Gravitation From Entanglement

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Based on:

1308.3716 w. Lashkari + McDermott

1312.7856 w. Faulkner, Guica, Hartman, Myers

1405.2933 w. Swingle

BACKGROUND

Intriguing connections between entanglement structure and dual spacetime structure/geometry in holographic CFTs.

e.g. Maldacena, Ryu-Takayanagi, Swingle, MVR

QUESTION: Can we understand spacetime dynamics from properties of entanglement?

Will Show:

$\delta S_B = \delta E_B$ "First Law of Entanglement"

$$\Downarrow$$

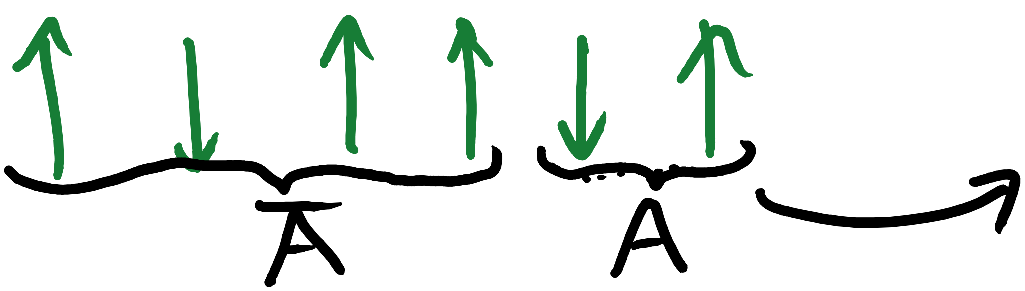
very weak version

Geometries dual to CFT states near $|\text{vacuum}\rangle$ satisfy linearized Einstein Equations (including source)

For any quantum system:

Define **ENTANGLEMENT ENTROPY** of a subsystem A by:

$$S_A = -\,\mathrm{tr}\left(\rho_A \log \rho_A\right)$$



$$\rho_A = \mathrm{tr}_{\bar{A}} |\psi\rangle\langle\psi|$$

If we vary the state

$$|\psi\rangle \to |\psi\rangle + \delta|\psi\rangle$$

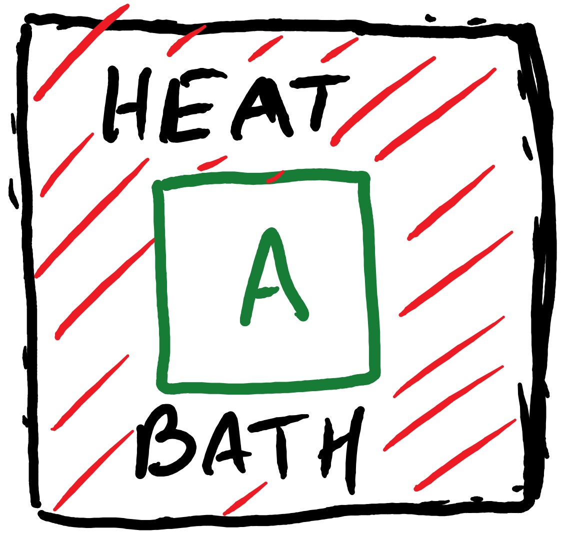
Then:

$$\boxed{\delta S_A = -\operatorname{tr}(\delta\rho_A \log \rho_A)}$$

Useful if we know ρ_A.

