BTZ/CFT

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in collaboration with
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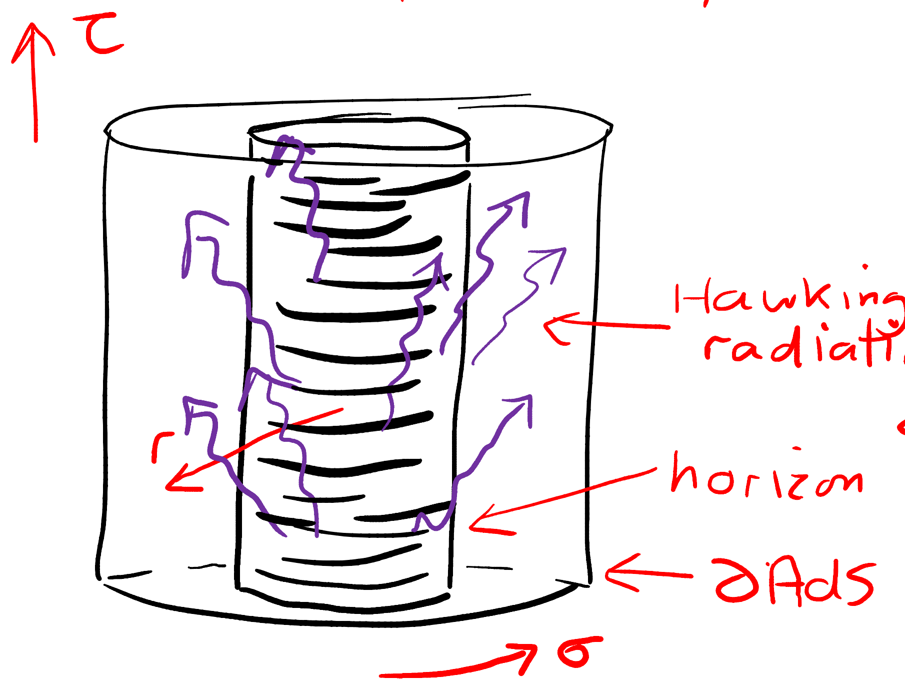
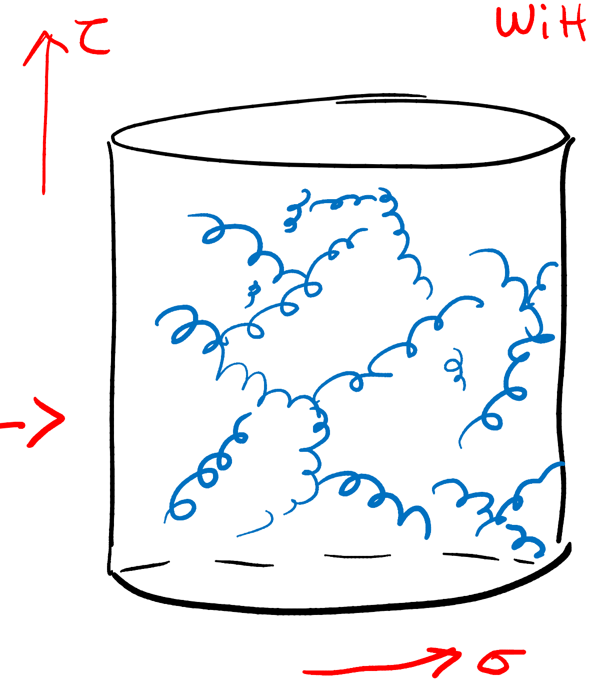
based on hep-th/1311.7738

BTZ = Eternal Black Hole in AdS_{2+1}

Banados, Teitelboim, Zanelli '92

Witten '98





$$ds^2_{\text{Schwarzchild}} = (r^2-1)d\tau^2 - \frac{dr^2}{r^2-1} - r^2 d\sigma^2$$

$CFT_{\text{time} \times S^1}$

$T \neq 0$

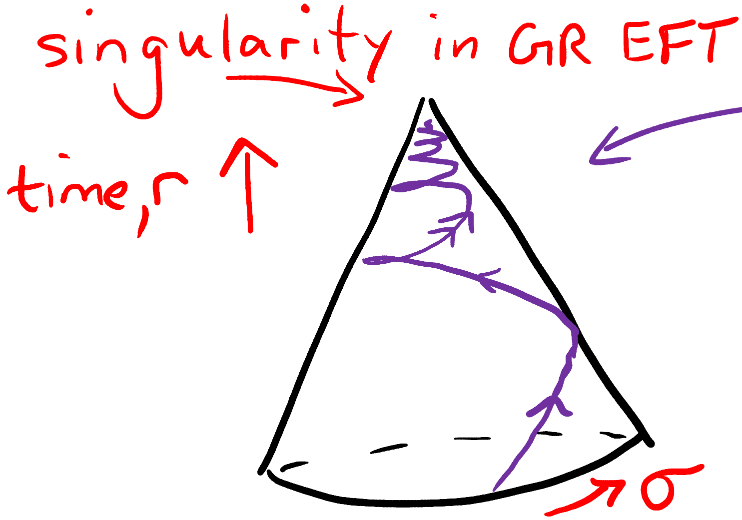
$R_{AdS} \equiv 1$, $\sigma \in (-r_s/2, r_s/2]$ periodic

2π in these units

Boundary Correlators $\longleftrightarrow \operatorname{tr} e^{-\beta H_{CFT}} \mathcal{O}_1 \ldots \mathcal{O}_n$

The Cosmic Accelerator Within Horizon

$$ds^2 \underset{r\sim 0}{\sim} -d\tau^2 + \underbrace{dr^2 - r^2 d\sigma^2}$$



CONSERVED angular momentum

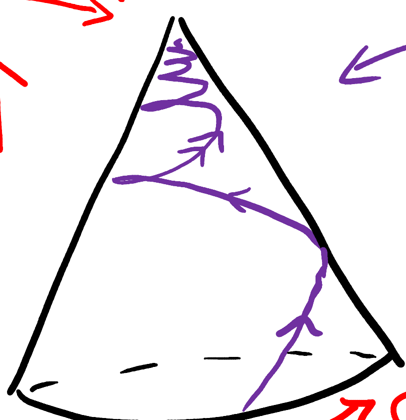
$\sim p \times r \Rightarrow p \to \infty$ as $r \to 0$

The Cosmic Accelerator Within Horizon

$$ds^2 \underset{r\sim 0}{\sim} -d\tau^2 + dr^2 - \underbrace{r^2 d\sigma^2}$$

singularity in GR EFT

time, r $\uparrow$

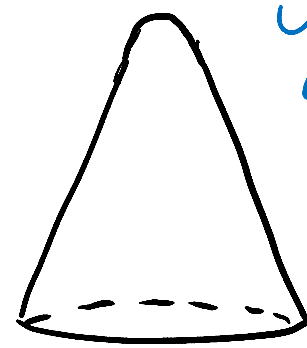


σ

CONSERVED angular momentum

$\sim p \times r \Rightarrow p \to \infty$ as $r \to 0$

smoothed in weakly-coupled string theory



McGreevy, Silverstein '05

GOAL

Given CFT Computer

provide explicit CFT correlators

to sharply probe inside horizon

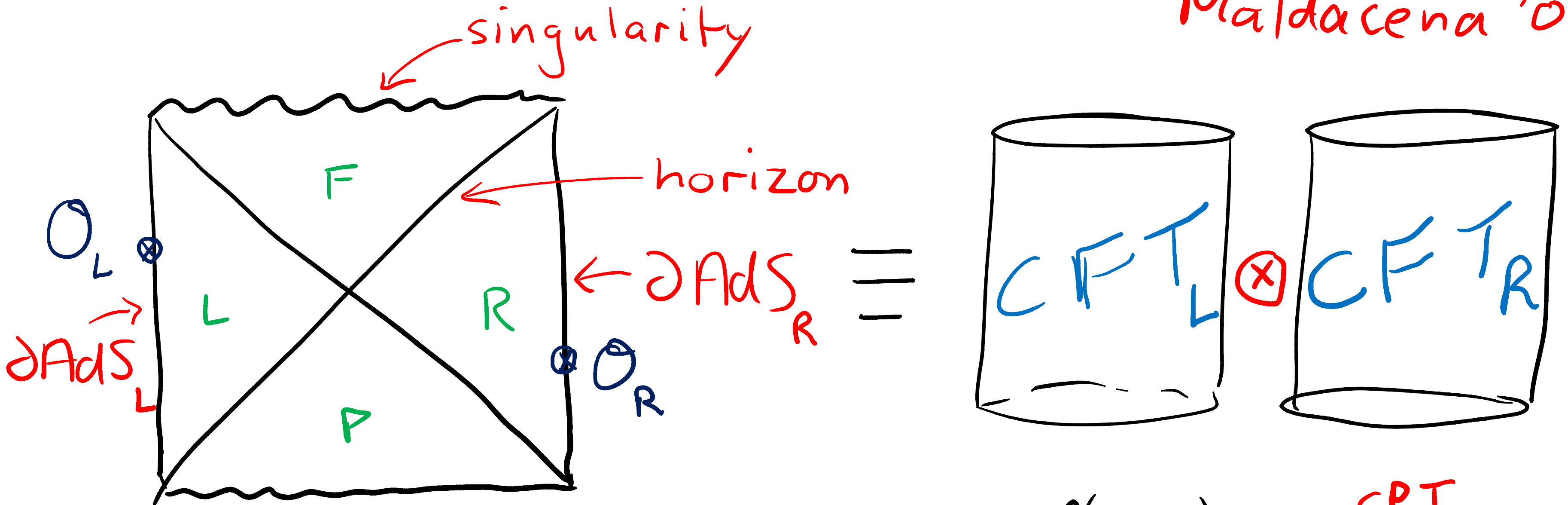
in UV-complete, non-perturbative manner,

thereby improving on GR EFT & perturbative string theory.

Extended Black Hole in Pure State

Israel '76

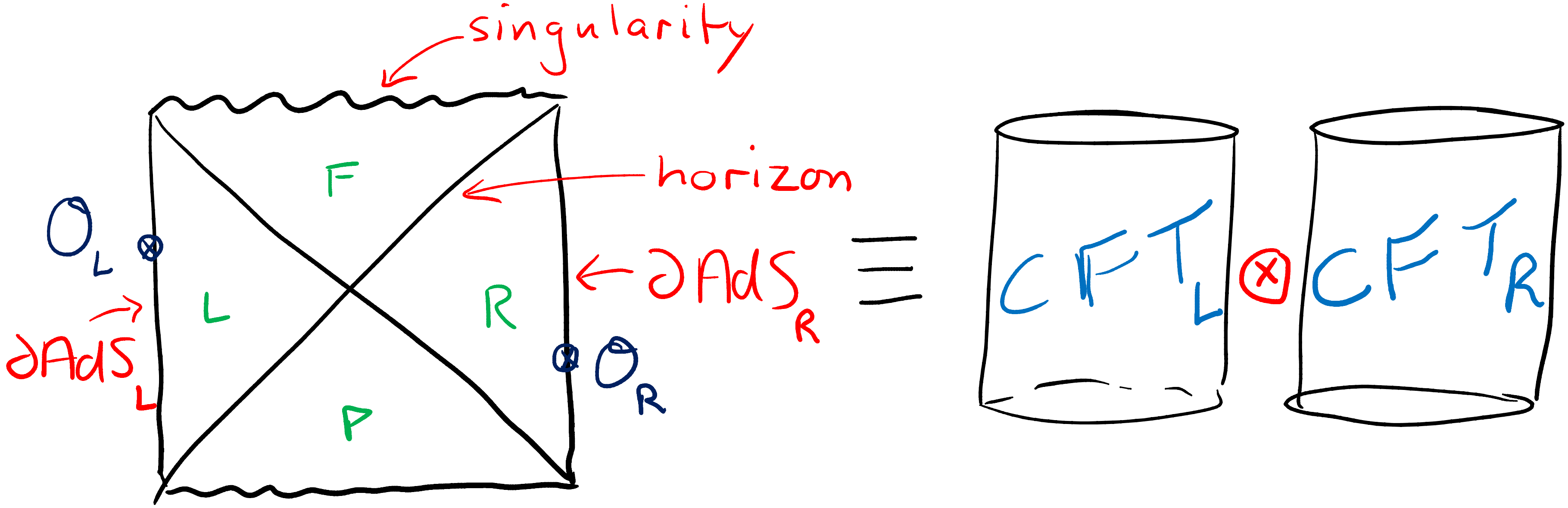
Maldacena '01



$$|\text{Hartle-Hawking}\rangle \equiv |\psi\rangle = \sum_n e^{-\beta(E_n+E_m)/2} \, |\overset{\text{CPT}}{\bar{n}}\rangle_L \otimes |n\rangle_R$$

