Carving out the space of EFTs

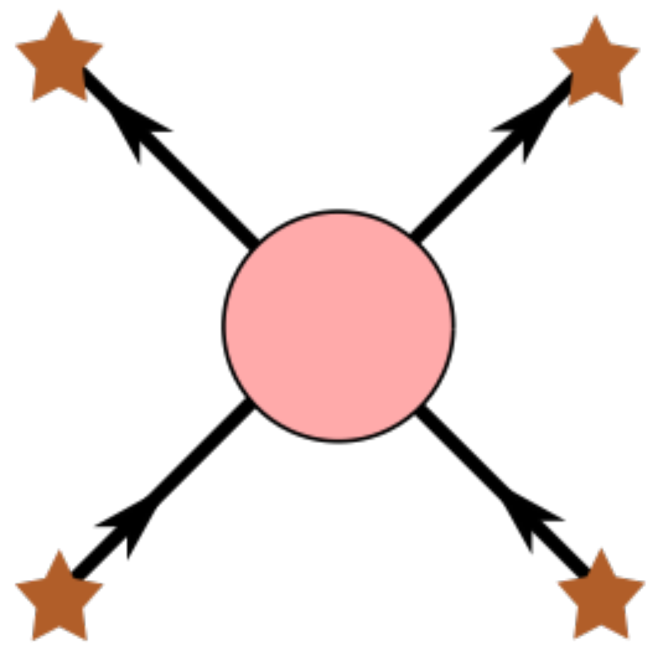
Review Talk at Strings 2021

Leonardo Rastelli
Yang Institute for Theoretical Physics, Stony Brook

Not anything goes in EFT

Effective field theory: universal framework to organize physics scale by scale

Best to define low-energy parameters from an on-shell process

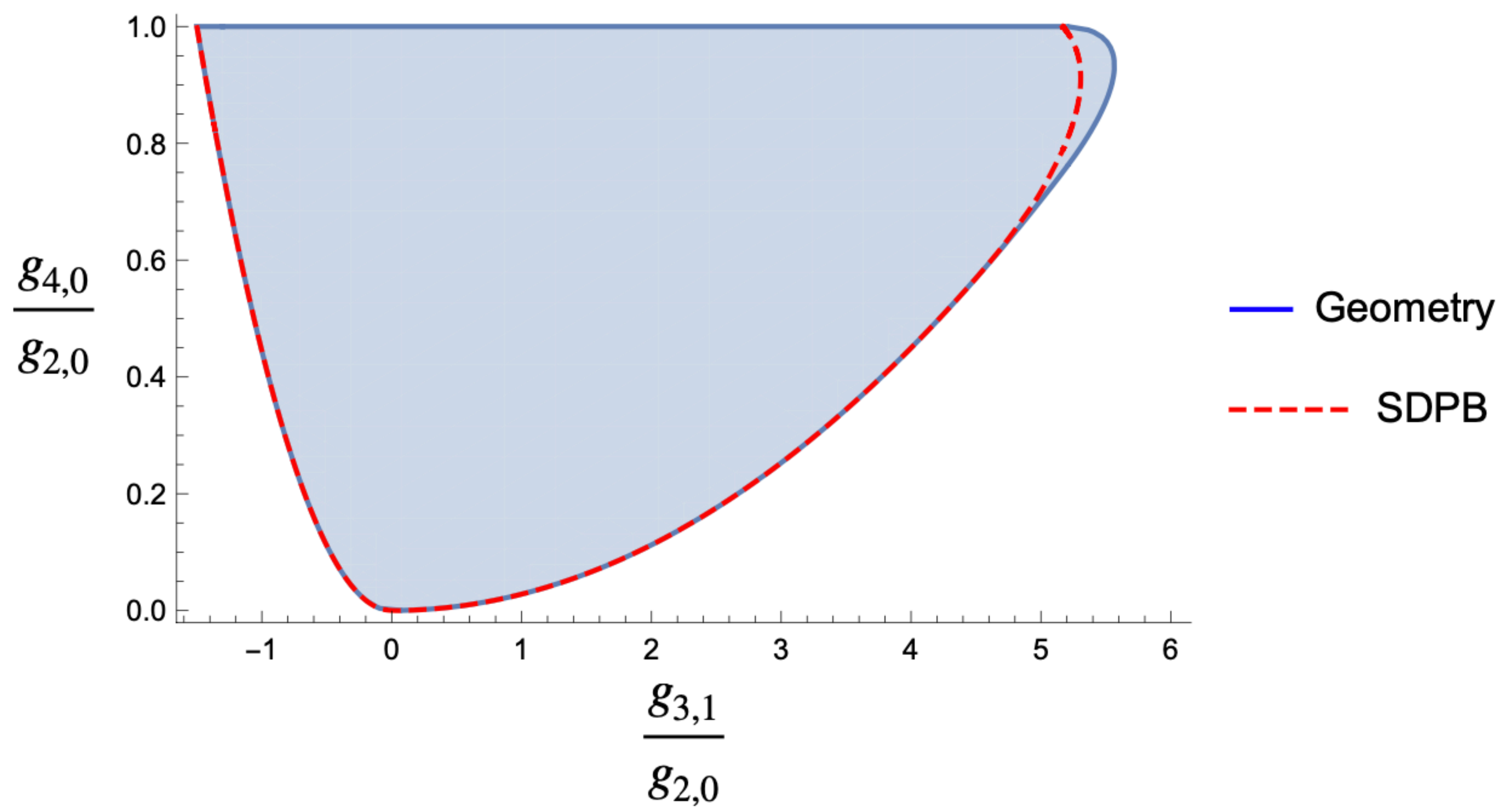


At energies ≪ EFT cut-off M,

$$\mathcal{M}_{\text{low}}(s,u) = -\lambda_3^2\left[\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right] - \lambda_4 + g_2(s^2+t^2+u^2) + g_3(stu) + \ldots$$

Are we just parametrizing our ignorance about the UV, and *anything goes* in the IR?

NO! If the EFT arises from a healthy (causal, unitary, Lorentz invariant) UV theory, low-energy parameters must obey certain inequalities.



Old wisdom from pion physics

Adams Arkani-Hamed Dubovsky Nicolis Rattazzi `06

Motivations

For $\Lambda = 0$, myriad phenomenological applications
For $\Lambda < 0$, AdS/CFT
(For $\Lambda > 0$, cosmology)

For $\Lambda \leq 0$, inclusion of gravity seems straightforward (at least superficially).
A conservative approach to quantum gravity. A quantitative swampland program.
Is string theory the unique perturbative theory of gravity?

Bootstrap approach: constrain observables (S-matrix or CFT correlator)
by general principles such as analyticity, unitarity, boundedness etc.

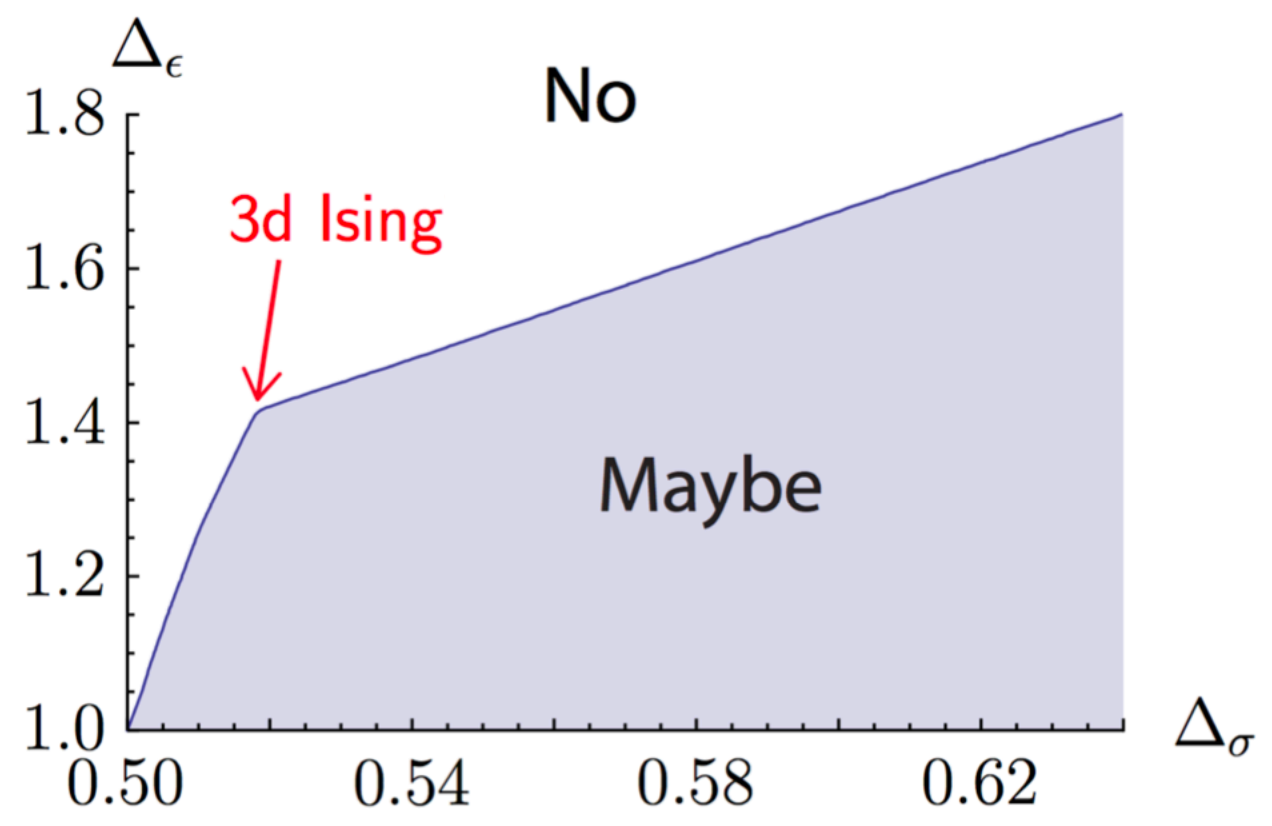
These ideas have a venerable history

Causality/analiticity connection since Kramers & Konig 1920s.
S-matrix bootstrap program for the strong interactions (Chew ...) in the 1960s.
Dual models →string theory (Veneziano 1968)

The program of systematically carving out EFT space has accelerated in recent years.