Example: canonical ensemble

$$\rho_A = \frac{1}{Z} e^{-H_A/T}$$

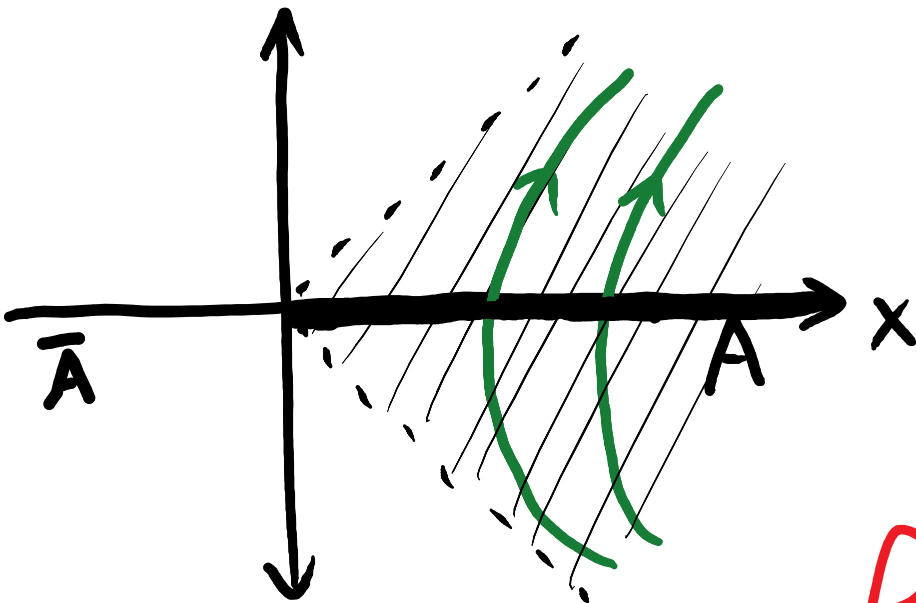


Get:

$$\delta S_A = \frac{1}{T} \delta \langle H_A \rangle$$

(Quantum) 1st Law of Thermodynamics

Another example: QFT on $\mathbb{R}^{d,1}$



$$|\psi\rangle = |\text{vacuum}\rangle$$

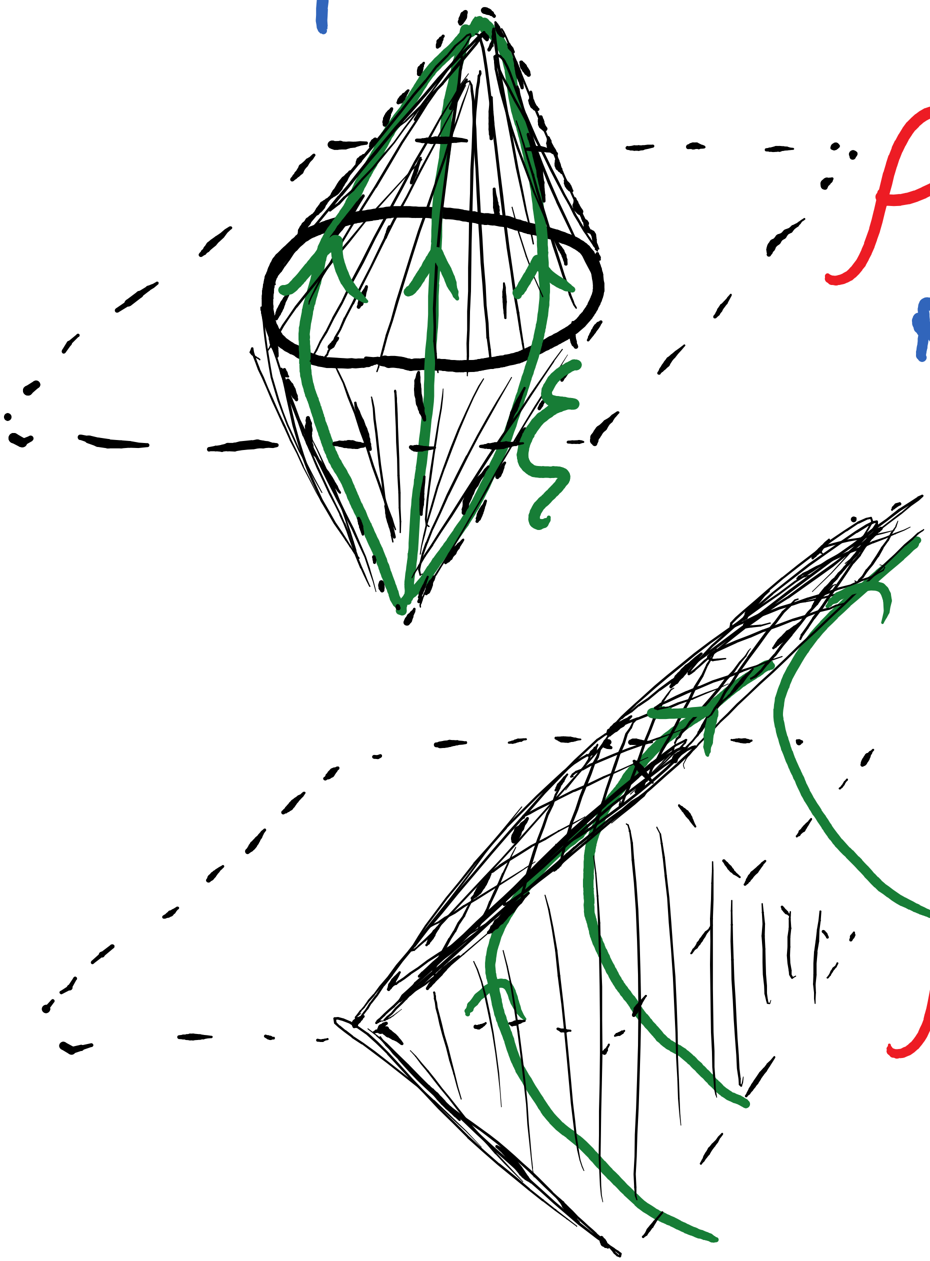
$$A = \{x > 0\}$$

$$\rho_A = \frac{1}{Z} e^{-2\pi H_{\text{RINDLER}}}$$

Unruh

$$\delta S_A = 2\pi \int_{x>0} d^d x \; x \langle T_{00}(x) \rangle$$

Example #3: CFT on $\mathbb{R}^{d,1}$

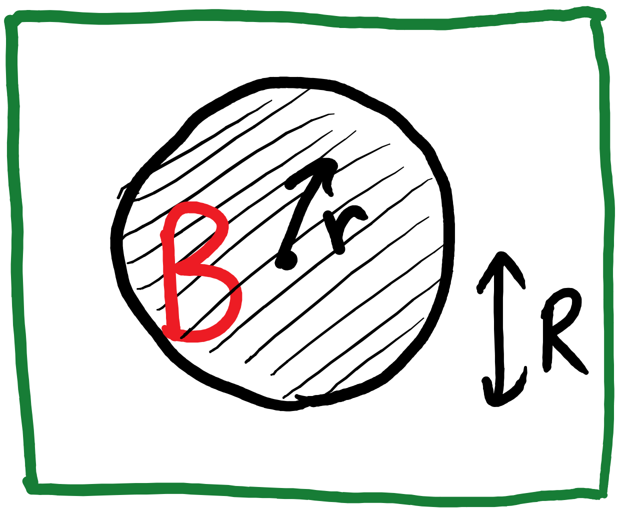


$$\rho_A = \frac{1}{Z} e^{-2\pi H_\xi}$$

conformal transform

$$\rho_A = \frac{1}{Z} e^{-2\pi H_{RIND}}$$

Have: $H_\xi = \int d^dx \, \frac{R^2 - r^2}{2R} T_{00}$



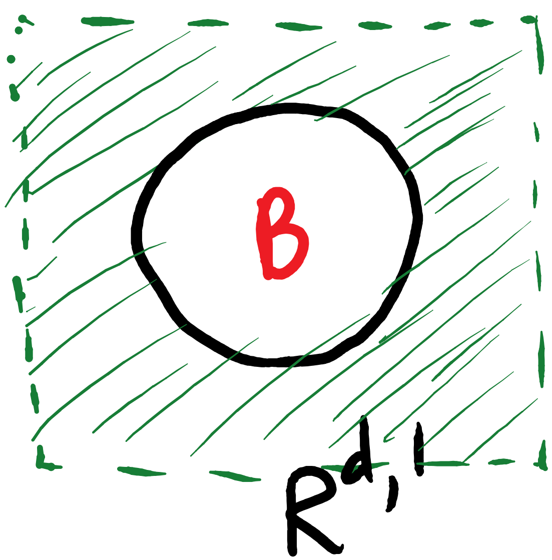
Result: For perturbation to CFT vacuum,

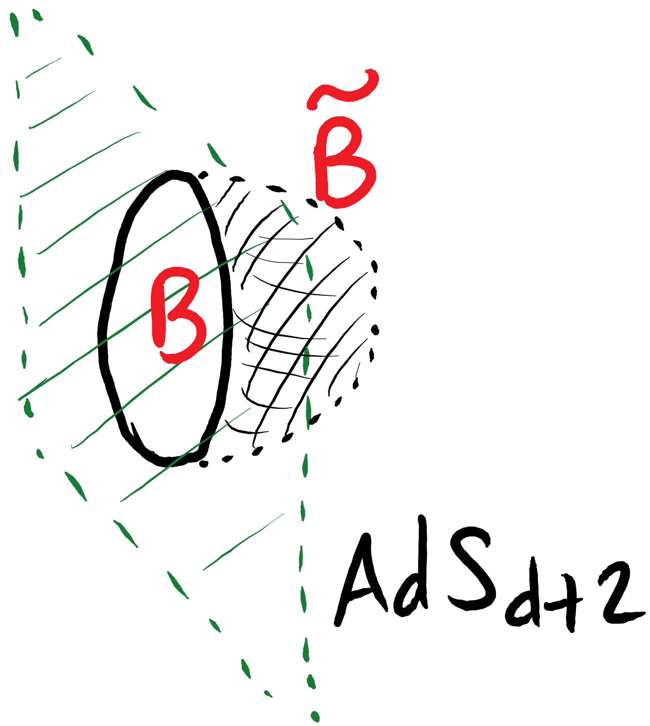
$$\delta S_B = 2\pi \int \frac{R^2 - r^2}{2R} \delta \langle T_{00} \rangle \equiv \delta E_B$$

Blanco, Casini, Hung, Myers

For some CFTs:

S_B for $|\text{vacuum}\rangle$ matches extremal surface areas in AdS





