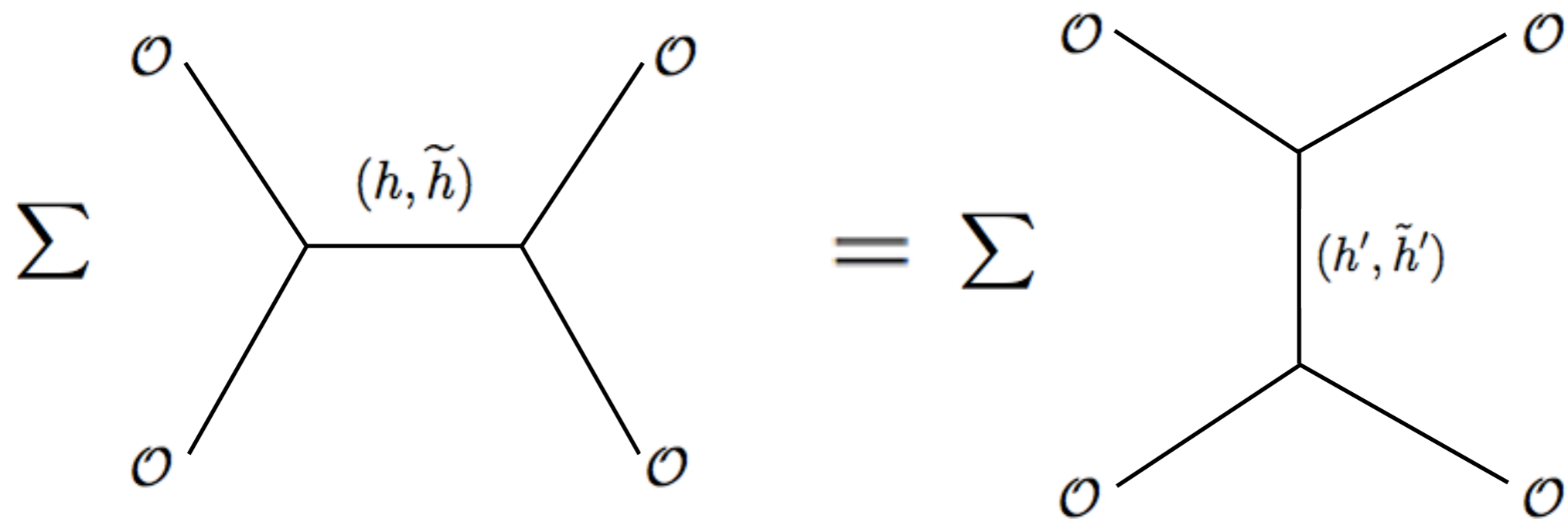
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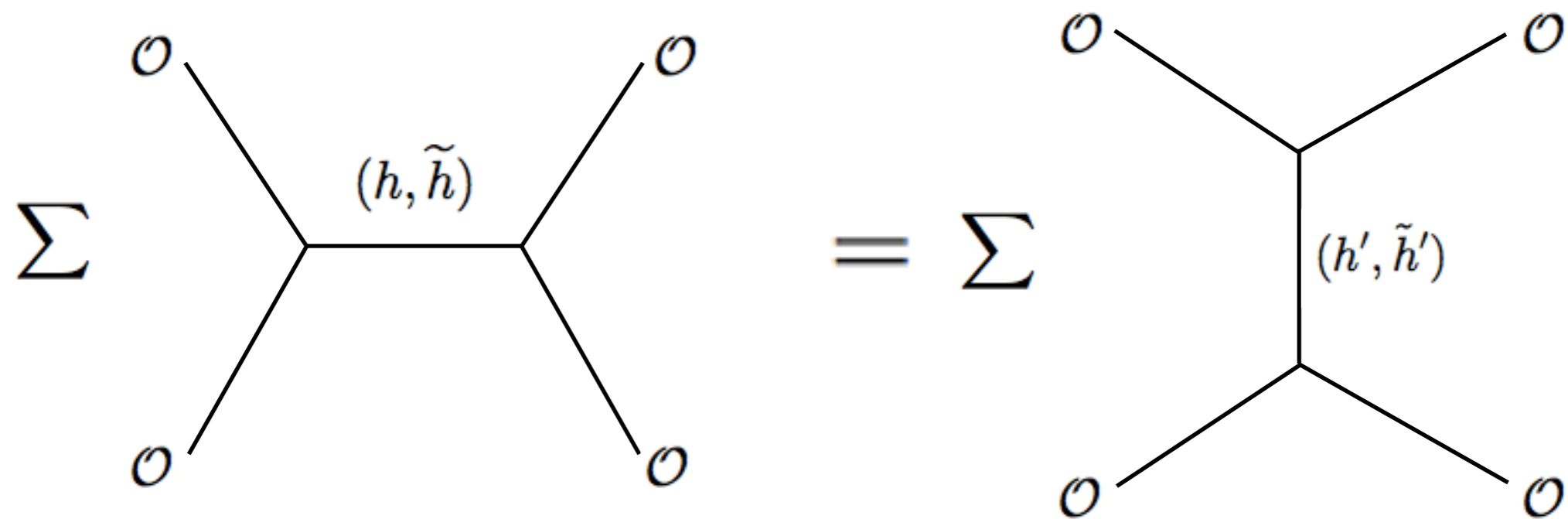
We revisit this problem in 2D, using modern numerical methods.

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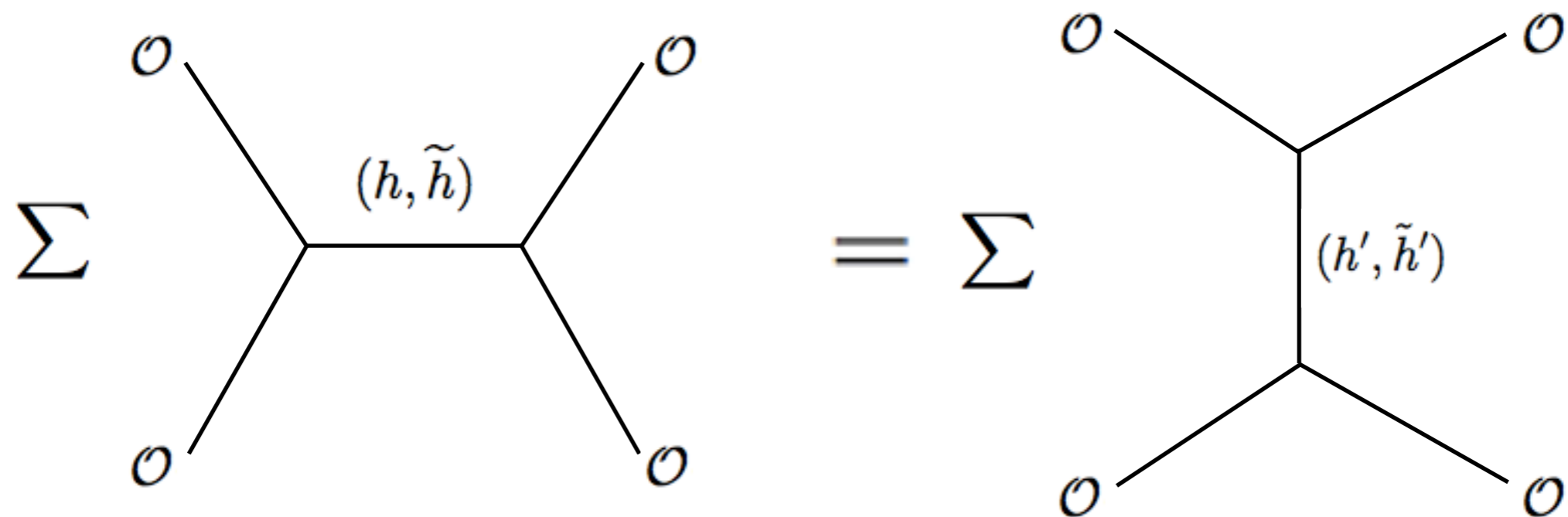
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and constrain the content of OPE from unitarity

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When there is a conformal manifold, we would like to understand the moduli dependence of the spectrum.

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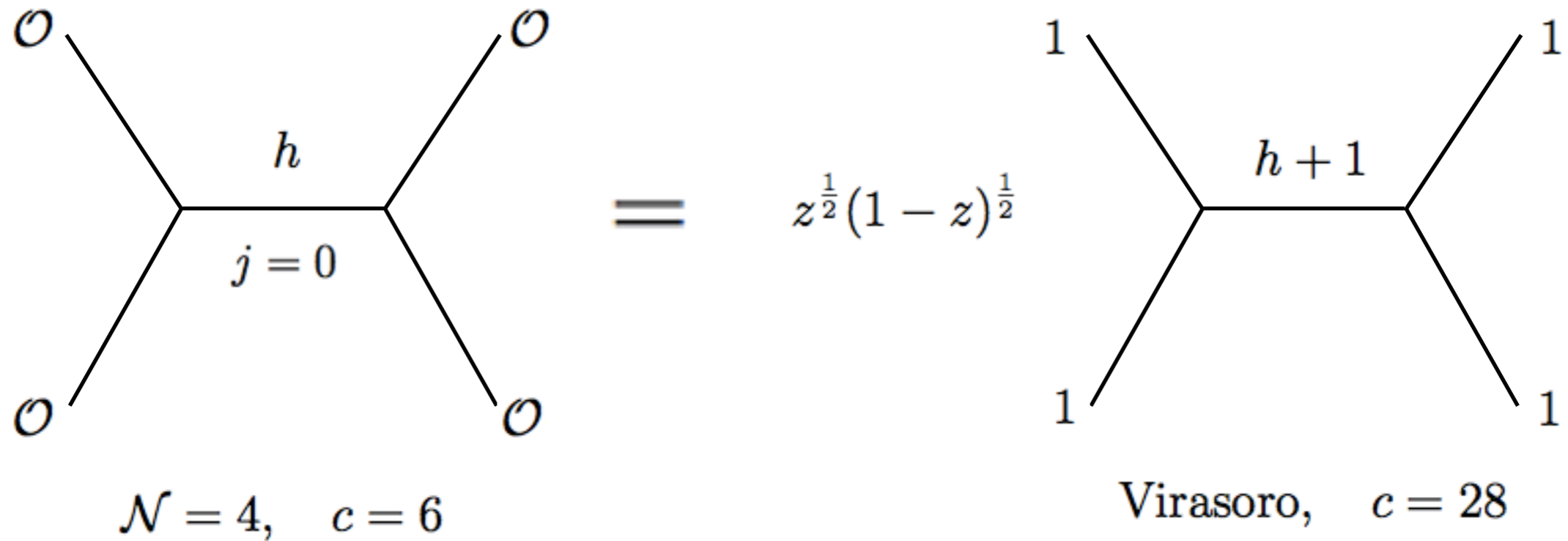
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4. study superconformal block decomposition of BPS  
4-point function



Key ingredient 1:  $N=4$  BPS superconformal block  
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# Key ingredient 1: N=4 BPS superconformal block (c=6, k=1 case)

$$\begin{array}{ccc}
 \begin{array}{c} \mathcal{O} \quad \mathcal{O} \\ \diagdown \quad \diagup \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ \diagup \quad \diagdown \\ \mathcal{O} \quad \mathcal{O} \\ h \\ j=0 \end{array} & = & z^{\frac{1}{2}}(1-z)^{\frac{1}{2}} \begin{array}{c} 1 \quad 1 \\ \diagdown \quad \diagup \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \\ \diagup \quad \diagdown \\ 1 \quad 1 \\ h+1 \end{array} \\
 \mathcal{N} = 4, \quad c = 6 & & \text{Virasoro, } c = 28
 \end{array}$$

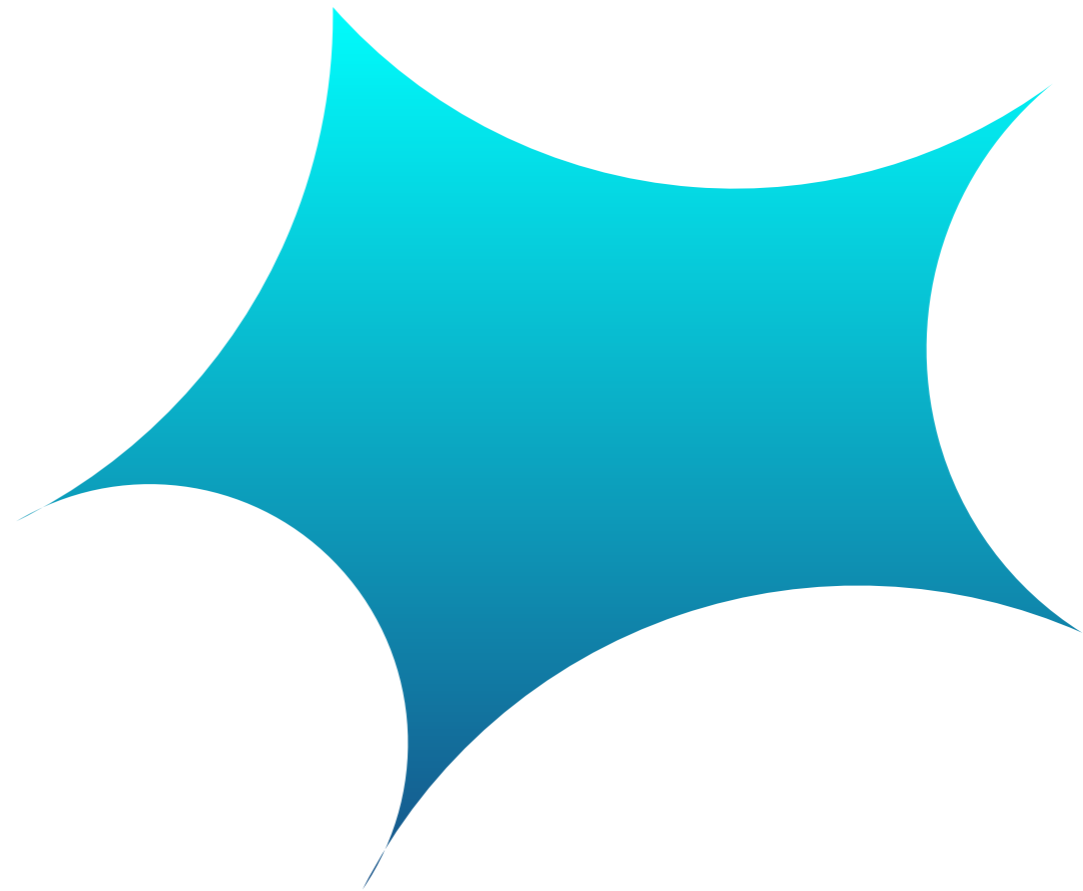
Comes from analyzing the N=2 cigar coset  $SL(2)/U(1)$ , combined with Ribault-Teschner relation between  $SL(2)$  WZW and bosonic Liouville.