Why now?

Modern emphasis on theory space
Success story of the conformal bootstrap
AdS/CFT
Modern computational methods



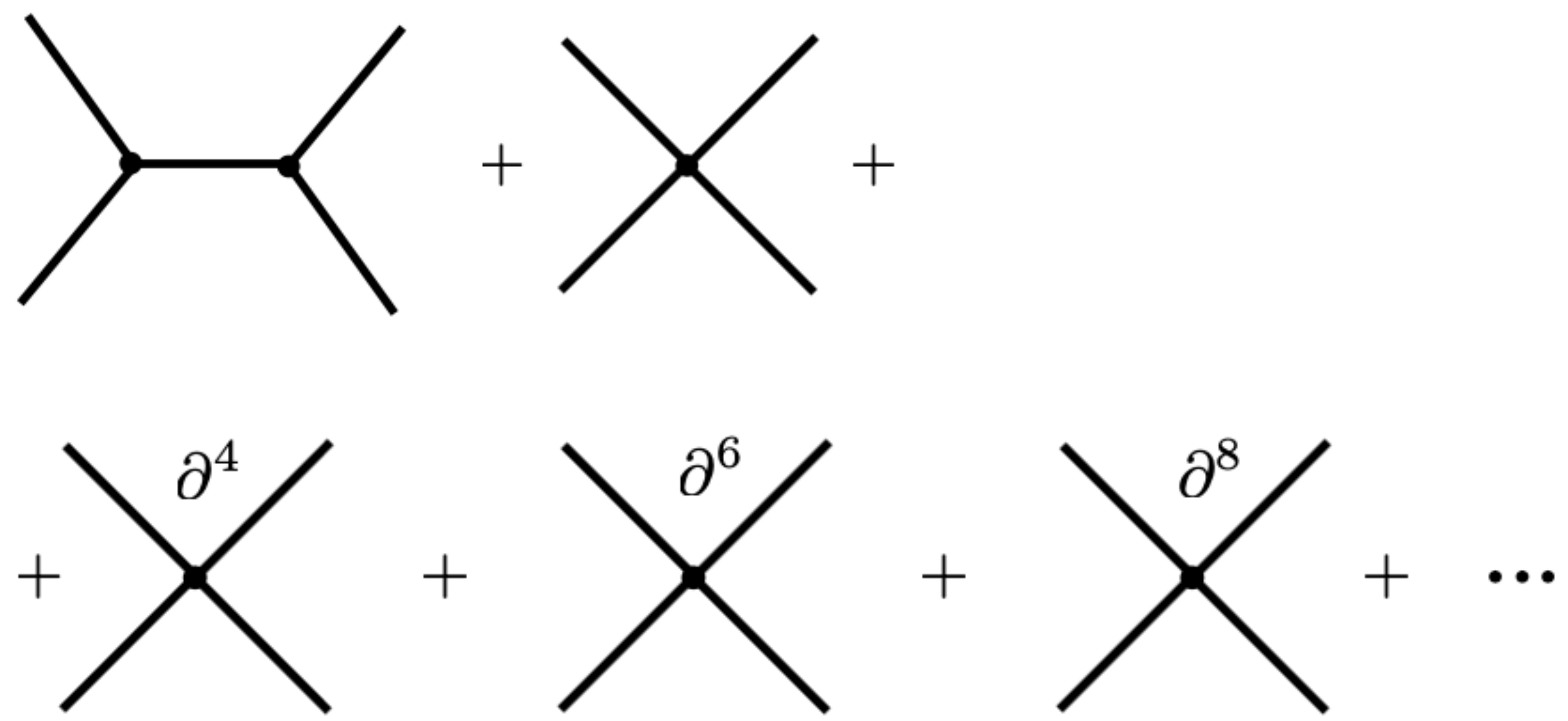
I will survey some of the progress in deriving sharp bounds for weakly coupled EFTs, both in flat space and Anti de Sitter space, and both with and without gravity.

A simple model

Massless scalar coupled to unknown massive states with energy $E \geq M$

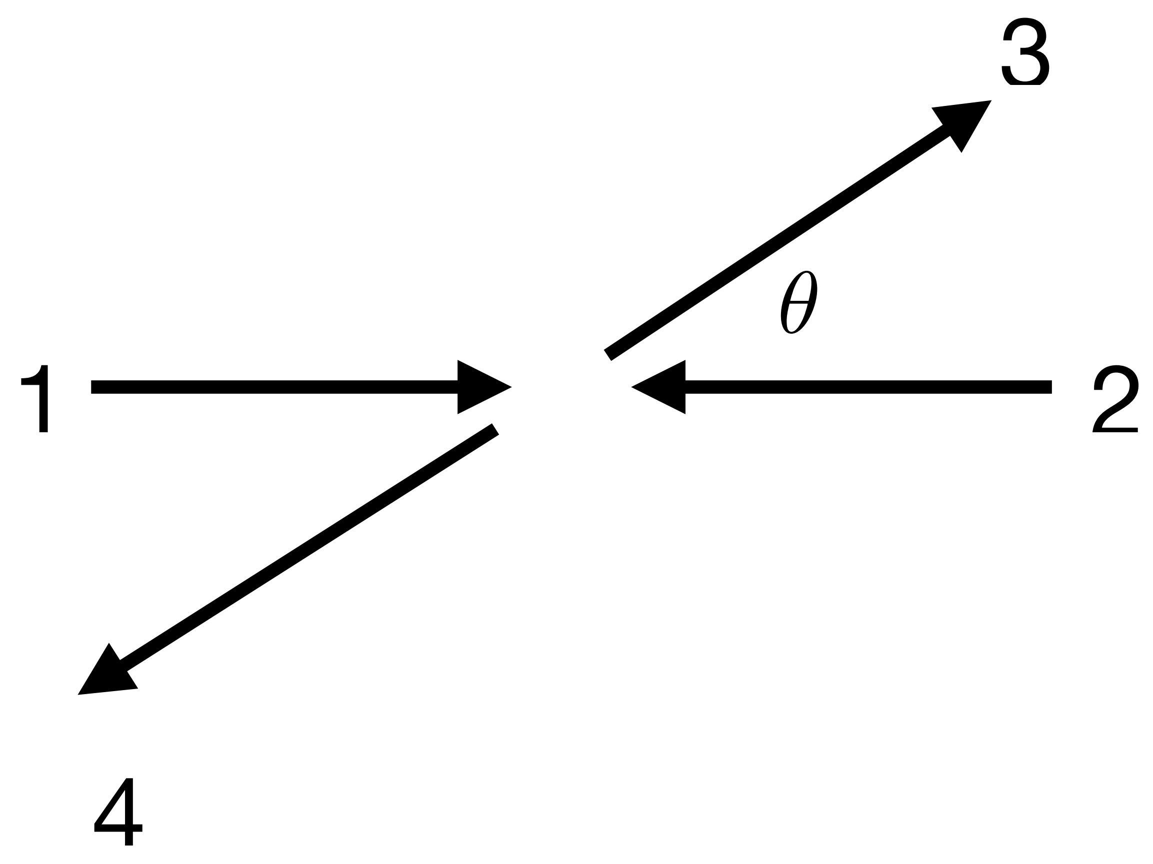
$$\mathcal{M}_{\text{low}}(s,u) = -\lambda_3^2\left[\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right] - \lambda_4$$

$$+g_2(s^2+t^2+u^2) + g_3(stu) + g_4(s^2+t^2+u^2)^2 + \ldots$$



Most general term: $(s^2+t^2+u^2)^a(stu)^b$, with $s+t+u=0$.

NB: in our conventions, $s = E^2$ and $u = -\vec{q}^{\,2} = -E^2\sin^2(\theta/2)$

