Mordell-Weil group and massless matter in F-theory

- arXiv: 1405.3656 with O. **Till**, C. **Mayrhofer**, D. **Morrison**
- arXiv: 1406.6071 with L. **Lin**
  arXiv: 1307.2902 with J. **Borchmann**, E. **Palti** and C. **Mayrhofer**
- arXiv: 1402.5144 with M. **Bies**, C. **Mayrhofer** and C. **Pehle**

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The beauty of F-theory

$$\textbf{Particle physics} \overset{\text{F-theory}}{\longleftrightarrow} \textbf{Geometry}$$

Fruitful connections to particle physics via F-theory GUTs have motivated a lot of recent progress in understanding of 4-dimensional F-theory vacua

Two examples of recent advances:

1) The geometry of elliptic fibrations with extra sections

- determines the abelian sector of gauge theories
- affects the structure of Yukawa selection rules
- is responsible for absence of proton decay operators in F-theory GUTs

2) The C_3-background

- determines matter multiplicities
- is a crucial input in 4-dimensional F-theory

Outline

I.) The Mordell-Weil group in F-theory

II.) Non-torsional sections & applications:

Standard Models in F-theory ('F-theory non-GUTs')

[Ling,TW'14]

III.) Torsional sections

[Mayrhofer,Morrison,Till,TW'14]

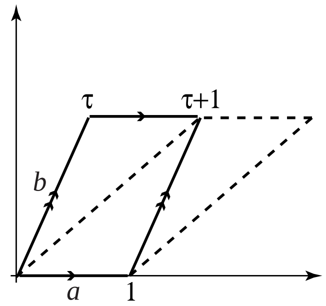
IV.) Gauge data$/C_3$ backgrounds and applications

[Bies,Mayrhofer,Pehle,TW'14]

Mordell-Weil group

1) Elliptic curve: $\mathcal{E} = \mathbb{C}/\Lambda$

$\leftrightarrow$ addition of points



Rational points:

- have $\mathbb{Q}$-rational coordinates (x, y, z) in Weierstrass model

$$y^2 = x^3 + fxz^4 + gz^6, \qquad [x : y : z] \in \mathbb{P}^2_{2,3,1}$$

- form an abelian group under addition = **Mordell-Weil group** E

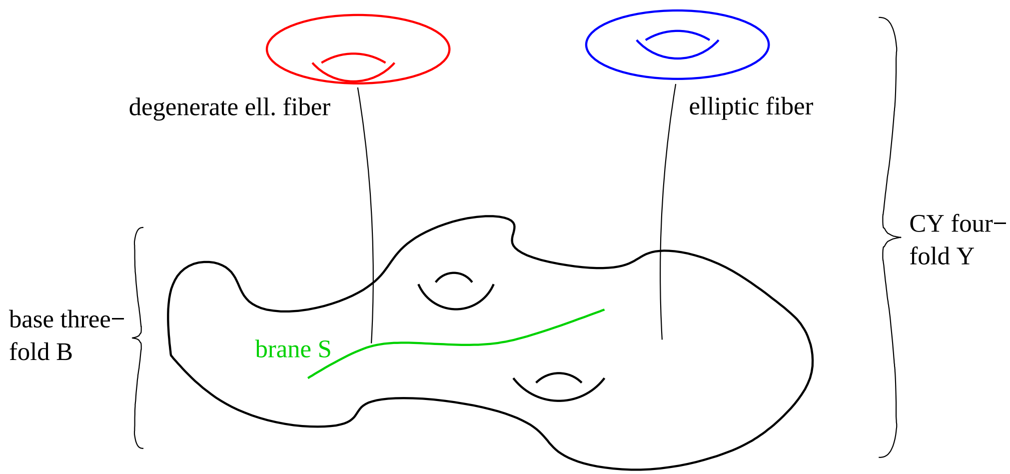
$$E = \mathbb{Z}^r \oplus \mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_n}$$

2) Elliptic fibration: $\pi : Y \to \mathcal{B}$

Rational section σ:

$$\mathcal{B} \ni b \mapsto \sigma(b) = [x(b) : y(b) : z(b)]$$

- $\sigma(b)$ is a K-rational point in fiber
- degenerations in codimension allowed



Mordell-Weil group

Mordell-Weil group $E(K)$= group of rational sections

- zero-element = zero-section $\sigma_0 : b \to [1:1:0]$ in $y^2 = x^3 + fxz^4 + gz^6$
- group law = fiberwise addition

$$E(K) = \underbrace{\mathbb{Z}^r}_{\text{free part}} \oplus \underbrace{\mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_n}}_{\text{torsion part}}$$