[Chang-Lin-Shao-Wang-XY, '14]

# Key ingredient 2: An exact result on the integrated BPS 4-point function (as function of moduli)

The moduli space of K3 CFT

$$\text{Aut}(\Gamma_{20,4}) \backslash SO(20,4) / SO(20) \times SO(4)$$

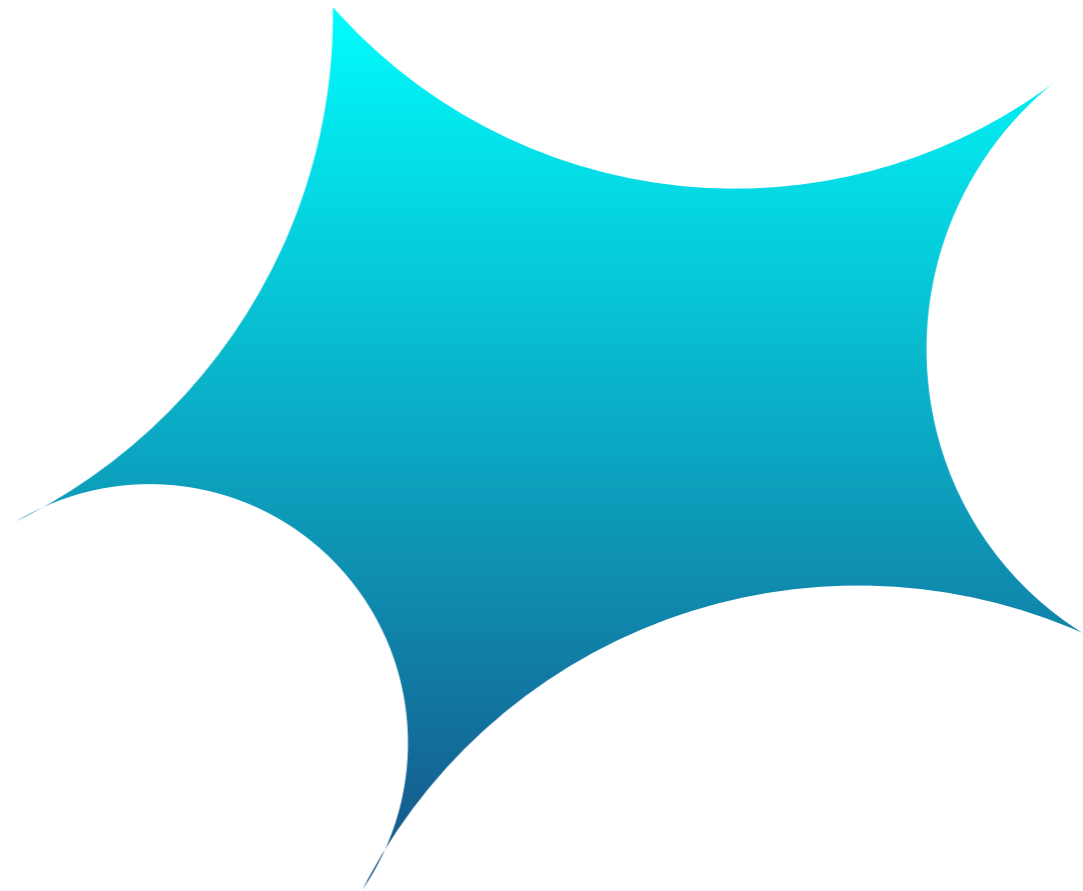


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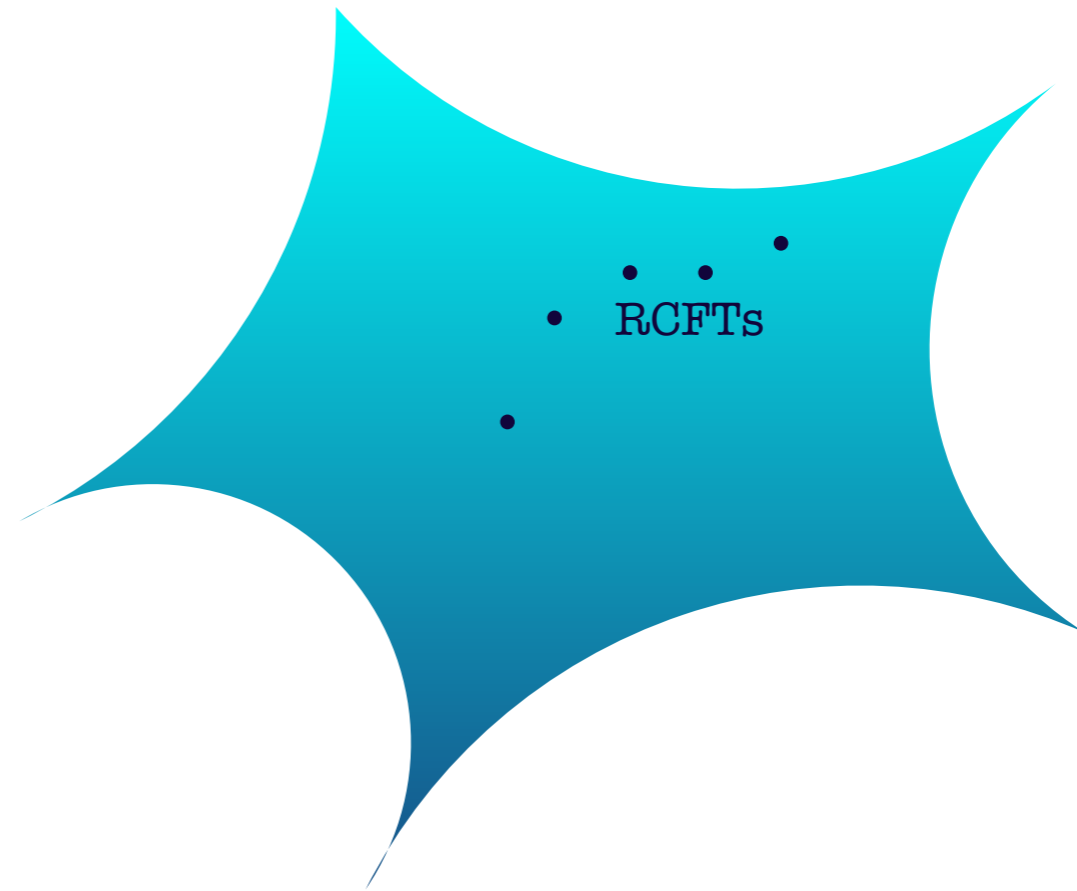


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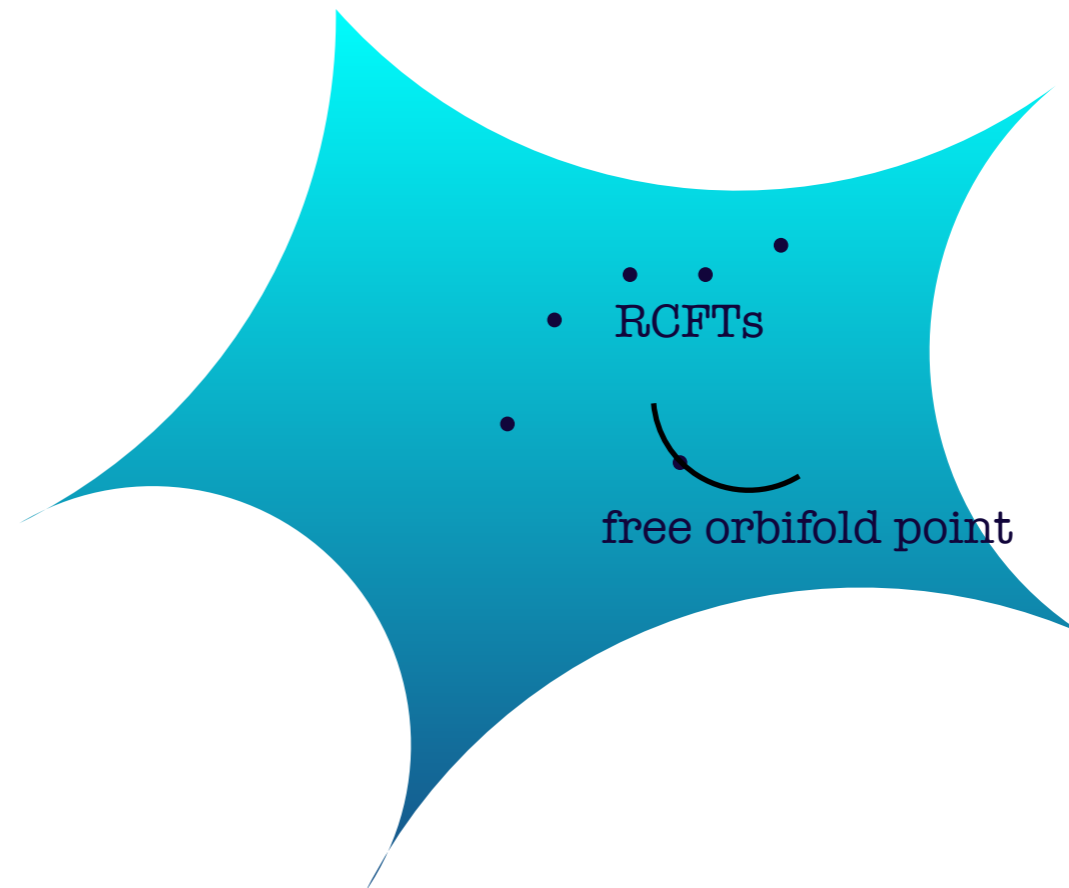


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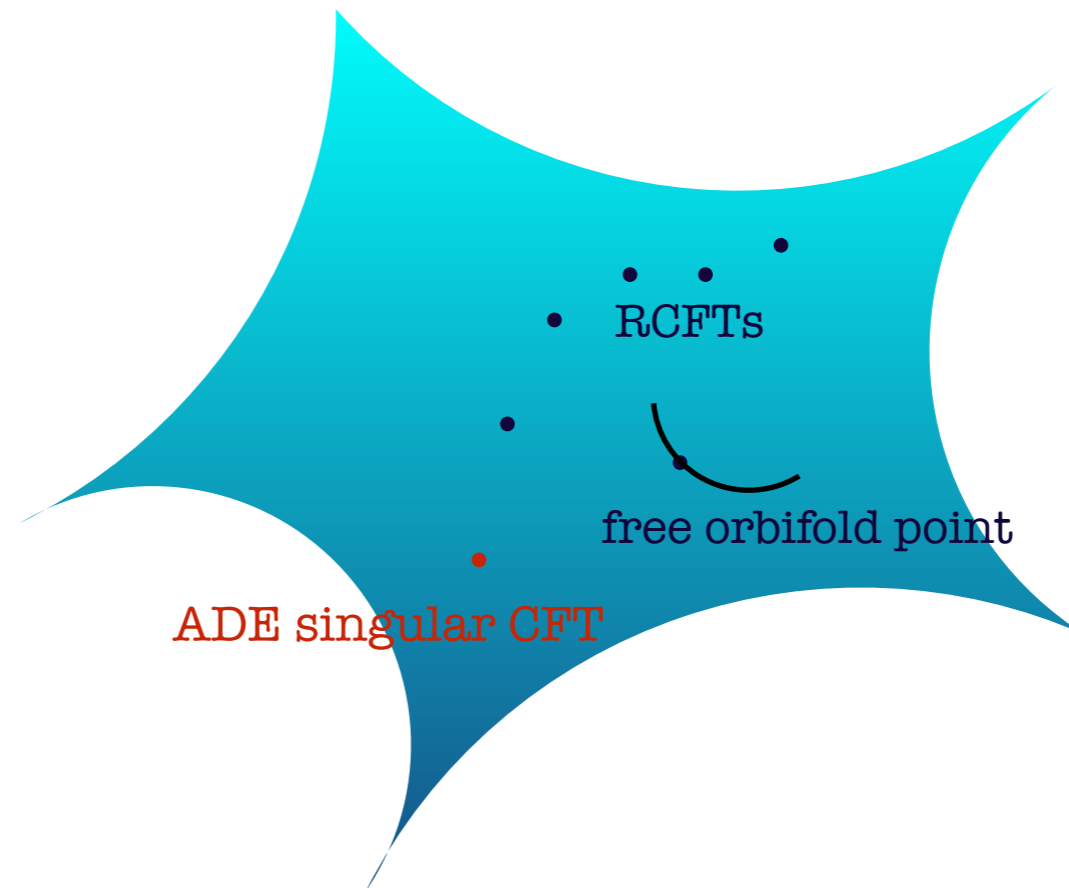


