Studies on Chern-Simons matter theories

Tomohisa Takimi (HRI)

cf) Jain-Minwalla-Sharma-T.T-Wadia-Yokoyama [JHEP09 (2013) 009]
T.T [JHEP07(2013)177]

Gong Show in Strings 2014, 26th Jun. 2014

1. What we did

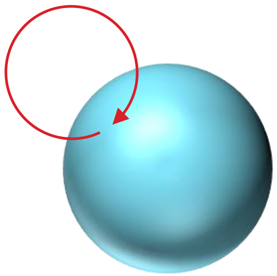
A. We investigated **phase structure** of Chern-Simons (CS) fundamental matter theories on $S^1 \times S^2$ in large N 't Hooft limit.

B. Based on A, we observed the **Bose-Fermi duality** between CS theory coupled to bosons and the one coupled to fermions

A. Investigating the phase structure

[Jain-Minwalla-Sharma-T.T-Wadia-Yokoyama 2013]

A-1. Outline of Path integration of a CS matter theory on $S^1 \times S^2$



$$Z_{\mathrm{CS}} = \int DA D\mu \; e^{i\frac{k}{4\pi}\mathrm{Tr}\int\left(AdA+\frac{2}{3}A^3\right)-S_{matter}}$$

Matter integration $D\mu$

$$Z_{\mathrm{CS}} = \int DA e^{i\frac{k}{4\pi}\mathrm{Tr}\int\left(AdA+\frac{2}{3}A^3\right)-S_{eff}}$$

Effective potential depending on gauge fields

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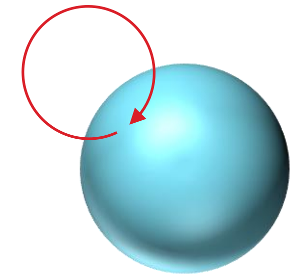

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$$Z_{\mathrm{CS}} = \int DA e^{i\frac{k}{4\pi}\mathrm{Tr}\int\boxed{\left(AdA+\frac{2}{3}A^3\right)}-\boxed{S_{eff}}}$$

$O(N^2)$ V.S $O(N^1)$ (fundamental)

In large N

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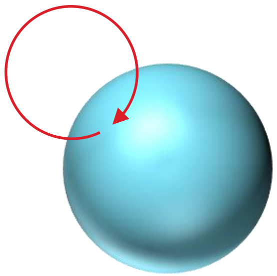

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$O(N^2)$ V.S $O(N^1)$ (fundamental)

In large N

If phase transition occur by the matter effect,
→ **transition point should be very high temperature $T^2 \sim N^1$**
so that the effective action becomes same order $O(N^2)$ as CS terms

A-2 Effective action in high termperature

In high temperature limit, the effective action will be simple one depending only on holonomy along S^1

$$S_{eff} = \int d^2x \, (T^2 v(U) +$$

We can calculate the large N free energy by the Blau-Thompson method.

Nucl.Phys. B408 (1993) 345-390.

A-3 Matrix integration form of the free energy by the Blau-Thompson method.

$$Z_{CS} = \sum_{n_m} \int \prod_m d\alpha_m \prod_{m \neq l, m, l=1}^{N} \left(2 \sin \frac{\alpha_l - \alpha_m}{2} \right)^{\frac{1}{2}\chi_{S_2}} \exp \left(i \frac{k}{2\pi} \sum_m \alpha_m n_m - S_{eff} \right)$$

Unitary matrix model in large N
governed by holonomy eigenvalue distribution

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Sum of the monopole along S^2

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Sum of the monopole

$$\sum_n e^{ik\alpha n} = \sum_{m \in \mathbb{Z}} \delta(\alpha - \frac{2\pi m}{k})$$

$$= \int \prod_{j=1}^{N} d\alpha_j \left(\prod_{m \neq l} 2 \sin \left(\frac{\alpha_m(n_m) - \alpha_l(n_l)}{2} \right) \right) e^{-N\zeta v(U)} \sum_{n \in \mathbb{Z}} \delta(k\alpha_j - 2\pi n)$$