CFT Thermofield state

Extended Black Hole in Pure State

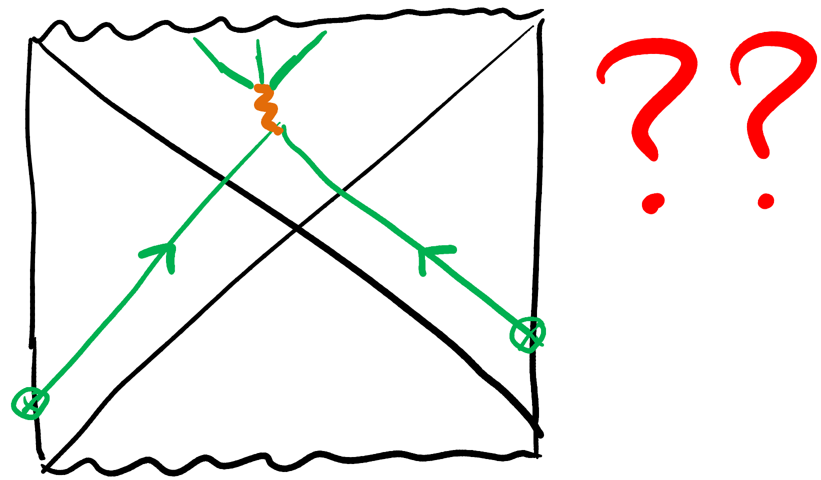


$$\langle\psi|\, T\{O_{L_1}\cdots O_{L_k}\}\otimes T\{O_{R_1}\cdots O_{R_\ell}\}\,|\psi\rangle$$

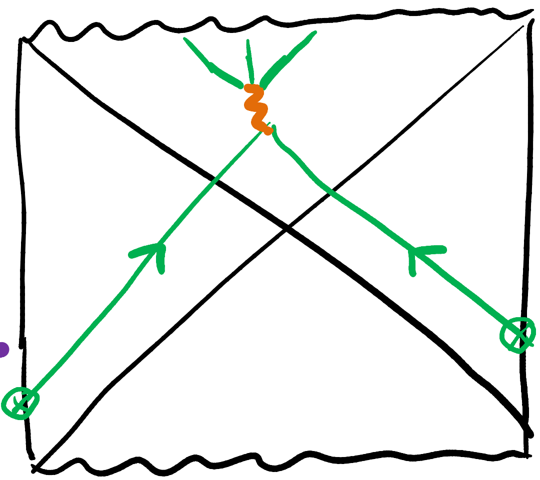
$$= \langle\psi|\, \mathbb{1}\otimes T\{\tilde{O}_{L_1}\cdots \tilde{O}_{L_k}\}\, T\{O_{R_1}\cdots O_{R_\ell}\}\,|\psi\rangle$$

where $\tilde{O} \equiv e^{\beta H/2}\, O_{local}\, e^{-\beta H/2}$ NON-LOCAL EPR TWIN

AMPLITUDES FOR SCATTERING INSIDE HORIZON?

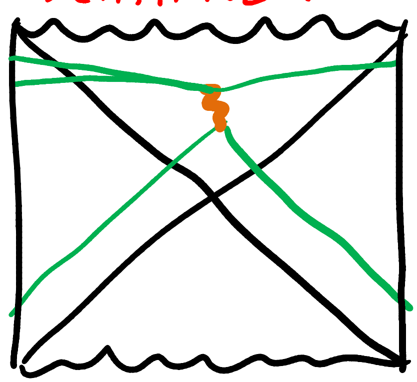


AMPLITUDES FOR SCATTERING INSIDE HORIZON?



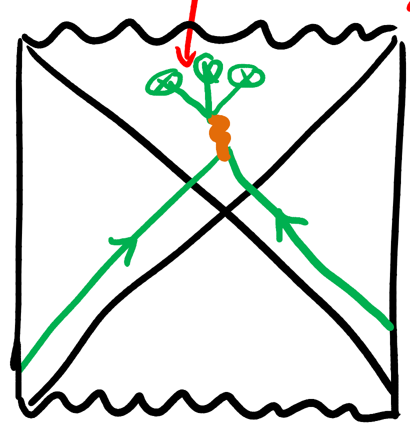
??

local boundary correlators spacelike sensitive to interior



See
Ooguri, Kraus, Shenker '03
Fidkowski, Hubeny, Kleban, Shenker '03

Local bulk operators can only be constructed in CFT by solving bulk dynamics (order by order in 1/N)



see
Hamilton, Kabat, Lifschytz, Lowe '07
Heemskerk, Marolf, Polchinski, Sully '12
Papadodimas, Raju '12

GENERALIZATION:

NON-LOCAL THERMOFIELD CORRELATORS

$$\langle \psi_F | T\{O_{L_1} \cdots\} \otimes T\{O_{R_1}, \cdots\} | \psi_P \rangle$$

anti-T

$$\equiv \langle \psi | T\{O_{L_1} \cdots\} \bar{T}\{\tilde{O}_{P_1} \cdots\} \otimes \bar{T}\{\tilde{O}_{F_1} \cdots\} T\{O_{R_1} \cdots\} | \psi \rangle$$

where $$|\psi_P\rangle \equiv \bar{T}\{\tilde{O}_{P_1} \cdots\} \otimes \mathbb{1} \, |\psi\rangle$$ ← Thermo-Field state

$$\langle \psi_F | \equiv \langle \psi | \, \mathbb{1} \otimes \bar{T}\{\tilde{O}_{F_1} \cdots\}$$

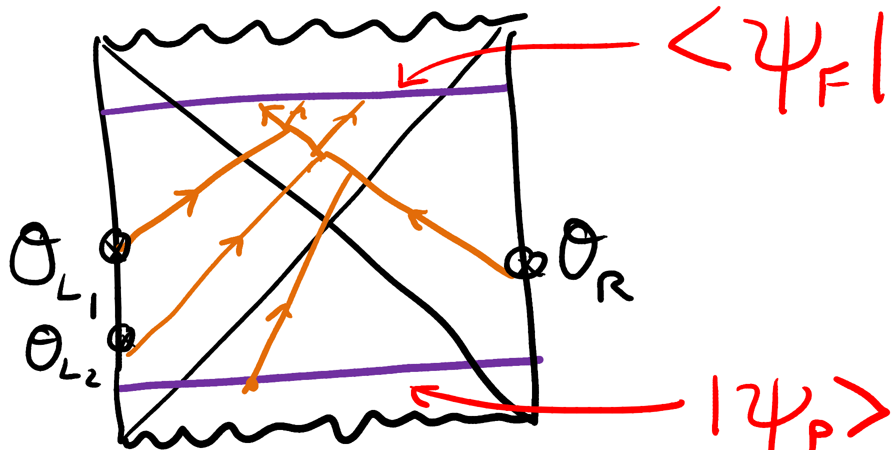
$$\tilde{O}_F(\tau,\sigma) \equiv e^{\frac{\beta}{4}(H_\tau - P_\sigma)} \, O_F(\tau,\sigma) \, e^{-\frac{\beta}{4}(H_\tau - P_\sigma)}$$

(angular) momentum

CENTRAL CLAIM

$$\langle \psi_F | T\{\mathcal{O}_{L_1} \cdots\} \otimes T\{\mathcal{O}_{R_1} \cdots\} | \psi_P \rangle$$

$\equiv$



are CFT-computable, & give (some) sharp probes inside horizon, effectively via non-local EPR-like correlations outside.

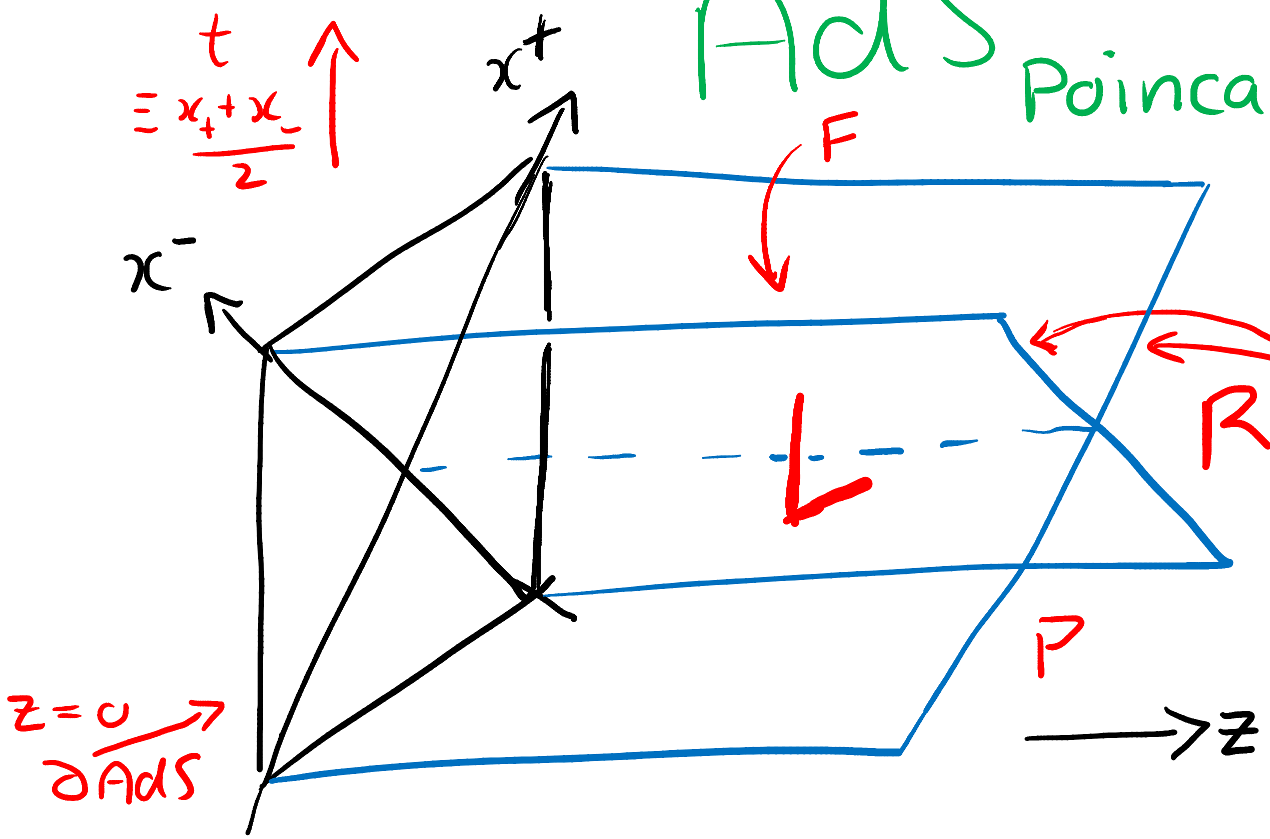
NON-COMPACT LIMIT $r_s \to \infty$

$$\sigma \in (-\infty, \infty)$$

BLACK HOLE $\longrightarrow$ BLACK "STRING"