Physical significance:

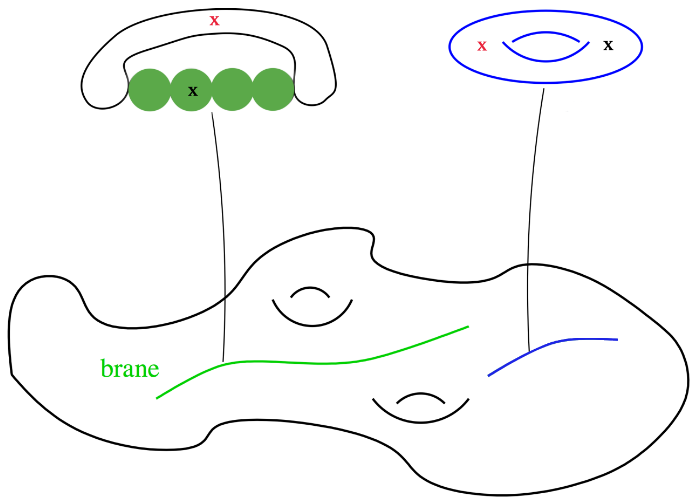
- Free part ↔ $U(1)$ gauge symmetries [Morrison,Vafa'96],[Klemm,Mayr,Vafa'98],...
- Torsion part ↔ Global structure of non-ab. gauge groups ($\pi_1(G)$)
  [Aspinwall,Morrison'98], [Aspinwall,Katz,Morrison'00], ..., [Mayrhofer,Till,Morrison,TW'14]

Systematic recent study of U(1)s via rational sections:

✓ extra selection rules, e.g. crucial in F-theory GUTs
✓ window to gauge fluxes and chirality
✓ general interest in any theory with massless U(1)s (landscape studies, ...)

Antoniadis,Anderson,Bizet,Borchmann,Braun,Braun,Choi,Collinucci,Cvetič,Etxebarria,Grassi,Grimm, Hayashi, Keitel,Klevers,Küntzler,Krippendorf,Oehlmann,Klemm,Leontaris,Lopes,Mayrhofer, Mayorga, Morrison, Park,Palti,Piragua,Rühle,S-Nameki,Song,Valandro,Taylor,TW,...

Mordell-Weil group



Divisors on elliptic 4-fold $\hat{Y}_4 \to \mathcal{B}$:

- zero-section Z
- pullback from base $\pi^{-1}(D_\mathrm{b})$
- resolution divisors F_m, $m = 1, \ldots, \mathrm{rk}(G)$
- **rational sections** S_i

Shioda map

- homomorphism $\varphi : \underbrace{E(K)}_{\text{group of sections}} \to \underbrace{NS(\hat{Y}_4) \otimes \mathbb{Q}}_{\text{group of divisors}}$ [Shioda'89]

$$\varphi(S - Z) = S - Z - \pi^{-1}(\delta) + \sum l_i F_i, \qquad l_i \in \mathbb{Q}$$

- transversality

$$\int_{\hat{Y}_4} [\varphi(S - Z)] \wedge [X] \wedge [\pi^{-1}\omega_4] = 0 \qquad X \in \{Z, F_l, \pi^{-1}(D_\mathrm{b})\}$$

Non-torsional sections

- $[\mathcal{S}] \equiv [\varphi(S - Z)]$ is non-trivial in $H^{1,1}(\hat{Y}_4)$
- $[\mathcal{S}]$ serves as the generator of $U(1)$ gauge group

$$C_3 = A \wedge [\mathcal{S}] \qquad A\text{: } U(1) \text{ gauge potential}$$

Engineering extra sections via restriction of complex structure of fibration

- Non-generic Weierstrass models, simplest example: [Grimm,TW'10]

  $y^2 - a_1 x\, y\, z - a_3\, y\, z^3 = x^3 + a_2\, x^2\, z^2 + a_4\, x\, z^4 + \cancel{a_6\, z^6}, \quad [x : y : z] \in \mathbb{P}_{2,3,1}$

  zero-section: $[1 : 1 : 0]$ extra section: $[0 : 0 : 1]$

  Generalisations in [Mayrhofer,Palti,TW'12]