Ryu-Takayanagi

Basic Assumption

Suppose $\exists$ CFT where this is true for family of states near $|\text{vac}\rangle$

$$|\text{vac}\rangle + \delta|\psi\rangle \Longleftrightarrow \text{AdS} + \delta M$$

s.t.

$$\delta S_B = c \cdot \delta \,\text{Area}(\tilde{B})$$

any ball

associated extremal surface

Describe $AdS_{d+2} + \delta M$ by

$$ds^2 = \frac{\ell^2}{z^2}\left(dz^2 + dx_\mu dx^\mu + z^{d+1} H_{\mu\nu}(z,x)\, dx^\mu dx^\nu\right)$$

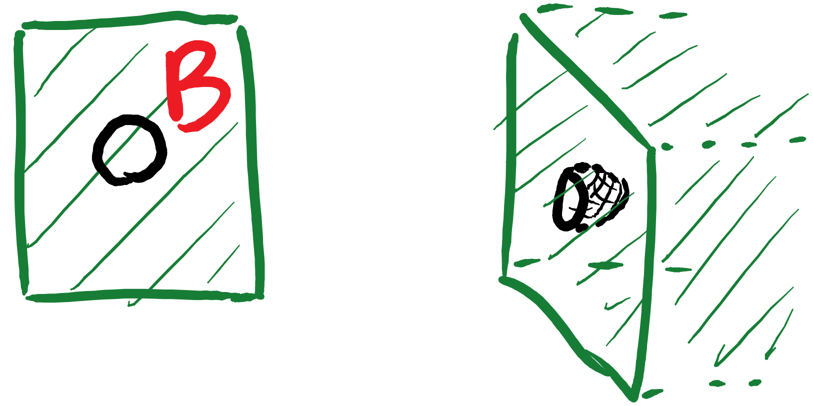
metric perturbation

What does

$$\delta S_B = 2\pi \int \frac{R^2 - r^2}{2R} \delta \langle T_{00} \rangle \equiv \delta E_B$$

tell us about $H_{\mu\nu}$?

Small B:



$$\lim_{R\to 0} \delta E_{B(R)} = \lim_{R\to 0} \delta S_{B(R)}$$

$$= \lim_{R\to 0} c \cdot \delta(\mathrm{Area}(\tilde{B}(R)))$$

$$\Rightarrow \langle T_{\mu\nu}(x)\rangle \propto H_{\mu\nu}(x, z\to 0)$$

Get holographic stress tensor

General B:

$$\delta S = 2\pi \int_B \frac{R^2 - r^2}{2R} \langle T_{00} \rangle$$

$$\Downarrow$$

$$\delta \text{Area} = 2\pi \int_B \frac{R^2 - r^2}{2R} H_{ii}(z=0)$$

$$\Downarrow$$

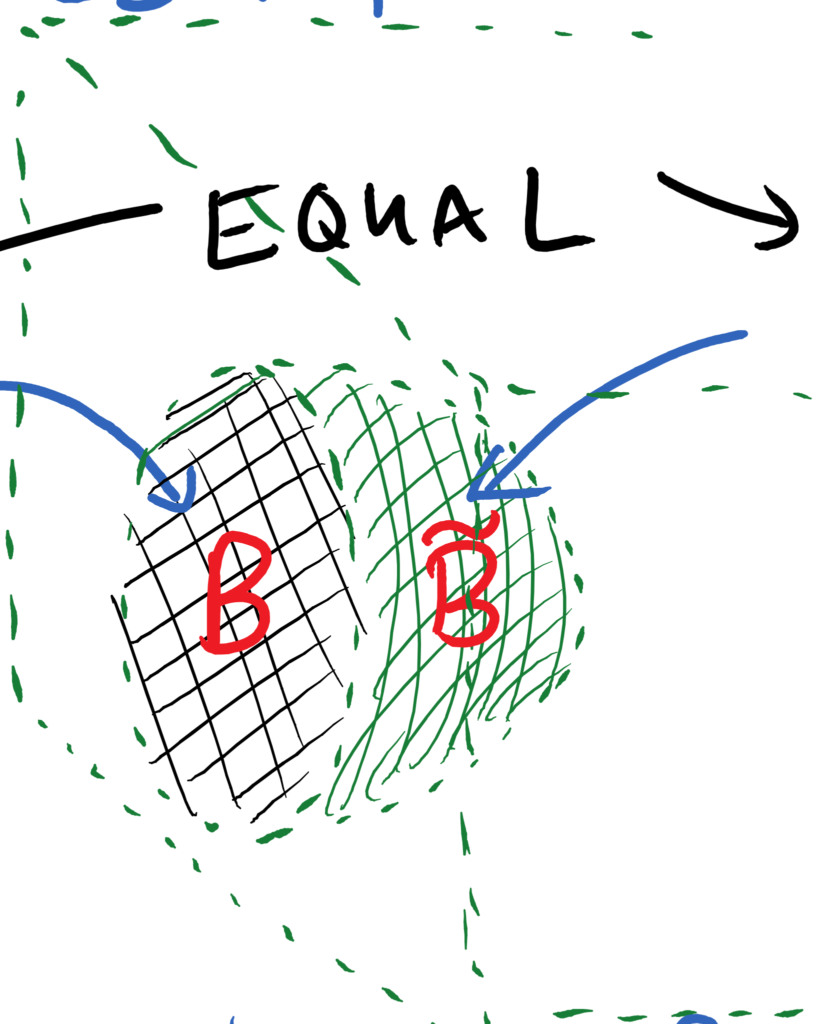
$$\int_{\tilde{B}} \left(R^2 H_{ii} - x^i x^j H_{ij} \right) = \int_B \pi \left(\frac{R^2 - r^2}{R} \right) H_{ii}$$

In pictures: for each B:

$\int f_1(H)$ on this surface

EQUAL

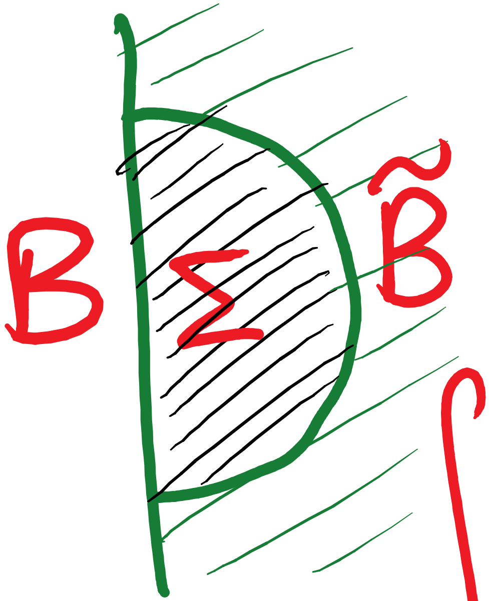
$\int f_2(H)$ on this surface



One constraint for each spacetime point

Can we turn these nonlocal constraints into a local constraint?