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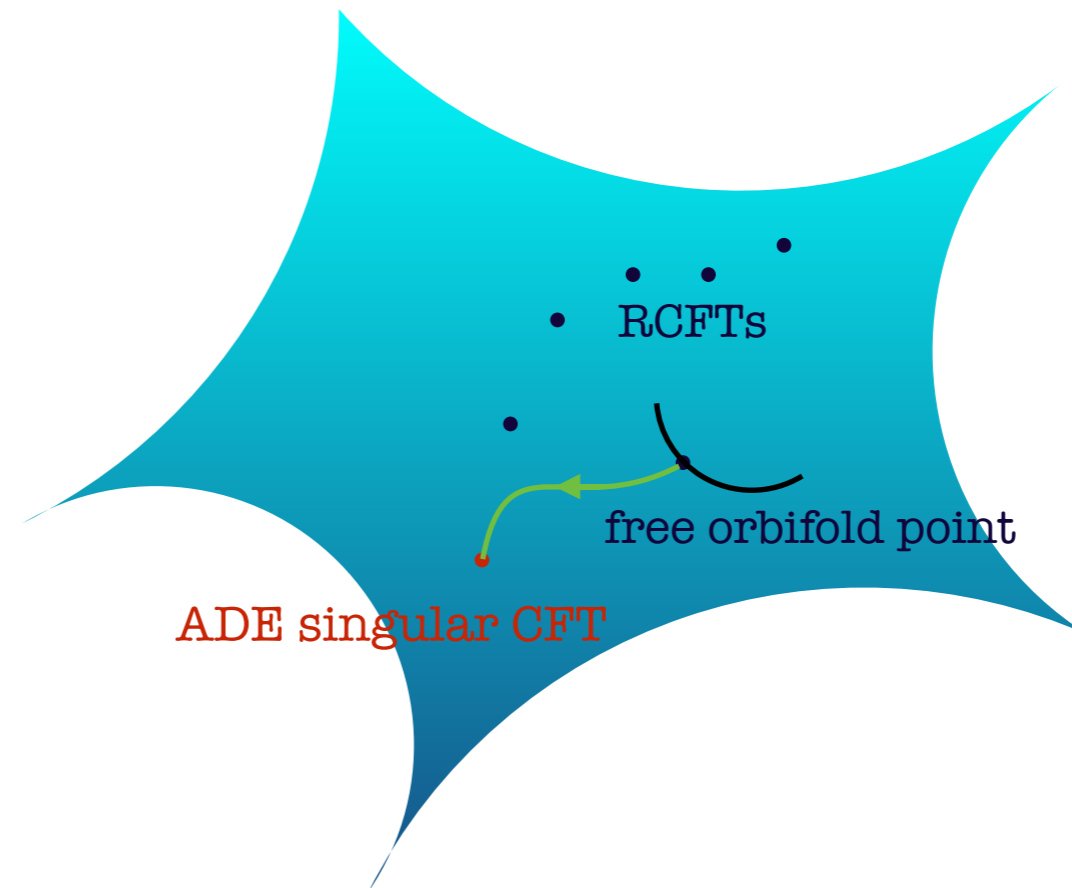


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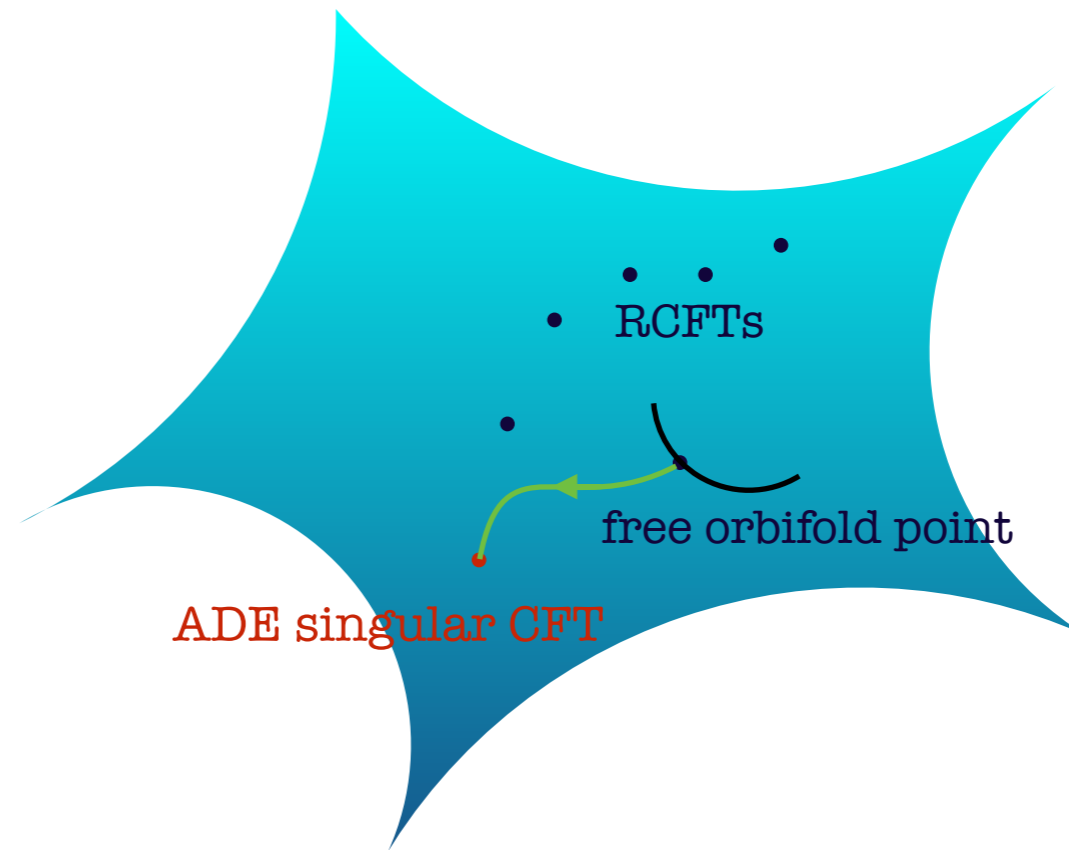


# Key ingredient 2: An exact result on the integrated BPS 4-point function (as function of moduli)

The moduli space of K3 CFT

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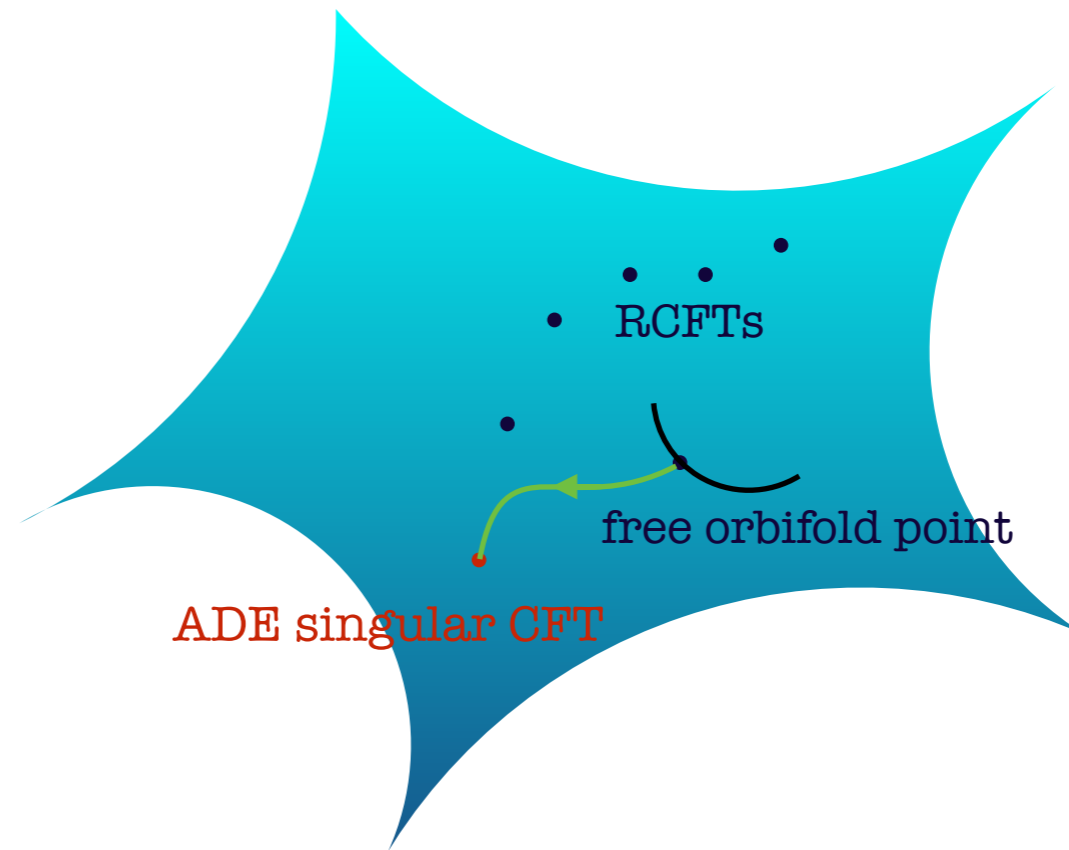
$$\int \frac{d^2 z}{|z(1-z)|} \langle \mathcal{O}_i(z, \bar{z}) \mathcal{O}_j(0) \mathcal{O}_k(1) \mathcal{O}_\ell(\infty) \rangle = \frac{\partial^4}{\partial y^i \partial y^j \partial y^k \partial y^\ell} \Big|_{y=0} \int_{\mathcal{F}} d^2 \tau \frac{\Theta_\Lambda(y|\tau, \bar{\tau})}{\eta(\tau)^{24}}$$

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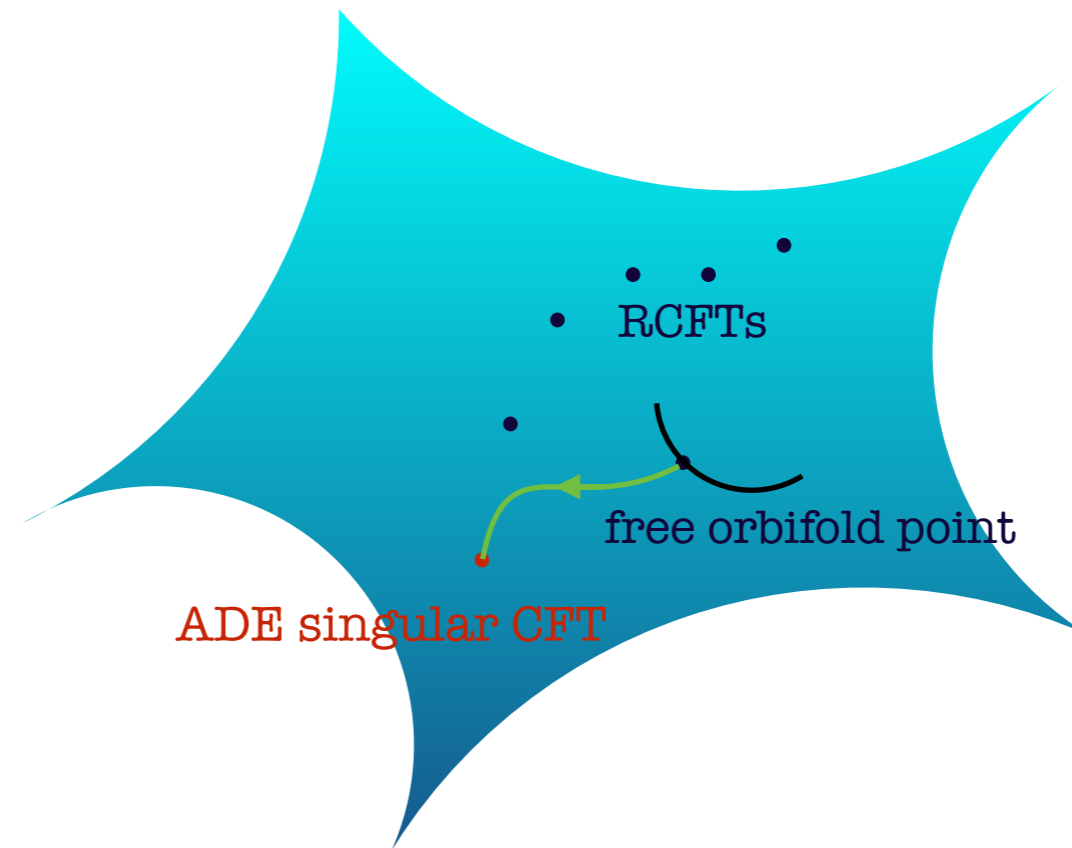
LHS = effective coupling of type IIB string on K3 at tree level