$\equiv$ Rindler View of AdS$_{\text{Poincare}}$

$$ds^2 = \frac{dx^+dx^- - dz^2}{z^2}$$



$$x^\pm \equiv \begin{cases} \pm\sqrt{1-1/r^2}\, e^{\sigma\pm\tau}, & r>1 \\ \sqrt{1/r^2-1}\, e^{\sigma\pm\tau}, & r<1 \end{cases}$$

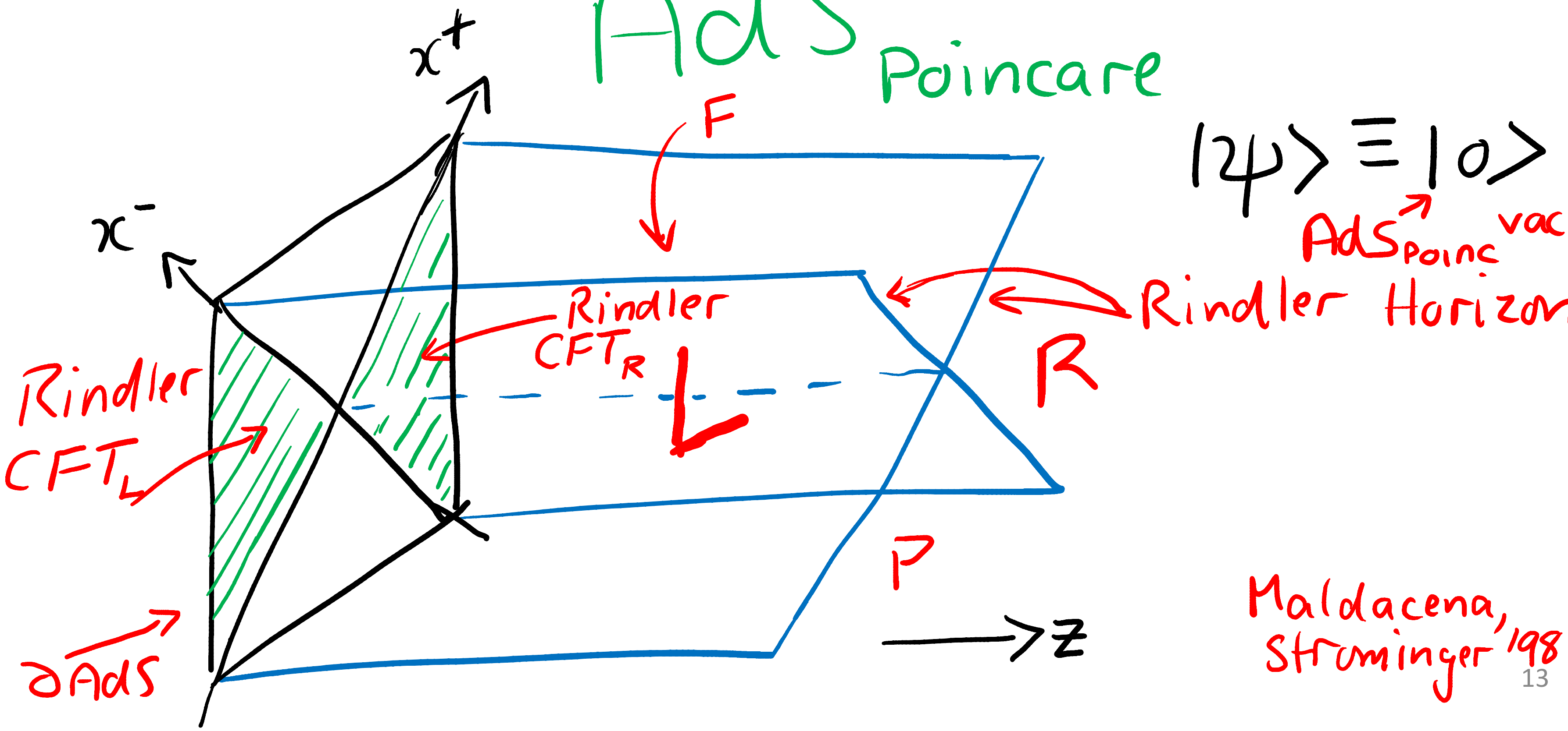
$$z \equiv e^{\sigma}/r$$

NON-COMPACT LIMIT $r_S \to \infty$

$$\sigma \in (-\infty, \infty)$$

BLACK HOLE $\longrightarrow$ BLACK "STRING"

$\equiv$ Rindler View of AdS Poincare



$|\psi\rangle \equiv |0\rangle$

AdS_{Poinc} vacuum

Maldacena, Strominger '98

PROBING BEHIND RINDLER HORIZON WITH RINDLER $CFT_L \otimes CFT_R$

$$\langle \psi | T_\tau\{\mathcal{O}_{L_1} \cdots\} \bar{T}_\tau\{\tilde{\mathcal{O}}_{P_1} \cdots\} \otimes \bar{T}_\tau\{\tilde{\mathcal{O}}_{F_1} \} T_\tau\{\mathcal{O}_{R_1} \} | \psi \rangle$$

$$= \langle 0 | T_t\{\mathcal{O}_{F_1}(\pm x_{1\pm}) \cdots \mathcal{O}_{L_1}(x'_{1\pm}) \cdots \mathcal{O}_{R_1}(x''_{1\pm}) \cdots \mathcal{O}_{P_1}(\pm x'''_{1\pm}) \cdots\} | 0 \rangle$$

$$\times\, e^{i\pi(h^-_{F_1}+1)} \cdots e^{i\pi(h^-_{P_1}+1)} \cdots$$

$h^\pm \equiv (\Delta \pm s)/2$

scaling dimension

spin

CONFORMAL WEIGHTS

PROBING BEHIND RINDLER HORIZON WITH RINDLER $CFT_L \otimes CFT_R$

$$\langle\psi| T_\tau\{O_{L_1}\ldots\} \bar{T}_\tau\{\tilde{O}_{P_1}\ldots\} \otimes \bar{T}_\tau\{\tilde{O}_{F_1}\} T_\tau\{O_{R_1}\}|\psi\rangle$$

$$= \langle 0| T_t\{O_{F_1}(\pm x_{1\pm})\ldots O_{L_1}(x'_{1\pm})\ldots O_{R_1}(x''_{1\pm})\ldots O_{P_1}(\pm x'''_{1\pm})\ldots\}|0\rangle$$

$$\times\, e^{i\pi(h^-_{F_1}+1)}\ldots e^{i\pi(h^-_{P_1}+1)}\ldots$$

related: Hemming, Keski-Vakkuri, Kraus '02

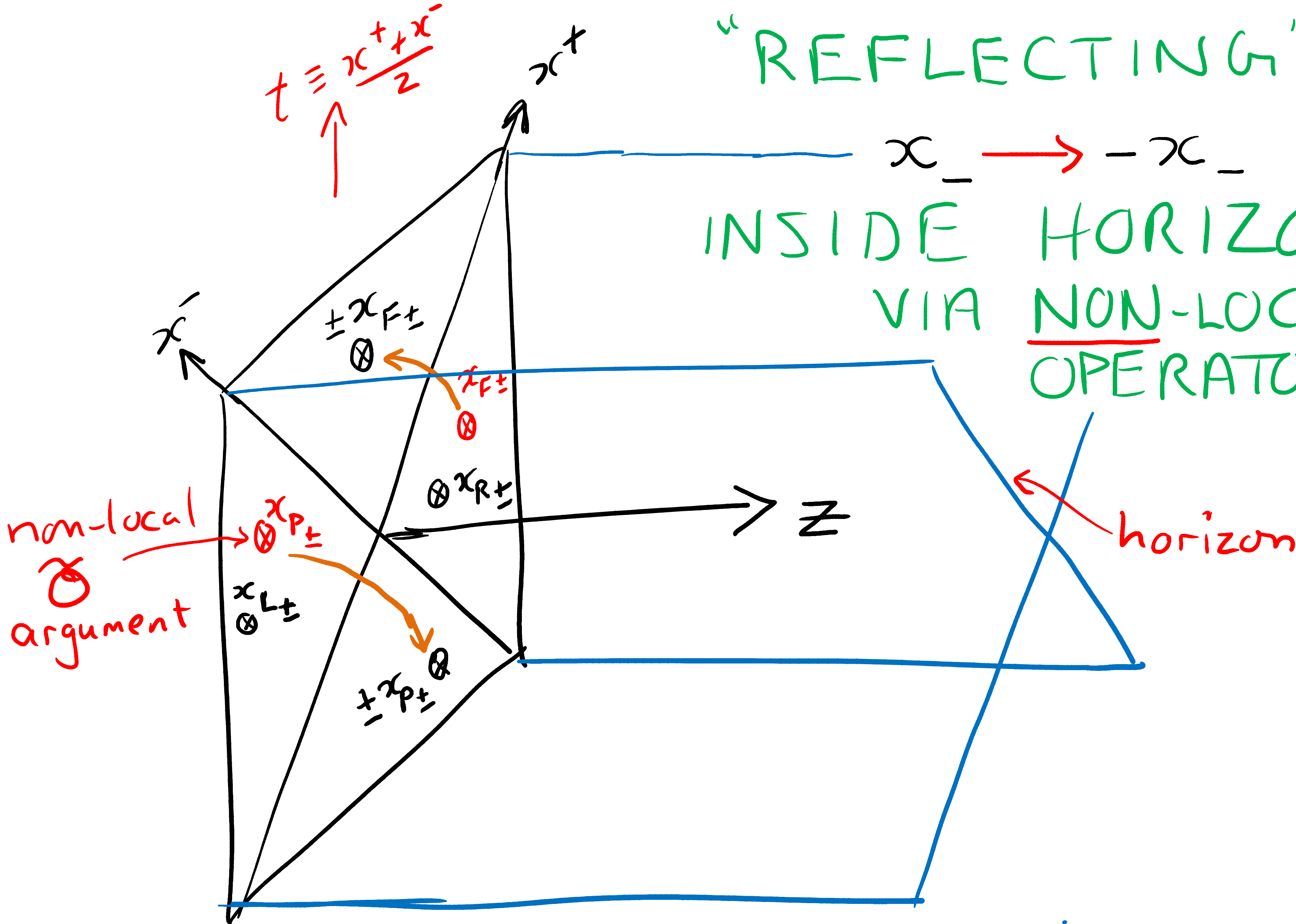
TECHNICALLY, $e^{\frac{\pi}{2}(H_\tau - P_\sigma)}\ldots e^{-\frac{\pi}{2}(H_\tau - P_\sigma)}$ induces analytic continuation $\tau,\sigma \to \tau + i\frac{\pi}{2}, \sigma \to \sigma - i\frac{\pi}{2}, x_\pm \to \pm x_\pm$

similar to thermofield context: $e^{\beta/2 H}\ldots e^{-\beta/2 H}$ induces $\tau \to \tau + i\beta/2$

"REFLECTING" $x_- \rightarrow -x_-$

INSIDE HORIZON

VIA NON-LOCAL OPERATORS



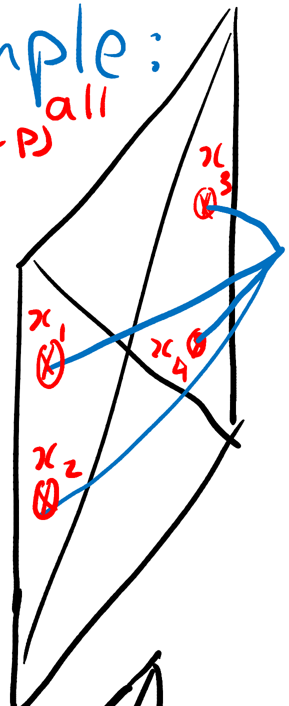
∃ other non-local CFT probes inside Rindler horizon

but need symmetry (operators, eg. $H_{\text{Minkowski}}$) broken in BTZ

PROOF is AdS-diagrammatic

Example:

where all $e^{\pm\frac{\pi}{2}(H-P)}$ cancel



ϕ^4 interaction

$$= \int d^2x_\pm \int_0^\infty \frac{dz}{z^3} \prod_{j=1}^{4} \left[\frac{z}{(x_j^+ - x^+)(x_j^- - x^-) - z^2 + i\varepsilon} \right]^{\Delta_j}$$