- Systematic approach via different fiber representations $\rightsquigarrow$ cf. talk by D. Klevers
  - rank 1: $\mathbb{P}_{1,1,2}[4]$ [Morrison,Park'12]
  - rank 2: $\mathbb{P}^2[3]$ [Borchmann,Mayrhofer,Palti,TW][Cvetič,(Grassi),Klevers,Piragua]'13
  - rank 3: complete intersections [Cvetič,Klevers,Piragua,Song'13]
  - toric hypersurfaces [Braun,Grimm,Keitel'13][Grassi,Perduca'12]

Standard Model applications

Explore U(1) selection rules in F-theory Standard Models [Ling,TW'14]

$SU(3) \times SU(2) \times U(1)_1 \times U(1)_2 \;\leftrightarrow\; U(1)_Y = a\,U(1)_1 + b\,U(1)_2$

- $U(1)_1 \times U(1)_2$ [Borchmann,Mayrhofer,Palti,TW][Cvetič,(Grassi),Klevers,Piragua]'13

$$\begin{aligned} 0 \quad = \quad & \mathrm{v\,w}(c_1\,\mathrm{w}\,s_1 + c_2\,\mathrm{v}\,s_0) + \mathrm{u}\,(b_0\,\mathrm{v}^2\,{s_0}^2 + b_1\,\mathrm{v\,w}\,s_0\,s_1 + b_2\,\mathrm{w}^2\,{s_1}^2) + \\ & + \mathrm{u}^2(d_0\,\mathrm{v}\,{s_0}^2\,s_1 + d_1\,\mathrm{w}\,s_0\,{s_1}^2 + d_2\,\mathrm{u}\,{s_0}^2\,{s_1}^2) \end{aligned}$$

- $SU(2) \leftrightarrow W_2 = \{w_2 = 0\}$ $\qquad SU(3) \leftrightarrow W_3 = \{w_3 = 0\}$

  $g_m \in \{b_i, c_j, d_k\}$, $\qquad g_m = g_{m,k,l} w_2^k w_3^l$ $\qquad 3 \times 3$ 'toric enhancements'

- fully resolved fibrations over suitable base $\mathcal{B}$

Types of matter representations:

$(\mathbf{3},\mathbf{2}) \leftrightarrow Q_L \qquad 3 \times (\mathbf{1},\mathbf{2}) \leftrightarrow (H_u, H_d, L) \qquad 5 \times (\overline{\mathbf{3}},\mathbf{1}) \leftrightarrow (u_R^c, e_R^c)$

$6 \times (\mathbf{1},\mathbf{1}) \leftrightarrow \nu_R^c$ or extra singlet extensions

Rich pattern of Yukawa couplings

$\leftrightarrow$ MSSM + extra dim-4 and dim-5 couplings

Standard Model identifications

[Ling,TW'14]