Delta function shows up

Constraining holonomy eigenvalue α to be ***discrete***

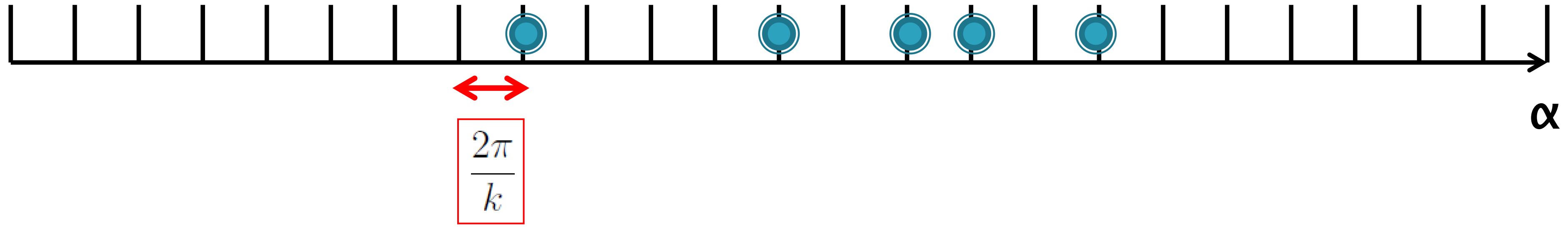
A-3.Figure for location of eigenvalue

●:Indicating the location of the eigenvalues α

By the effect of the delta functions

● **must be skewed by comb located at (2πn/k)**

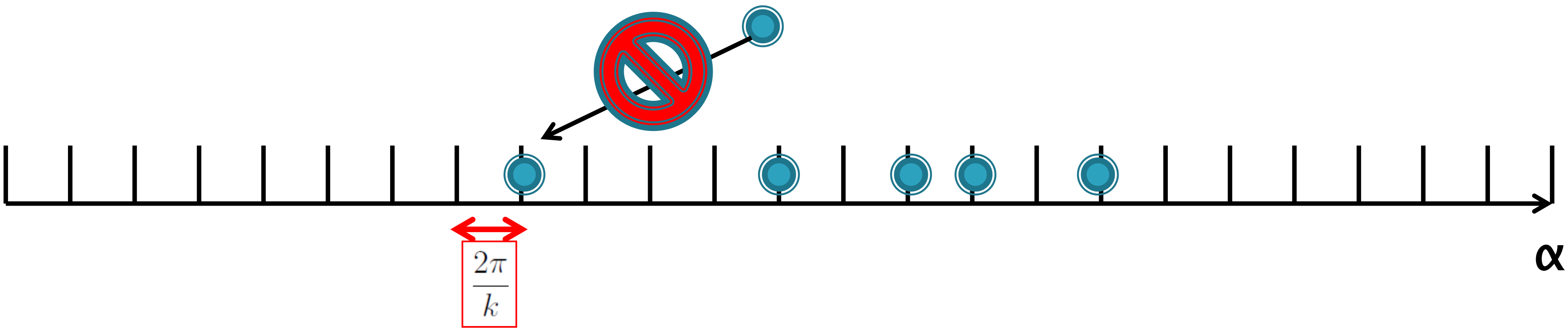
n:integer



Due to vandermond determinant causing the repulsive force between eigenvalues

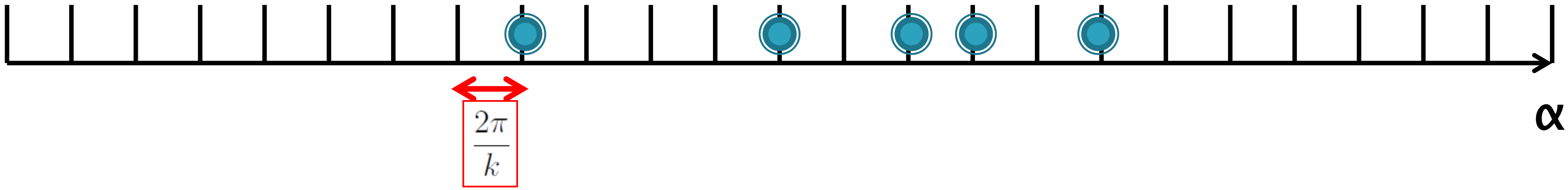
Only one eigenvalue can be skewed with one comb,

And then only one eigenvalue reside within the interval $2\pi/k$



Eigenvalue density is saturated from above

$$\rho(\alpha) \leq \frac{k}{2\pi} \times \frac{1}{N} = \frac{1}{2\pi\lambda}$$



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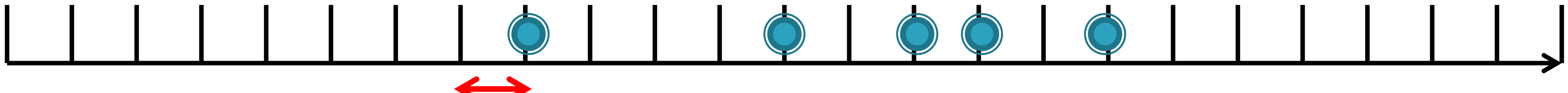
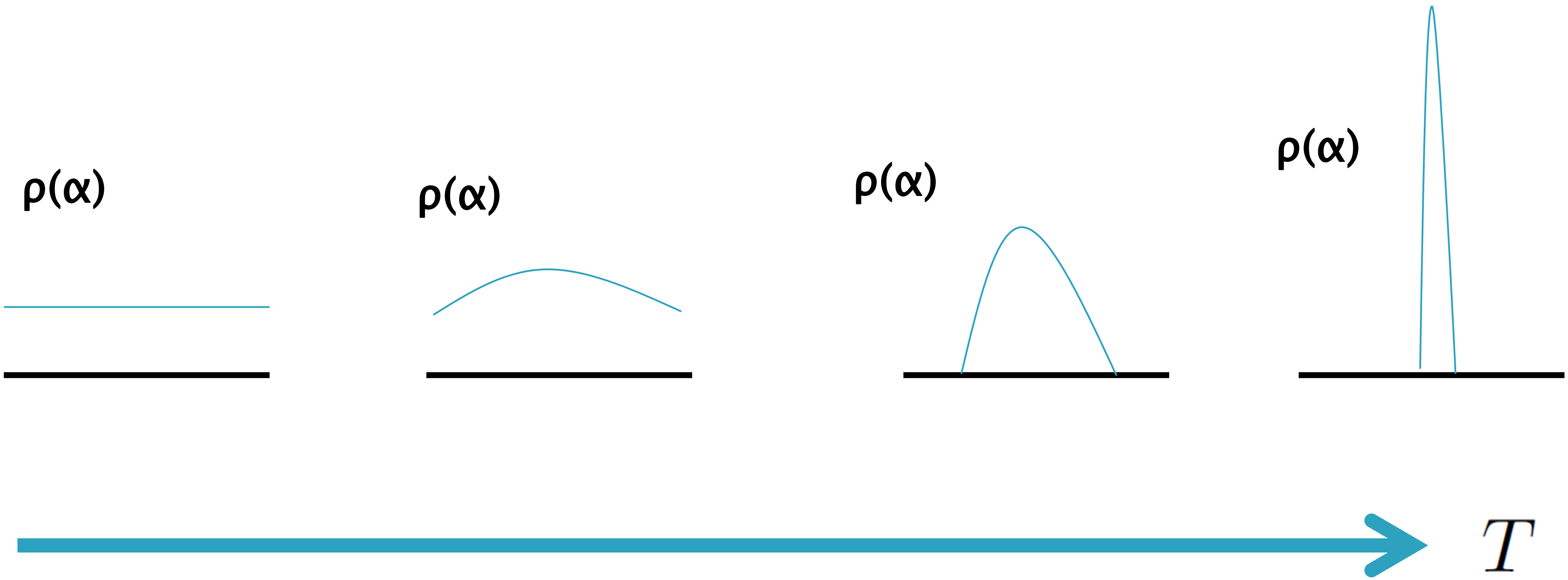
't Hooft coupling