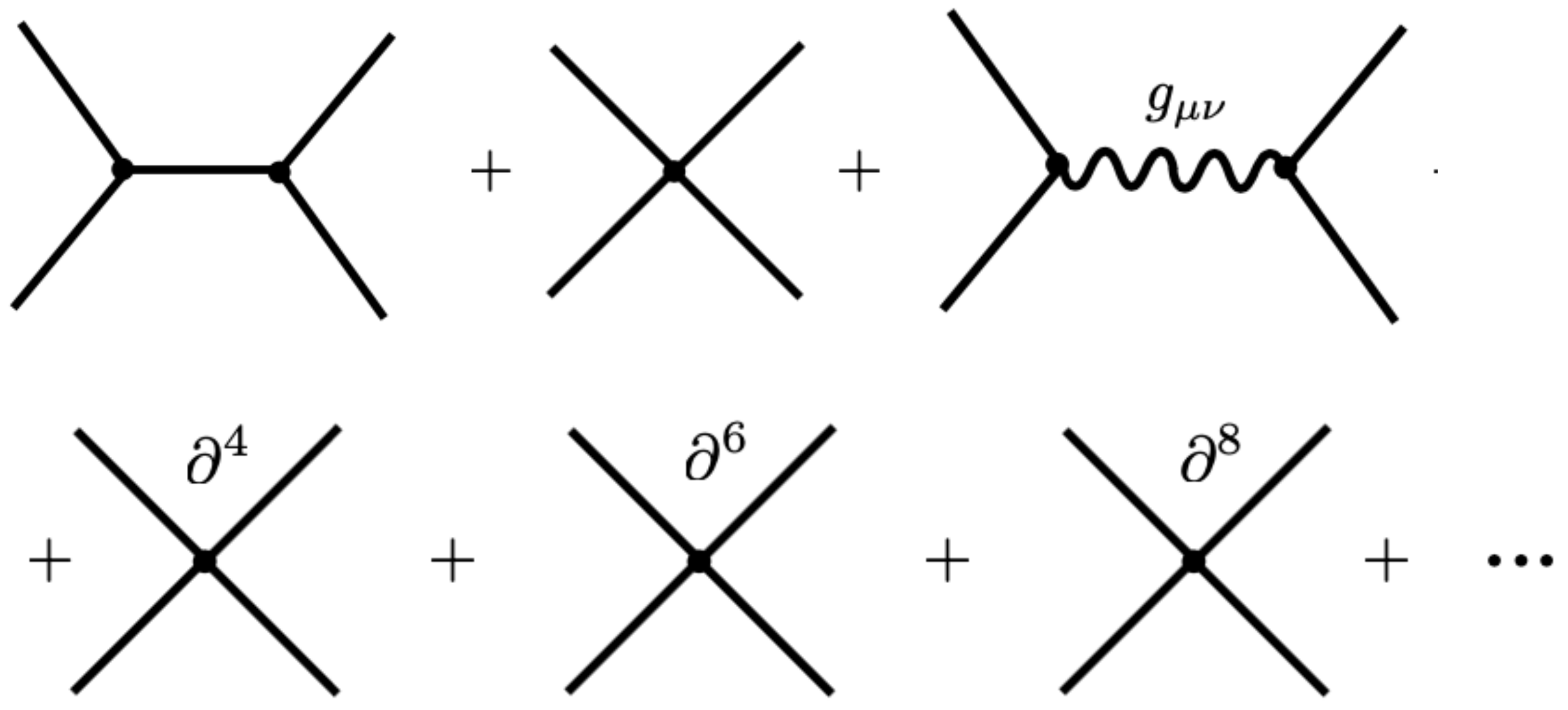


A simple model

Massless scalar coupled to gravity and to unknown massive states with energy $E \geq M$

$$\mathcal{M}_{\text{low}}(s,u) = -\lambda_3^2\left[\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right] - \lambda_4 + 8\pi G\left[\frac{st}{u}+\frac{su}{t}+\frac{tu}{s}\right]$$

$$+g_2(s^2+t^2+u^2) + g_3(stu) + g_4(s^2+t^2+u^2)^2 + \ldots$$



With gravity, need spacetime dimension $D > 4$ to avoid IR divergence from soft gravitons

A simple model

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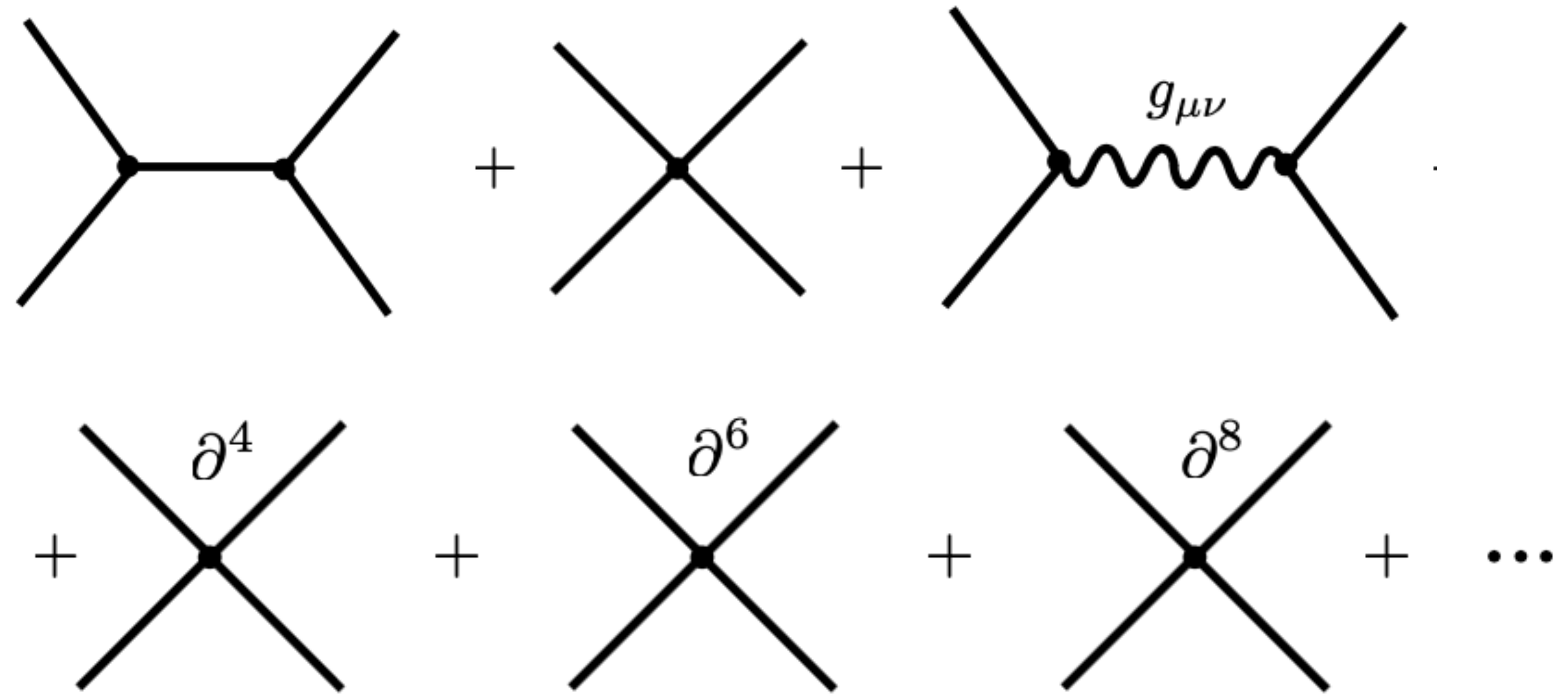
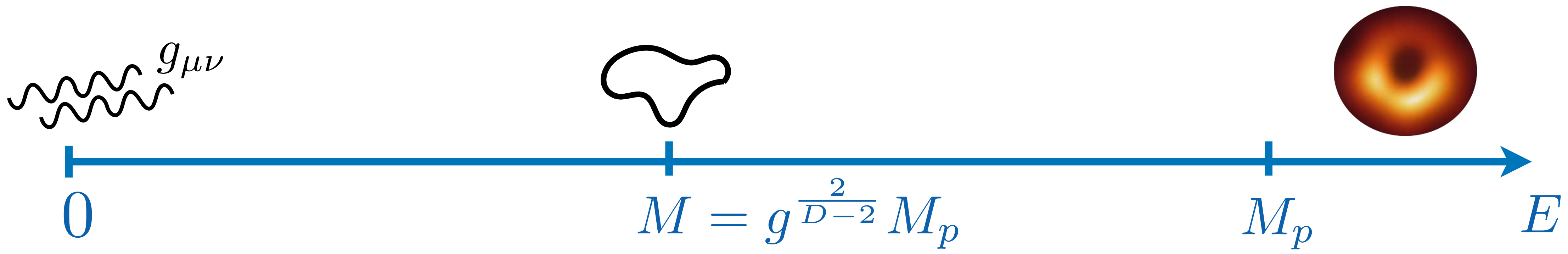


Assume EFT is weakly coupled: all low-energy couplings $\propto \epsilon \ll 1$. To leading order in ϵ: tree-level EFT

The theory *can* be strongly coupled for $E \gg M$. E.g., string theory with *fixed but small* $g_s = \epsilon \ll 1$



A simple model

$$\mathscr{M}_{\text{low}}(s,u) = -\lambda_3^2\left[\frac{1}{s}+\frac{1}{t}+\frac{1}{u}\right] - \lambda_4 \quad + 8\pi G\left[\frac{st}{u}+\frac{su}{t}+\frac{tu}{s}\right]$$