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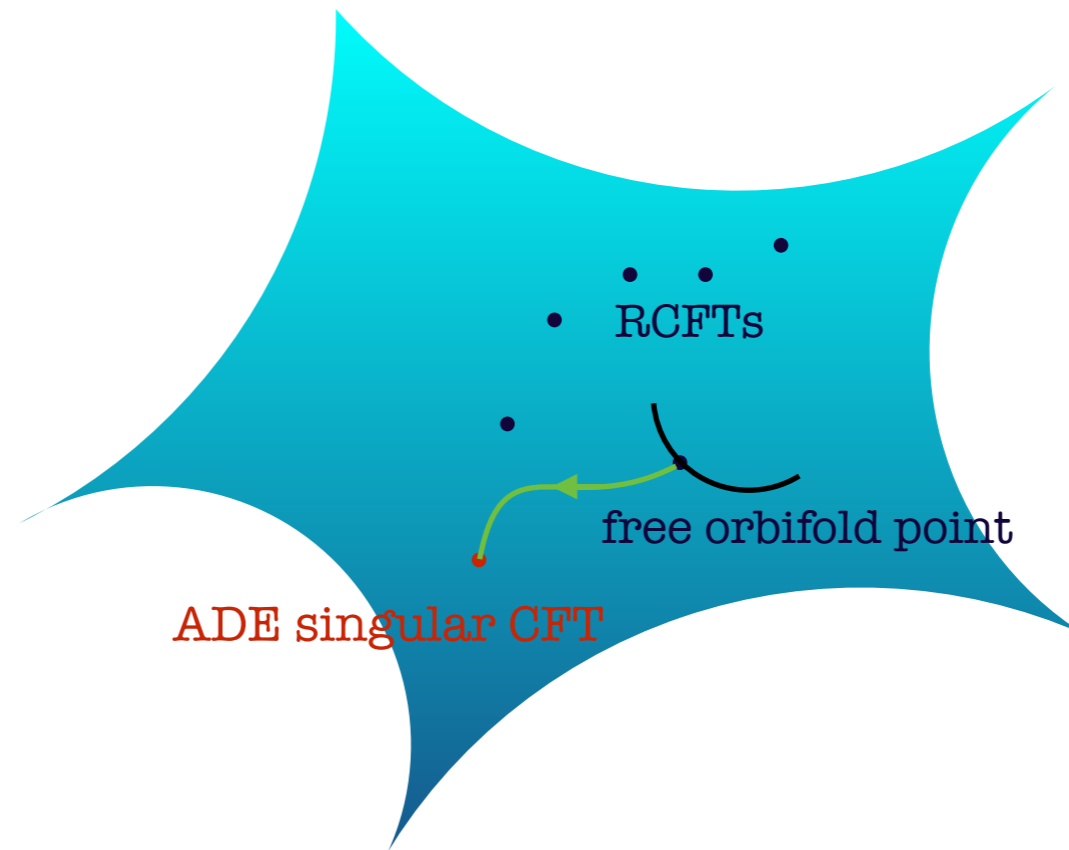
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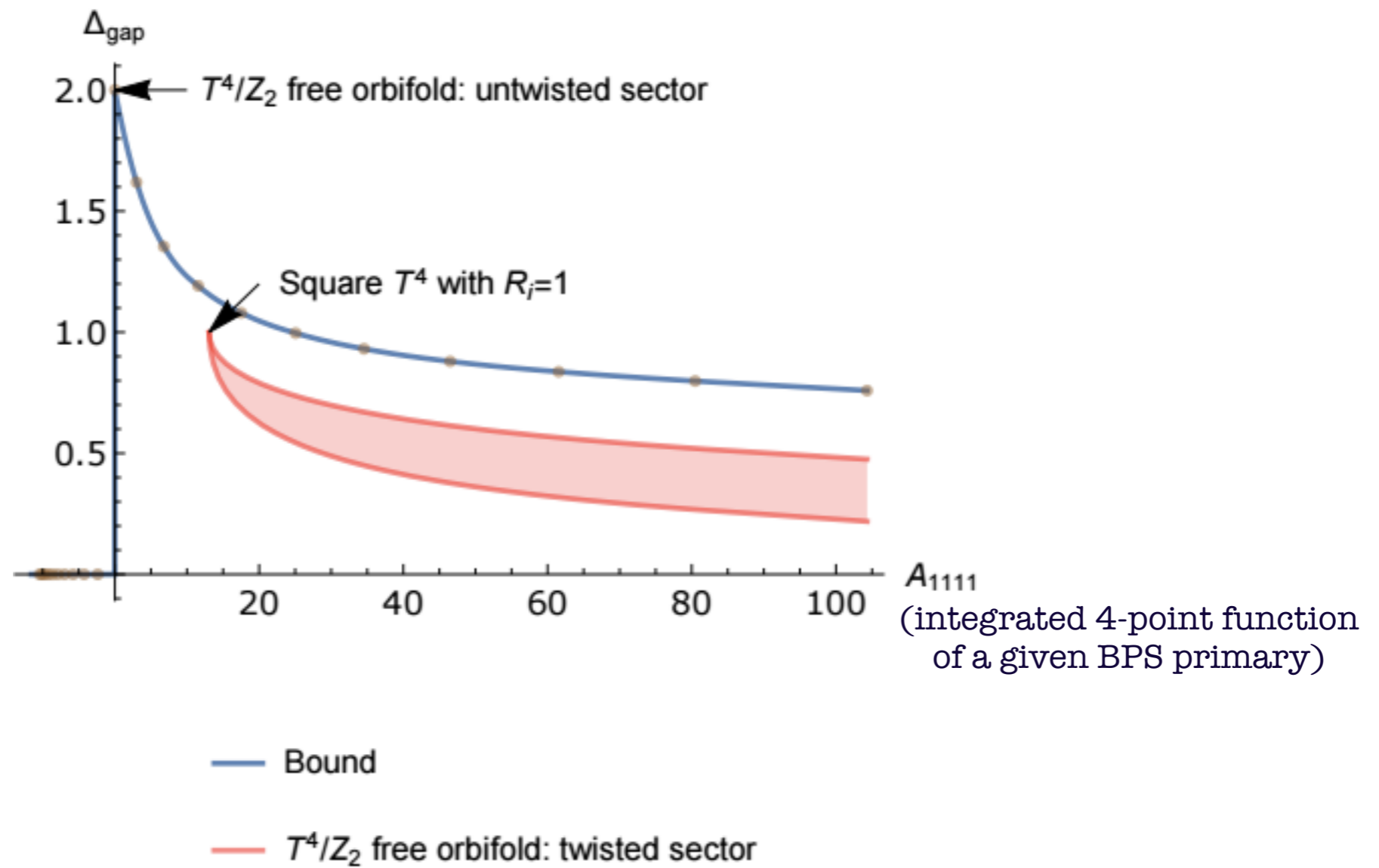
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[Kiritsis-Obers-Pioline, '00, Lin-Shao-Wang-XY, '15]

Feed into the bootstrap machine...

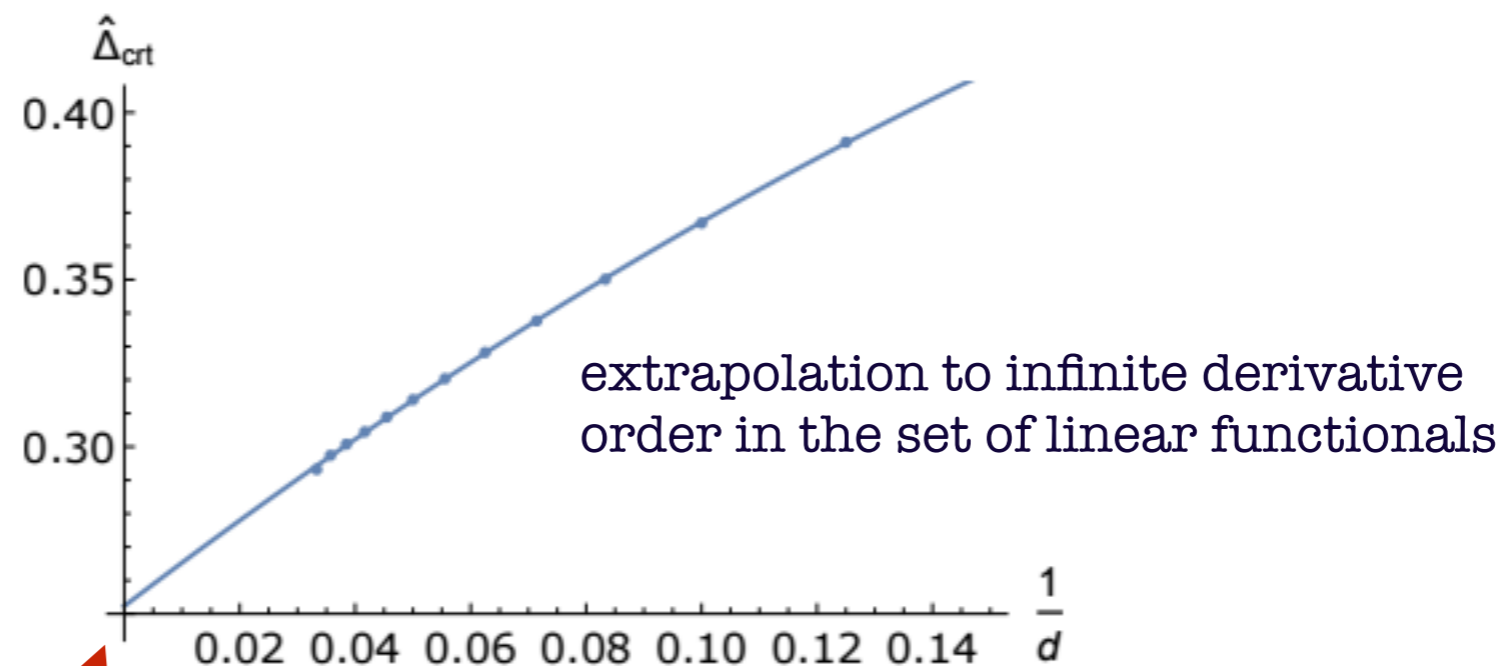


# The gap in the non-BPS operator spectrum of the K3 CFT



# The development of continuum

(when the integrated BPS 4-point function diverges)



quadratic fit	0.252
$A_1$ cigar	0.25



A taste of  $N=2$  superconformal bootstrap...

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Let us study the OPE of a conjugate pair of BPS operators (chiral primary  $h=q/2$  and anti-chiral primary  $h=-q/2$ ), and bound the gap in terms of chiral ring coefficients.

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A warm up:  $c=3$ ,  $q=1/3$

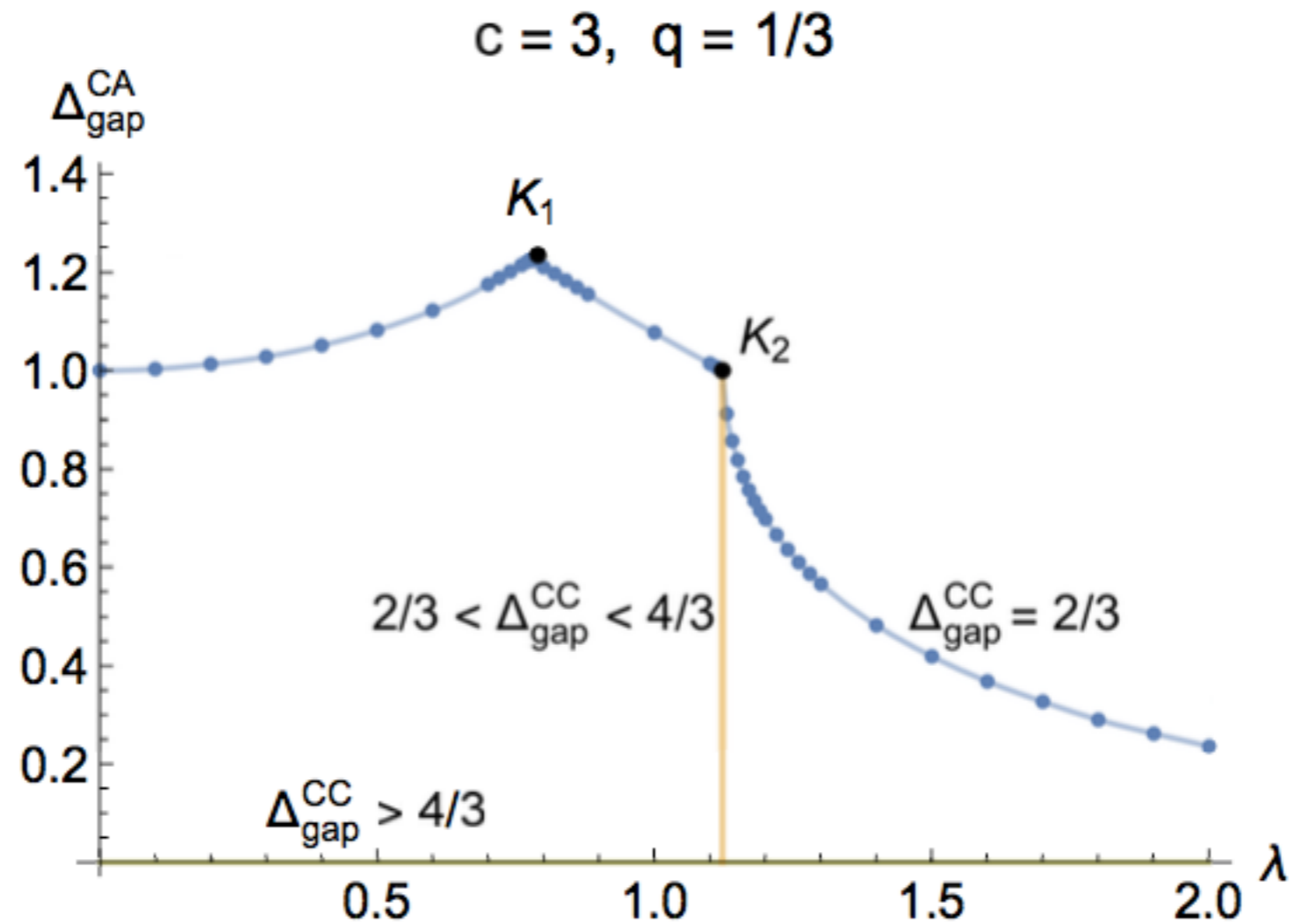
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A warm up:  $c=3$ ,  $q=1/3$  (realized by twist fields of  $T^2/Z_3$ )