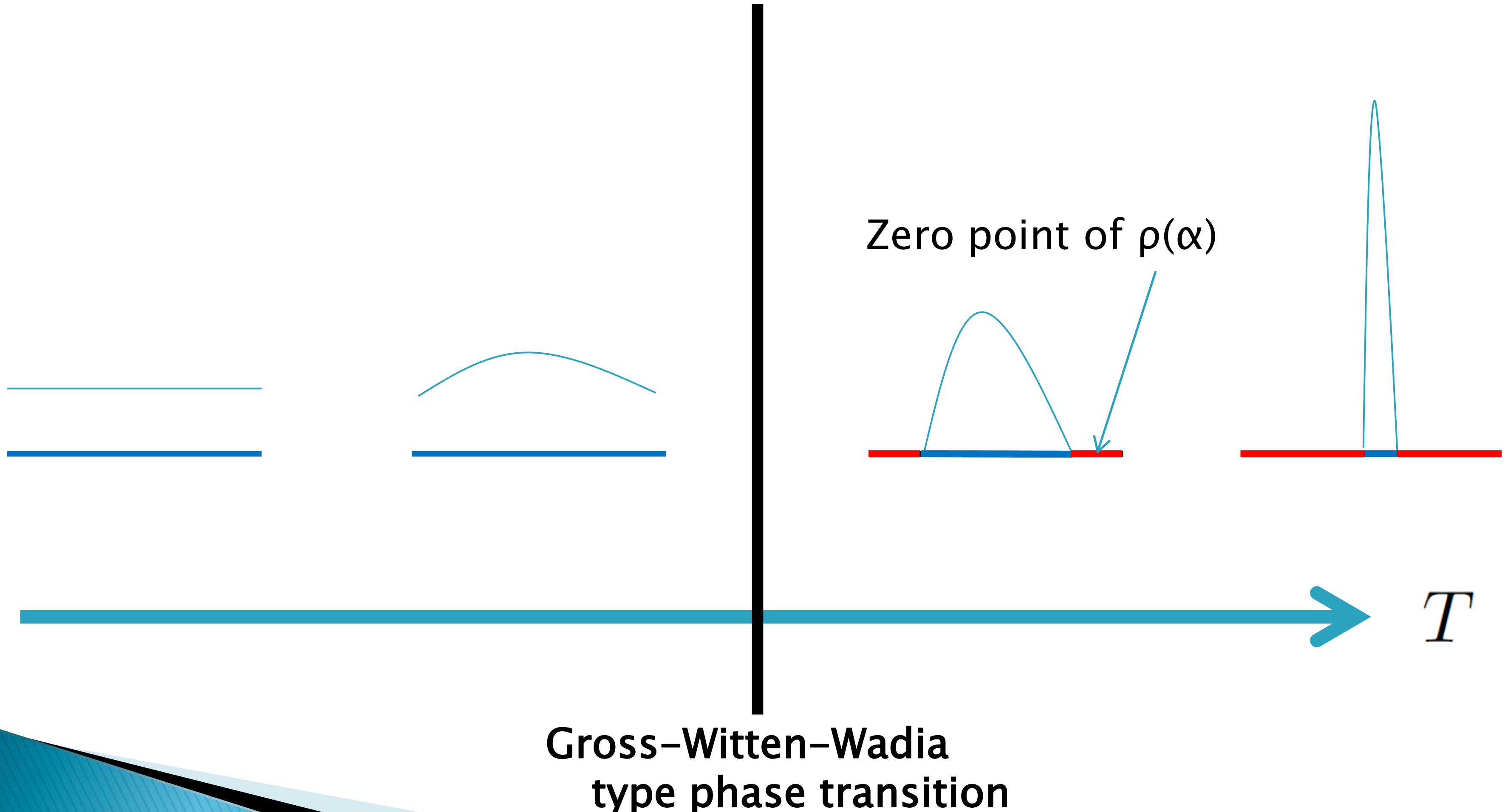
$\frac{2\pi}{k}$

Behavior of eigenvalue density ρ(α)

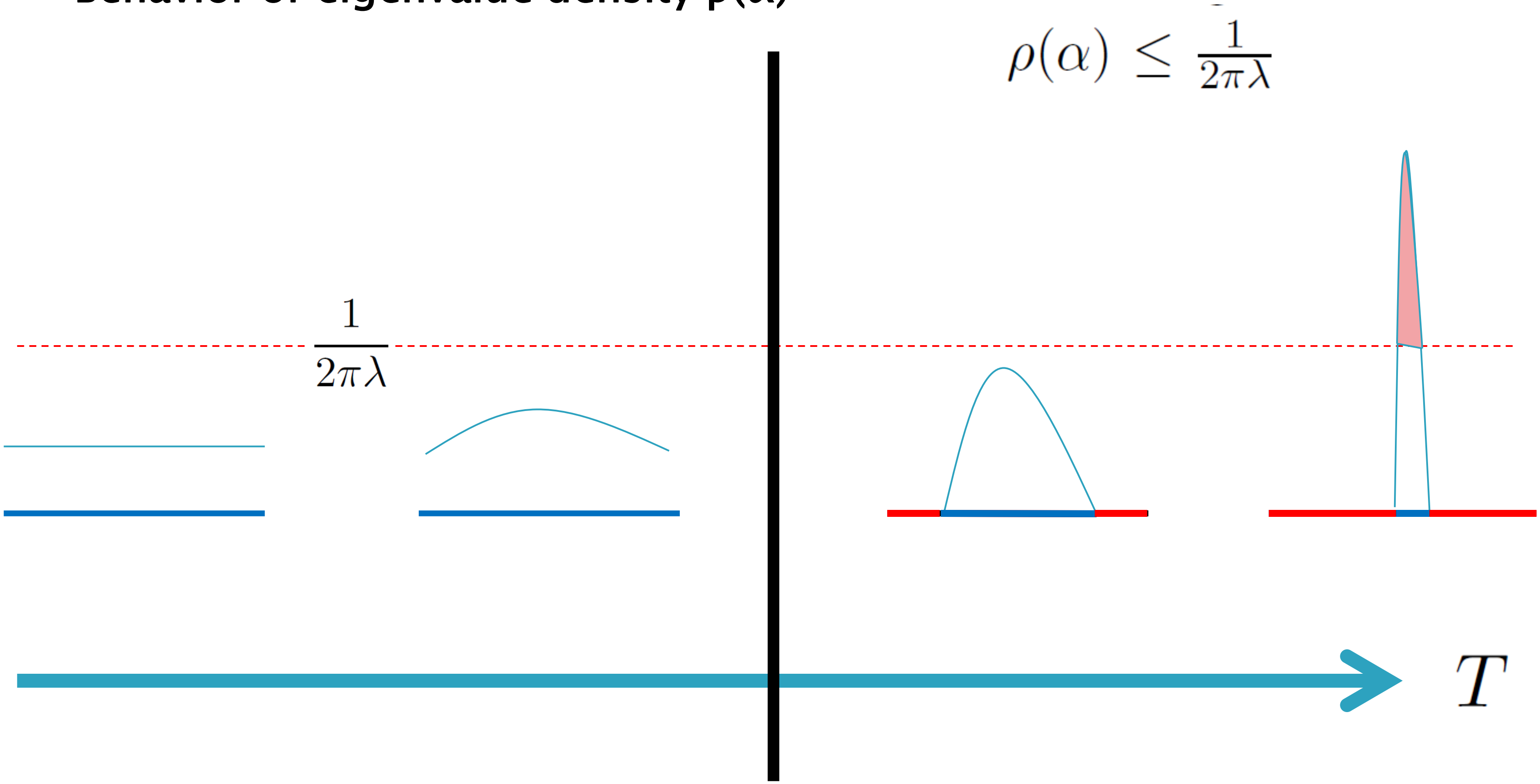


ρ(α) clump in the higher temperature by the stronger attractive force by the effective potential.

