Linking Dark Matter to Dark Energy in a String Theory Scenario

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This talk is based on work in progress with **Sam Wong**

黃崇清

which is based on earlier work with **Yoske Sumitomo**:

arXiv:1204.5177,
arXiv:1209.5086,
arXiv:1211.6858 and
arXiv:1305.0753 (also with Sam Wong)

Puzzle

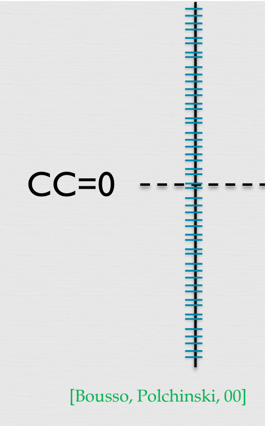
- Taking the dark energy to be the vacuum energy density, or the cosmological constant,

$$\Lambda \sim +10^{-122} M_P^4$$

where $G_N = M_P^2$.

- This vanishingly small Λ value poses a puzzle in physics.
- Since Λ is calculable in string theory, string theory is the place to search for an explanation/resolution to this puzzle.

Bousso and Polchinski observed that fluxes in string theory are quantized. E.g., J types of quantized 4-form fluxes $F^{i}_{\mu\nu\rho\sigma}$ contribute to the Λ.



$$\lambda = \lambda_{\mathrm{bare}} + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2 \ .$$

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Our Proposal : Introduce interactions. Then there is a **statistical preference** for a very small Λ due to the string dynamics.
That is, most classically stable vacua have very small Λ.
We have illustrated this point with some examples in Type IIB string theory.

Very Light Scalar Fields

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Hu, Barkana, Gruzinov (2000), Harko et.al. (2011-2015), Schive, Chiueh, Broadhurst (2014-2016), Marsh et al. (2015), Fan (2016), Calabrese, Spergel (2016), ...

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• We find that such light bosons can appear (order of magnitude) naturally in scenarios where an observed Λ is statistically preferred.

- Consider a string model with a set of moduli $\{u_i\}$ and 2-form fields C_2 and B_2. The 3-form fluxes $F_3 = dC_2$ and $H_3 = dB_2$ wrap cycles in a Calabi-Yau like manifold. The quantized fluxes lead to a set of discrete values labelled as $\{n_j\}$, yielding $V(B_2, C_2, u_i) \to V(n_j, u_i)$,
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  where each n_j can take a discretum of values.
- Solve $V(n_j, u_i)$ for all the meta-stable vacua. For every meta-stable vacuum with a given set $\{n_j\}$, each u_i is determined in terms of $\{n_j\}$: $u_{i,min}(n_j)$.
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So $\Lambda(n_j) = V_{min}(n_j, u_{i,min})$.
- Treat each $\{n_j\}$ as a flux parameter with some (typically smooth or uniform) probability distribution $P_j(n_j)$. Find the probability distribution $P(\Lambda)$ for $\Lambda(n_j)$ as we sweep through allowed $\{n_j\}$.

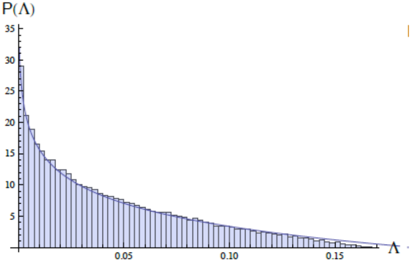
Toy Model : $V(x_i, c, \phi) = x_1\phi - x_2\phi^2/2 + c\phi^3/3!$

(No constant or a disconnected sector)

For fixed x_1, x_2 and c, we find the minimum at ϕ_{min},

$$\Lambda(x_1, x_2, c) = V_{min}(x_1, x_2, c, \phi_{min}) \simeq \frac{3x_1x_2}{2c}(1 - 3x_2^2/8x_1c)$$

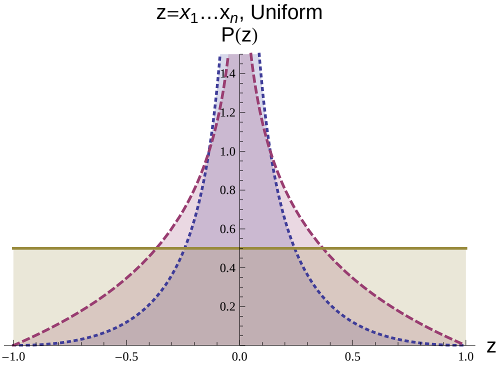
Now, we sweep through discrete values for x_1, x_2 and c. Given probability distributions $P_1(x_1)$, $P_2(x_2)$ and $P_c(c)$, we obtain $P(\Lambda)$.