# $N=(2,2)$ Bootstrap

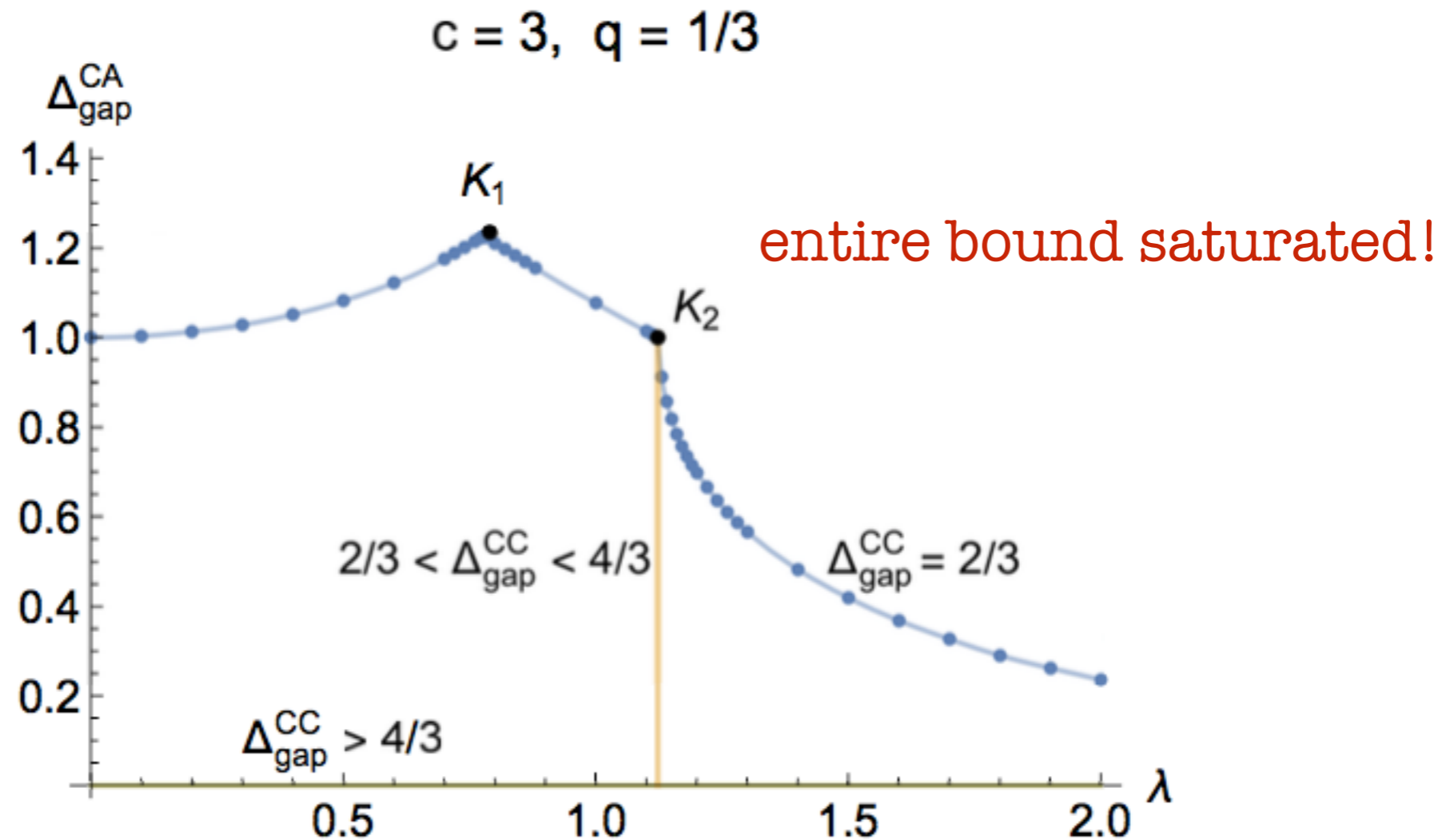
$c=3$ , chiral-anti-chiral OPE,  $q=1/3$



OPE coefficient of  $q=2/3$  BPS primary

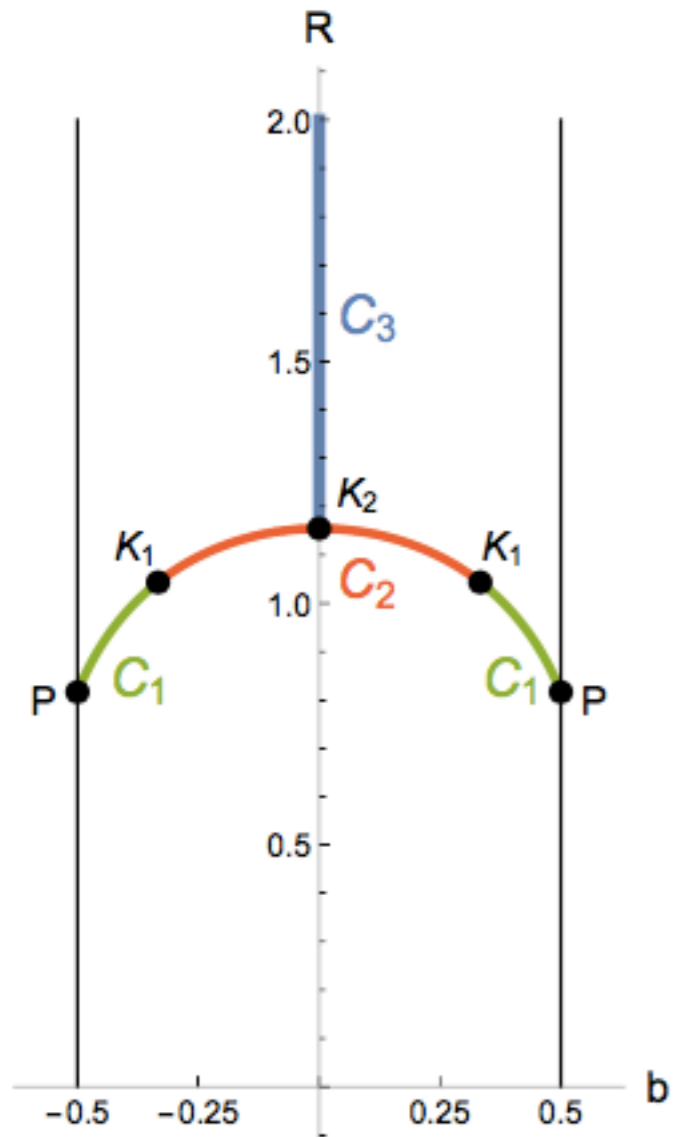
# $N=(2,2)$ Bootstrap

$c=3$ , chiral-anti-chiral OPE,  $q=1/3$

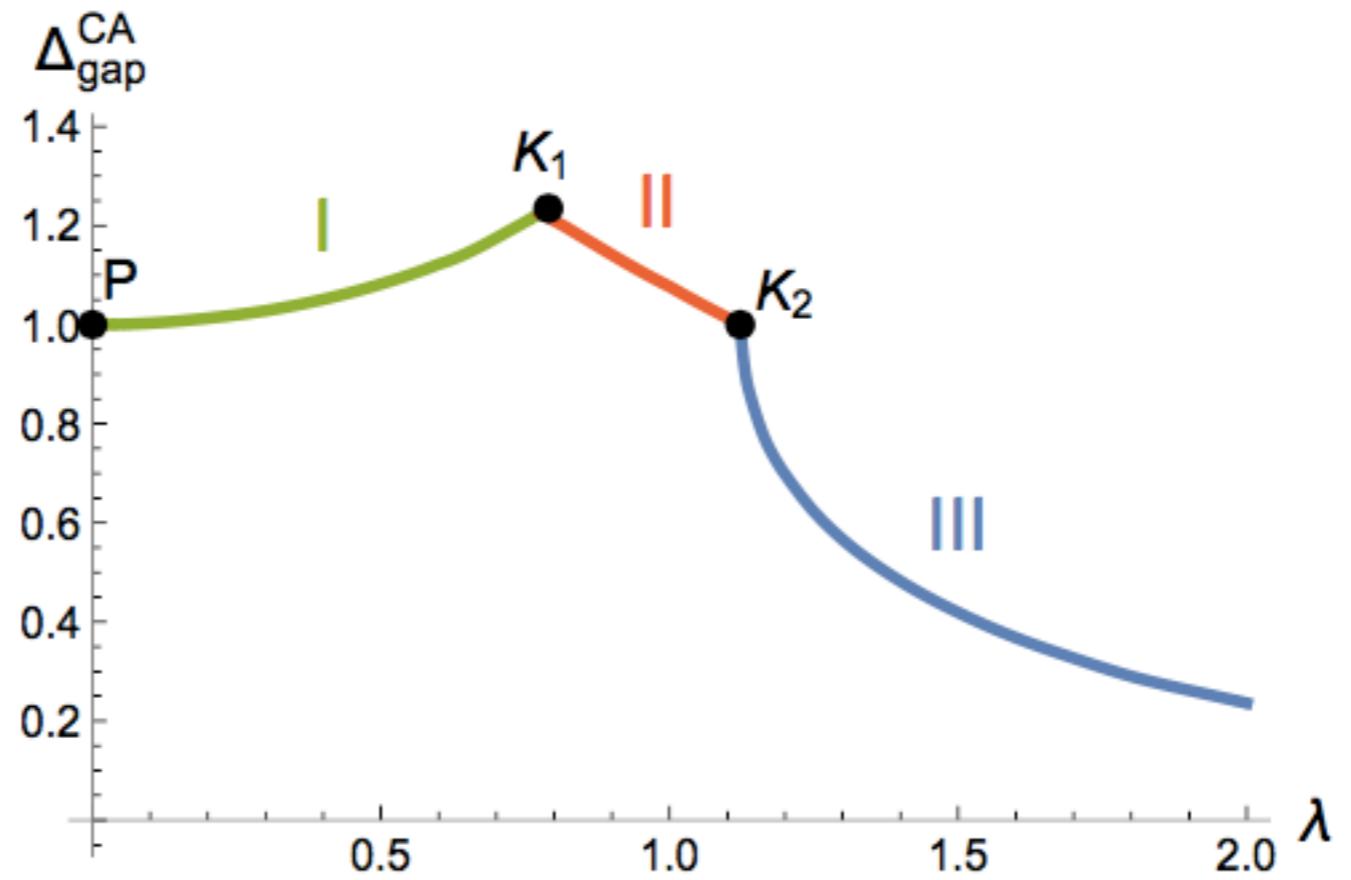


OPE coefficient of  $q=2/3$  BPS primary

# $N=(2,2)$ Bootstrap



moduli space of  $T^2/Z_3$



bootstrap bound

Now for Calabi-Yau 3-fold model...

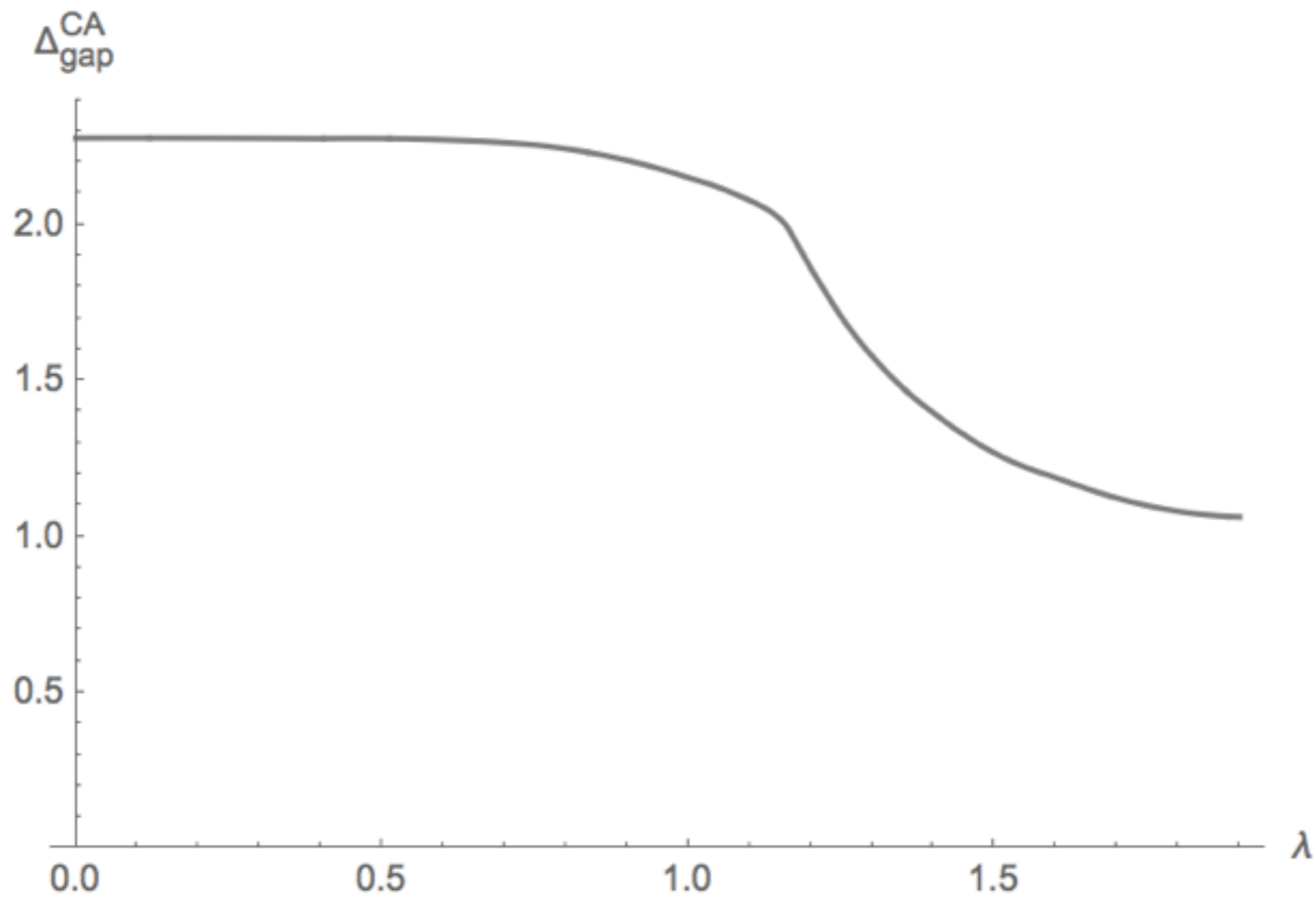


Now for Calabi-Yau 3-fold model...