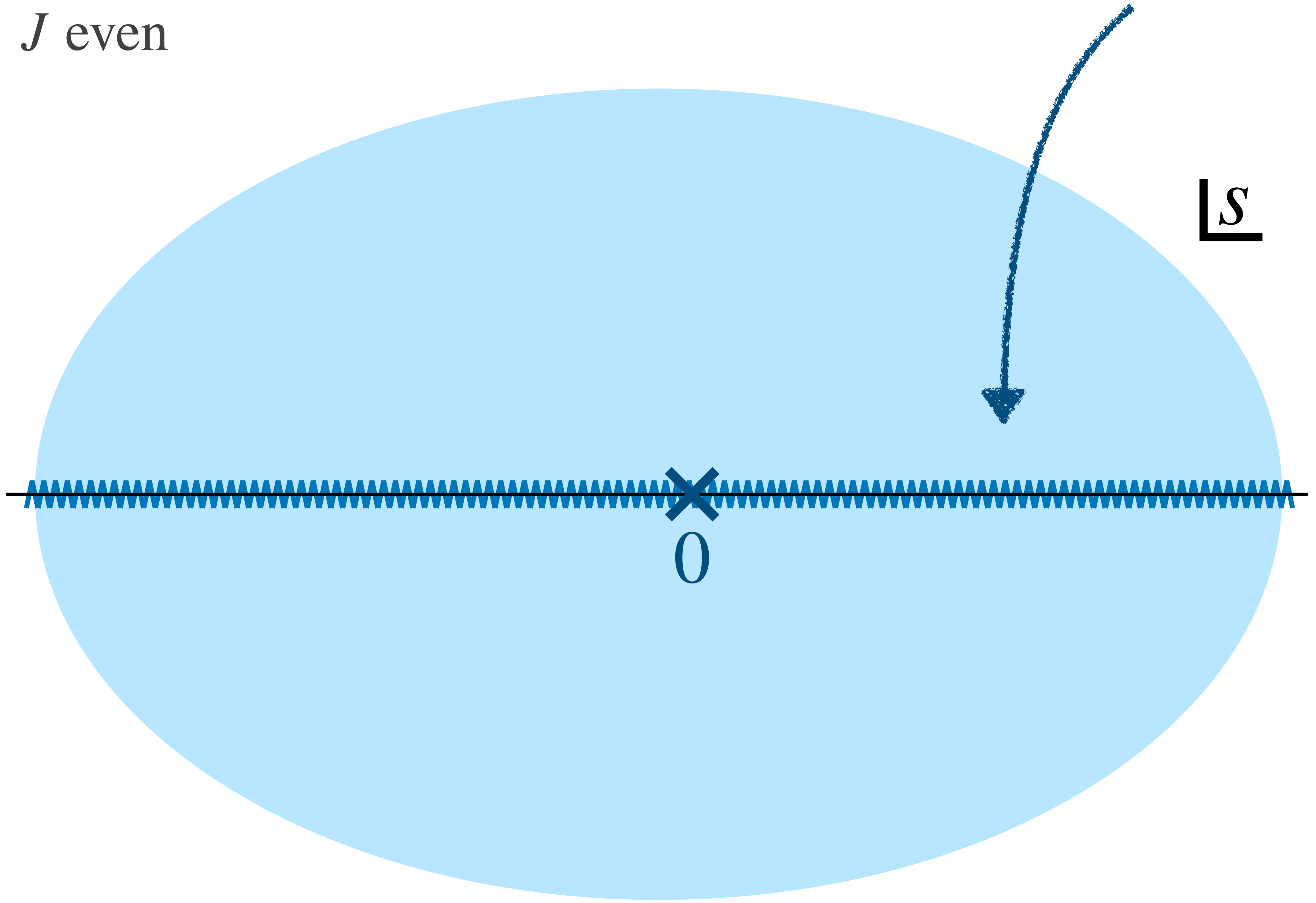
$$+ g_2(s^2+t^2+u^2) + g_3(stu) + g_4(s^2+t^2+u^2)^2 + \dots$$

Goal: derive sharp bounds for dimensionless ratios such as $\dfrac{g_n M^{2n-2}}{8\pi G}$

Some Assumptions about $\mathscr{M}$

Positive partial wave decomposition: on physical s-channel cut,

$$\mathrm{Im}\,\mathscr{M}(s,u) = \sum_{J\ \text{even}} \rho_J(s)\, P_J(\cos\theta) \qquad 0 \le \rho_J(s) \le 2$$



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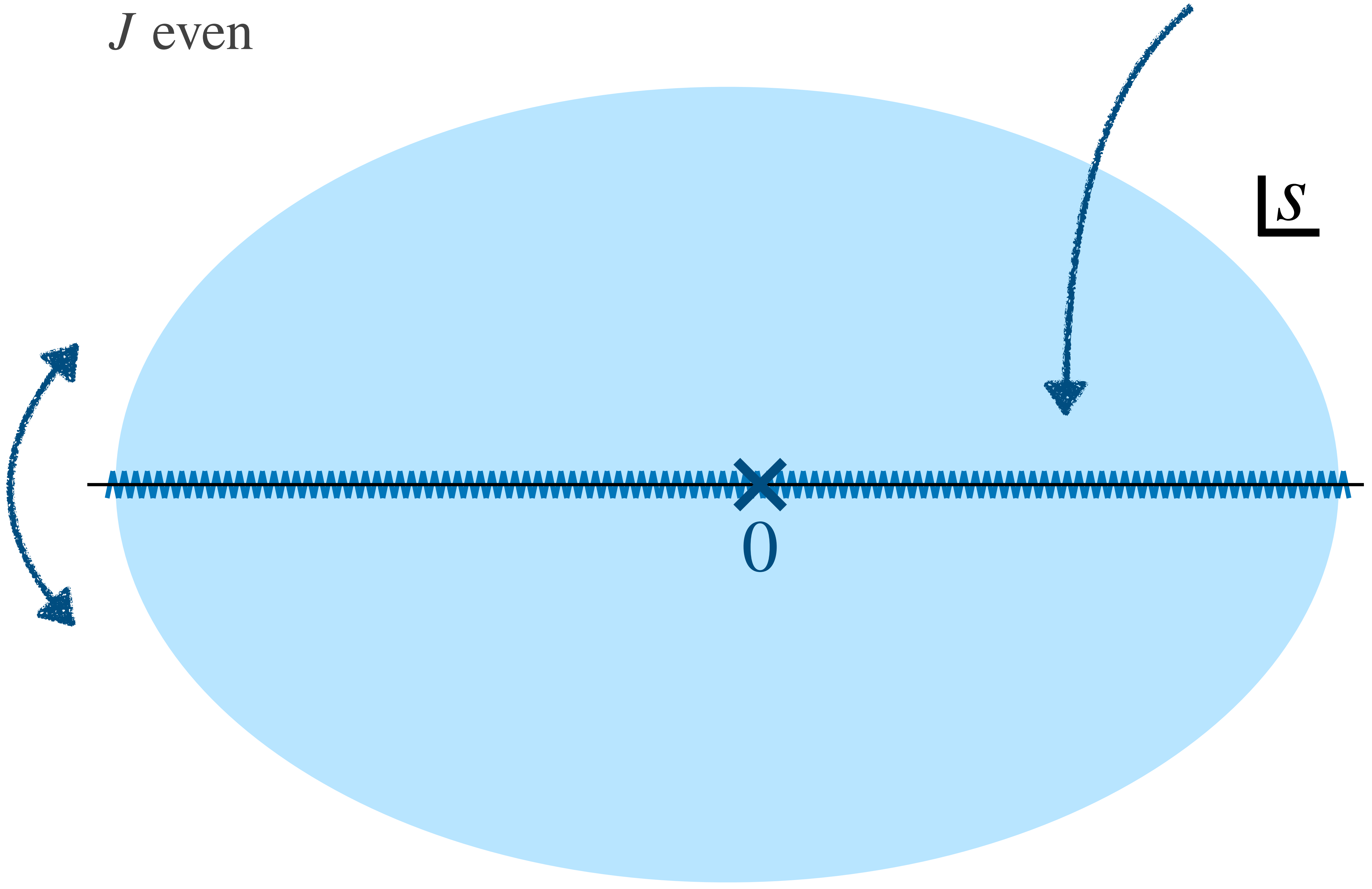
Real analyticity:

$$\mathscr{M}(s^*, u^*) = \mathscr{M}^*(s,u)$$



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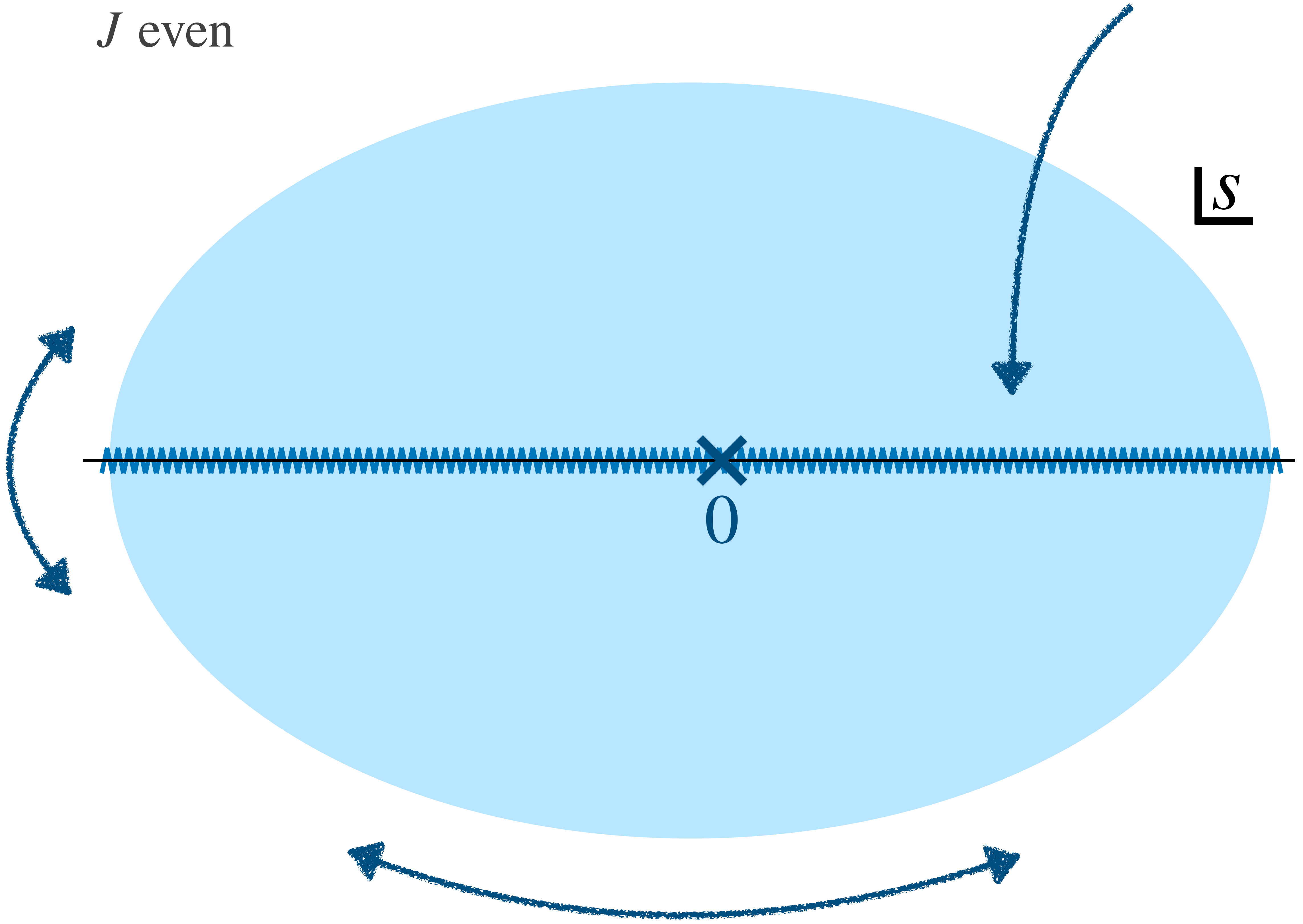
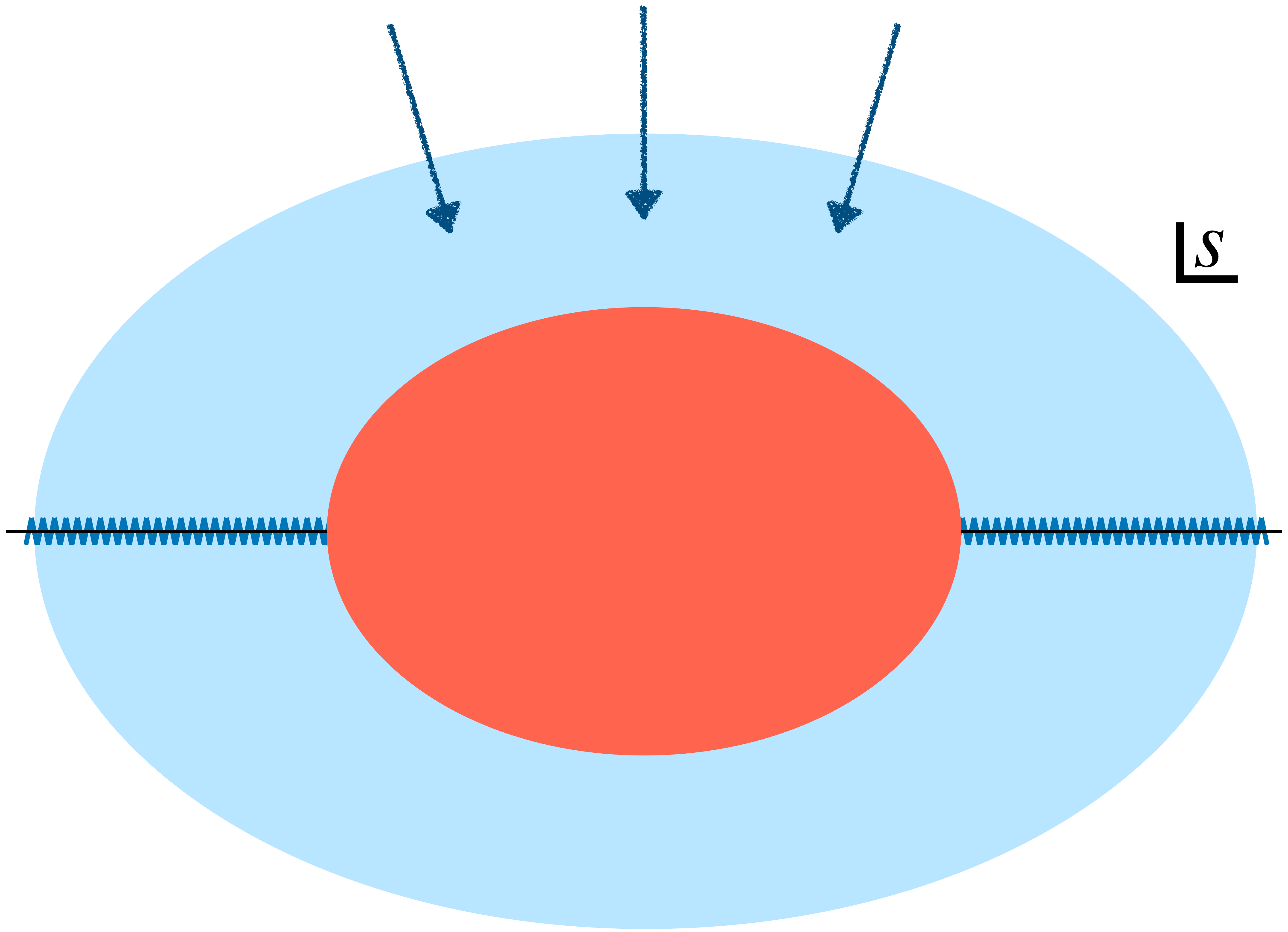


Crossing symmetry: $\mathscr{M}(s,u) = \mathscr{M}(u,s) = \mathscr{M}(t,u)$ [See Mizera's talk]

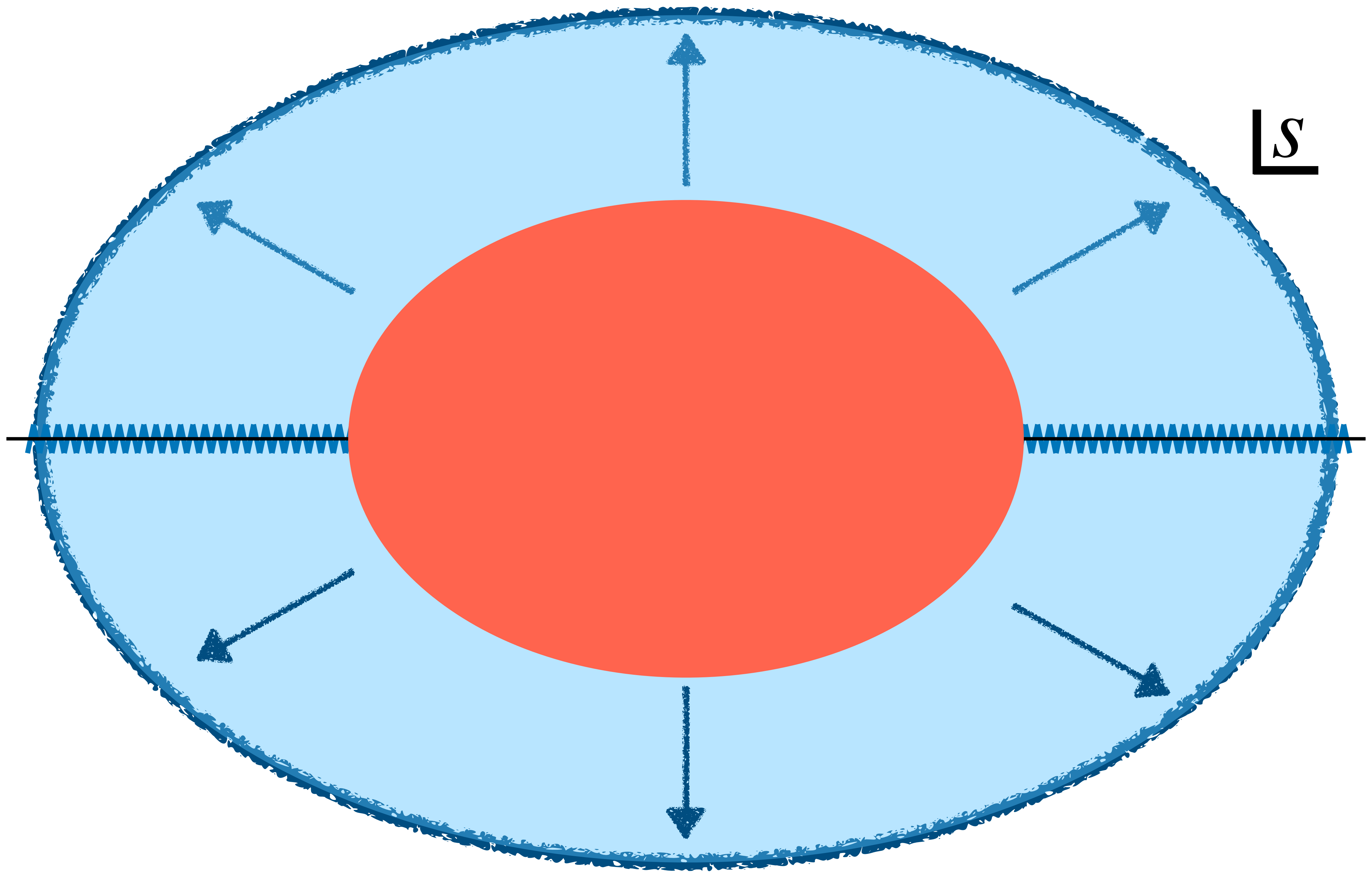
Some Assumptions about $\mathscr{M}$

Extended analyticity. Needed at least for large enough $|s|$ at fixed u



Some Assumptions about $\mathscr{M}$

(Strong) spin-2 Regge boundedness: $\lim_{|s|\to\infty} \frac{\mathscr{M}(s,u)}{s^2} = 0$ for fixed $u < 0$ along any ray



Two subtractions suffice

Caveat:
These properties have not been fully established even in ordinary QFT!

Working hypothesis:
They are conservative assumptions encoding
(asymptotic) causality and unitarity, even with dynamical gravity.

Regge Boundedness

$O(s^{2-\delta})$ Regge behavior: better than *Classical Regge Growth* $O(s^2)$

Chowdhury et al.