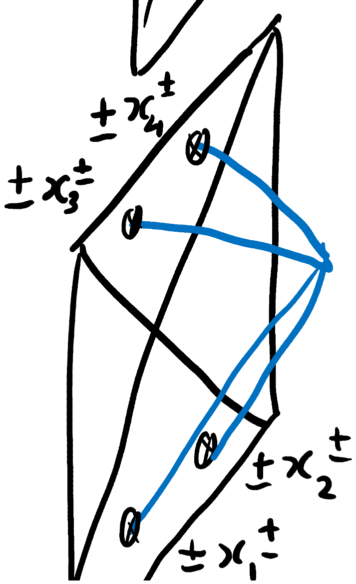
rotate contour $z \to iz$

$$= \int d^2x_\pm \int_0^\infty \frac{dz}{z^3} \prod_{j=1}^{4} \left[\frac{iz}{(x_j^+ - x^+)(x_j^- - x^-) + z^2 + i\varepsilon} \right]^{\Delta_j}$$

$x_- \to -x_-$

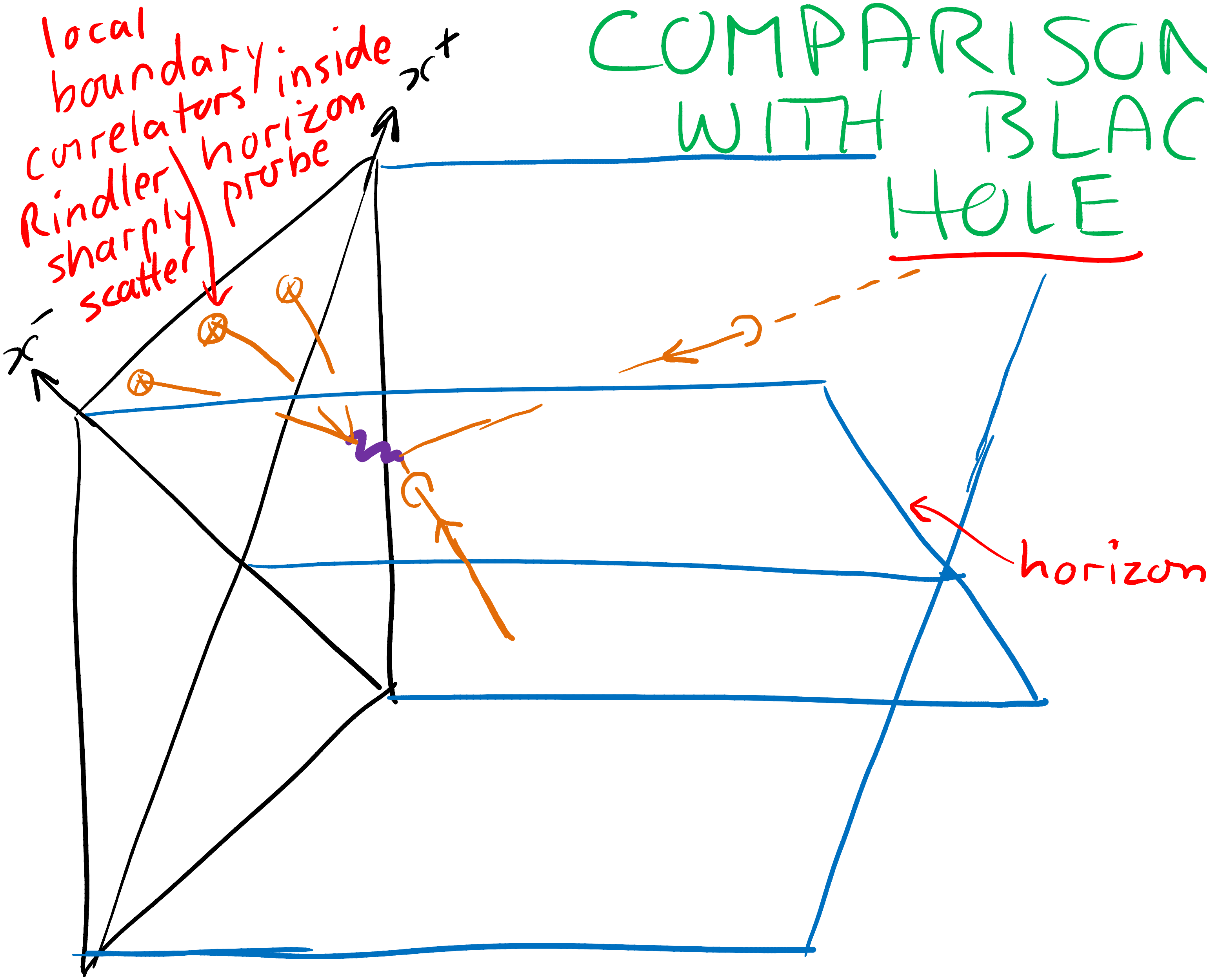
$$= \int d^2x_\pm \int_0^\infty \frac{dz}{z^3} \prod_{1}^{4} \left[\frac{-iz}{(x_j^+ - x^+)(-x_j^- - x^-) - z^2 - i\varepsilon} \right]^{\Delta_j}$$

=

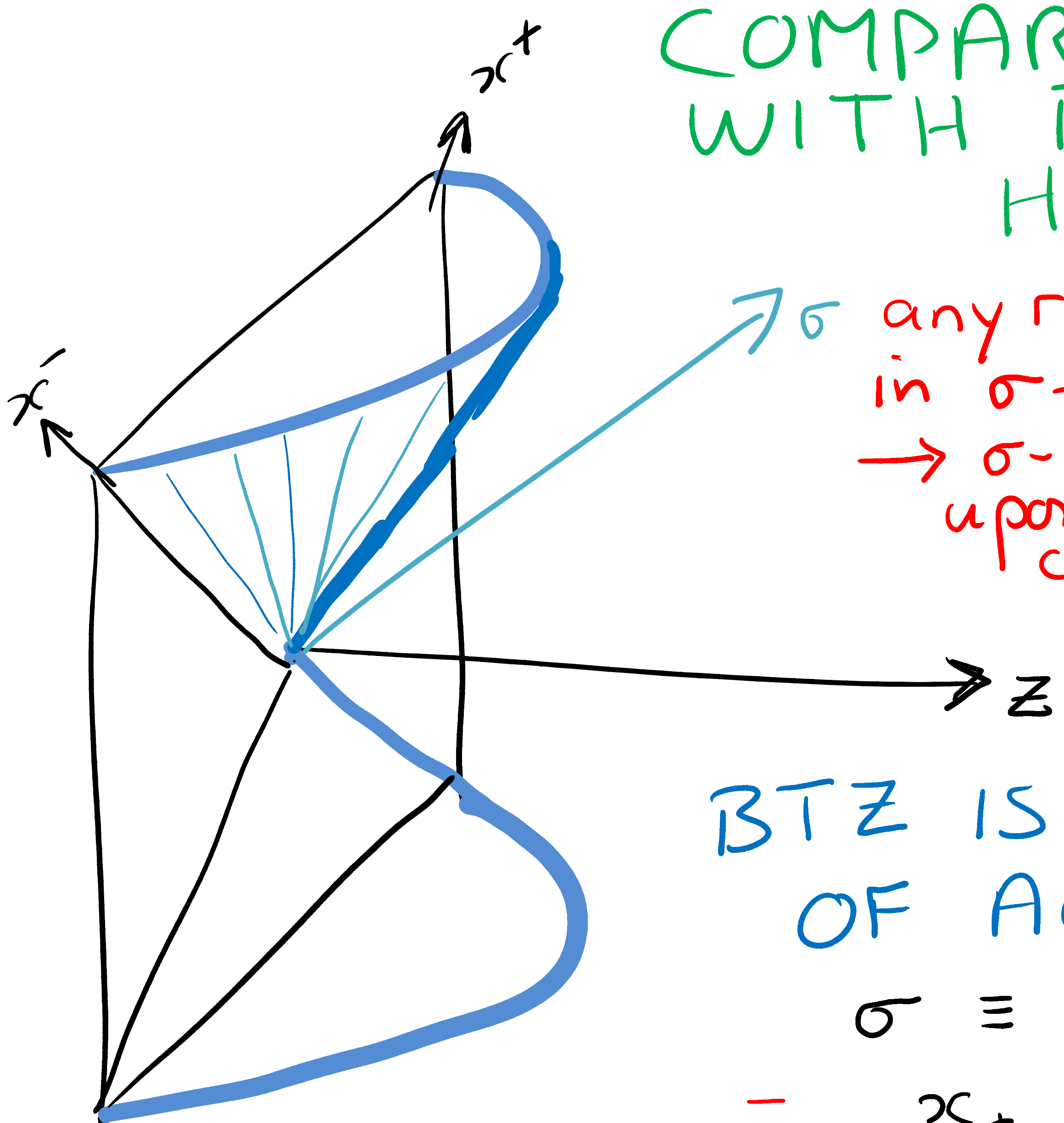


$$\times\ e^{i\frac{\pi}{2}\Delta_1} \cdots e^{i\frac{\pi}{2}\Delta_4}$$

COMPARISON WITH BLACK HOLE



COMPARISON WITH BLACK HOLE



any radial line is in σ-direction

$\rightarrow$ σ-circle upon compactification

BTZ IS QUOTIENT OF AdS_3:

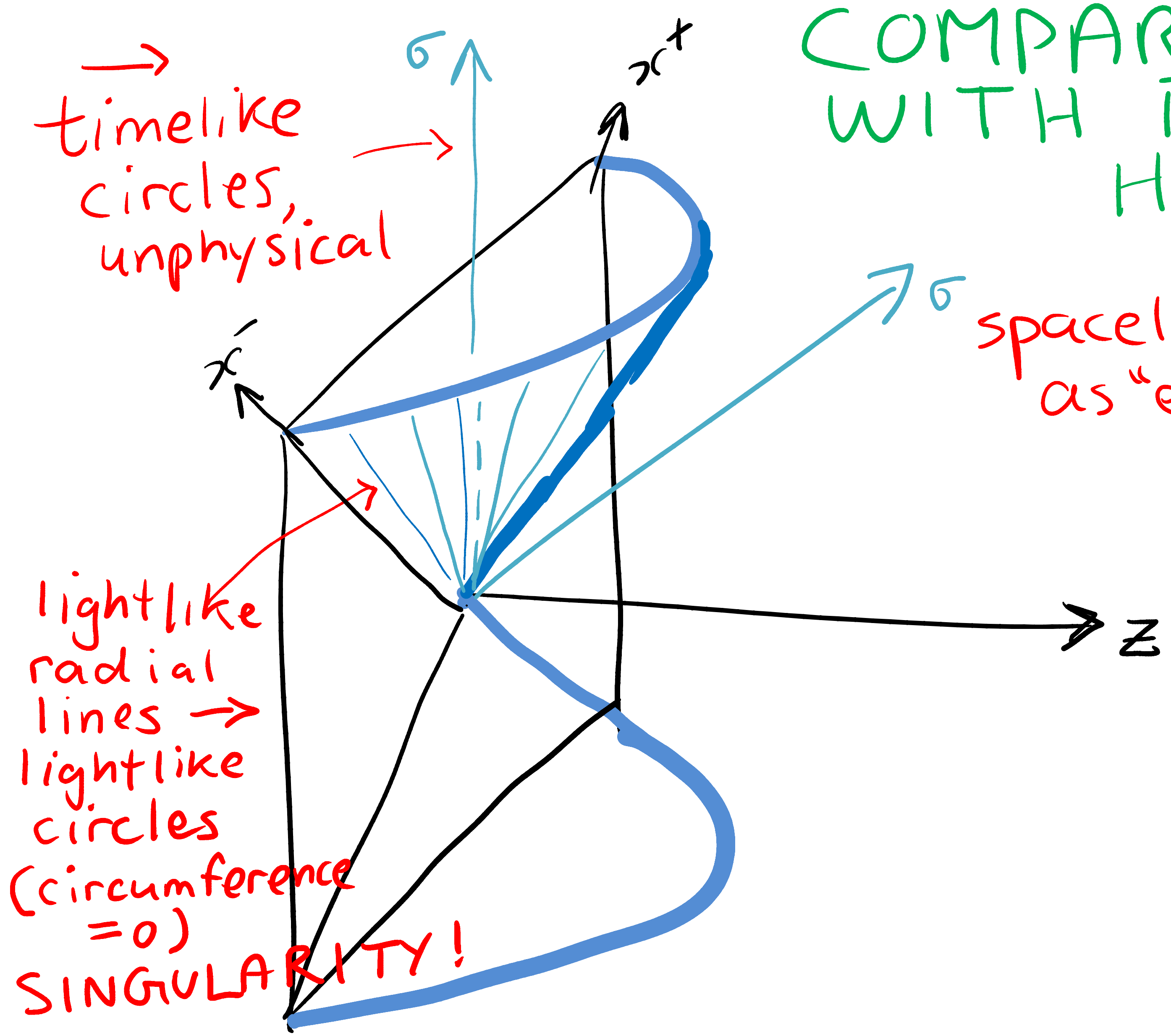
$$\sigma \equiv \sigma + r_s$$

$$\equiv \quad x_\pm \equiv \lambda x_\pm$$

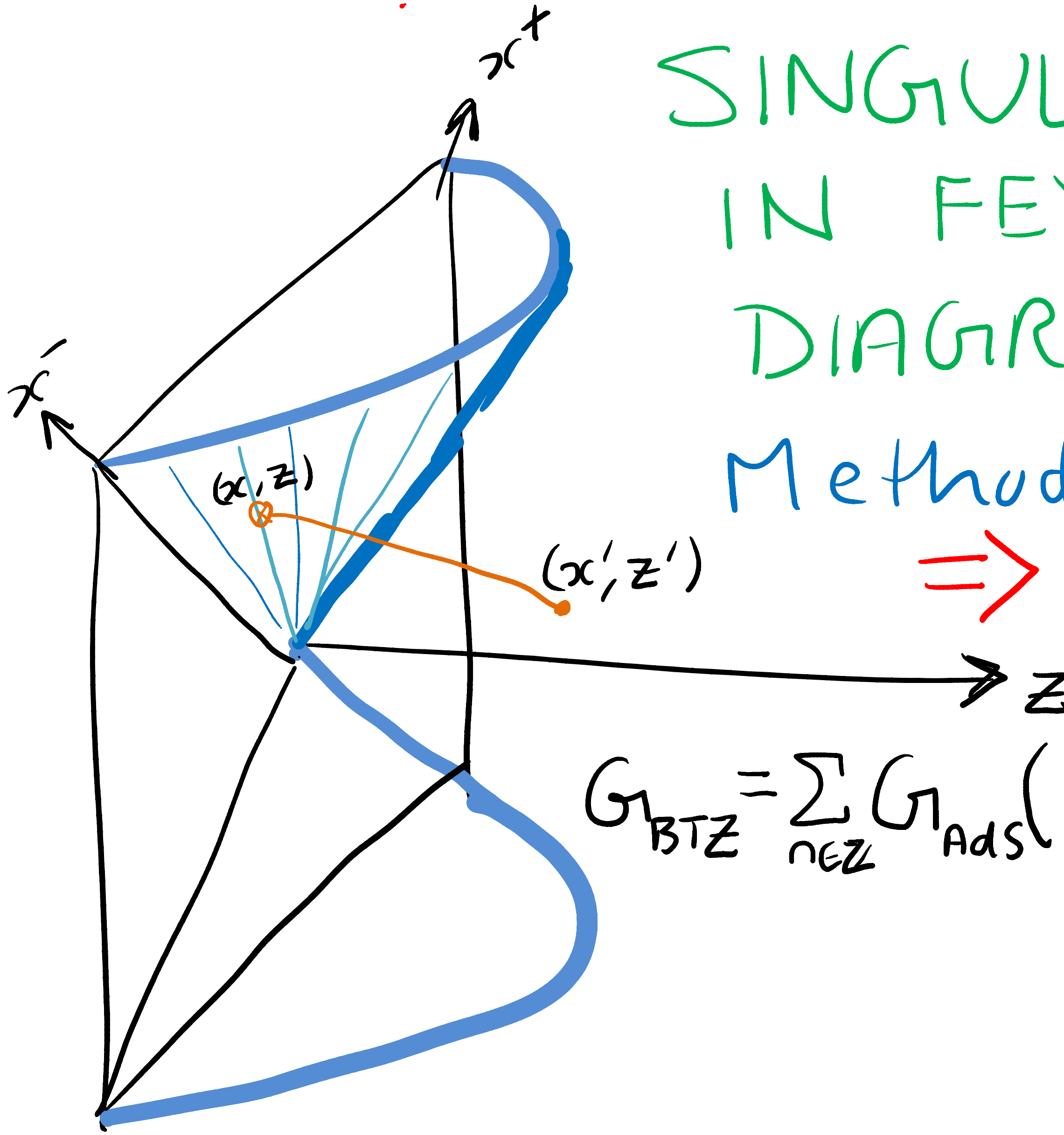
$$z \equiv \lambda z \leftarrow e^{r_s}$$

Banados, Henneaux, Teitelboim, Zanelli '93

COMPARISON WITH BLACK HOLE.



SINGULARITY IN FEYNMAN DIAGRAMS

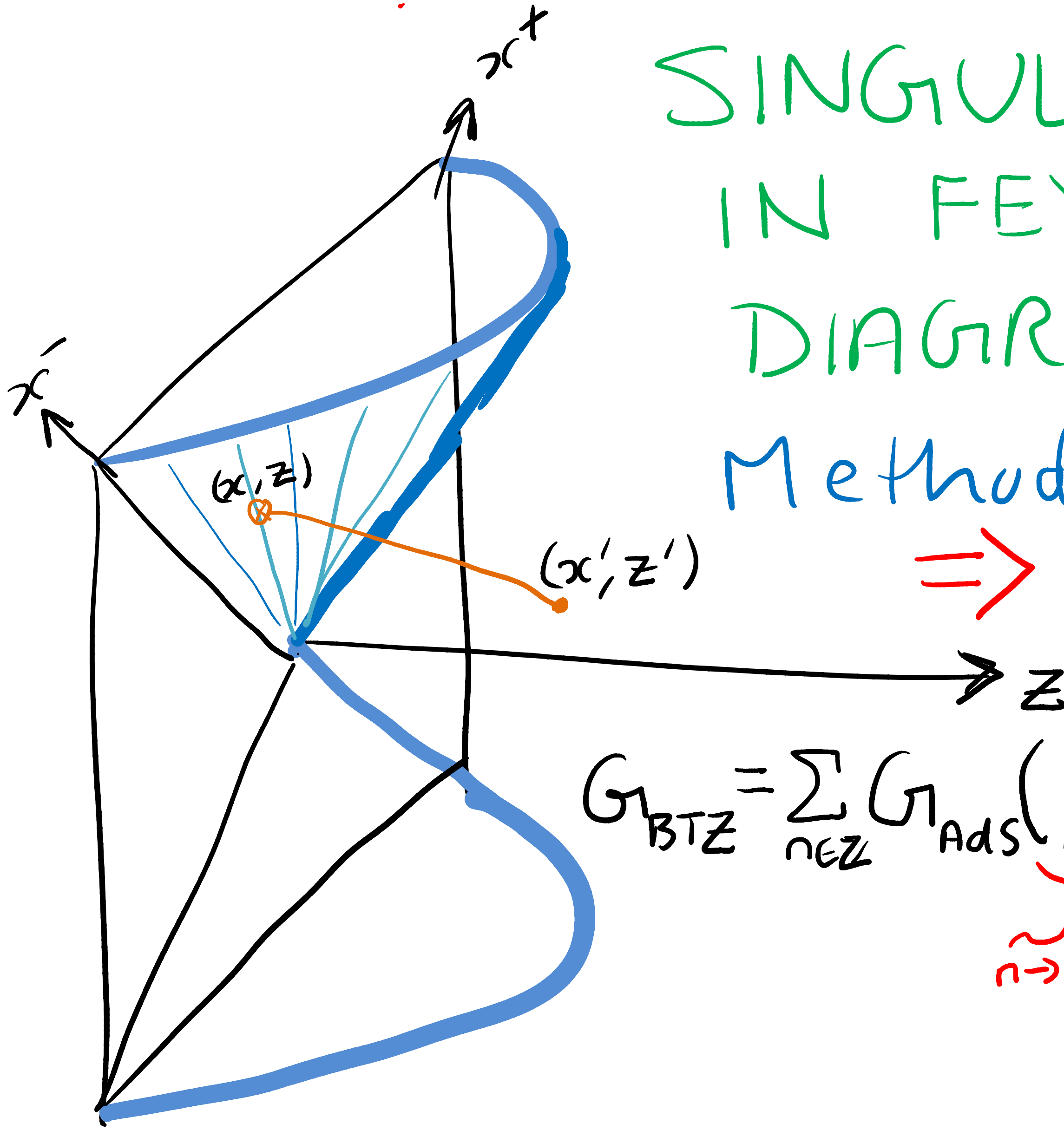


Method of images $\Rightarrow$

$$G_{BTZ} = \sum_{n\in\mathbb{Z}} G_{AdS}\left(\frac{\lambda^n z z'}{\lambda^{2n} z^2 + z'^2 - (\lambda^n x - x')^2}\right)$$