$c=9$ , chiral-anti-chiral OPE,  $q=1$

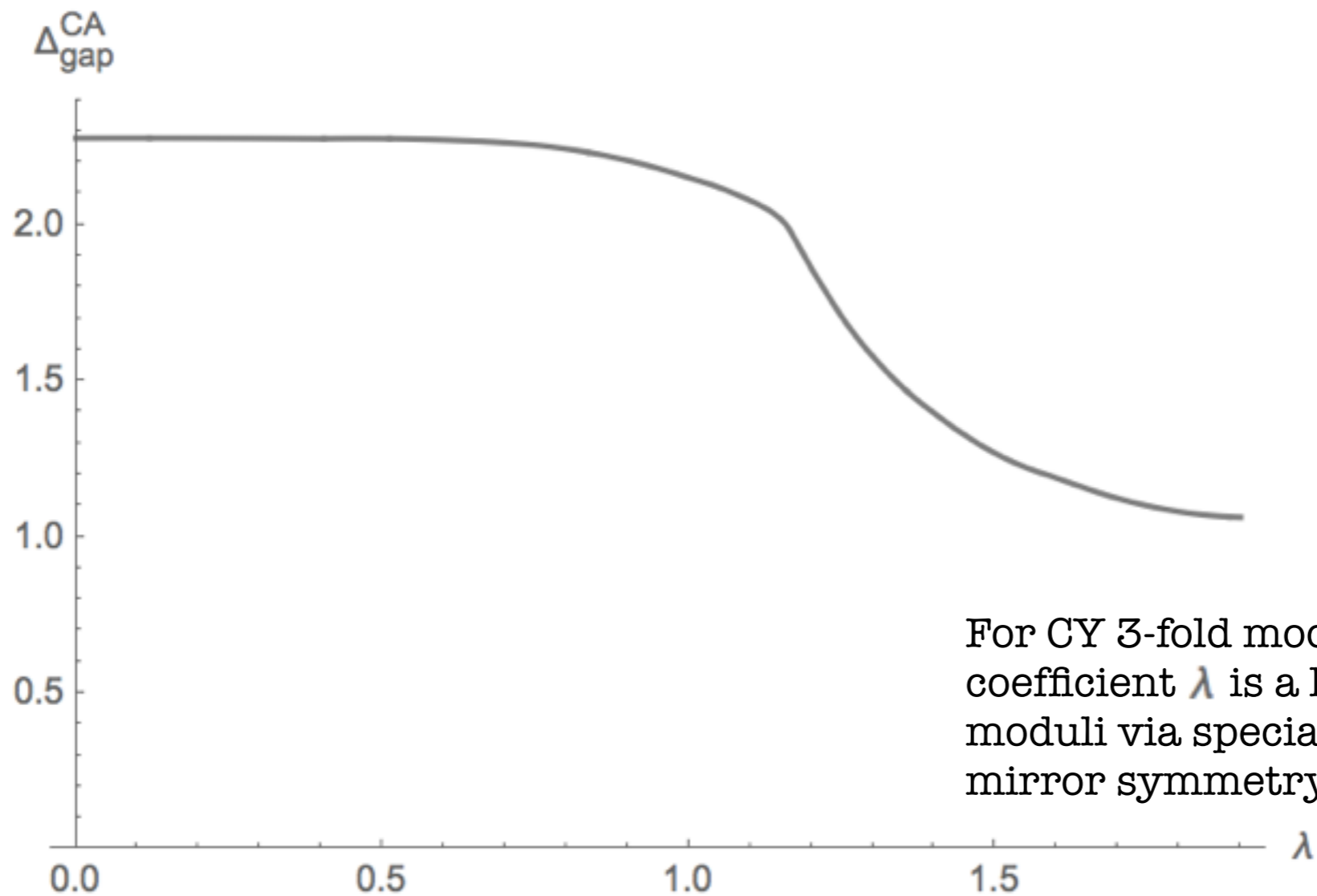
# $N=(2,2)$ Bootstrap

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# $N=(2,2)$ Bootstrap

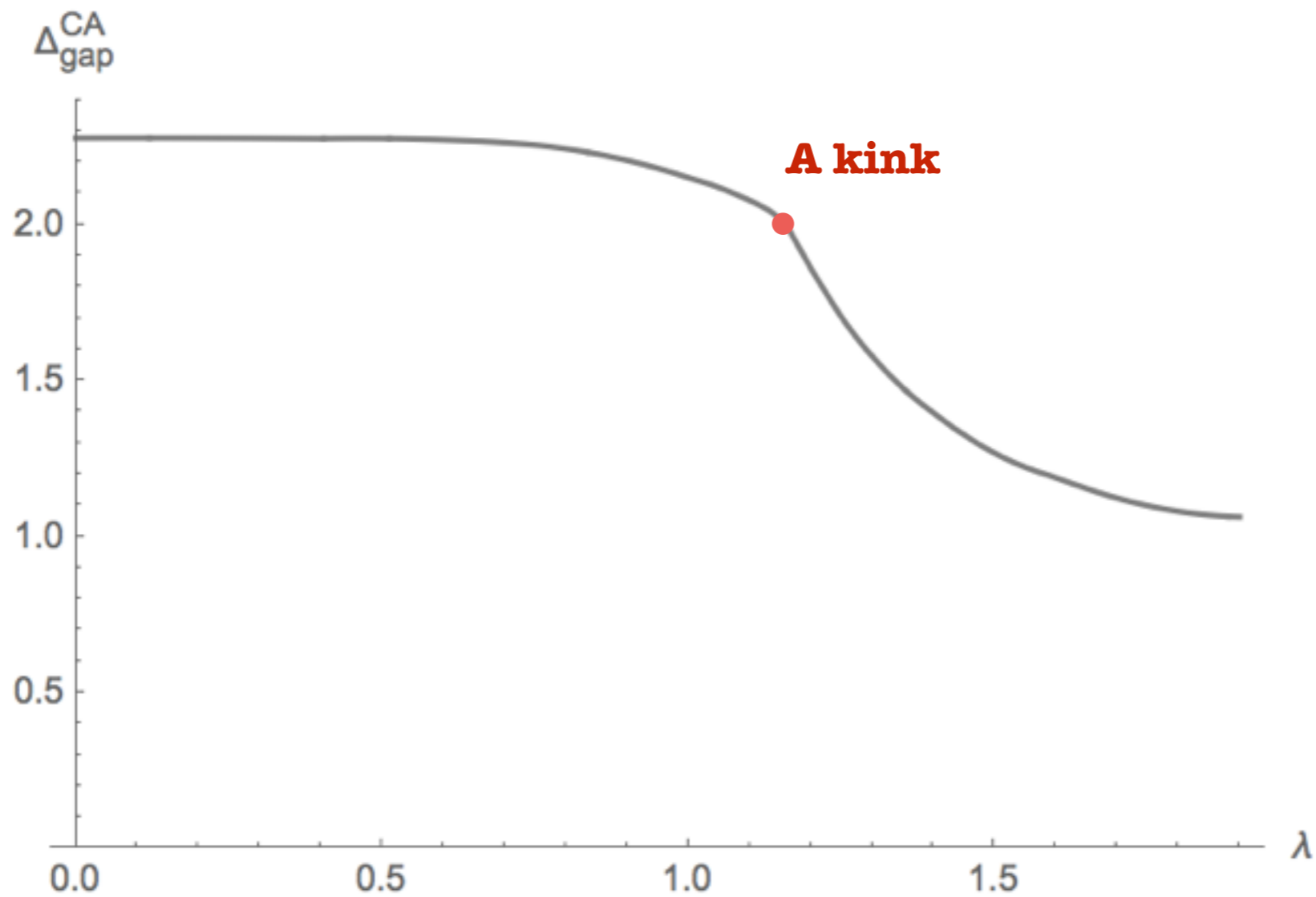
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For CY 3-fold models, the BPS OPE coefficient  $\lambda$  is a known function of moduli via special geometry and mirror symmetry.