Chandokar Choudhury Kundu Minwalla

In tree-level string theory, from Reggeization of the graviton $\sim s^{2+\frac{\alpha' u}{2}}$

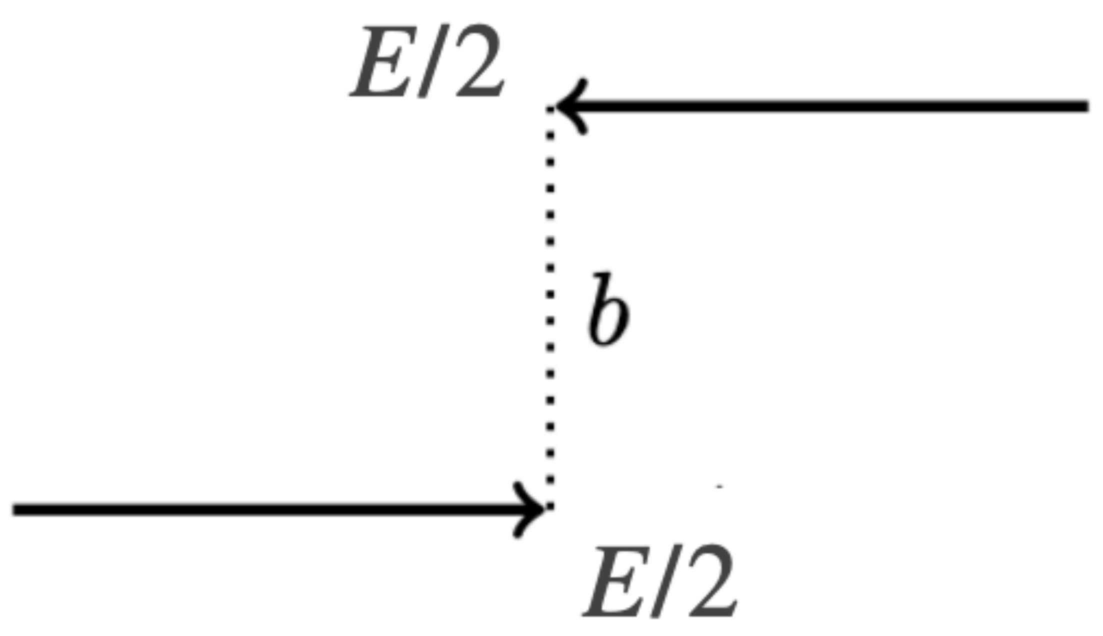
Seems safe, at least for large enough D

Impact parameter $\vec{b} \equiv$ Fourier conjugate to momentum transfer $\vec{q} \in \in \mathbb{R}^{D-2}$.

Gravity is weakly coupled for $b > (GE^2)^{\frac{1}{D-4}}$

Amati Ciafaloni Veneziano

Giddings Porto



$$b = \frac{2J}{E}$$

[See Gross-Veneziano discussion session for large energy scattering in string theory]

Regge Boundedness

$O(s^{2-\delta})$ Regge behavior: better than *Classical Regge Growth* $O(s^2)$

Chowdhury et al.

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In tree-level string theory, from Reggeization of the graviton $\sim s^{2+\frac{\alpha' u}{2}}$

Seems safe, at least for large enough D:

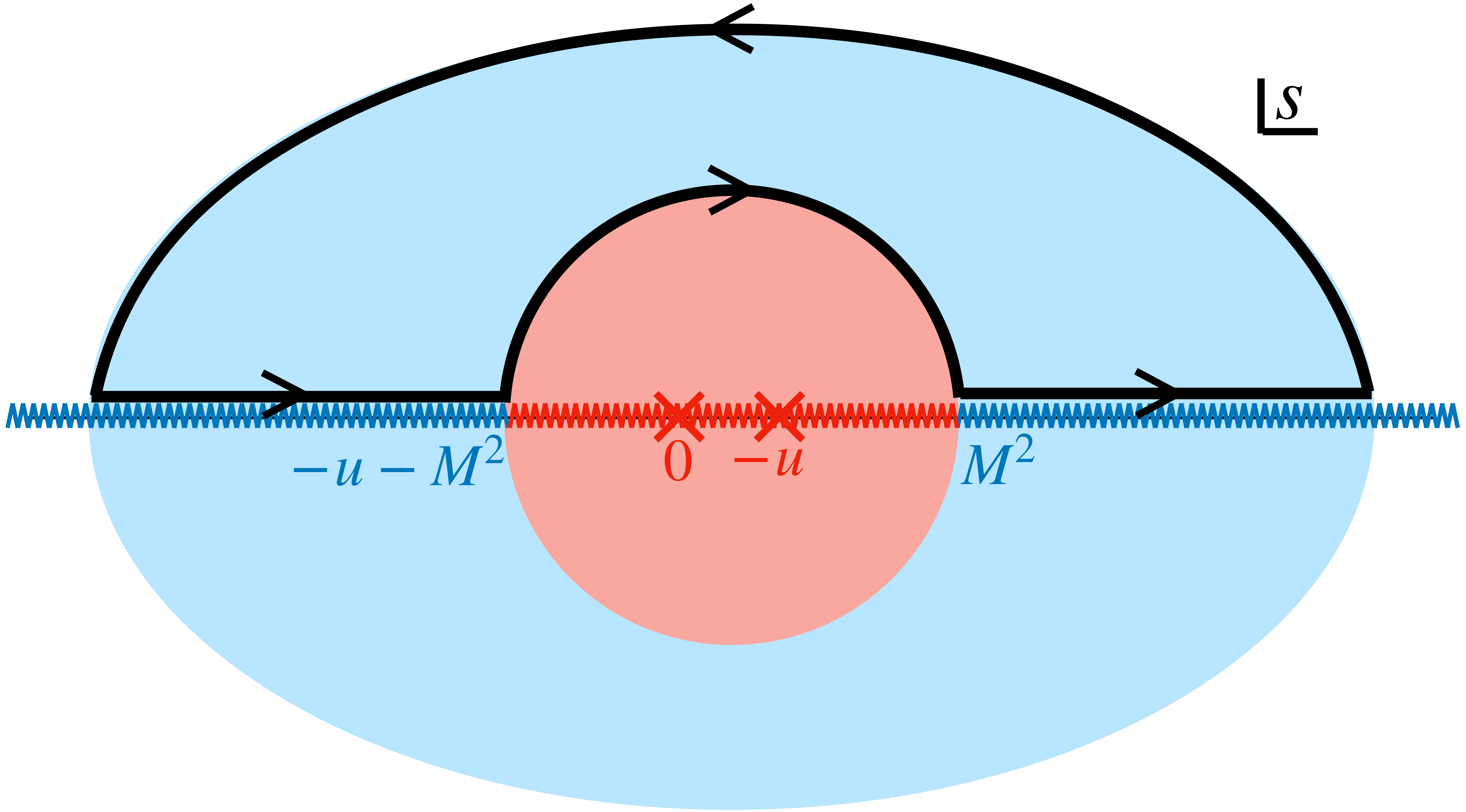
For $s \to +\infty$ on real axis,

$|\mathcal{M}(s,u)| < s^{2-\frac{D-7}{2(D-4)}}$ [Born] $|\mathcal{M}(s,u)| < s^{2-\frac{D-5}{2(D-4)}}$ [tidal+eikonal]

Extend to $s \in$ UHP by Phragmén-Lindelöf, assuming sub-exponential growth

Häring & Zhiboedov, private communication

Connect IR and UV via dispersion relation



Arkani-Hamed T-C Huang Y-t Huang

Chiang Y-t Huang Li Rodina Weng

Bellazzini Mirò Rattazzi Riembau Riva

Tolley Wang Zhou

Caron-Huot van Duong

Sinha Zahed

Recently, several equivalent systematic formalisms for $2 \to 2$ scattering that extend previous work

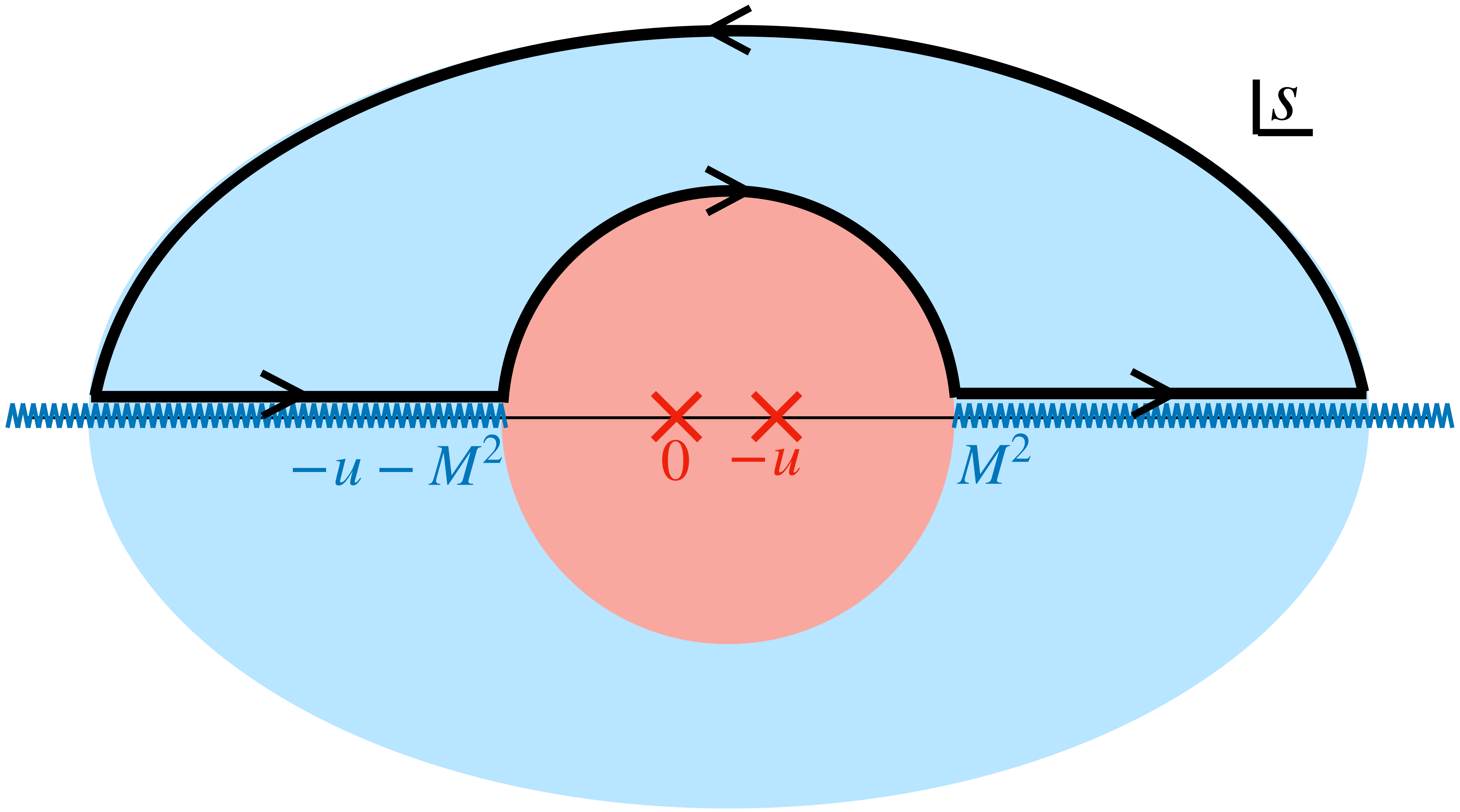
(Initiated by Adams Arkani-Hamed Dubovsky Nicolis Rattazzi `06)