Some math:



$\exists$ d-form $\chi_B(H)$ s.t.

$$\int_B \chi = \delta E_B \qquad \int_{\tilde{B}} \chi = \delta S_B$$

$$d\chi = f(x) \cdot \delta E_{tt} \cdot \mathrm{vol}_\Sigma$$

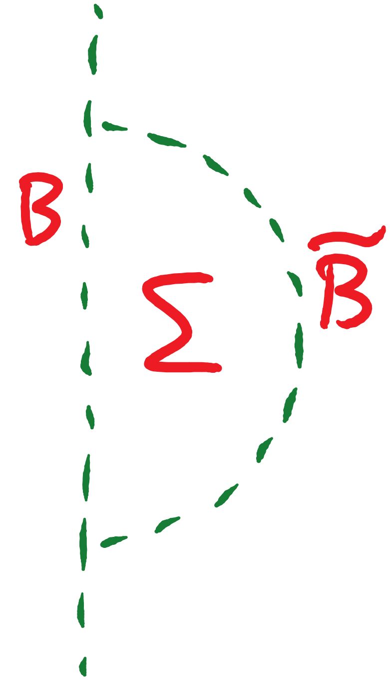
time-time-component of Einstein's Equations

$$\delta S_B^{grav} = \delta E_B^{grav}$$

$$\Leftrightarrow \int_B \chi = \int_{\tilde{B}} \chi$$

$$\Leftrightarrow \int_{\partial\Sigma} \chi = 0$$

$$\Leftrightarrow \int_\Sigma d\chi = 0$$

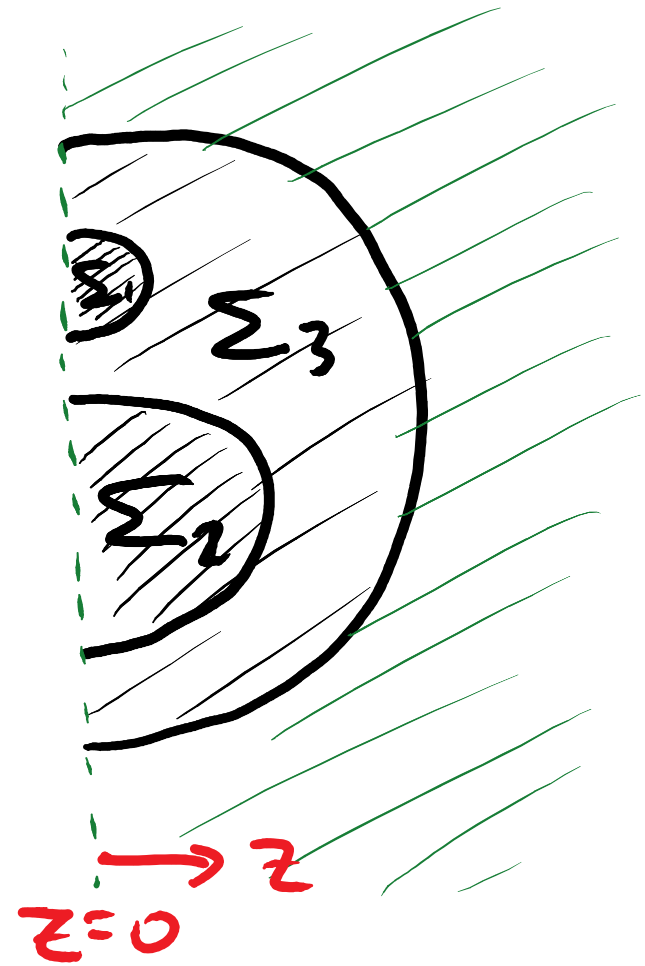


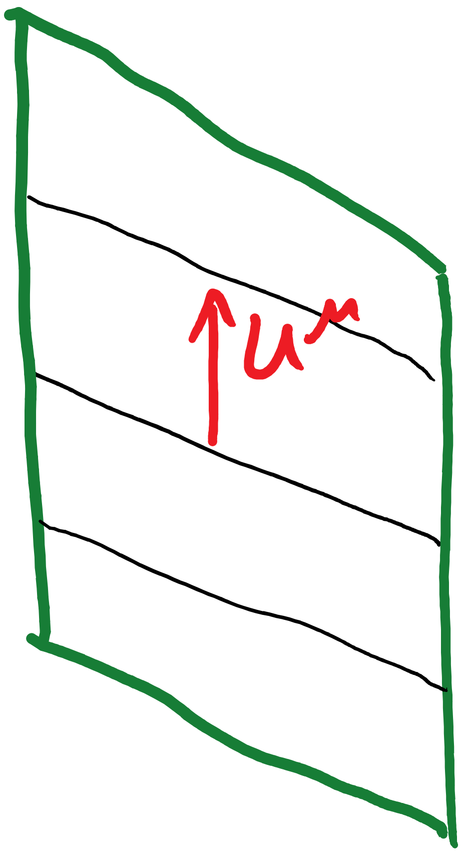
$$\int_\Sigma \big(f(x)\cdot \delta E_{tt}\big)\cdot \mathrm{vol}_\Sigma = 0 \quad \text{for all } \Sigma$$

$$\Downarrow$$

$$\delta E_{tt} = 0$$

everywhere





So far: used

$\delta S_B = \delta E_B$

in frame associated with $u^\mu = (1,0,0,0)$

to show $u^\mu u^\nu \, \delta E_{\mu\nu} = 0$

Same argument $\Rightarrow$ true for all u^μ