Lifshytz, Ortiz '93

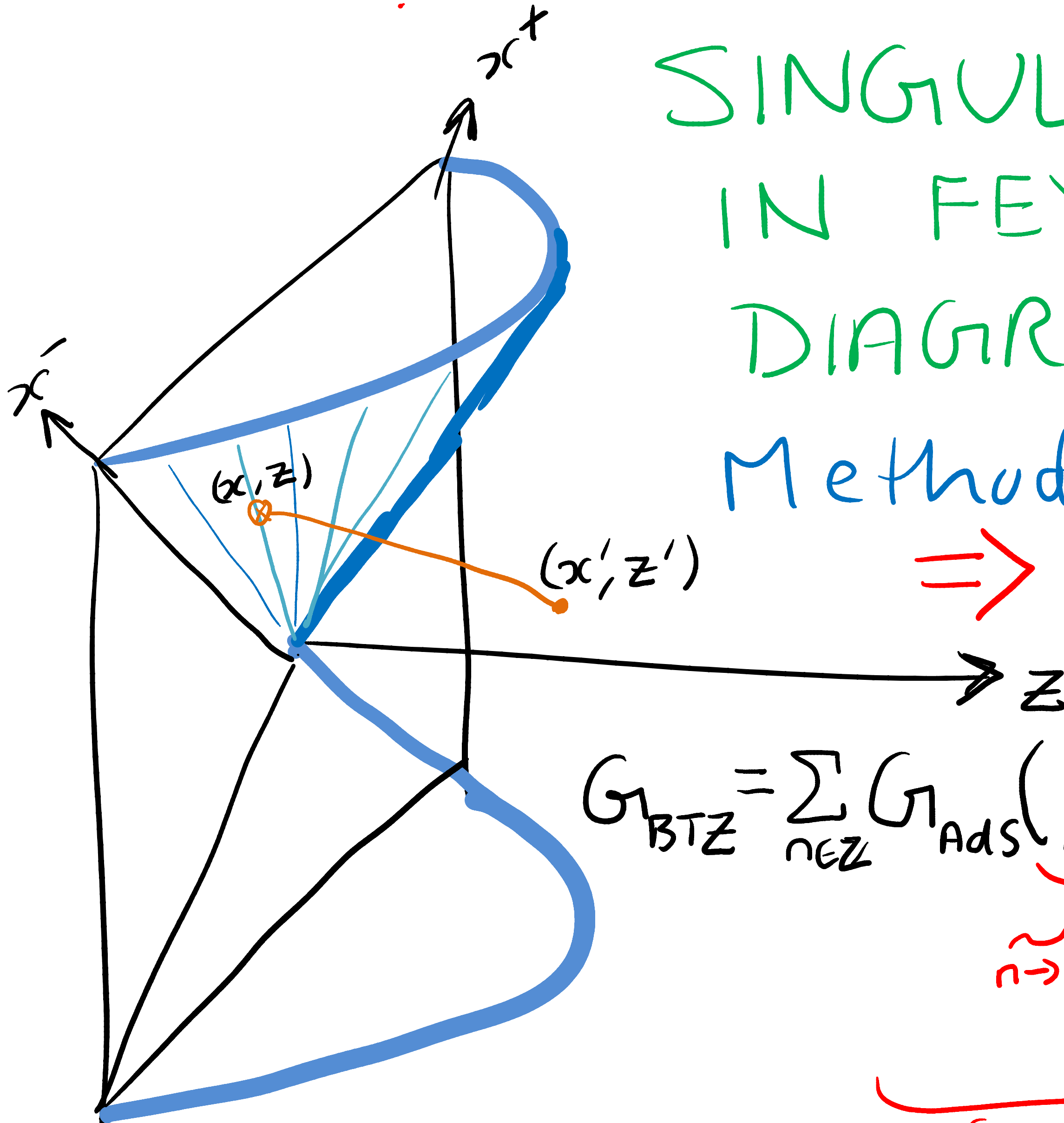
SINGULARITY IN FEYNMAN DIAGRAMS



Method of images

$\Rightarrow$

$$G_{BTZ} = \sum_{n \in \mathbb{Z}} G_{AdS}\Bigg(\underbrace{\frac{\lambda^n z z'}{\lambda^{2n} z^2 + z'^2 - (\lambda^n x - x')^2}}_{\underset{n\to\pm\infty}{\sim} \begin{cases} \mathcal{O}(\lambda^{-|n|}) & \text{generically} \\ \mathcal{O}(1), & x_+ x_- = z^2 \end{cases}}\Bigg)$$



SINGULARITY IN FEYNMAN DIAGRAMS

Method of images

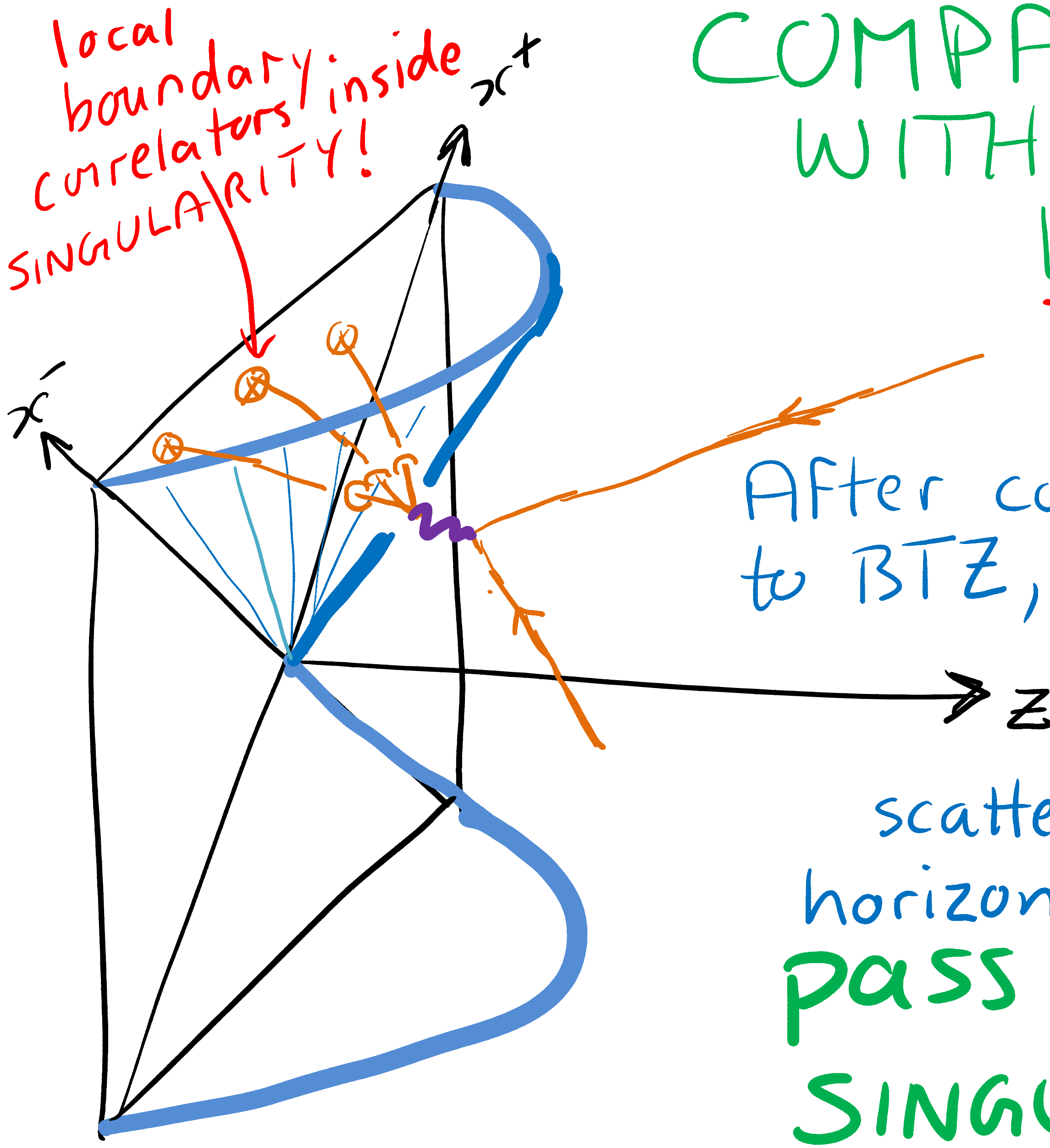
$\Rightarrow$

$$G_{BTZ}=\sum_{n\in\mathbb{Z}} G_{AdS}\left(\frac{\lambda^n z z'}{\lambda^{2n}z^2+z'^2-(\lambda^n x-x')^2}\right)$$

$$\underset{n\to\pm\infty}{\sim}\begin{cases}\mathcal{O}(\lambda^{-|n|}) & \text{generically}\\ \mathcal{O}(1), & x_+x_-=z^2\end{cases}$$

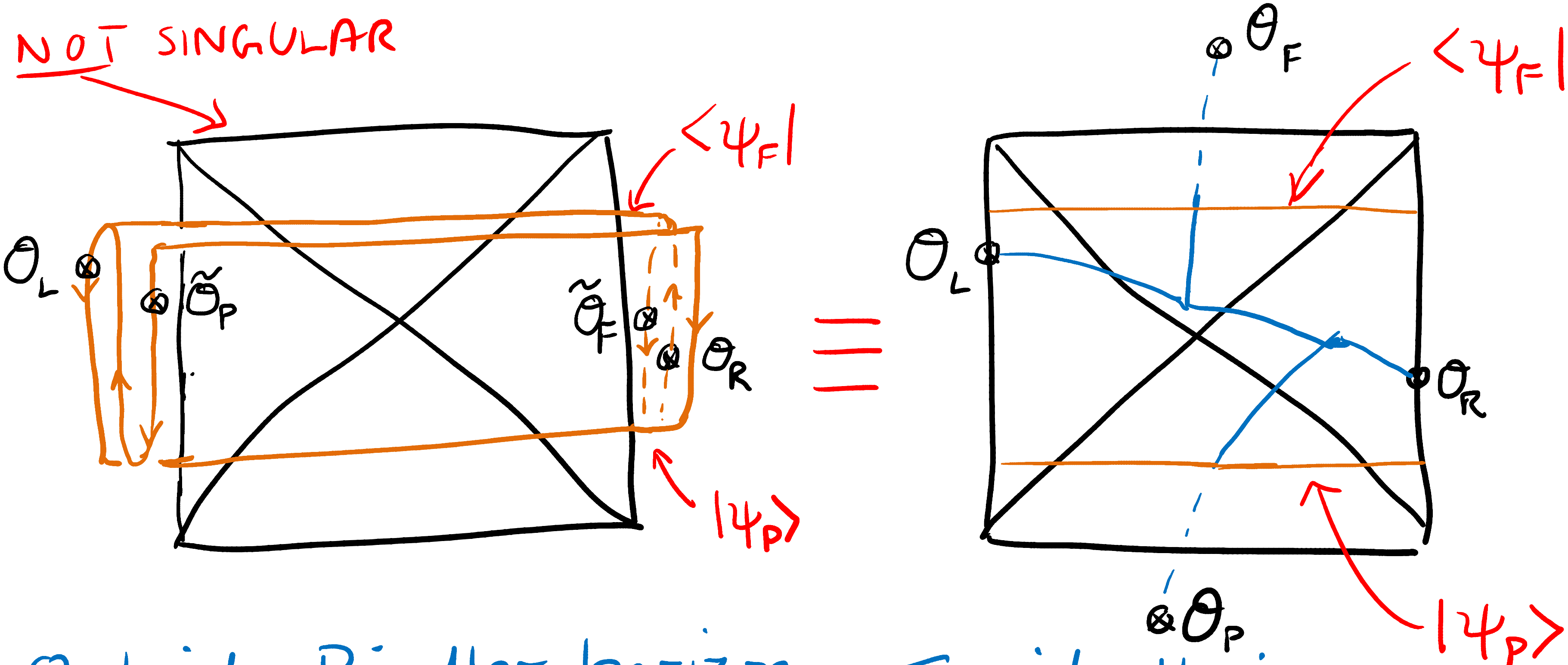
$$\sim\begin{cases}\mathcal{O}(\lambda^{-|n|\Delta}) & \text{generically}\\ \mathcal{O}(1), & x_+x_-=z^2\end{cases}$$

COMPARISON WITH BLACK HOLE



After compactification to BTZ, looks like correlators probing scattering inside horizon must **pass through SINGULARITY!?**