Behavior of eigenvalue density ρ(α)

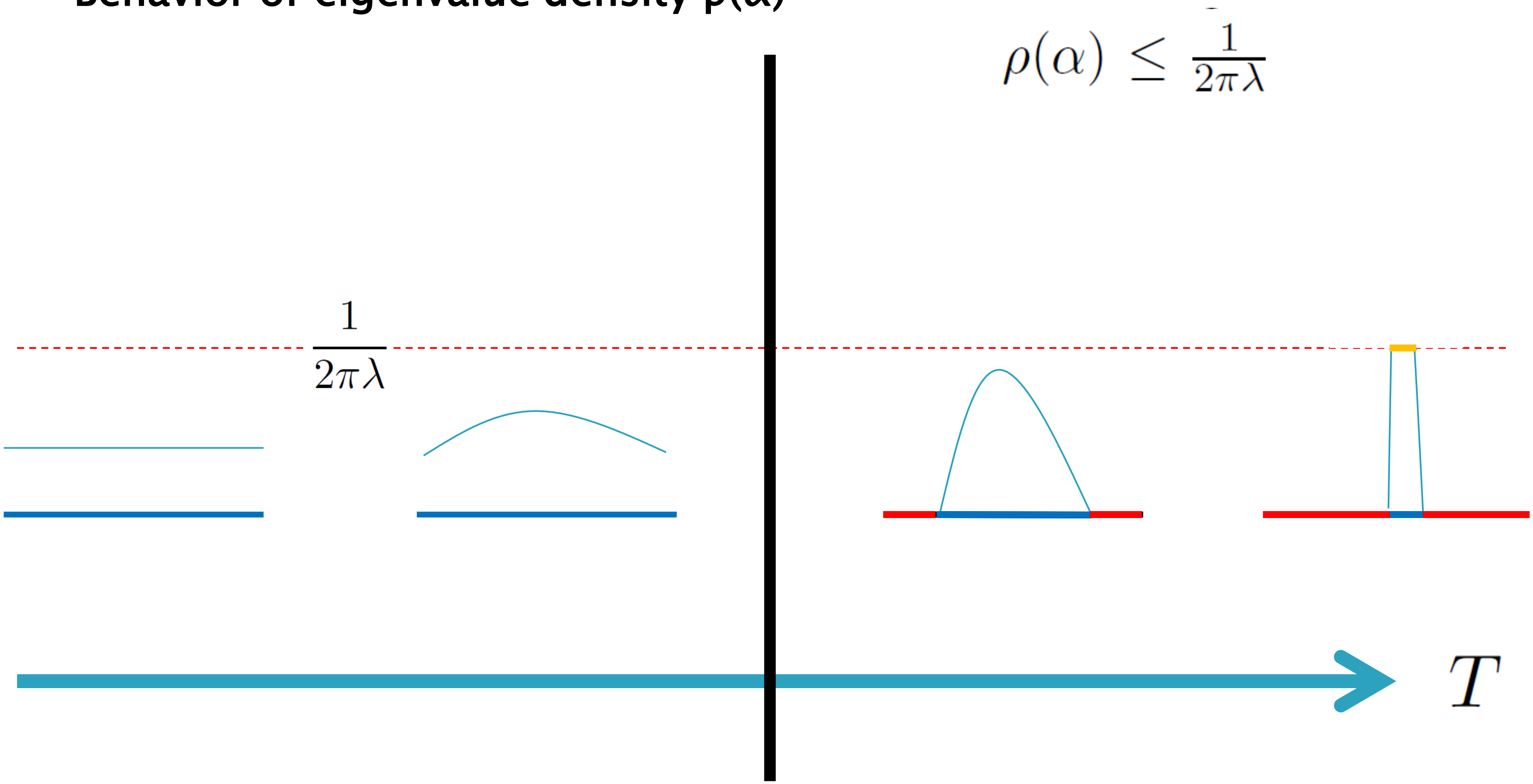


Behavior of eigenvalue density ρ(α)



Gross–Witten–Wadia
type phase transition

Behavior of eigenvalue density ρ(α)



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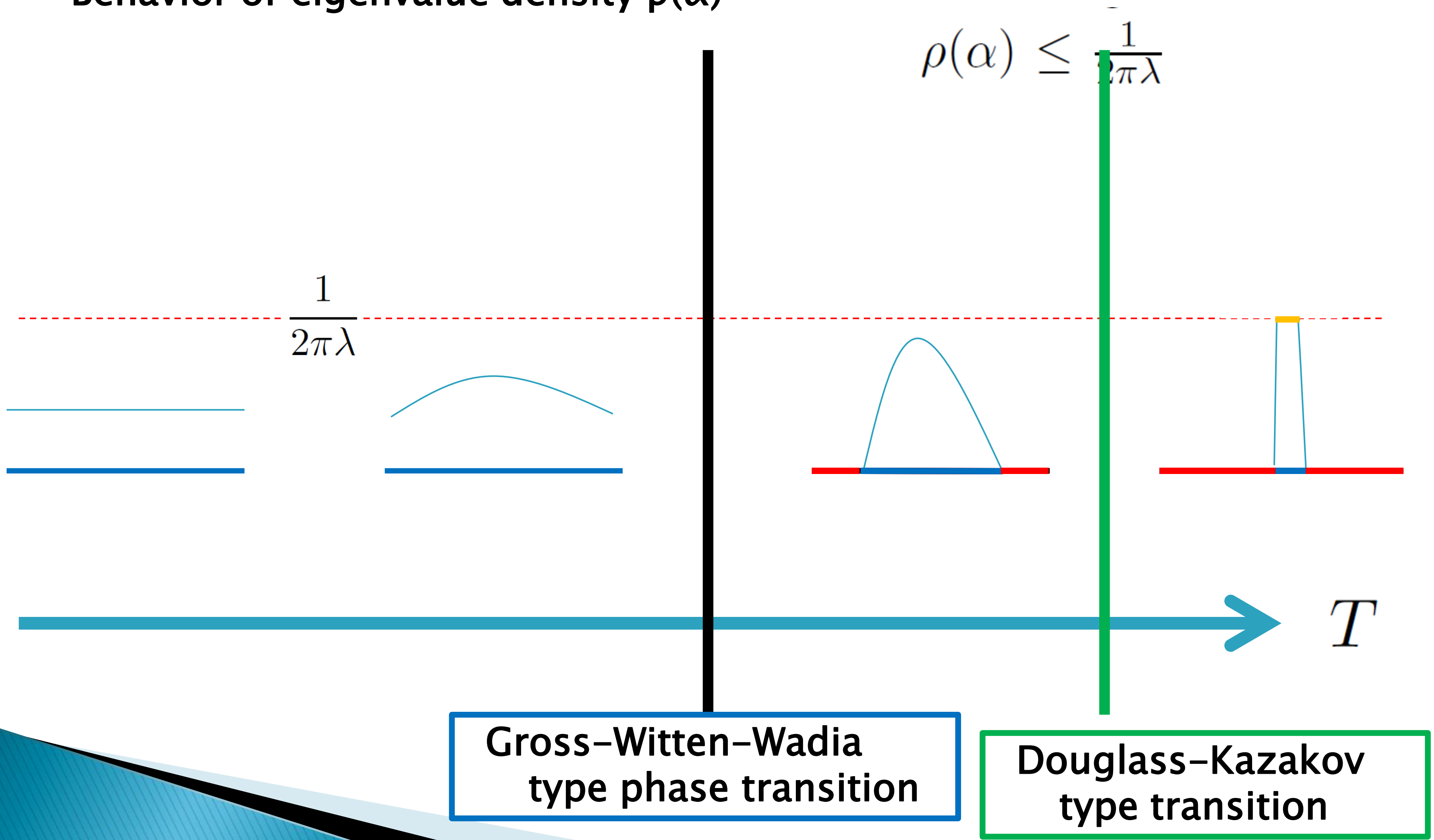

