Argyres-Douglas Matter and New $\mathcal{N} = 2$ dualities

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- The space of such (Argyres-Douglas) theories is much larger than those with integral scaling dimensions.
- Previous studies on S duality mainly focus on SCFTs with integral scaling dimension and mainly use class $\mathcal{S}$ construction. We will find a two discrete parameter generalization of duality (in fact class $\mathcal{S}$ sits at the bottom). These new examples should provide us more insights in understanding duality.

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What is the S duality behavior for those general class of theories? We knew very few examples, here we will provide an answer for those SCFTs engineered using 6d $(2,0)$ theory.

Generalities of $\mathcal{N}=2$ conformal deformations

The $\mathcal{N}=2$ preserving conformal deformations are classified by operator $\mathcal{E}_{r,(0,0)}$ with $r=2$ (Dolan, Osborn, 02, Argyres, Lotito, Lu, Martone, 15, Cordorva, Dumitrescu, Intriligator 16):

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- The duality group and the weakly coupled gauge theory description, i.e. decomposing the theory into gauge groups and matters with flavor symmetries.

Models

Let's consider 6d A_{N-1} $(2,0)$ theory, and compactify it on a sphere with following irregular singularity (DX, 2012)

$$\Phi = \frac{T_n}{z^n} + \frac{T_{n-1}}{z^{n-1}} + \ldots + \frac{T_1}{z} + \ldots \quad (2)$$

here T_i are diagonal matrices whose eigenvalue degeneracy is labeled by a Young Tableaux Y_i, we have the following constraint

$$Y_n \subset Y_{n-1} \subset \ldots \subset Y_1 \quad (3)$$

We can add an extra regular singularity which is also labeled by a Young Tableaux Y_0.

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- These theories all have Lagrangian 3d mirror (Boalch 10; DX 12).

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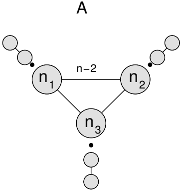

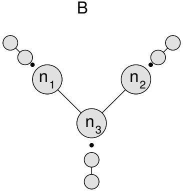

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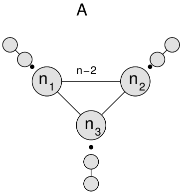

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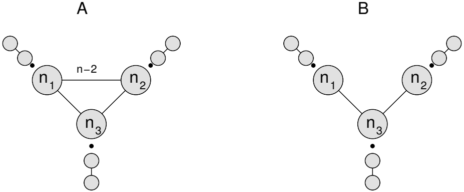

- They can have three non-abelian flavor symmetries: an infinite sequence of generalization of T_N theory.
- The flavor central charges for three non-abelian favor symmetries are:

$$k_{SU(n_1)} = n_1 + \frac{1}{n-1}, \quad k_{SU(n_2)} = n_2 + \frac{1}{n-1}, \quad k_{SU(n_3)} = n_3 + \frac{n-2}{n-1}$$

SCFT

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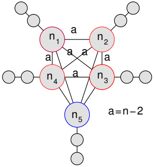

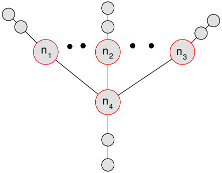

S duality

Interestingly, we found an infinite sequence of matter and SCFT labeled by an integer n. $n = 2$ case are nothing but the Class $\mathcal{S}$ theory. The S duality of class $\mathcal{S}$ theory is interpreted as the degeneration of a punctured Riemann surface into three punctured sphere which represents the T_N matter.