Probability distribution of $z = x_1 x_2$ and $z = x_1 x_2 x_3$

In the above example, $P(\Lambda) \sim P(z)$, where $z = x_1 x_2$.

Let us see how sensitive is $P(z)$ on $P_i(x_i)$?



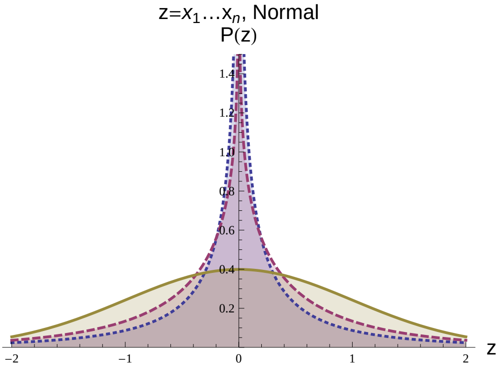
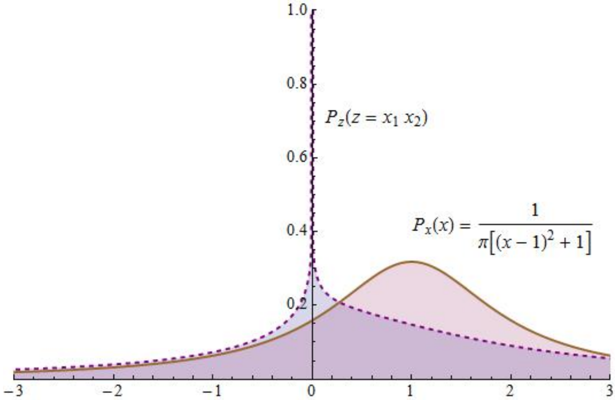
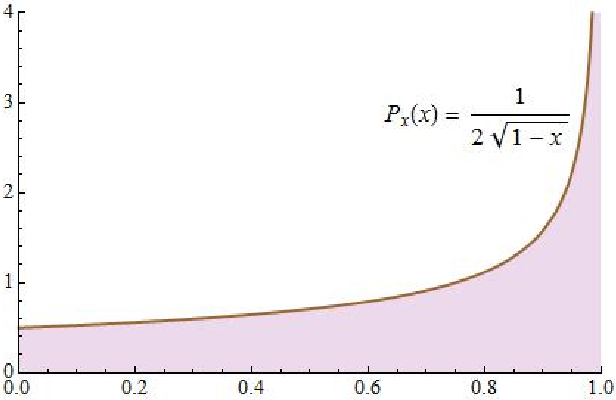
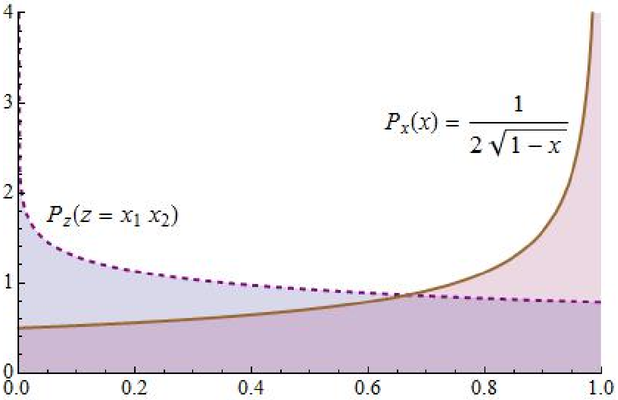


Figure: The product distribution $P(z)$ is for $z = x_1$ (solid brown curve for normal distribution), $z = x_1 x_2$ (red dashed curve), and $z = x_1 x_2 x_3$ (blue dotted curve), respectively. In general, the curves are given by the Meijer-G function.