(unquotiented) RINDLER PENROSE DIAGRAM

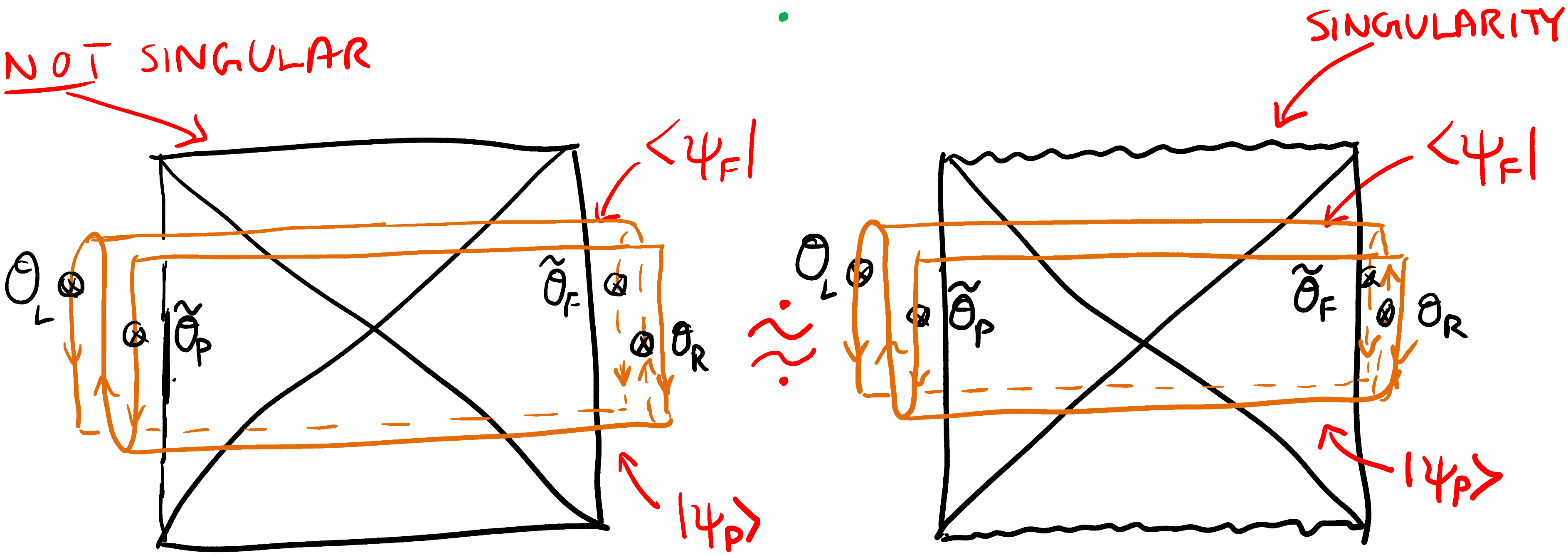


Outside Rindler horizon correlators = Inside Horizon correlators

via EPR-like entanglement

COMPARISON WITH BTZ far from singularity

RINDLER $\approx$ BLACK HOLE

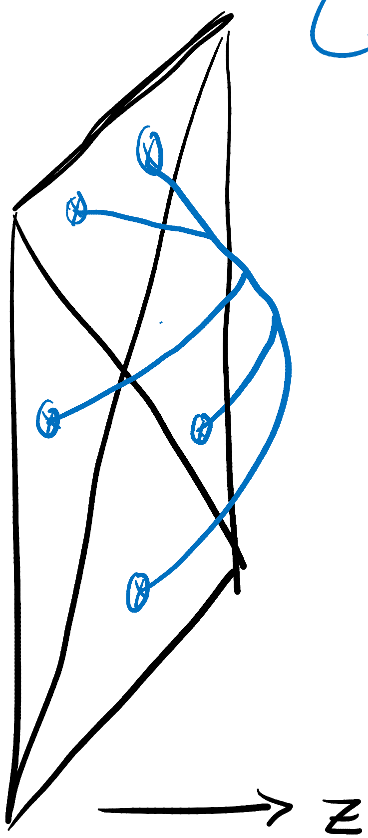


$e^{\pm\pi/2(H_z - P_\sigma)}$ weights in $\tilde{O}$ similar for compact & non-compact σ in "folded" evolution, where σ-circumference is large.

INSENSITIVITY TO SINGULARITY OF BTZ ~~WITTEN~~ DIAGRAMS

∂BTZ correlators

Contour rotate $z \to e^{i\theta} z$ in all interaction vertex integrals

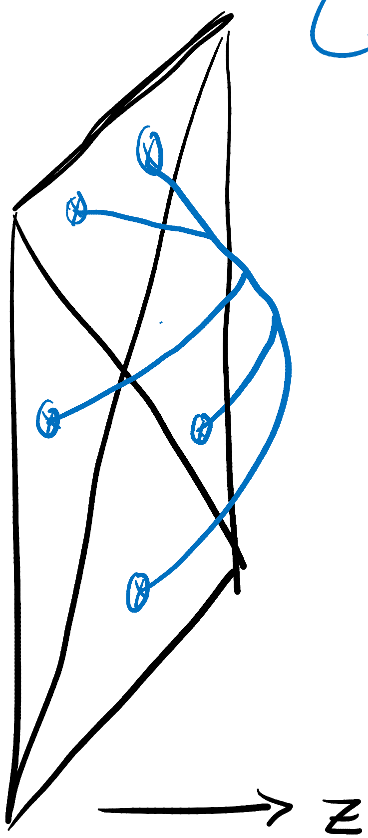


$$G_{BTZ} \longrightarrow \underbrace{\sum_{n \in \mathbb{Z}} G_{AdS}\left(\underbrace{\frac{e^{2i\theta} z z' \lambda^{n}}{e^{2i\theta}(z^2 + z'^2 \lambda^{2n}) - (x - \lambda^{n} x')^2 + i\varepsilon}}_{O(\lambda^{-|n|}),\ \text{always}} \right)}_{\underset{\substack{n \to \pm\infty \\ e^{2i\theta} = i}}{\sim} O(\lambda^{-|n|\Delta})} < \infty$$

INSENSITIVITY TO SINGULARITY OF BTZ WITTEN DIAGRAMS

∂BTZ correlators

Contour rotate $z \to e^{i\theta} z$ in all interaction vertex integrals



$$G_{BTZ} \longrightarrow \sum_{n\in\mathbb{Z}} G_{AdS}\left(\underbrace{\frac{e^{2i\theta} z z' \lambda^n}{e^{2i\theta}(z^2 + z'^2\lambda^{2n}) - (x - \lambda^n x')^2 + i\varepsilon}}_{\mathcal{O}(\lambda^{-|n|}),\ \text{always}}\right) < \infty$$

$$\underset{\substack{n\to\pm\infty \\ e^{2i\theta} = i}}{\sim} \mathcal{O}(\lambda^{-|n|}), \text{ always}$$

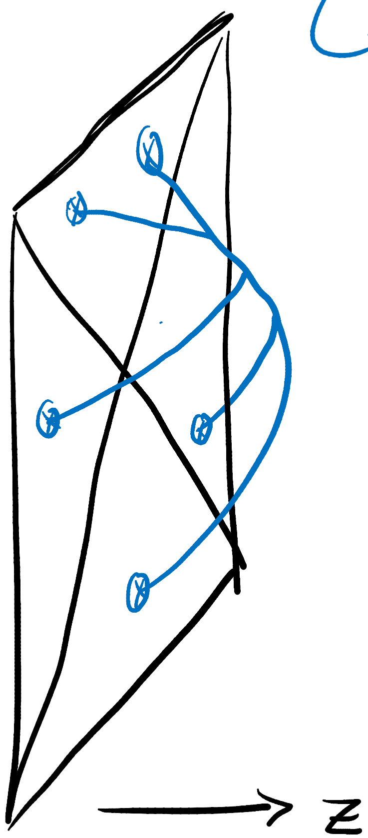
$$\sim \mathcal{O}(\lambda^{-|n|\Delta})$$

← ultimately cancelations inside & outside singularity

related: Kraus, Ooguri, Shenker '03

INSENSITIVITY TO SINGULARITY OF BTZ WITTEN DIAGRAMS

∂BTZ correlators



Contour rotate $z \to e^{i\theta} z$ in all interaction vertex integrals

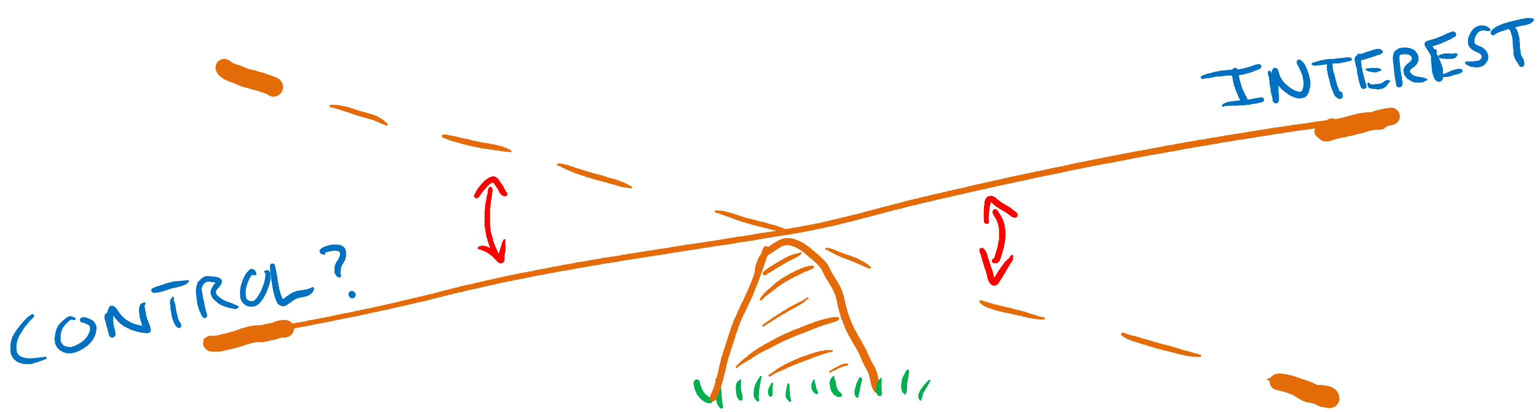
$$G_{BTZ} \longrightarrow \underbrace{\sum_{n\in\mathbb{Z}} G_{AdS}\Bigg(\underbrace{\frac{e^{2i\theta} z z' \lambda^n}{e^{2i\theta}(z^2+z'^2\lambda^{2n}) - (x-\lambda^n x')^2 + i\varepsilon}}_{O(\lambda^{-|n|}),\ \text{always}}\Bigg)}_{\underset{\substack{n\to\pm\infty \\ e^{2i\theta}=i}}{\sim} O(\lambda^{-|n|\Delta})}$$

In fact $n=0$ dominates for large $\lambda \equiv e^{r_s}$

$\Rightarrow$ explicit $CFT^2_{S^1}$ correlators sharply probing inside BTZ horizon

many of which $\doteq$ scattering inside Rindler horizon of AdS_3

"Sharpness" $\equiv$ that of local boundary correlators restricted to POINCARE PATCH