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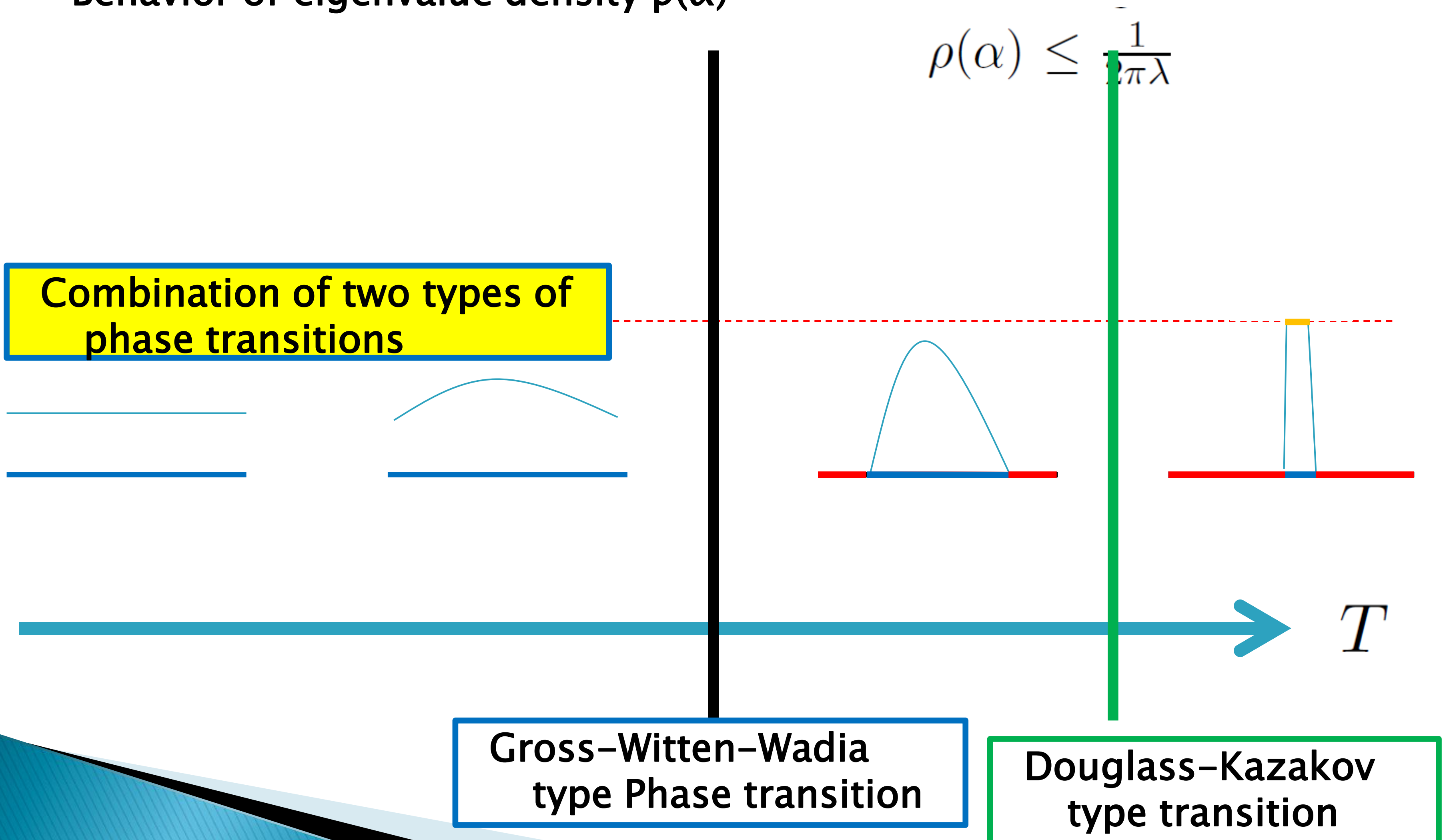
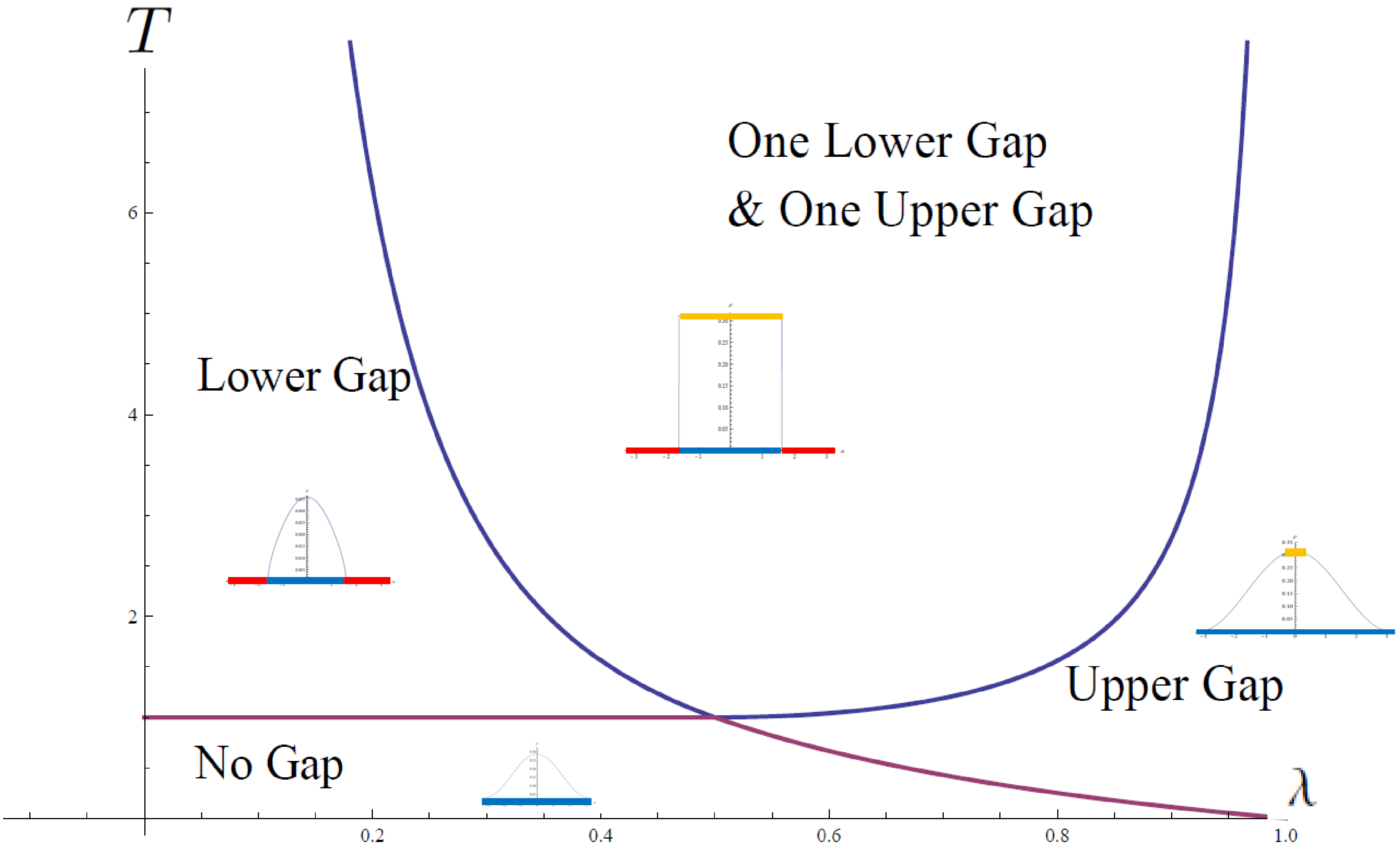