1.0
0.8
0.6
0.4
0.2
$P_z(z = x_1 x_2)$
$P_x(x) = \frac{1}{\pi[(x-1)^2+1]}$
−3
−2
−1
0
1
2
3


$P_x(x) = \frac{1}{2\sqrt{1-x}}$
0
1
2
3
4
0.0
0.2
0.4
0.6
0.8
1.0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Suppose x_1 and x_2 are related in
$V(x_i, c, \phi) = x_1\phi - x_2\phi^2/2 + c\phi^3/3!$.

What happens if $x_1 = x_2$?

Suppose x_1 and x_2 are related in
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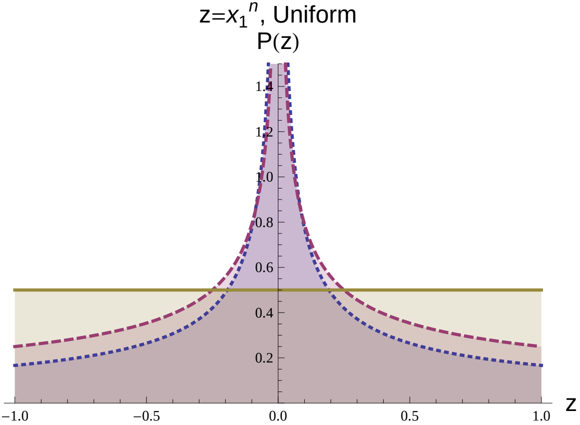
What happens if $x_1 = x_2$?

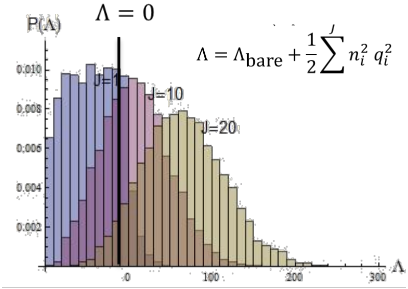


$P(z) \sim z^{-1+1/n}$

• The peaking behavior of $P(\Lambda)$ is not sensitive to the input probability distributions of the discrete spectra of fluxes.

• Constraint on flux values for a minimum: $x_2^2 > 2x_1 c$

• No peaking behavior at $\Lambda = 0$ for $P(\Lambda)$ if Λ is a sum of independent terms. Example : Bousso-Polchinski case :



Type IIB String Theory ($M_P = 1$)

Consider the superpotential W_0 (Gukov-Vafa-Witten)

$$W_0(U_i, S) = \frac{1}{2\pi} \sum_{cycles} \int G_3 \wedge \Omega = (F_3 - iSH_3) \cdot \Pi(U_i)$$

$$= (f_{3j} - iSh_{3j}) \mathcal{F}_j(U_i)$$

$$\simeq c_1 + \sum_j b_j U_j - S(c_2 + \sum_j d_j U_j)$$

where f_{3j} and h_{3j} take discrete flux values.

E.g., Only linear terms in U_j in orientifolded toroidal orbifolds
(Font, ..., Lust, Reffert, Schulgin, Stieberger, ...).

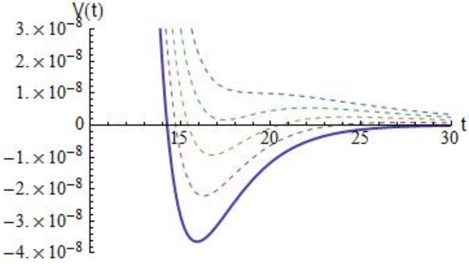
MODEL : An orientifolded Calabi-Yau 3-fold in Type IIB :

$$\begin{aligned} V =& e^K \left(K^{J\bar{I}} D_J W D_{\bar{I}} \bar{W} - 3|W|^2 \right), \\ K =& -2\ln(\mathcal{V} + \xi/2) - \ln(S + \bar{S}) - \sum_j \ln(U_j + \bar{U}_j) \\ \mathcal{V} =& Vol/\alpha'^3 = \gamma(T + \bar{T})^{3/2}, \\ W =& W_0(U_j, S) + Ae^{-aT}, \\ W_0(U_j, S) =& c_1 + \sum_j b_j U_j - S(c_2 + \sum_j d_j U_j) \end{aligned}$$

where ξ is the α' correction (Becker, Becker, Haack and Louis: Pedro, Rummel and Westphal) that can provide the Kähler uplift to de Sitter solutions (Rummel and Westphal, deAlwis and Givens.)