$$\delta E_{\mu\nu} = 0$$

Remaing components:

$$\langle T_\mu{}^\mu \rangle = 0\,, \quad \langle \partial_\mu T^{\mu\nu} \rangle = 0$$

$$\Downarrow$$

$$H_\mu{}^\mu(z=0) = 0\,, \quad \partial_\mu H^{\mu\nu}(z=0) = 0$$

$$\Downarrow$$

$$\delta E_{zz}(z=0) = 0\,, \quad \delta E_{z\mu}(z=0) = 0$$

$$\Downarrow \text{ using } \delta E_{\mu\nu} = 0$$

$$\delta E_{zz} = \delta E_{z\mu} \text{ everywhere}$$

Summary so far:

$$\delta S_B^{CFT} = \delta E_B^{CFT}$$ (true for any CFT)

$\Big\Downarrow$ Assume δS_B matches $\delta(\text{Area})$ for some geometry

$$\delta S_B^{grav} = \delta E_B^{grav} \iff \delta E_{ab} = 0$$

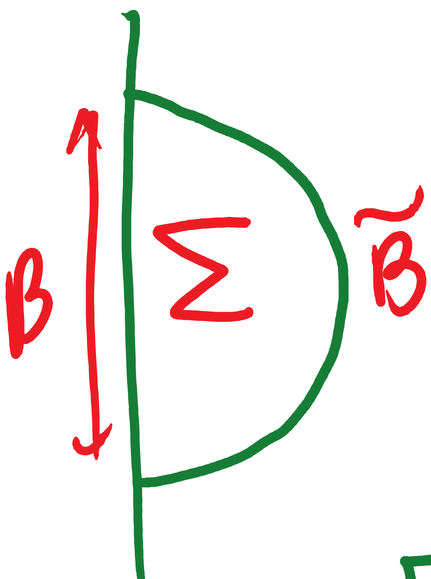
nonlocal constraint on geometry

local Einstein Eqns

So far: leading order in large N

$\frac{1}{N}$ correction:

$$\delta S^{CFT}_{B} = \frac{\delta \text{Area}}{4G_N} + \delta S^{bulk}_{\Sigma}$$