Matter spectrum	*Baryon- and Lepton number violation*
possibility no. 5 $a=1, b=0$; $(H_u,H_d)=(\mathbf{2}^{\rm I}_2,\overline{\mathbf{2}}^{\rm I}_1)$ heavy (u^c_R,d^c_R): $(\overline{\mathbf{3}}^{\rm A}_4,\overline{\mathbf{3}}^{\rm A}_3)$ light u^c_R: $\overline{\mathbf{3}}^{\rm A}_1$; light d^c_R: $\overline{\mathbf{3}}^{\rm A}_2$, $\overline{\mathbf{3}}^{\rm A}_5$ heavy generations of (L,ν^c_R,e^c_R): $(\overline{\mathbf{2}}^{\rm I}_1,\overline{\mathbf{1}}^{(5)},-)$, $(\overline{\mathbf{2}}^{\rm I}_2,-,\mathbf{1}^{(2)})$, $(\overline{\mathbf{2}}^{\rm I}_3,\overline{\mathbf{1}}^{(6)},\mathbf{1}^{(1)})$ light ν^c_R: $\mathbf{1}^{(5)}$, $\mathbf{1}^{(6)}$; light e^c_R: $\mathbf{1}^{(3)}$, $\mathbf{1}^{(4)}$ $\mathbf{1}_\mu$: $\overline{\mathbf{1}}^{(5)}$	α: $(\overline{\mathbf{2}}^{\rm I}_1,\overline{\mathbf{3}}^{\rm A}_3)$, $(\overline{\mathbf{2}}^{\rm I}_2,\overline{\mathbf{3}}^{\rm A}_2)$, $(\overline{\mathbf{2}}^{\rm I}_3,\overline{\mathbf{3}}^{\rm A}_5)$; β: $\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_3\overline{\mathbf{3}}^{\rm A}_2$, $\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_5\overline{\mathbf{3}}^{\rm A}_5$, $\overline{\mathbf{3}}^{\rm A}_1\overline{\mathbf{3}}^{\rm A}_3\overline{\mathbf{3}}^{\rm A}_5$; δ: $\overline{\mathbf{1}}^{(5)}\mathbf{1}^{(6)}\mathbf{1}^{(6)}$, $\mathbf{1}^{(5)}\overline{\mathbf{1}}^{(6)}\overline{\mathbf{1}}^{(6)}$; γ: $\overline{\mathbf{2}}^{\rm I}_1\overline{\mathbf{2}}^{\rm I}_2\mathbf{1}^{(2)}$, $\overline{\mathbf{2}}^{\rm I}_1\overline{\mathbf{2}}^{\rm I}_3\mathbf{1}^{(1)}$, $\overline{\mathbf{2}}^{\rm I}_2\overline{\mathbf{2}}^{\rm I}_2\mathbf{1}^{(3)}$, $\overline{\mathbf{2}}^{\rm I}_2\overline{\mathbf{2}}^{\rm I}_3\mathbf{1}^{(4)}$, $\overline{\mathbf{2}}^{\rm I}_3\overline{\mathbf{2}}^{\rm I}_3\mathbf{1}^{(2)}$; λ_1: $\mathbf{2}^{\rm I}_1$; λ_3: ✓ ; λ_6: $-$; λ_8: $\mathbf{1}^{(2)}$; λ_{10}: $(\overline{\mathbf{3}}^{\rm A}_3)^*$; λ_4: $(\overline{\mathbf{3}}^{\rm A}_4,\mathbf{1}^{(2)})$, $(\overline{\mathbf{3}}^{\rm A}_1,\mathbf{1}^{(1)})$; λ_5: $(\overline{\mathbf{2}}^{\rm I}_2,\overline{\mathbf{2}}^{\rm I}_2)$; λ_9: $(\overline{\mathbf{3}}^{\rm A}_4,(\overline{\mathbf{2}}^{\rm I}_2)^*)$, $(\overline{\mathbf{3}}^{\rm A}_1,(\overline{\mathbf{2}}^{\rm I}_3)^*)$; λ_7: $\overline{\mathbf{3}}^{\rm A}_4(\overline{\mathbf{3}}^{\rm A}_3)^*\mathbf{1}^{(2)}$, $\overline{\mathbf{3}}^{\rm A}_4(\overline{\mathbf{3}}^{\rm A}_2)^*\mathbf{1}^{(3)}$, $\overline{\mathbf{3}}^{\rm A}_4(\overline{\mathbf{3}}^{\rm A}_5)^*\mathbf{1}^{(4)}$, $\overline{\mathbf{3}}^{\rm A}_1(\overline{\mathbf{3}}^{\rm A}_3)^*\mathbf{1}^{(1)}$, $\overline{\mathbf{3}}^{\rm A}_1(\overline{\mathbf{3}}^{\rm A}_2)^*\mathbf{1}^{(4)}$, $\overline{\mathbf{3}}^{\rm A}_1(\overline{\mathbf{3}}^{\rm A}_5)^*\mathbf{1}^{(2)}$; λ_2: $\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_3\mathbf{1}^{(3)}$, $\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_2\mathbf{1}^{(2)}$, $\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_5\mathbf{1}^{(4)}$, $\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_1\overline{\mathbf{3}}^{\rm A}_3\mathbf{1}^{(4)}$, $\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_1\overline{\mathbf{3}}^{\rm A}_2\mathbf{1}^{(1)}$, $\overline{\mathbf{3}}^{\rm A}_4\overline{\mathbf{3}}^{\rm A}_1\overline{\mathbf{3}}^{\rm A}_5\mathbf{1}^{(2)}$, $\overline{\mathbf{3}}^{\rm A}_1\overline{\mathbf{3}}^{\rm A}_1\overline{\mathbf{3}}^{\rm A}_3\mathbf{1}^{(2)}$, $\overline{\mathbf{3}}^{\rm A}_1\overline{\mathbf{3}}^{\rm A}_1\overline{\mathbf{3}}^{\rm A}_5\mathbf{1}^{(1)}$

Red choice: no dim-4 ~~R-parity~~, no $Q\,Q\,Q\,L$, $u^c_R\,u^c_R\,d^c_R\,e^c_R$

Next steps include:

- Analysis of chiral spectrum via gauge fluxes (see later)
- Analyse distinctive non-pert. features

Torsional sections

[Mayrhofer,Morrison,Till,TW'14]