Phase structure of CS matter theories



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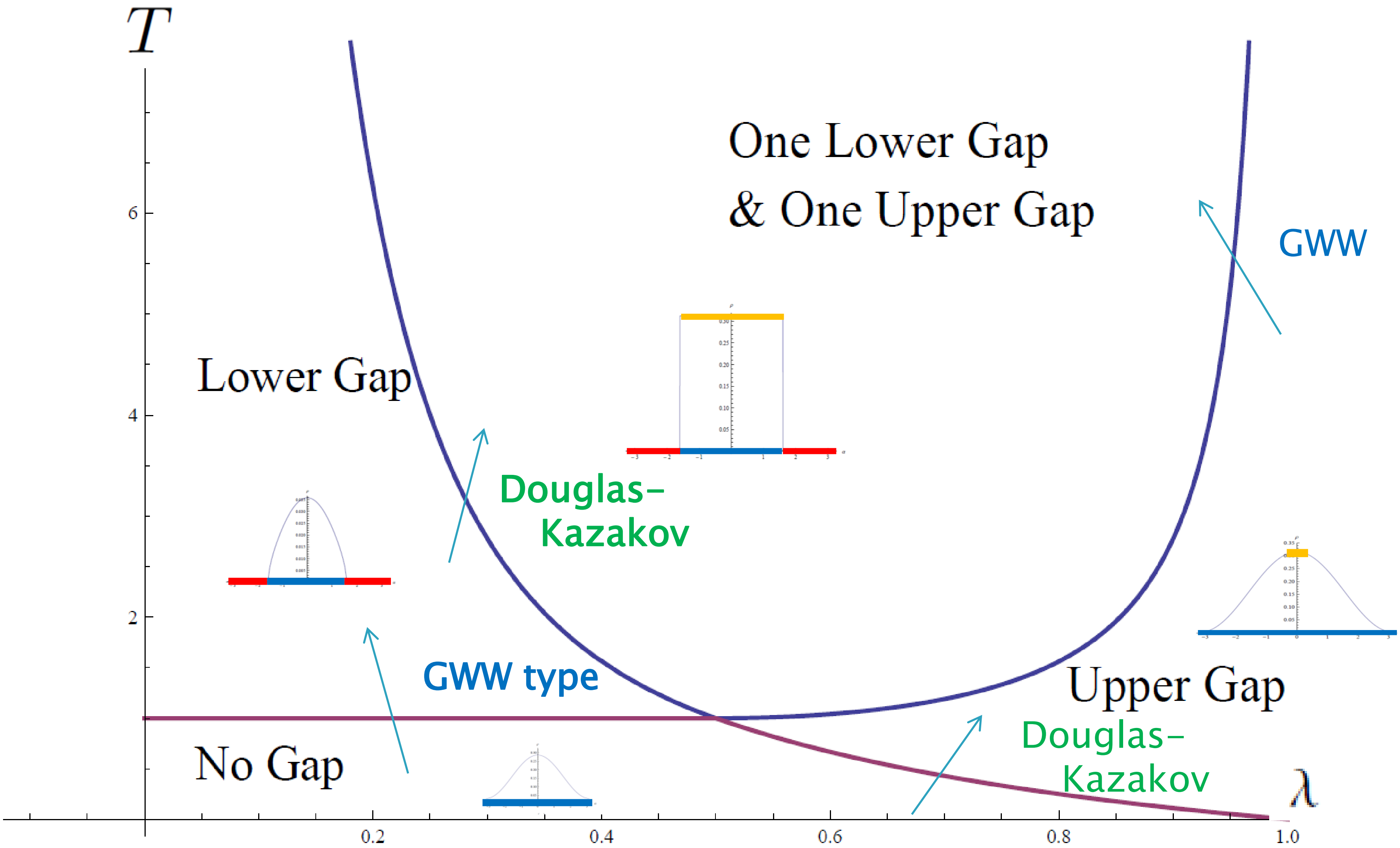

B.Bose-Fermi duality

B. Bose–Fermi duality

CS theory coupled to bosons
(critical boson)

See the duality

CS theory coupled to fermions
(regular fermion)

B-2. Free energy of both theory

- Free energy for critical boson theory

$$F^N_{c.b} = V^{c.b}[\rho, N] - N^2 \mathcal{P} \int_{-\pi}^{\pi} d\alpha \int_{-\pi}^{\pi} d\beta \; \rho(\alpha)\rho(\beta) \log \left| 2 \sin \frac{\alpha - \beta}{2} \right|$$
$$= V^{c.b}[\rho, N] + F_2[\rho, N].$$

$$V(U) = -\frac{N^2 \zeta}{6\pi} \sigma^3 + \frac{N^2 \zeta}{2\pi} \int_{\sigma}^{\infty} dy \int_{-\pi}^{\pi} d\alpha \; y \rho(\alpha) \left(\ln(1 - e^{-y + i\alpha}) + \ln(1 - e^{-y - i\alpha}) \right)$$
$$\equiv V^{c.b}[\rho, N],$$

- Free energy for regular fermion theory

$$F^N_{r.f} = V^{r.f}[\rho, N] - N^2 \mathcal{P} \int_{-\pi}^{\pi} d\alpha \int_{-\pi}^{\pi} d\beta \; \rho(\alpha)\rho(\beta) \log \left| 2 \sin \frac{\alpha - \beta}{2} \right|$$
$$= V^{r.f}[\rho, N] + F_2[\rho, N].$$

$$V(U) = -\frac{N^2 \zeta}{6\pi} \left(\frac{\tilde{c}^3}{\lambda} - \tilde{c}^3 + 3 \int_{-\pi}^{\pi} d\alpha \rho(\alpha) \int_{\tilde{c}}^{\infty} dy \; y (\ln(1 + e^{-y - i\alpha}) + \ln(1 + e^{-y + i\alpha})) \right)$$
$$\equiv V^{r.f}[\rho, N; \tilde{c}, \zeta],$$

B-2. Matching of the free energy

- We have checked that they are matched

$$F_{c.b}^{N} = F_{r.f}^{k-N}$$

under the following relationship

$$\lambda_{r.f} = 1 - \lambda_{c.b}, \qquad \zeta_{r.f} = \frac{\lambda_{c.b}}{1-\lambda_{c.b}}\zeta_{c.b},$$

$$\lambda_{r.f}\rho_{r.f}(\alpha) + \lambda_{c.b}\rho_{c.b}(\pi + \alpha) = \frac{1}{2\pi}$$

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I will explain that

the combination of the two types of phase transition is crucial to make the duality valid.

particularly the comb. is important for this relationship.