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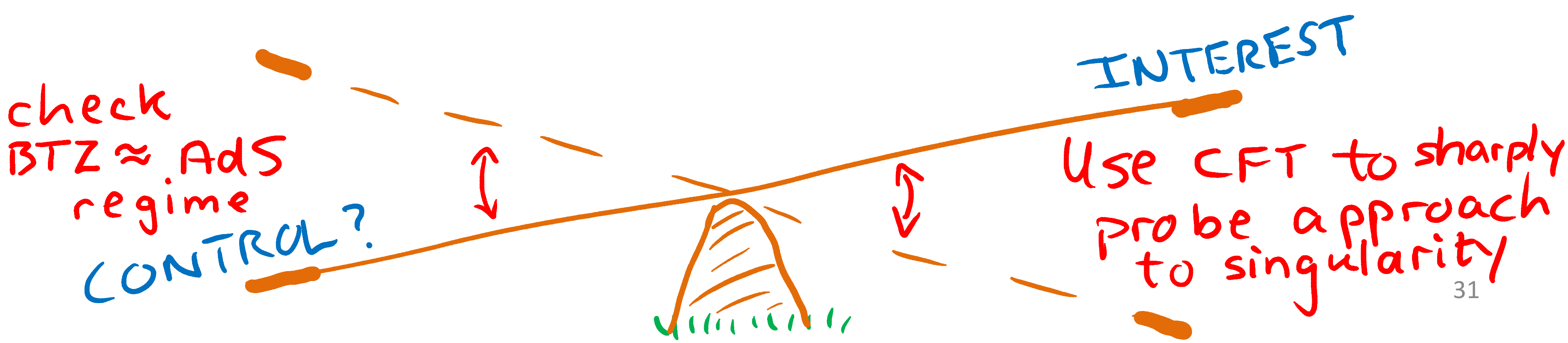
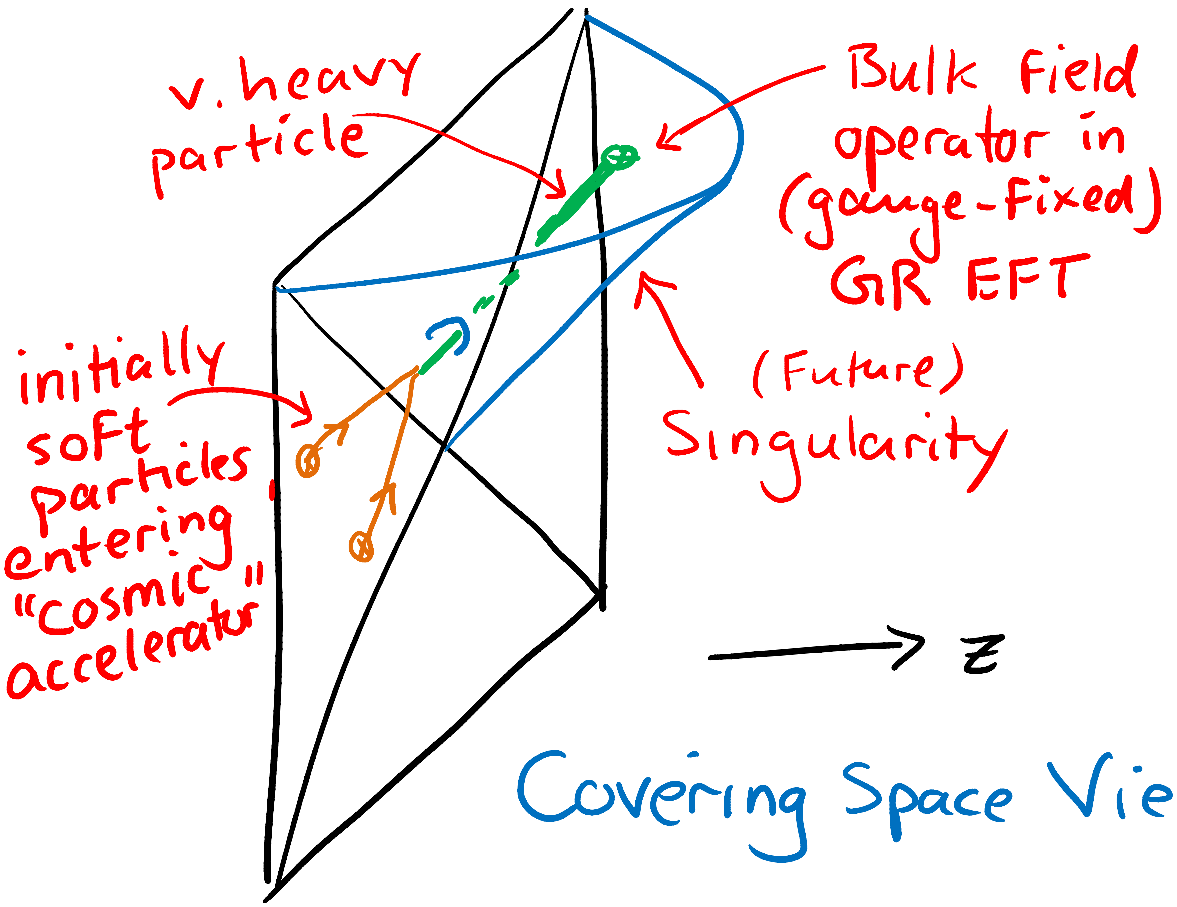


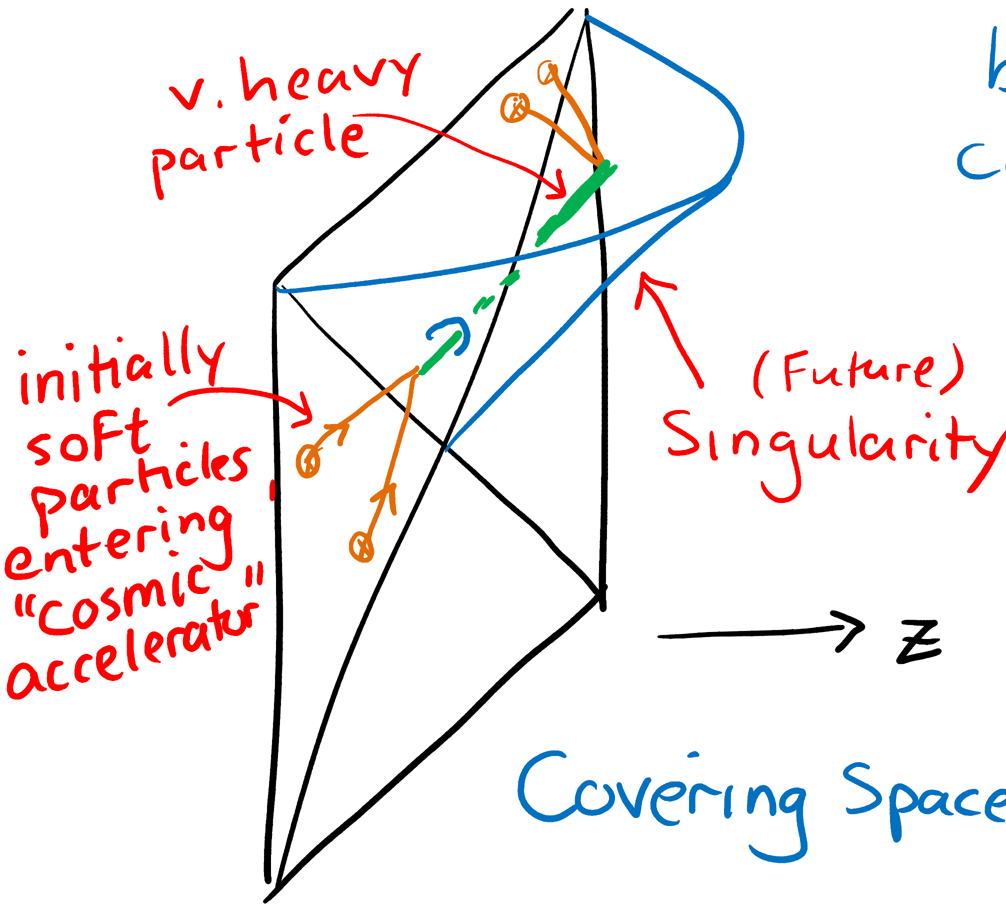
Bulk (+ boundary) Correlators ARE SENSITIVE to UV physics

but mathematically finite (again due to cancelation across singularity)



Covering Space View

BOUNDARY CORRELATORS ALONE (in $AdS_{\text{Poincare}}/\text{isometry}$) ARE INSENSITIVE to UV physics



Covering Space View

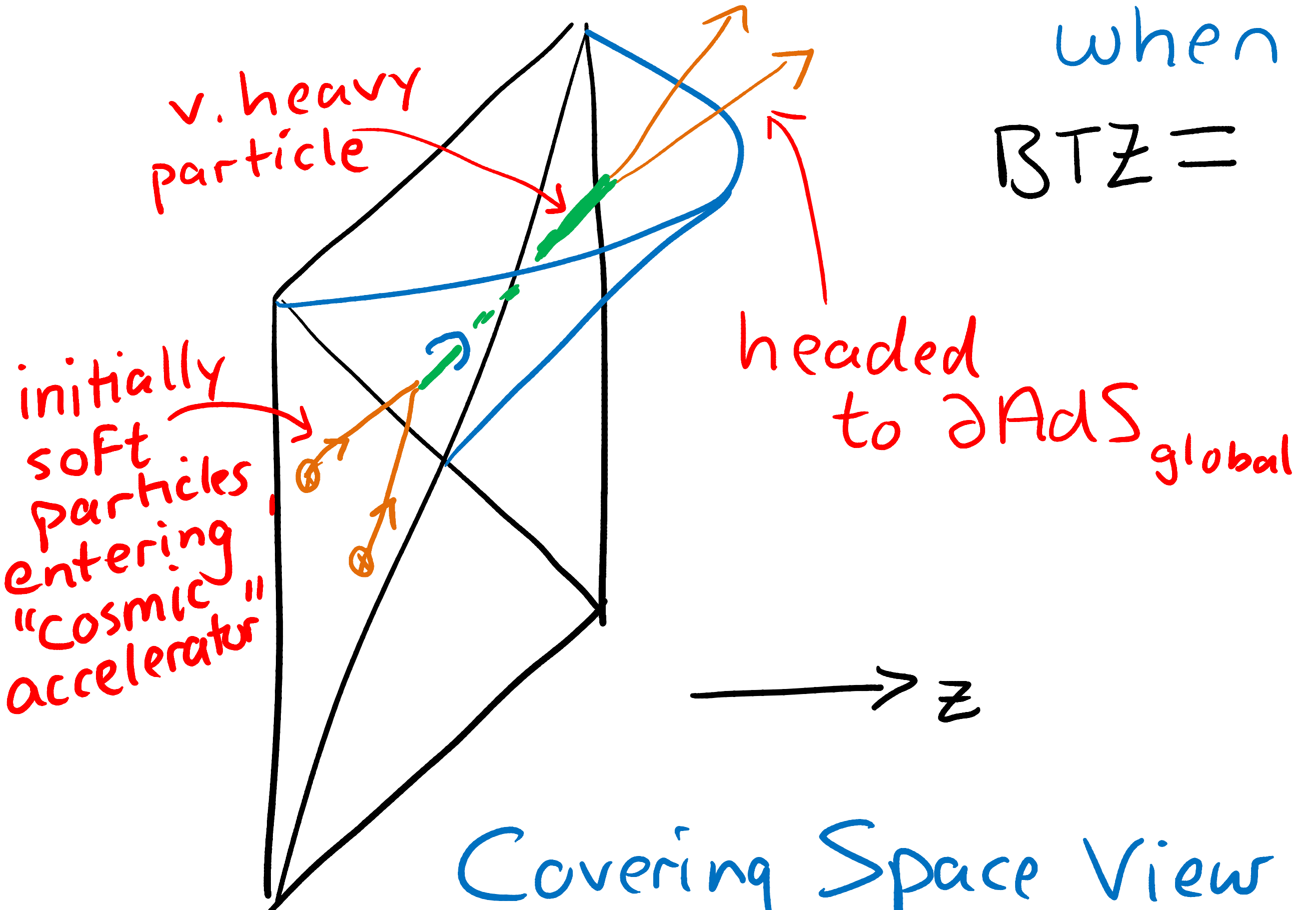
because z-momentum conserved up to soft warp factor effects (large R_{AdS}).

But UV sensitivity $\ni$ subleading corrections arising from compactness (image terms)

IN PROGRESS EXPECT Local Boundary Correlators ARE SENSITIVE to UV physics,

when

$$BTZ = AdS_{global} / \text{discrete isometry}$$



Covering Space View

CONCLUSIONS

Local ∂AdS $\equiv$ CFT correlators give non-perturbative, diffeo invariant "S-matrix" for AdS quantum gravity.

Such an "S-matrix" seems impossible in cosmological spacetimes $\ni$ bang/crunch singularities, such as inside black hole horizons

However, we have argued local ∂BTZ $\equiv$ CFT correlators do give a non-perturbative "S-matrix" for BTZ, even inside horizon, but requires boundary inside singularity.

Complementarily, = non-local CFT correlators outside horizon in standard thermofield formulation