$$\mathcal{V} \gg \xi = -\frac{\zeta(3)}{4\sqrt{2}(2\pi)^3} \chi(M) \left(S + \bar{S}\right)^{3/2} > 0$$

- Choose any A so vacuum solution exits via the GKP-KKLT mechanism. [1]
  For fixed values of c_i, b_i, d_i and A, we solve for $\Lambda(c_i, b_i, d_i, A)$.



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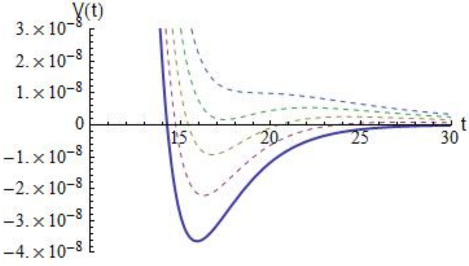

- All flux parameters b_i, c_i, d_i (and A) are treated as real random variables with some smooth probability distributions $P(c_i), P(b_j), P(d_j) \to P(\Lambda)$.

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From $P(\Lambda)$, we obtain the median $\Lambda = \Lambda_{50\%}$ and the average $< \Lambda >$, as a function of the number $h^{2,1}$ of complex structure moduli, for $\Lambda \geq 0$ and $h^{2,1} > 5$,

$$\Lambda_{50\%} \simeq 10^{-h^{2,1}-2} M_P^4$$
$$< \Lambda > \simeq 10^{-0.03h^{2,1}-6} M_P^4$$
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- We see that the average $<\Lambda>$ does not drop much, since a few relatively large Λ dominate its value.
- Here we have also given $\Lambda_{10\%}$, that is 10% of the vacua have a Λ smaller than it.
- For $h^{2,1} \sim \mathcal{O}(100)$, $\Lambda \sim 10^{-122} M_P^4$ is well within range.
- Similar (but somewhat different) pileup of vacua around $\Lambda \sim 0$ happens for $\Lambda < 0$.

Typical Manifolds Studied

$$\chi(M) = 2(h^{1,1} - h^{2,1})$$

Manifold	$h^{1,1}$	$h^{2,1}$	χ
$\mathcal{P}^4_{[1,1,1,6,9]}$	2	272	-540
$\mathcal{F}_{11}$	3	111	-216
$\mathcal{F}_{18}$	5	89	-168
$\mathcal{CP}^4_{[1,1,1,1,1]}$	1	$\mathcal{O}(100)$	$\mathcal{O}(-200)$

A manifold has $h^{1,1}$ number of Kähler moduli and $h^{2,1}$ number of complex structure moduli.

Bosonic mass matrix

No-scale structure : $i, j = 0, 1, 2, \cdots, h^{2,1} = n$,

$$V = e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} \right)$$

For $\Lambda \geq 0$, the $(h^{2,1} + 1) \times (h^{2,1} + 1)$ mass matrix $M^2 = e^K \omega_0^2 H$,

$$H_{ss} = 1 + \sum_{k=1}^{n} p_k^2,$$
$$H_{si} = p - 3p_i,$$
$$H_{ij} = 4\delta_{ij} + (n-4) + p_i p_j,$$
$$p = \sum_{k=1}^{n} p_k, \quad p_i = 2(b_i + sd_i)u_i/\omega_0,$$

One finds that bosons come in the following types :

• $(h^{2,1}-2)$ number of moduli have degenerate mass equal to the gravitino mass : $m^2 \sim \Lambda$ or $m^2 \sim 10^{-120} M_P^2$

• $(h^{2,1}-2)$ number of the corresponding axions are massless. They can get masses from non-perturbative terms.