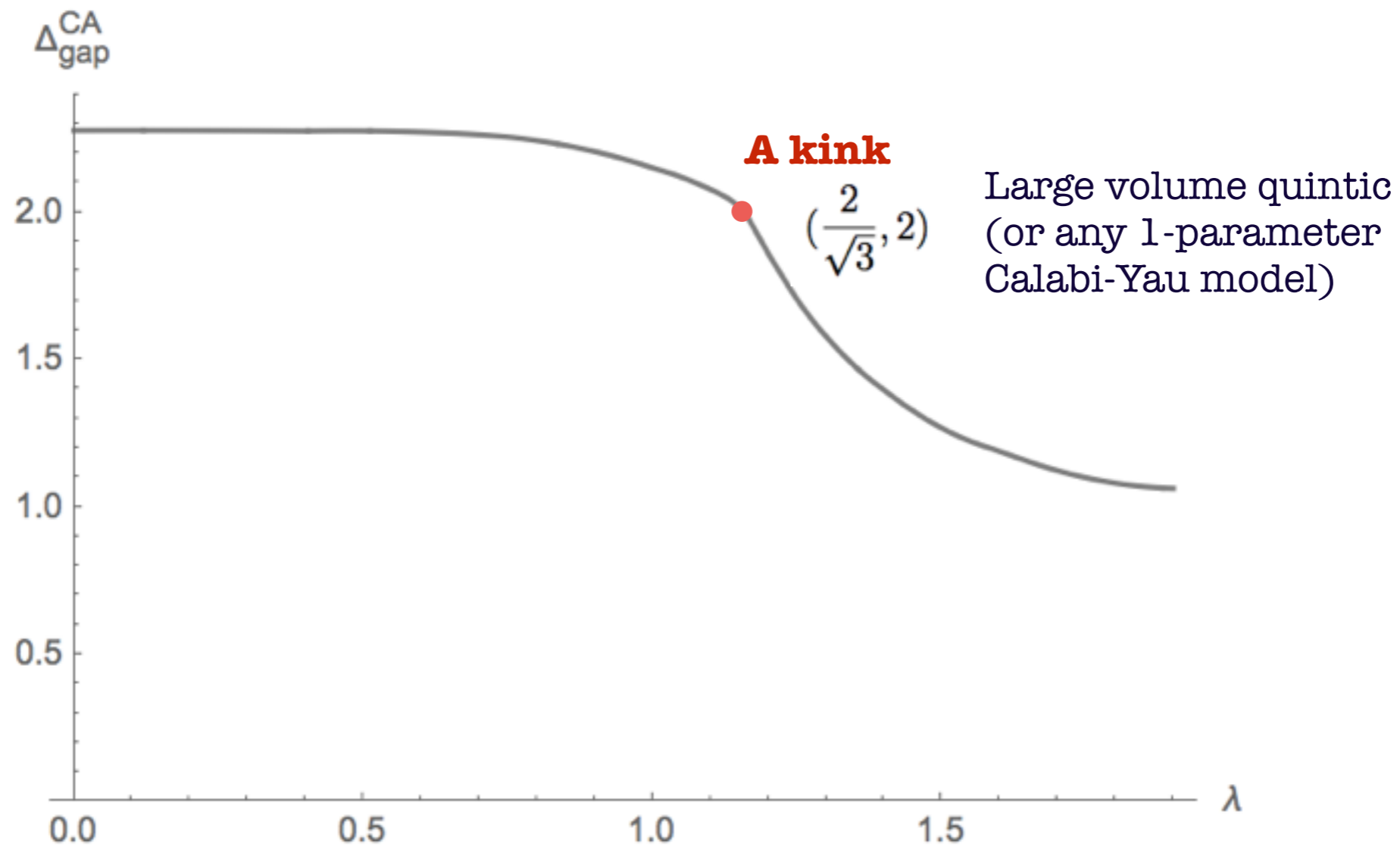
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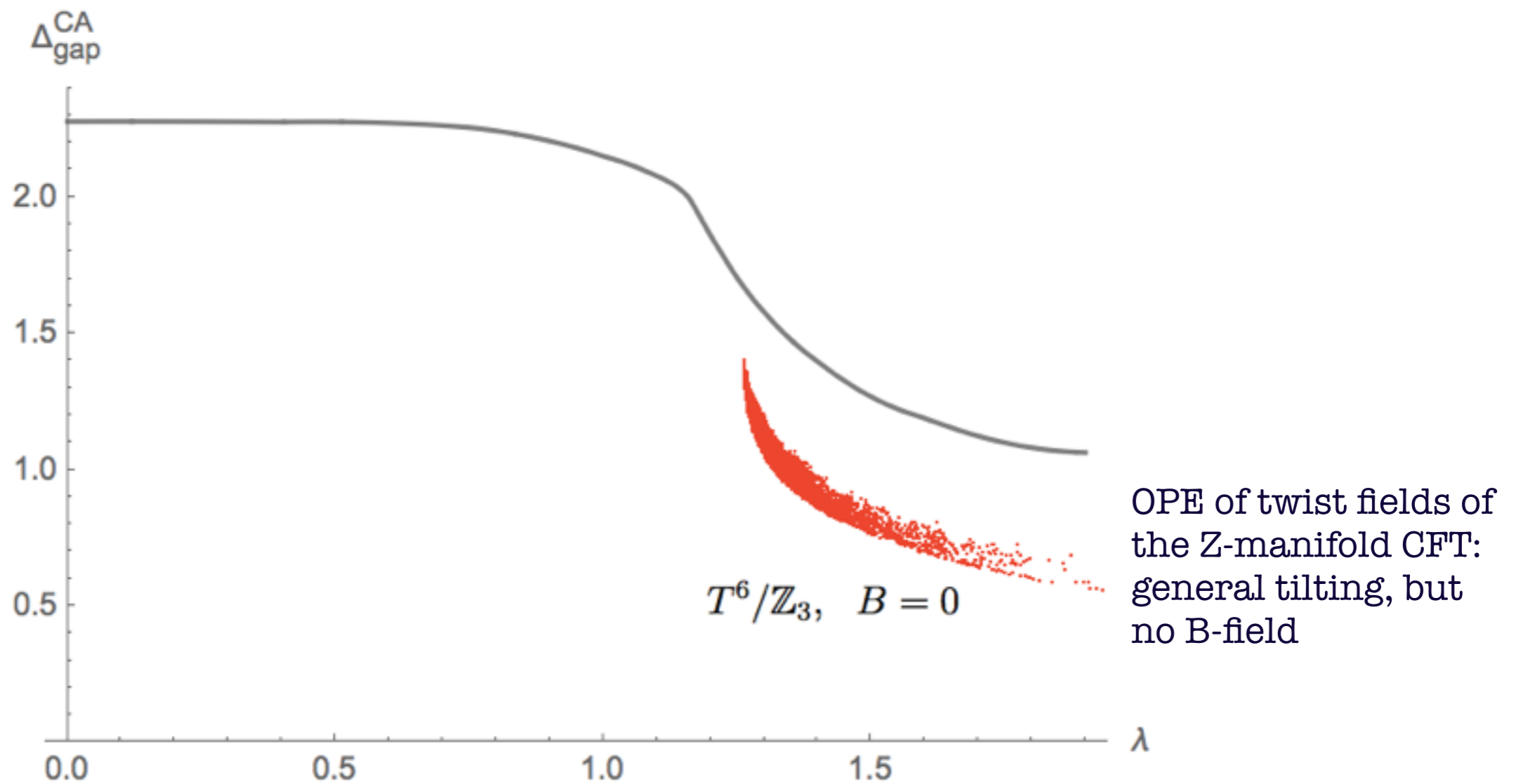
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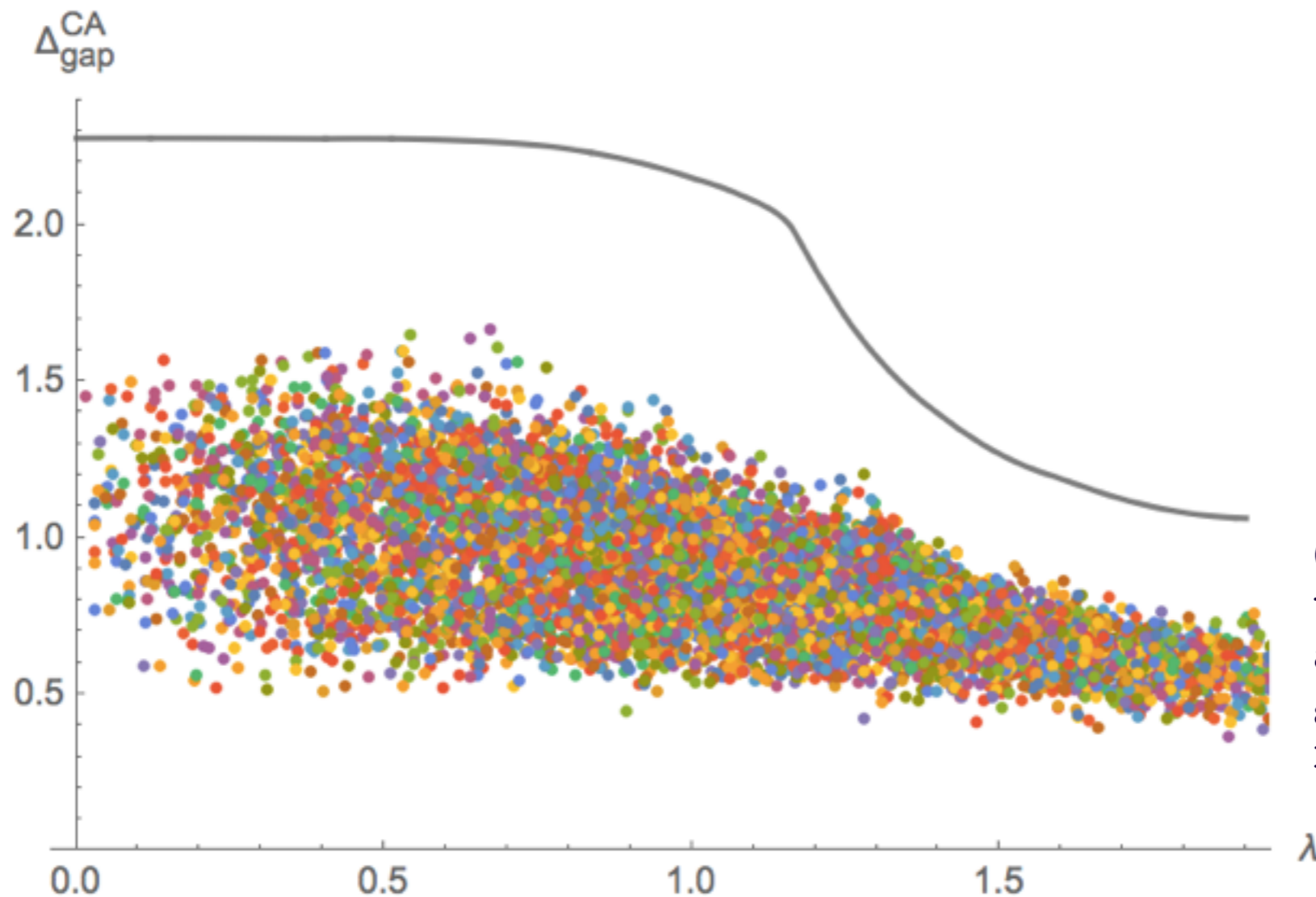
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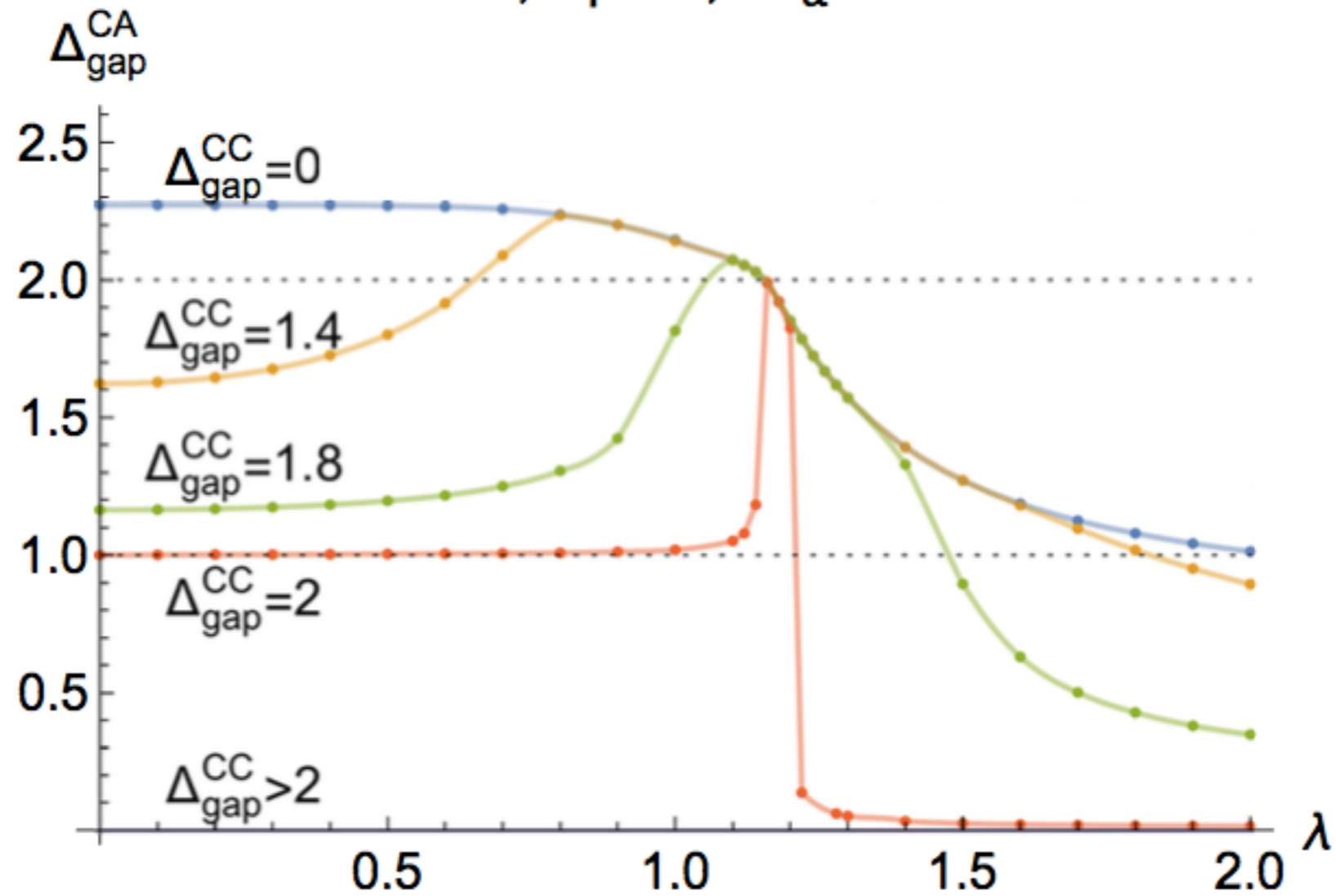


OPE of twist fields of the Z-manifold CFT: a sampling over general tilting and flat B-field

# $N=(2,2)$ Bootstrap

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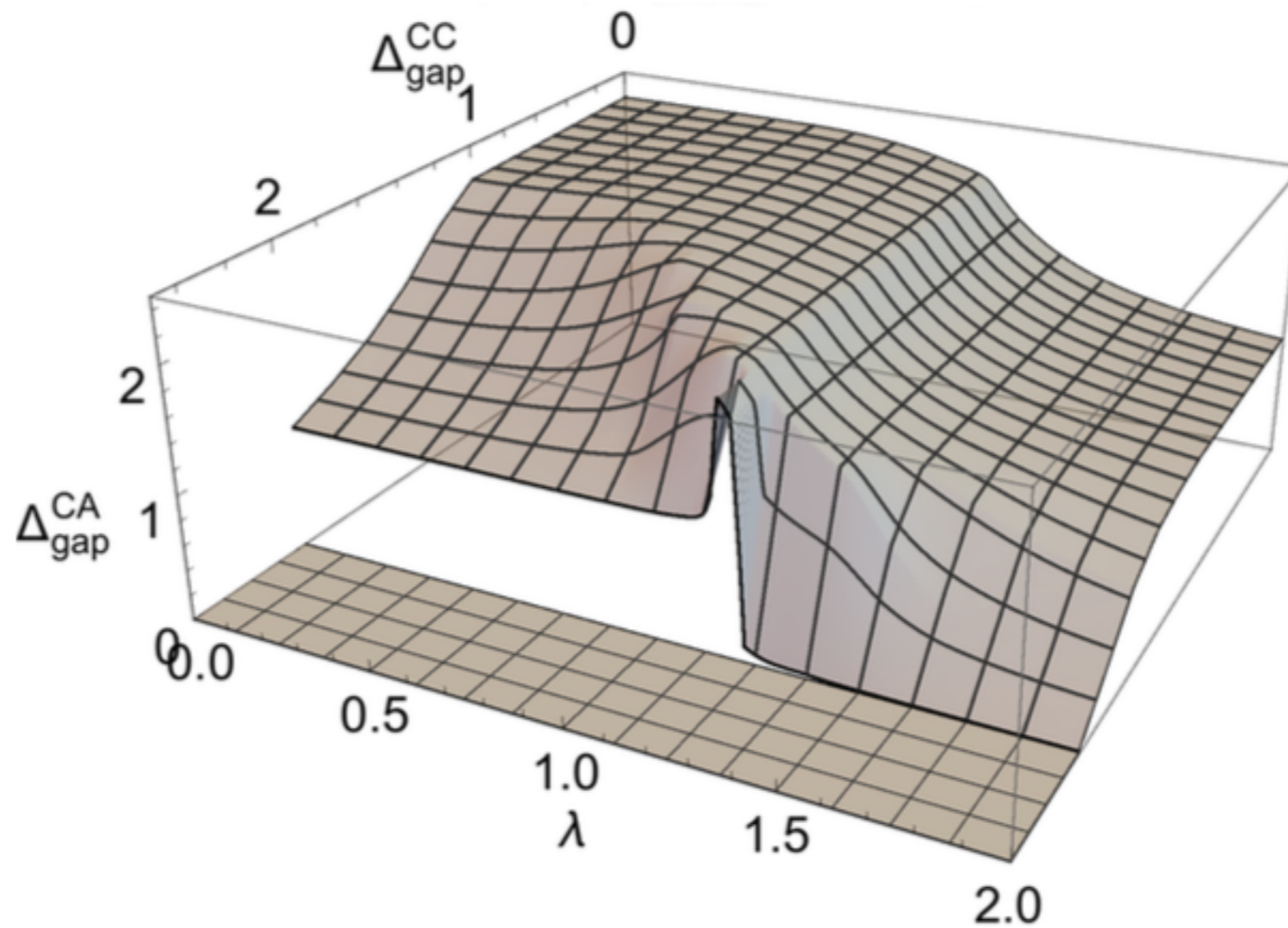
$c = 9, q = 1, N_\alpha = 24$