Nicolis Rattazzi Trincherini

de Rham Melville Tolley Zhou

Bellazzini Vecchi

Connect IR and UV with dispersion relation



For simplicity, treat EFT at tree level: only low-energy poles

Positive sum rules for IR parameters

[Setup of Caron-Huot van Duong]

$$\oint_\infty \frac{ds'}{2\pi i}\frac{1}{s'}\frac{\mathcal{M}(s',u)}{[s'(s'+u)]^{k/2}} = 0$$ gives sum rules $\mathcal{C}_{k,u}$, for k=2, 4, … and $u<0$:

$$\mathcal{C}_{2,u}\ :\quad \frac{8\pi G}{-u} + 2g_2 - g_3 u + 8 g_4 u^2 + \ldots = \sum_{J\text{ even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2)\ F_2(J,m^2;u)$$

$$\mathcal{C}_{4,u}\ :\quad 4g_4 + \ldots = \sum_{J\text{ even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2)\ F_4(J,m^2;u)$$

where $F_k(J,m^2;u)$ are explicitly known functions and $\rho_J(m^2) \geq 0$.

k = #of subtractions: $\mathcal{C}_{k,u} \supset$ EFT interactions growing at least as $O(s^k)$ in Regge limit

Null Constraints

Low-energy $s \leftrightarrow u$ symmetry implies infinitely many null constraints on heavy data, e.g.

$$\sum_{J \text{ even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2) \, \frac{\mathcal{J}^2(2\mathcal{J}^2 - 5D + 4)}{m^8} = 0 \qquad \text{where } \mathcal{J}^2 = J(J + D - 3)$$

Relevant dimensionless combination is $JM/m \sim bM$

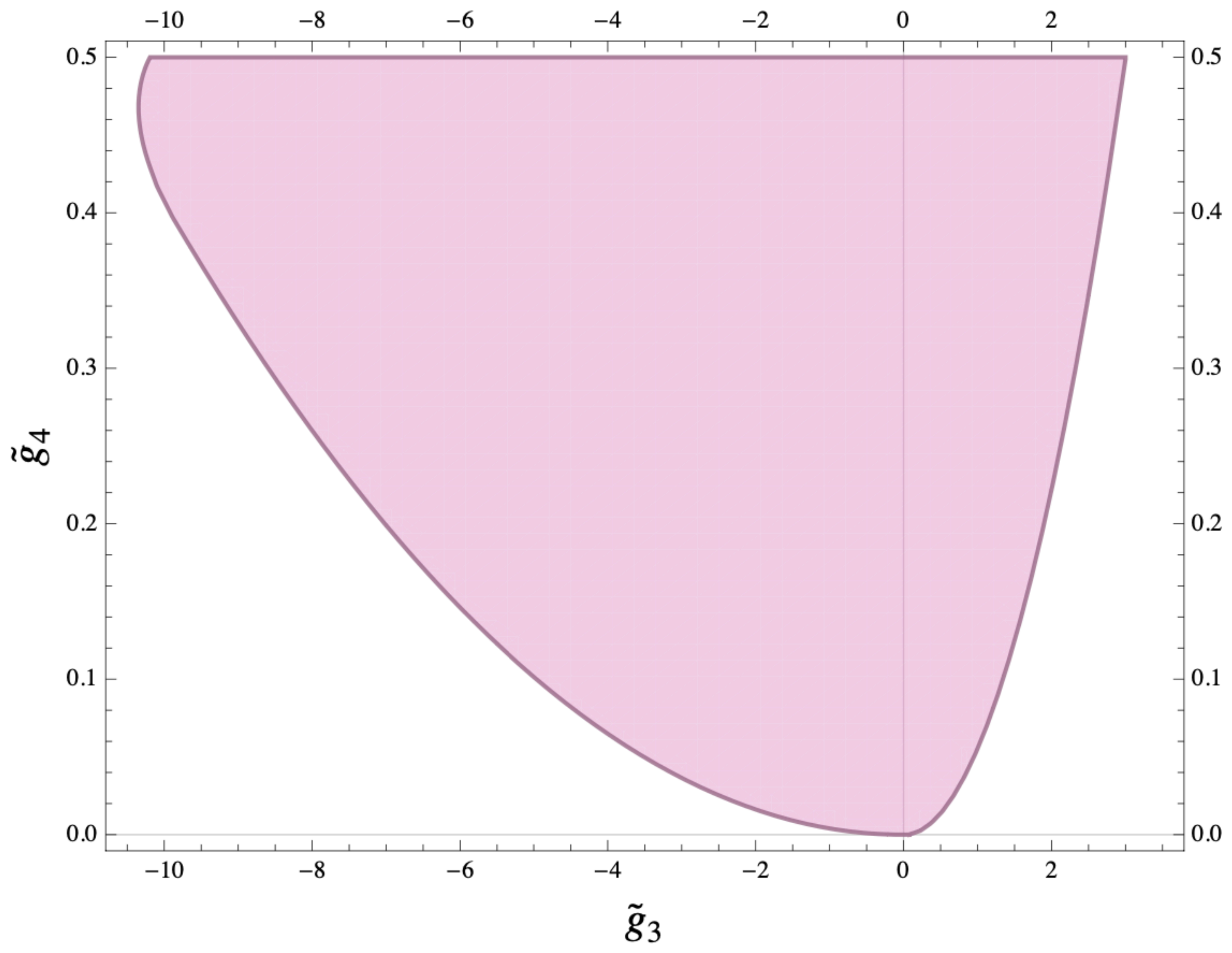
Causality implies EFT power counting

Tolley Wang Zhou

Caron-Huot van Duong

Without gravity $(G = 0)$ can Taylor expand sum rules in forward limit $u \to 0$

Carve out the space of $\{g_n\}$ using semidefinite programming



Caron-Huot van Duong

Double-sided bounds for the dimensionless ratios $\tilde{g}_n = \dfrac{g_n\, M^{2(n-2)}}{g_2}$

Theory space as a convex hull

Arkani-Hamed T-C Huang Y-t Huang

Chiang Y-t Huang Li Rodina Weng

Parametrize EFT couplings as $\mathcal{M}_{\text{low}} = \sum_{k,q} g_{k,q}\, s^{k-q} u^q$

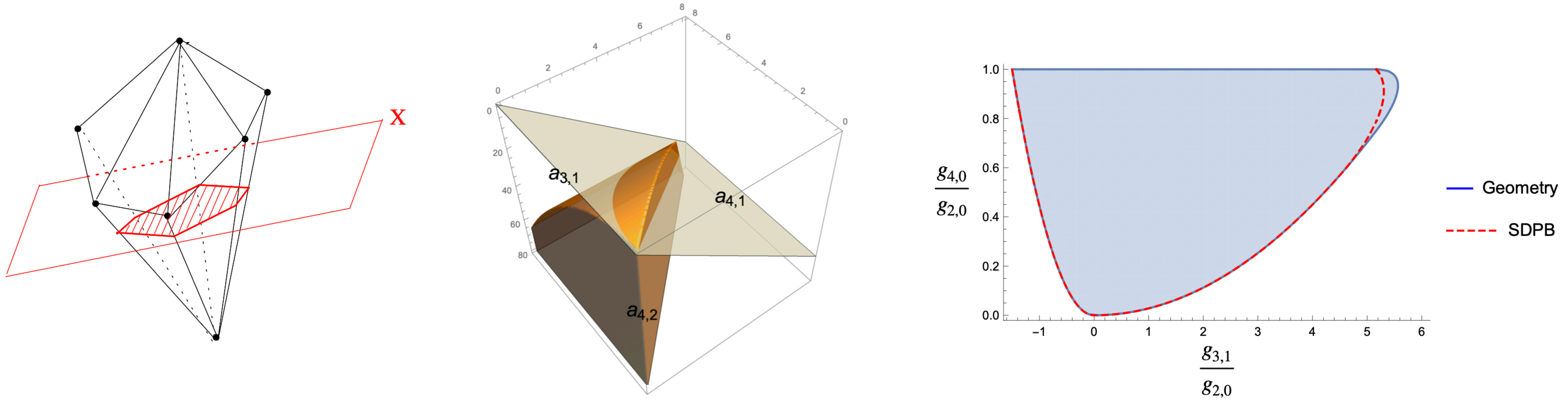
$g_{k,q} = \sum_i p_i \frac{1}{m_i^{2k+2}} X_{\ell_i,k,q}$ sum over heavy spectral data of mass m and spin ℓ, with $p_i \geq 0$