B-3 duality relationship and the phase structure

$$\lambda_{r.f}\rho_{r.f}(\alpha) + \lambda_{c.b}\rho_{c.b}(\pi + \alpha) = \frac{1}{2\pi}$$

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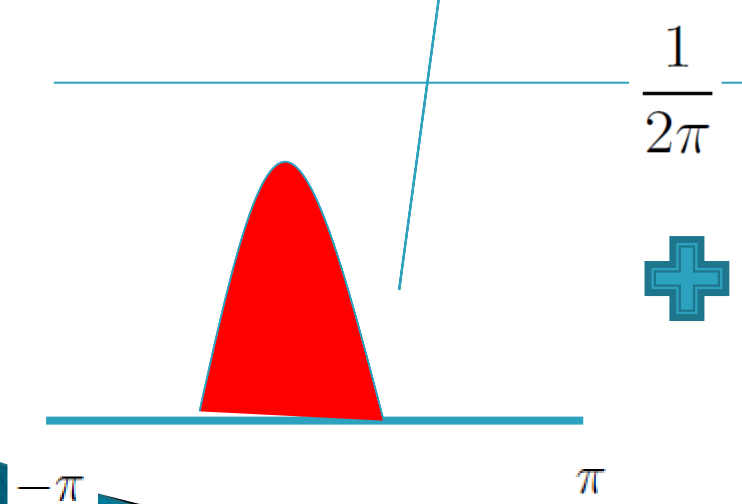

GWW phase transition in Fermion side

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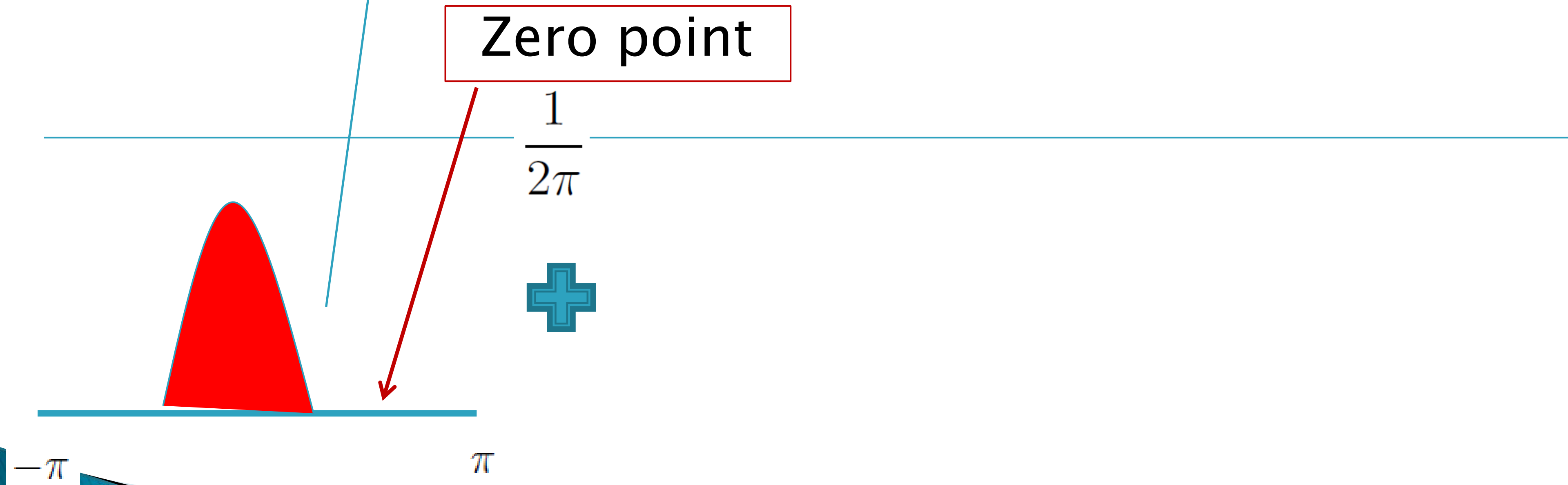

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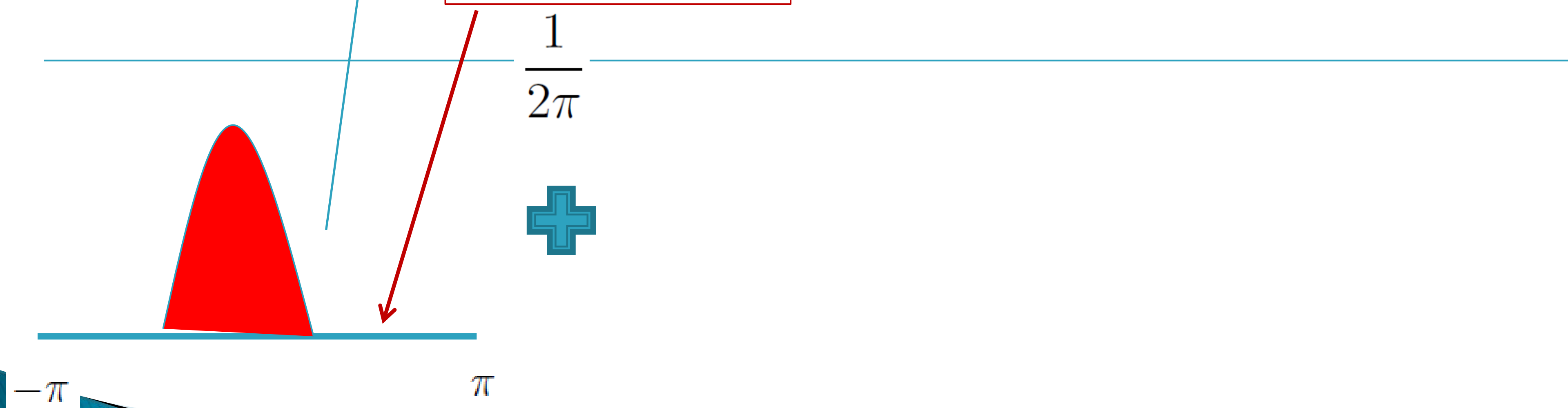
To satisfy the above equation, we need to prepare