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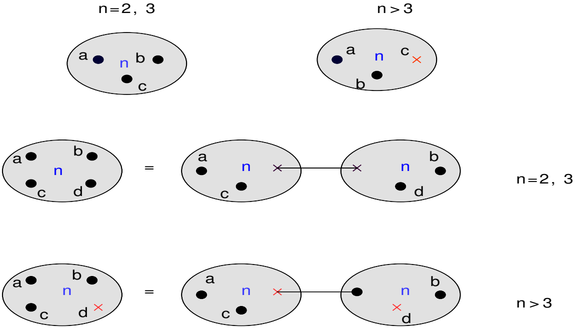
Interestingly, for $n > 2$, things also works perfectly in the same way: SCFT is represented by a punctured sphere, and matter is represented by a three punctured sphere. S duality is interpreted as different decompositions.

Matter



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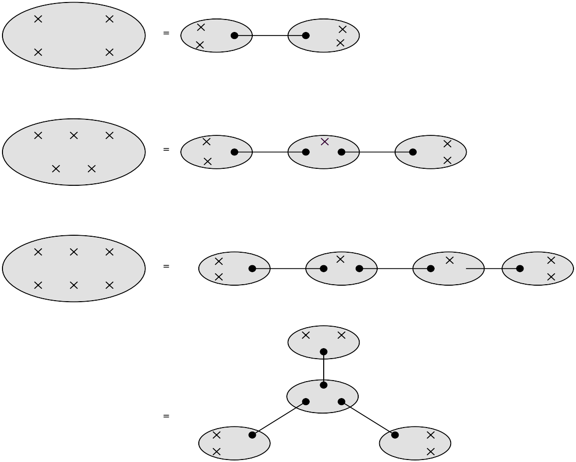
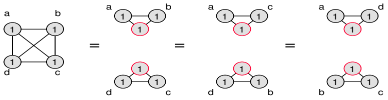
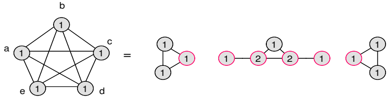
The SCFT can be represented by a sphere with N marked points, and different S duality frame is interpreted as different decomposition:



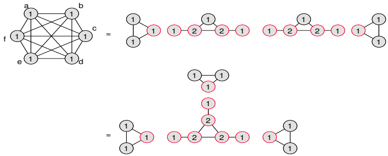
N=4



N=5



N=6



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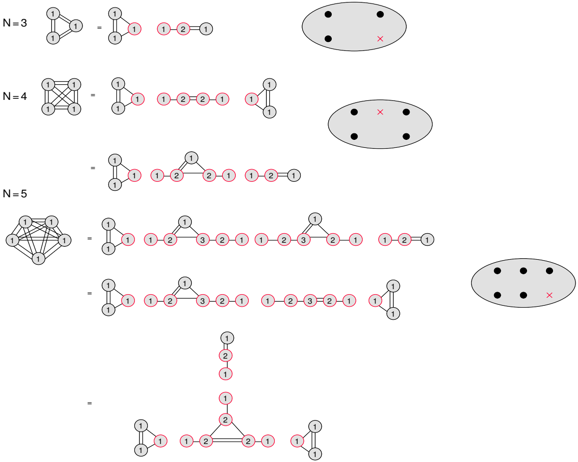

Checks

There are some checks one can do:

- Vanishing of β function.
- Coulomb branch spectrum, Higgs branch dimension, etc.
- Central charges a, c.
- Index computation (D.X, W.B.Yan, S.T. Yau).
- The number of cusp singularities of the moduli of singularity is equal to that of duality frame.

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- The theory can be generalized to SCFTs engineered using other type of 6d $(2, 0)$ theory.

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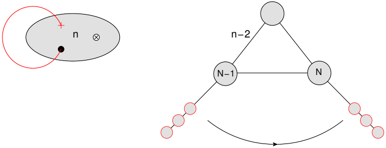

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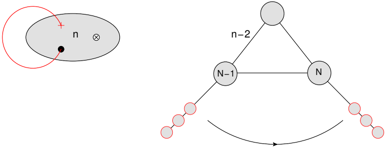

- We can gauge more than two AD matters for a single gauge group.

The classification of SCFTs and S duality of those theories deserve further study.

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- Better understanding of duality. Still very mysterious.