- If R is $\mathbb{Z}_k$-torsional section: $k(R-Z)=0$
- Shioda map is homomorphism: $0=\varphi(k(R-Z))=k\varphi(R-Z)\in NS(\hat{Y}_4)\otimes\mathbb{Q}$
- Since $NS(\hat{Y}_4)\otimes\mathbb{Q}$ is torsion free: $\varphi(R-Z)=R-Z-\pi^{-1}(\delta)+\sum_i \frac{a_i}{k}F_i$ is trivial

$$\Xi_k \equiv R-Z-\pi^{-1}(\delta)=-\mathbf{\frac{1}{k}}\sum_i a_iF_i,\quad a_i\in\mathbb{Z}$$ is **integer** divisor

Relation to group theory:

- Lie algebra $\mathfrak{g}\leftrightarrow$ coroot lattice $Q^\vee=\langle F_i\rangle_{\mathbb{Z}}$ $\quad F_i$: resol. divisors $\leftrightarrow$ codim.-1
- Representation content $\leftrightarrow$ weight lattice $\Lambda\leftrightarrow$ codimension-2 fibers
- Integer pairing $\Lambda^\vee\times\Lambda\to\mathbb{Z}$ gives weights $\quad$ coweight lattice $\Lambda^\vee\supseteq Q^\vee$
- If $\Lambda^\vee=Q^\vee=\langle F_i\rangle_{\mathbb{Z}}\to$ gauge group G_0: $\quad \pi_1(G_0)\approx\frac{\Lambda^\vee}{Q^\vee}=\emptyset$
- Integer $\Xi_k=-\frac{1}{k}\sum_i a_iF_i\to$ **'k-fractional refinement' of $\Lambda^\vee$**
- dual weight lattice becomes coarser $\to$ **smaller representation content**

gauge group $G=G_0/\mathbb{Z}_k$ $\qquad$ with $\qquad$ $\pi_1(G)=\mathbb{Z}_k$

Fibrations with MW torsion

Hypersurface fibrations with MW torsion

- $\mathbb{Z}_k$ for $k = 2, 3, 4, 5, 6$, $\mathbb{Z}_2 \oplus \mathbb{Z}_n$ with $n = 2, 4$ and $\mathbb{Z}_3 \oplus \mathbb{Z}_3$ [Morrison,Aspinwall'98]
- Toric models: $\mathbb{Z}_2$, $\mathbb{Z}_3$, $\mathbb{Z} \oplus \mathbb{Z}_2$ [Braun,Grimm,Keitel'13]
- Previous analysis from perspective of $SL(2, \mathbb{Z})$-monodromies: [Berglund,Klemm,Mayr,Theisen'98]
- Exemplification of general structure for toric models [Mayrhofer,Morrison,Till,TW'14]

General pattern

- Codimension-one fibers:
  $\mathbb{Z}_k$-torsion automatically produces non-ab. gauge group factor $G_0/\mathbb{Z}_k$
- Codimension-two fibers:
  matter spectrum greatly restricted due to reduced center of G
- Codimension-three fibers:
  no further selection rules at the level of Yukawa couplings

Example: $\mathbb{Z}_2$-torsional sections

[Mayrhofer,Morrison,Till,TW'14]

$$y^2 - a_1 x\, y\, z - \cancel{a_3\, y\, z^3} = x^3 + a_2\, x^2\, z^2 + a_4\, x\, z^4 + \cancel{a_6\, z^6}, \qquad [x:y:z] \in \mathbb{P}_{2,3,1}$$

Step 1: $a_6 \equiv 0 \Longrightarrow E(K) = \mathbb{Z}$ $\qquad G = U(1)$ [Grimm,TW'10]

- matter curve with $\mathbf{1}_{\pm 1}$ at $\{a_3 = 0\} \cap \{a_4 = 0\} \to SU(2)$ fibers in codim. 2

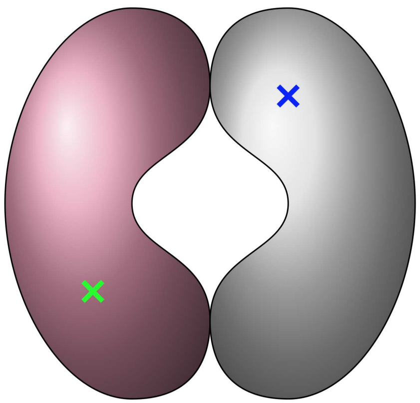
Step 2: $a_6 \equiv 0$ and $a_3 \equiv 0 \Longrightarrow E(K) = \mathbb{Z}_2$ [Aspinwall,Morrison'98]