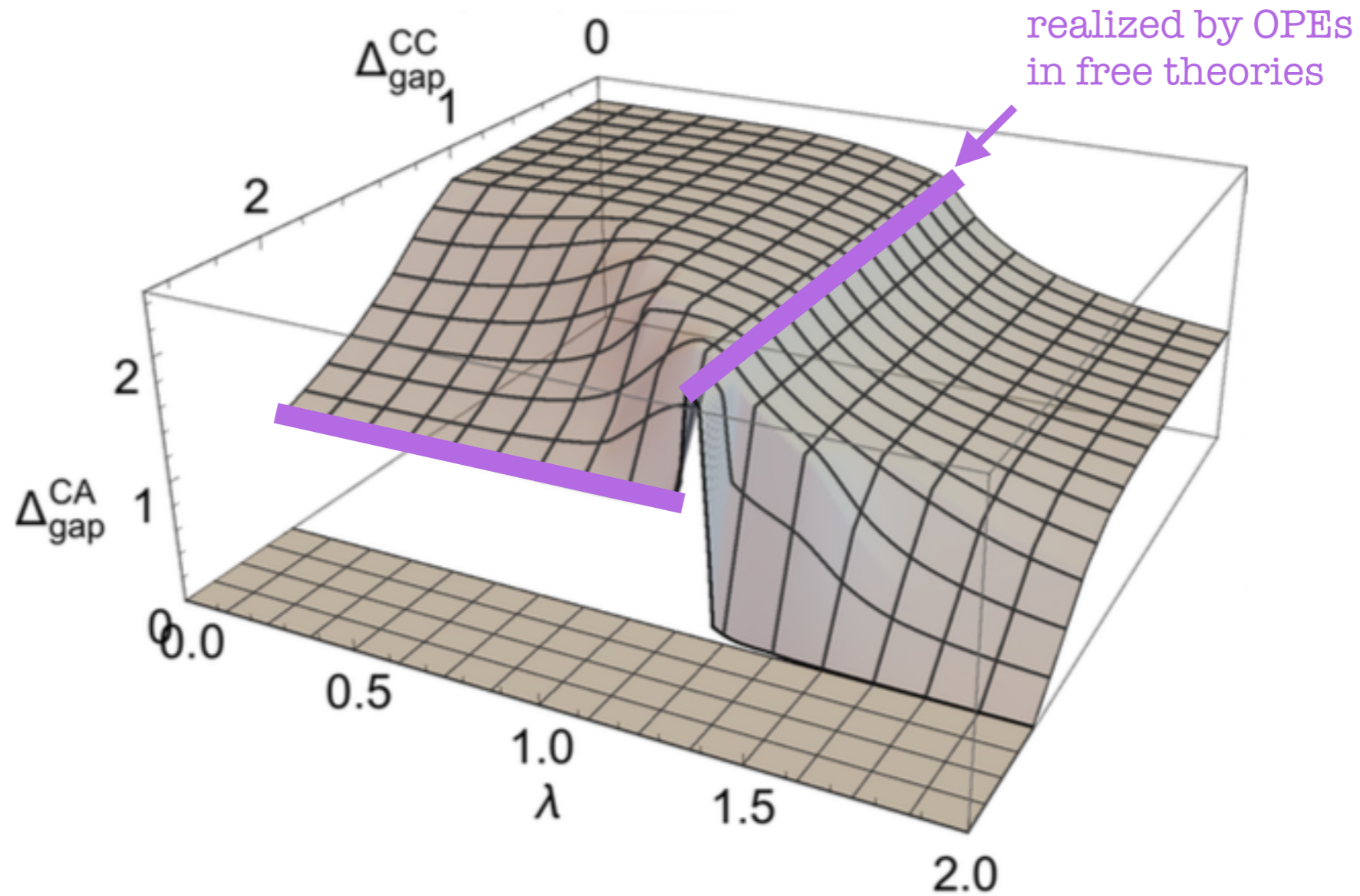
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$c=9$ , gaps in chiral-anti-chiral (CA) vs  
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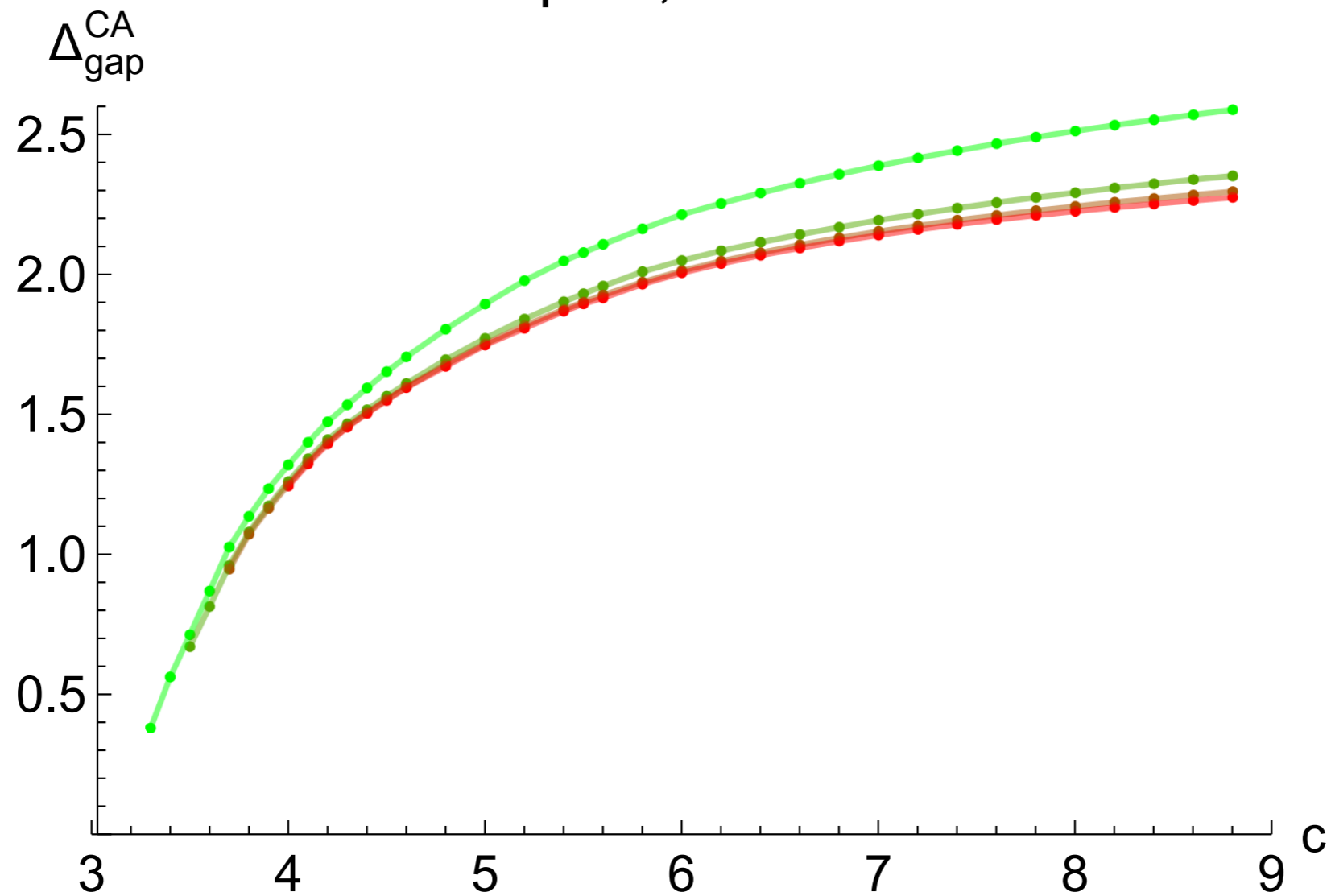
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# $(2,2)$ with marginal deformation

$3 < c < 9$ , OPE of marginal chiral and anti-chiral primaries

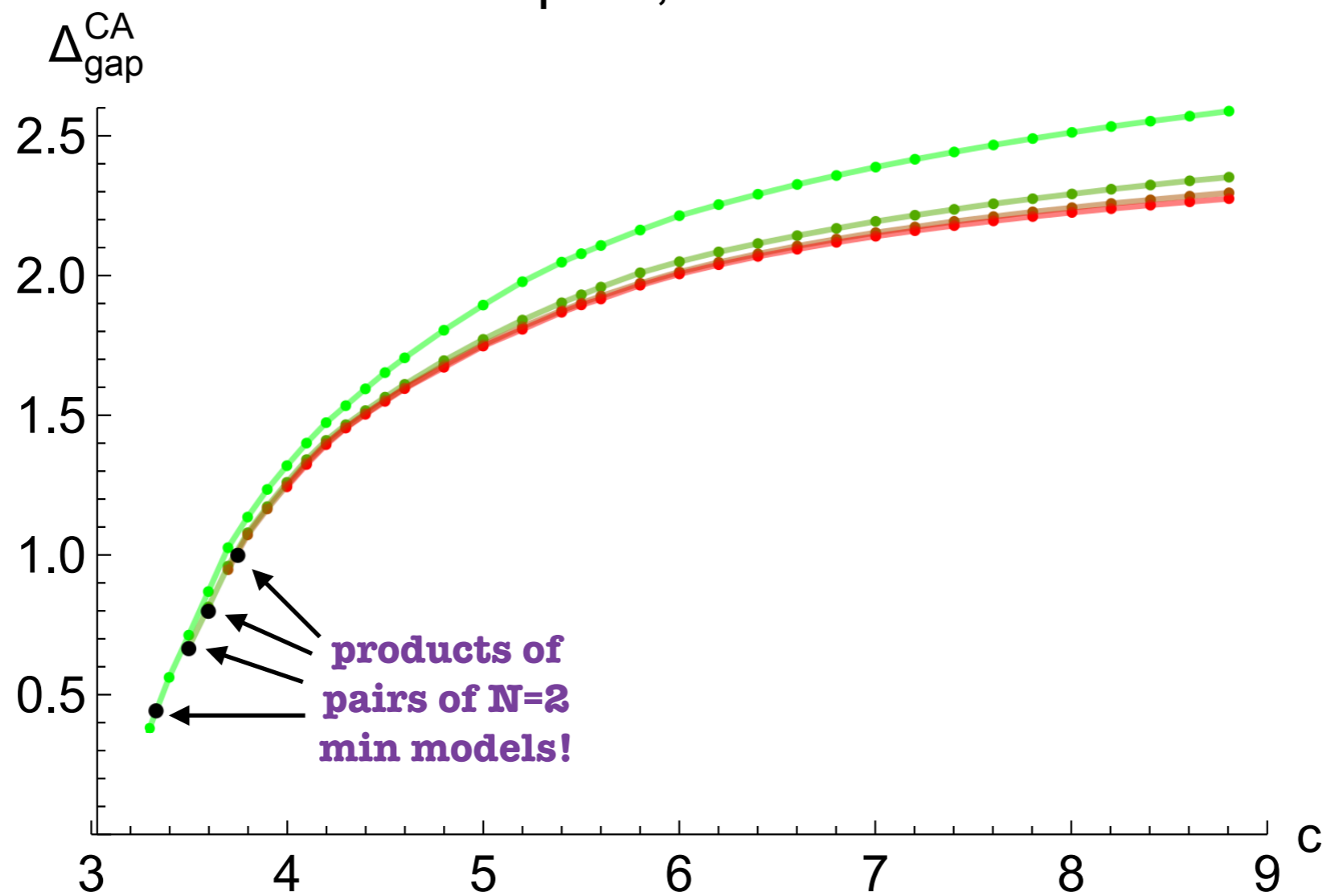
$$q = 1, \lambda = 0$$



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**To be continued...**