Zero point



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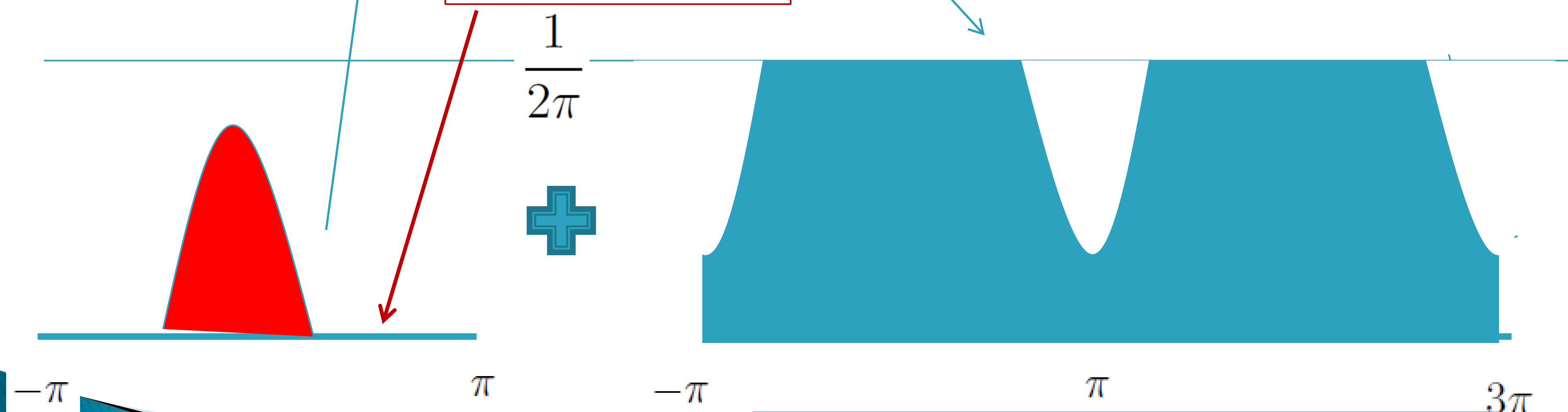

GWW phase transition in Fermion side

Douglass-Kazakov type phase transition in bosonic side

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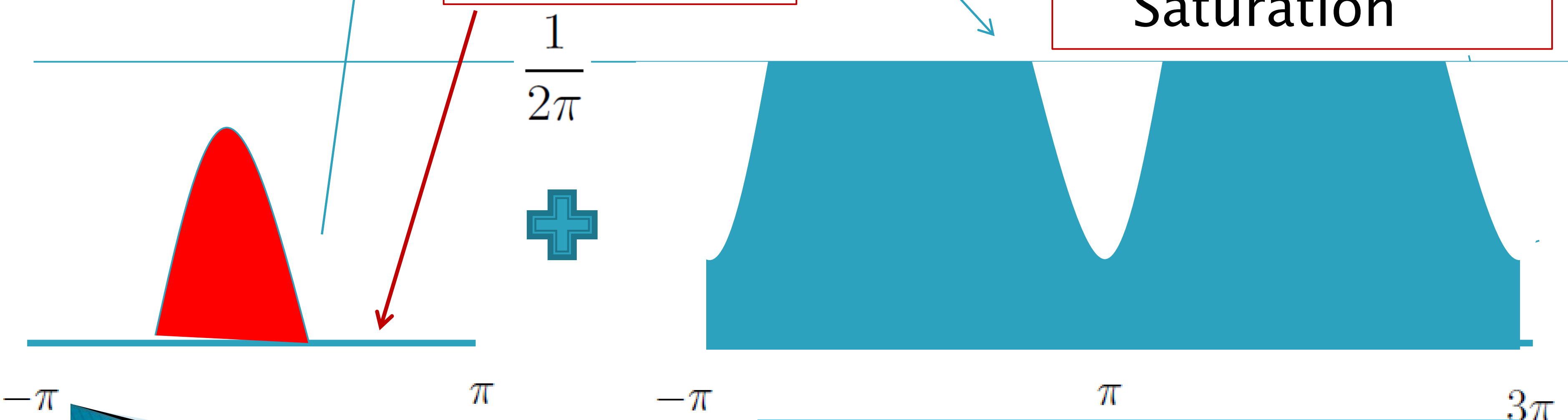
With upper limit Saturation



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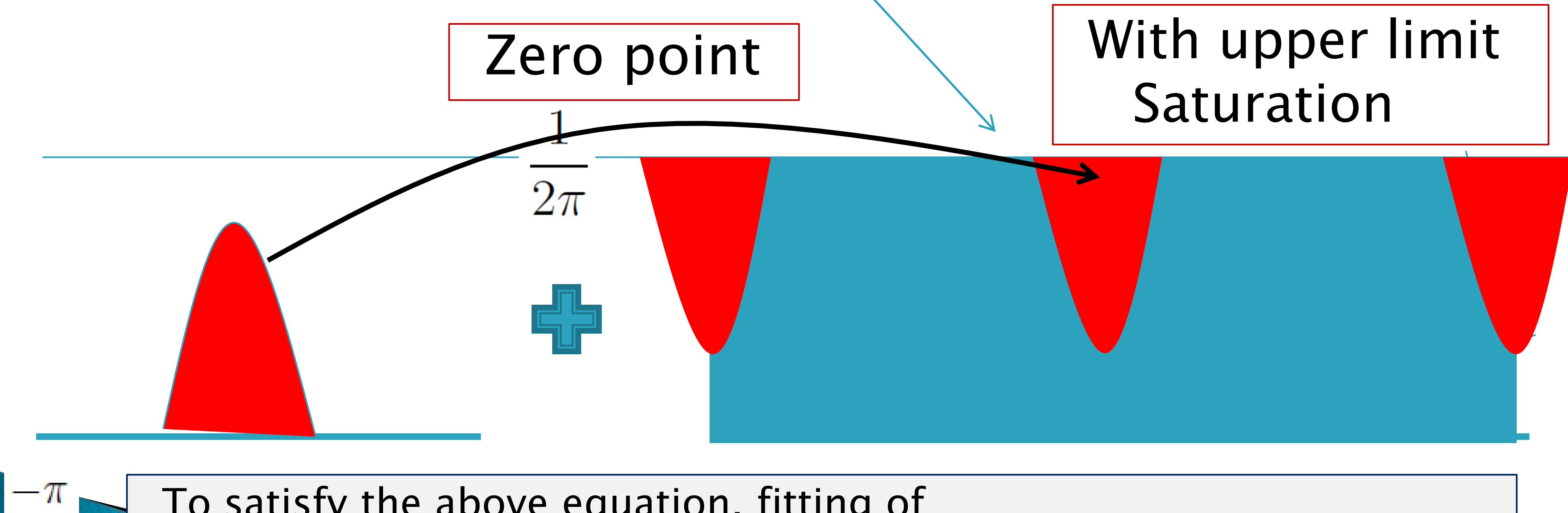

To satisfy the above equation, fitting of
(Zero point) & (upper limit saturation) is crucial.→crucial for duality

Thank you so much
ご清聴ありがとうございます。