- $SU(2)$ enhancement promoted to divisor $\{a_4 = 0\}$ with 'W-boson' $\mathbf{1}_1$
- no sources for fundamental fields
- enhancement $G = U(1) \to \mathbf{G = SU(2)/\mathbb{Z}_2}$

Resolved fibration

$$\hat{P} = -y^2\, \mathbf{s} - a_1 y\, z\, \mathbf{s}\, \mathbf{t} + \mathbf{s}^2\, \mathbf{t}^4 + a_2\, z^2\, \mathbf{s}\, \mathbf{t}^2 + a_4\, z^4$$

- $T : \{\mathbf{t} = 0\}$: $\mathbb{Z}_2$-section cf. [Grimm,Braun,Keitel'13]
  $S : \{\mathbf{s} = 0\}$: $SU(2)$ resolution divisor
- $\Xi_2 := T - Z - \bar{\mathcal{K}} = -\frac{1}{2} S$ is integer coweight
- restricted $SL(2,\mathbb{Z})$-monodromy: [Berglund et al. '98]



Example: $\mathbb{Z}_3$-torsion

$$y^2 - a_1 x\, y\, z - a_3\, y\, z^3 = x^3 + \cancel{a_2\, x^2\, z^2} + \cancel{a_4\, x\, z^4} + \cancel{a_6\, z^6}, \qquad [x:y:z] \in \mathbb{P}_{2,3,1}$$

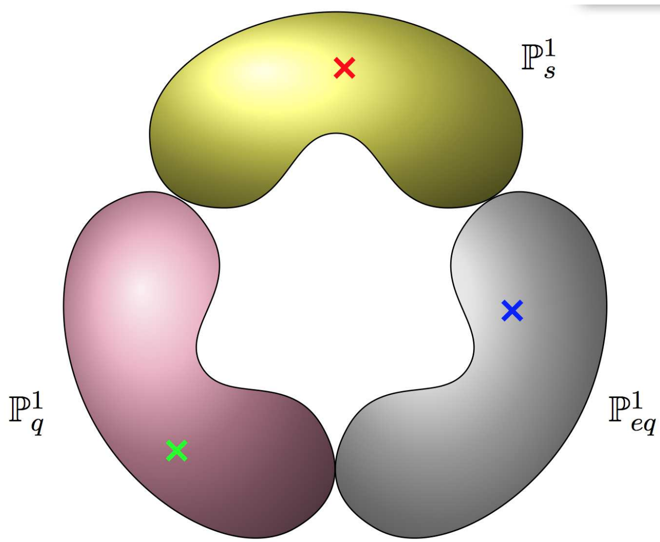
$\Delta \sim a_3^3(a_1^3 - 27a_3)$ $\qquad G = SU(3)/\mathbb{Z}_3$ $\qquad$ (no further matter)

Consider e.g. further enhancement with algebra $\mathfrak{g} = \mathfrak{su}(3) \oplus \mathfrak{su}(6)$:

$$\Delta \sim w^6 a_3^3(a_1^3 - 27w^2 a_3)$$

Representations from explicit resolution:

- no$(\mathbf{3}, 1)$
- no $(1, \mathbf{6})$
- no $(1, \mathbf{15})$,
- but: $(\mathbf{3}, \mathbf{6})$ and $(1, \mathbf{20})$



$$G = (SU(3) \times SU(6))/\mathbb{Z}_3$$

Gauge data in F/M-theory

Further defining data in F-theory via M-theory duality:

- $C_3 \simeq C_3 + d\Lambda_2$ is higher form gauge potential
- $G_4 = dC_3$, $\langle G_4 \rangle \neq 0$ 'flux' [Becker,Becker][Sethi,Vafa,Witten]'96 [Dasgupta,Rajesh,Sethi]'99,...

Gauge fluxes

- are described by certain $\mathbf{G_4 \in H^{2,2}(\hat{Y}_4)}$ with **'1 leg along fiber'**

  $\int_{\hat{Y}_4} G_4 \wedge \pi^{-1} D_a \wedge \pi^{-1} D_b = 0 \quad \int_{\hat{Y}_4} G_4 \wedge \pi^{-1} D_a \wedge Z = 0 \quad \forall D_i \in H^2(B)$

- determine the chiral index of charged matter in repr. R:

  $\chi_R = \int_{\mathcal{C}_R} G_4 \qquad \mathcal{C}_R : \mathbb{P}^1$-fibration over matter curve $C_R \subset \mathcal{B}$

  [Donagi,Wijnholt]'08,[Braun,Collinucci,Valandro],[Marsano,S-Nameki],[Krause,Mayrhofer,TW], [Grimm,Hayashi]'11, ...