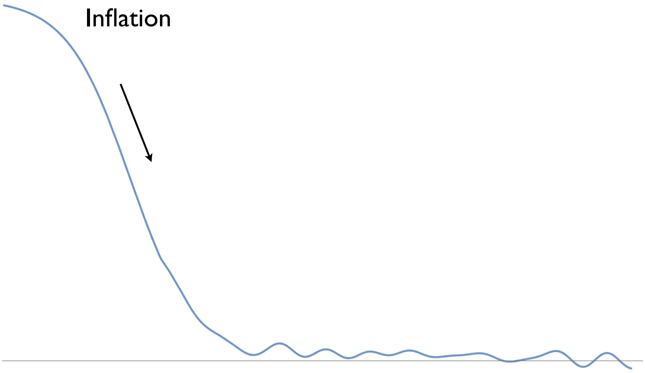
• One modulus has mass $m^2 \sim (h^{2,1}-2)^2\Lambda$

• 2 bosons have $m^2 \sim 10^{(0.12\pm 0.05)h^{2,1}+(3\pm 2)}\Lambda$.

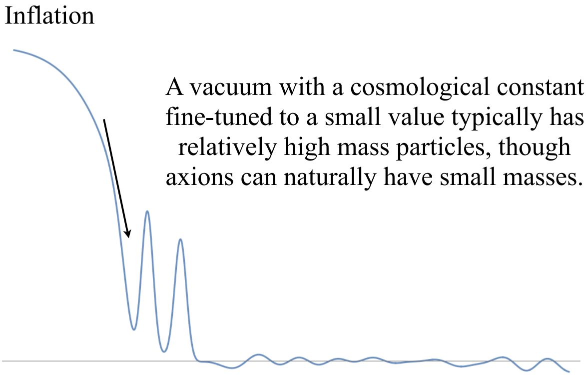
These heavy ones have mass scale in the right range when $h^{2,1} \sim \mathcal{O}(100)$, so $\Lambda \sim 10^{-122} M_P^2$ and $m^2 \sim 10^{22}\Lambda$.

If we turn on $\alpha_{ij} U_i U_j$ in W_0, many of the complex structure moduli masses get raised to values comparable to the heavy ones, suitable as dark matter candidates.

"Cartoon Picture"



"Cartoon Picture 2"

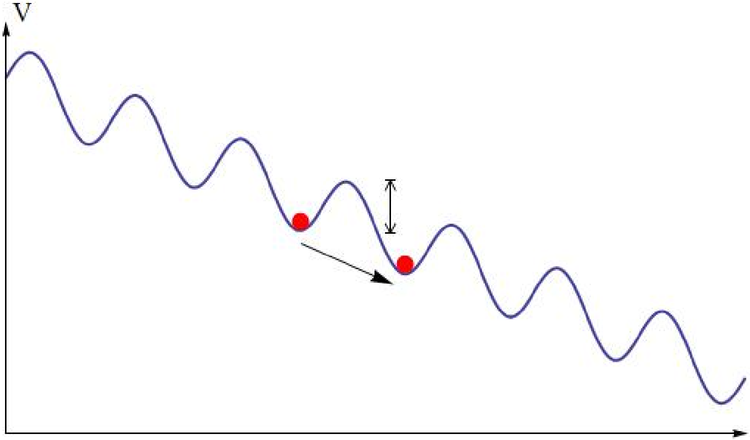


Summary and Remarks

- In this approach, most vacua accumulate around $\Lambda = 0$. Hardly any meta-stable vacua exist at higher vacuum energies.
- Some barrier heights are very low, comparable to the Λ value; still tunneling is very much suppressed.
- Requiring Λ to the observed value without fine-tuning will generically lead to very light bosons with very small self-couplings, where $m \sim 10^{-22}$ eV is within reach.
- The particular model we study here is too simple, since it seems hard to accommodate the "very heavy" Higgs Boson ?

Thank you !

If a flux direction has mass scale of order of Higgs mass ?



$$S = \int dx^4 \sqrt{-g}[\Lambda + \frac{M_P^2}{16\pi} R - \frac{m_H^2}{2} \Phi^2 + ...]$$