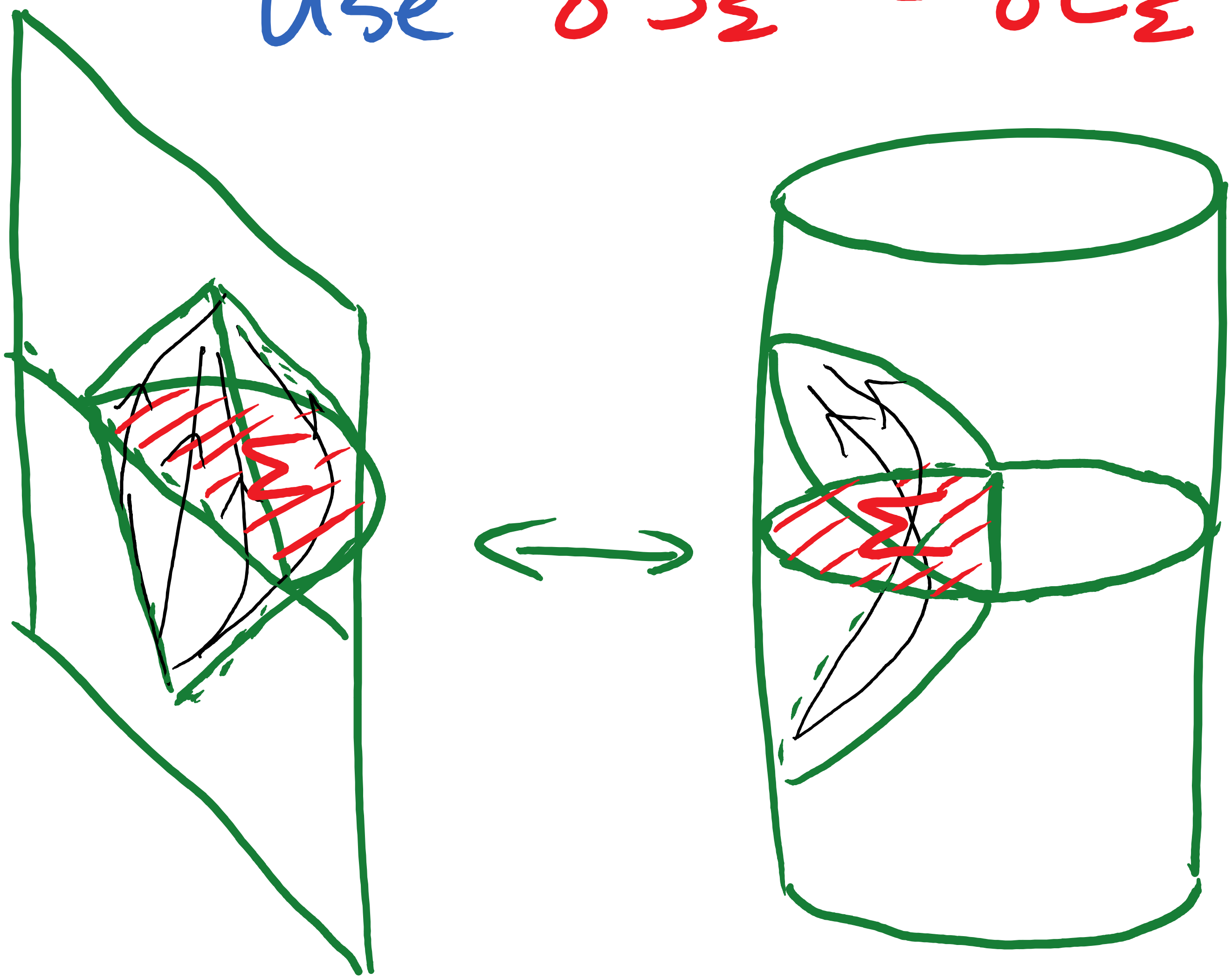


entanglement entropy of bulk quantum fields across $\tilde{B}$

Faulkner, Lewkowycz, Maldacena

Magic: Σ defines a Rindler wedge of AdS $\therefore$ know ρ_Σ

- Use $\delta S_\Sigma^{bulk} = \delta E_\Sigma^{bulk}$ to write S_Σ^{bulk} in terms of $\langle T_{\mu\nu}^{bulk} \rangle$



Final result with $\frac{1}{N}$ correction:

$$\delta E_{ab} = G_N \, \delta \langle T_{ab}^{\text{bulk}} \rangle$$

☆ all bulk fields contribute since they all contribute to entanglement ☆

- No other local operator $S_{\mu\nu}$ with $\delta\langle S_{\mu\nu}\rangle = \delta\langle T_{\mu\nu}\rangle$ for all perturbations to vacuum

- $\therefore T^{bulk}_{\mu\nu}$ is the source for the gravitational equations assuming locality

linearized E.E. + $T_{\mu\nu}$ source $\Rightarrow$ nonlinear?