Question:

- What specifies the 'gauge bundle' data beyond its field strength?
- What counts the actual number of $\mathcal{N} = 1$ chiral multiplets?

Intuition: Gauge data in IIB

- **Gauge data** on 7-brane $\Sigma_i \leftrightarrow$ **Picard group** $\mathrm{Pic}(\Sigma_i)$

  $=$ equivalence class of holo. line bundles L_i modulo gauge transformations

- Massless $\mathcal{N}=1$ chiral multiplets at intersection curves $C_{ab}=\Sigma_a \cap \Sigma_b$:

$$H^i(C_{ab}, (L_a \otimes L_b^*)|_{C_{ab}} \otimes \sqrt{K_{C_{ab}}}), \qquad i=0,1$$

Description of Picard group

$$0 \longrightarrow \underbrace{J^1(X)}_{\text{'Wilson line } \oint A'} \to \mathrm{Pic}(X) \xrightarrow{c_1} \underbrace{H^{1,1}_{\mathbb{Z}}(X)}_{\text{'curvature } F=dA'} \to 0$$

- $c_1(L) = \frac{1}{2\pi}F$ is linear map onto $H^{1,1}_{\mathbb{Z}}(X)$
- $J^1(X) = H^{0,1}(X,\mathbb{C})/H^1(X,\mathbb{Z})$: Jacobian of X space of flat connections

Picard group $\mathrm{Pic}(X) \cong$ **divisors modulo linear equivalence**

- A divisor on X is formal linear combination $D=\sum_i n_i f_i$ of $(d-1)$-cycles
- $D_1 \sim D_2$ if $D_1 - D_2 =$ zero/pole of globally def. merom. function on X

line bundle L up to gauge $\leftrightarrow$ divisor D mod linear equiv.

Bundle data in F/M-theory

C_3-background in F/M-theory [Curio,Donagi'98][(Diaconescu),Freed,Moore'03/4]

[Donagi,Wijnholt'12/3][Anderson,Heckman,Katz'13][Intriligator et al'12]

$$0 \longrightarrow J^2(\hat{Y}_4) \longrightarrow H^4_D(\hat{Y}_4, \mathbb{Z}(2)) \xrightarrow{\hat{c}_2} H^{2,2}_{\mathbb{Z}}(\hat{Y}_4) \longrightarrow 0$$

- Deligne cohomology $H^4_D(\hat{Y}_4, \mathbb{Z}(2))$ $\leftrightarrow$ equivalence classes of gauge data
- $H^{2,2}_{\mathbb{Z}}(\hat{Y}_4) \leftrightarrow$ field strength G_4
- $J^2(\hat{Y}_4) \simeq H^3(\hat{Y}_4, \mathbb{C})/(H^{3,0}(\hat{Y}_4) + H^{2,1}(\hat{Y}_4) + H^3(\hat{Y}_4, \mathbb{Z})) \leftrightarrow C_3$

Concrete specification of gauge data in examples via rational equivalence class of 4-cycles $\gamma_i \in Z_2(\hat{Y}_4)$ [Bies,Mayrhofer,Pehle,TW'14]

- Rational equivalence: $C_1 \cong C_2 \in Z_n(X)$ if $C_1 - C_2$ is zero/pole of a meromorphic function defined on an $(n+1)$-dimensional irreducible subvariety of X
- Chow group $\mathrm{CH}^k(X)$ = group of rational equivalence classes of codim. k-cycles
- Rational equivalence is finer than homological equivalence.
- Special case: $\mathrm{CH}^1(X) = \mathrm{Pic}(X)$

A cohomology formula

Math input: existence of $\hat{\gamma}_2 : \mathrm{CH}^2(X) \to H^4_D(X, \mathbb{Z}(2))$ (homomorphism)

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \mathrm{CH}^2_{\mathsf{hom}}(X) & \longrightarrow & \mathrm{CH}^2(X) & \xrightarrow{\gamma_2} & H^{2,2}_{alg}(X) & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle AJ} & & \downarrow{\scriptstyle \hat{\gamma}_2} & & \downarrow & & \\ 0 & \longrightarrow & J^2(X) & \longrightarrow & H^4_D(X, \mathbb{Z}(2)) & \xrightarrow{\hat{c}_2} & H^{2,2}_{\mathbb{Z}}(X) & \longrightarrow & 0 \end{array}$$

Strategy: [Bies,Mayrhofer,Pehle,TW'14]

1) Fix 4-cycle class $\alpha_G \in \mathrm{CH}^2(X)$: $[\alpha_G] = \hat{c}_2 \circ \hat{\gamma}_2(\alpha_G) = G_4$

2) **Manipulations mod rational equivalence preserve C_3 modulo gauge equivalence**

Upshot:

The map $\hat{\gamma}_2$ may not be surjective, but every α_G does give gauge data mod. gauge equivalence

A cohomology formula

Aim: Extract bundle data relevant for matter representation R

- Fix element $\alpha_G \in \mathrm{CH}^2(\hat{Y}_4)$ with $[\alpha_G] = G_4 \in H^{2,2}(\hat{Y}_4)$
- matter surface $\mathcal{C}_R \in Z_2(\hat{Y}_4)$ with projection $\pi_R : |\mathcal{C}_R| \to C_R$
- $\mathcal{C}_R \cdot_r \alpha_G \in \mathrm{CH}^2(|\mathcal{C}_R|)$ = Chow class of points on $|\mathcal{C}_R|$
- Projection to base $\mathcal{B}$ gives points on matter curve C_R:

$$\pi_{R*}(\mathcal{C}_R \cdot_r \alpha_G) \in \mathrm{CH}^1(C_R) \cong \mathrm{Pic}(C_R)$$

- Pick a representative $A_{R,G} \in Z_0(C_R)$ and $L = \mathcal{O}_{C_R}(A_{R,G})$
- **Proposal**: [Bies,Mayrhofer,Pehle,TW'14]

  massless $\mathcal{N} = 1$ chiral multiplets counted by

$$H^i(C_R, L_{G,R} \otimes K_{C_R}^{1/2}), \qquad i = 0, 1$$

  - chiral index $\chi(R) = \deg(L_{R,G}) = [\mathcal{C}_R] \cdot [\alpha_G] \equiv \int_{\mathcal{C}_R} G_4$
  - Computation of $\pi_{R*}(\mathcal{C}_R \cdot_r \alpha_G)$ proceeds within Chow ring
  - Related work: [Intriligator,Jockers,Mayr,Morrison,Plesser'12][Donagi,Wijnholt'09/10]

Application to $U(1)$-fluxes

$U(1)$ with $C_3 = A \wedge [\mathcal{S}]$

- $\mathcal{S} \in \mathrm{CH}^1(\hat{Y}_4)$ via Shioda map $\qquad [\mathcal{S}] \in H^{1,1}(\hat{Y}_4)$
- $U(1)$ bundle data $\alpha = \pi^* f \cdot \mathcal{S} \in \mathrm{CH}^2(\hat{Y}_4) \quad f \in \mathrm{CH}^1(\mathcal{B})$
- $G_4 = [\alpha] = [\pi^* f] \wedge [\mathcal{S}]$

matter in repr. R on curve C_R with $U(1)$ charge q_R:

$$H^i(C_R, L_R \otimes \sqrt{K_{C_R}}), \qquad L_R = [\mathcal{O}_{C_R}(C_R \cdot_r f)]^{\otimes q_R}$$

Explicit toy model $SU(5) \times U(1)$: via cohomCalg by Blumenhagen et al.'10]

curve	$H^0(C, \mathcal{L}\|_C)$		$H^1(C, \mathcal{L}\|_C)$	
	$h^0(C, \mathcal{L}\|_C)$	representation	$h^1(C, \mathcal{L}\|_C)$	representation
$C_{\mathbf{10}}$	4	$\mathbf{10}_{-1}$	1	$\overline{\mathbf{10}}_{+1}$
$C_{\bar{\mathbf{5}}_{\mathbf{m}}}$	6	$\bar{\mathbf{5}}_{\mathbf{3}}$	3	$\mathbf{5}_{-\mathbf{3}}$
$C_{\mathbf{5}_{\mathbf{H}}}$	9	$\mathbf{5}_{\mathbf{2}}$	9	$\bar{\mathbf{5}}_{-\mathbf{2}}$
$C_{\mathbf{1}}$	585	$\mathbf{1}_{\mathbf{5}}$	0	$\bar{\mathbf{1}}_{-\mathbf{5}}$

Conclusions

1) Mordell-Weil group of rational sections in F-theory:

- free piece: U(1) gauge groups
- torsion piece: global structure of non-abelian gauge groups

2) Application:

Geometries for direct embedding of Standard Model into F-theory

3) Specification of gauge data via Chow groups

- Proposal for computation of massless matter spectrum
- Many conceptual open questions and lots of work to systematize this