By a GL transformation $g_{k,q} \to a_{k,q}$ $\qquad a_{k,q} = \sum_i p_i \frac{1}{m_i^{2k+2}} J_i^{2q}$

Boundary of the "a-geometry" has a simple characterization in the infinite dimensional limit

$$\mathcal{M}_{\text{low}} = \sum_{k,q} g_{k,q}\, s^{k-q} u^q \qquad\qquad g_{k,q} = \sum_i p_i \frac{1}{m_i^{2k+2}} X_{\ell_i,k,q}$$

Crossing symmetry is imposed by slicing the EFThedron by symmetry planes (= null constraints)



In infinite dimensional limit, geometry agrees with semidefinite programming

Bounds with G

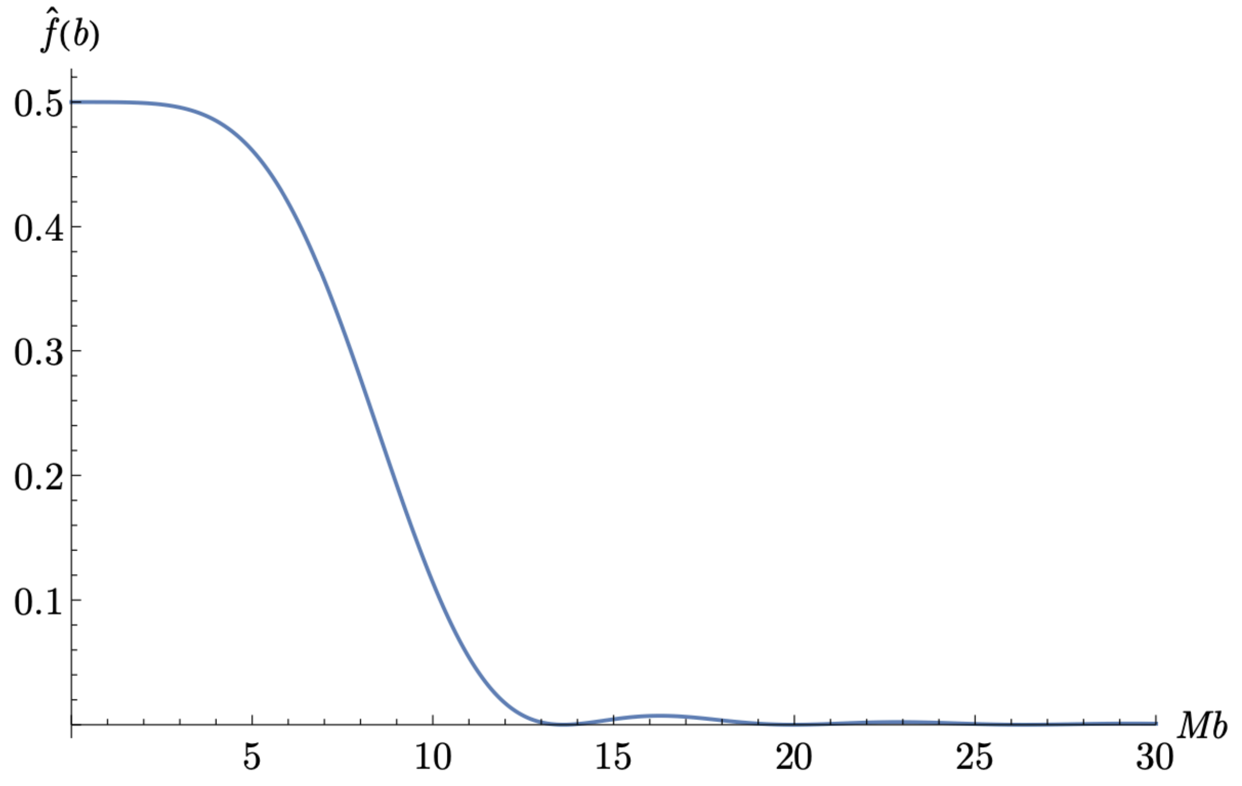
Caron-Huot Mazáč LR Simmons-Duffin

Graviton contribution to EFT $\frac{8\pi G}{-u}$ is singular in the forward limit $u \to 0$

Resolution: find improved sum rule whose LHS depends *only* on first few couplings,

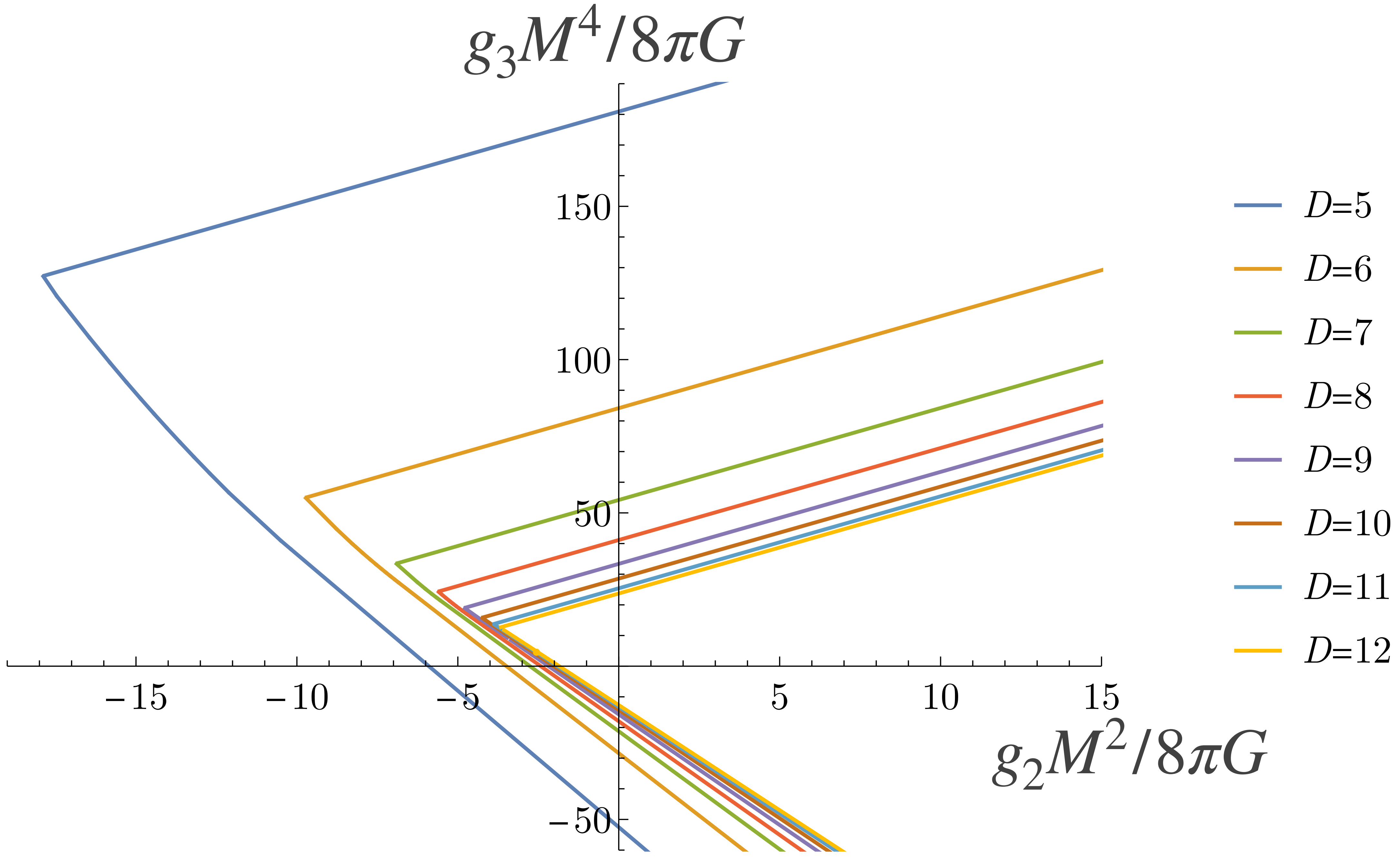
$$\frac{8\pi G}{-u} + 2g_2 - g_3 u = \sum_{J \text{ even}} \int_{M^2}^{\infty} dm^2 \rho_J(m^2)\ F_2^{\text{improved}}(J, m^2; u)$$

Then convolve with suitable $f(u)$ to derive bounds.



Physically, this amounts to measuring couplings at small impact parameter $b \lesssim 1/M$

Same kinematics as Camanho Edelstein Maldacena Zhiboedov but now with sharp bounds



$g_3 M^4/8\pi G$
150
100
50
−50
−15
−10
−5
5
10
15
$g_2 M^2/8\pi G$
D=5
D=6
D=7
D=8
D=9
D=10
D=11
D=12

Maximal sugra: graviton scattering

Caron-Huot Mazáč LR Simmons-Duffin

Factoring out helicity dependence, $\mathcal{M}_{\text{susy}}(s,u) = \dfrac{8\pi G}{stu} + g_0 + g_2(s^2+t^2+u^2) + \ldots$

Improved Regge behavior, $s^2\mathcal{M}_{\text{susy}}(s,u) \to 0 \quad \text{as} \quad s \to \infty$

$$0 \leq g_0 \leq 3.000\frac{8\pi G}{M^6} \quad \text{in } D = 10$$

All interactions $\to 0$ as $G \to 0$!

Compatible with type II string theory: $\dfrac{g_0 M^6}{8\pi G} = 2\zeta(3) \cong 2.40$

A *lower* bound for g_0 in Planck units $\dfrac{g_0 M_{\text{pl}}^6}{8\pi G} \geq c > 0$ Guerrieri Penedones Vieira

[See Penedones-Zhiboedov discussion session for more S-matrix bootstrap at strong coupling]

An application: Galileons

$$\phi(x) \to \phi(x) + b + b_\mu x^\mu$$

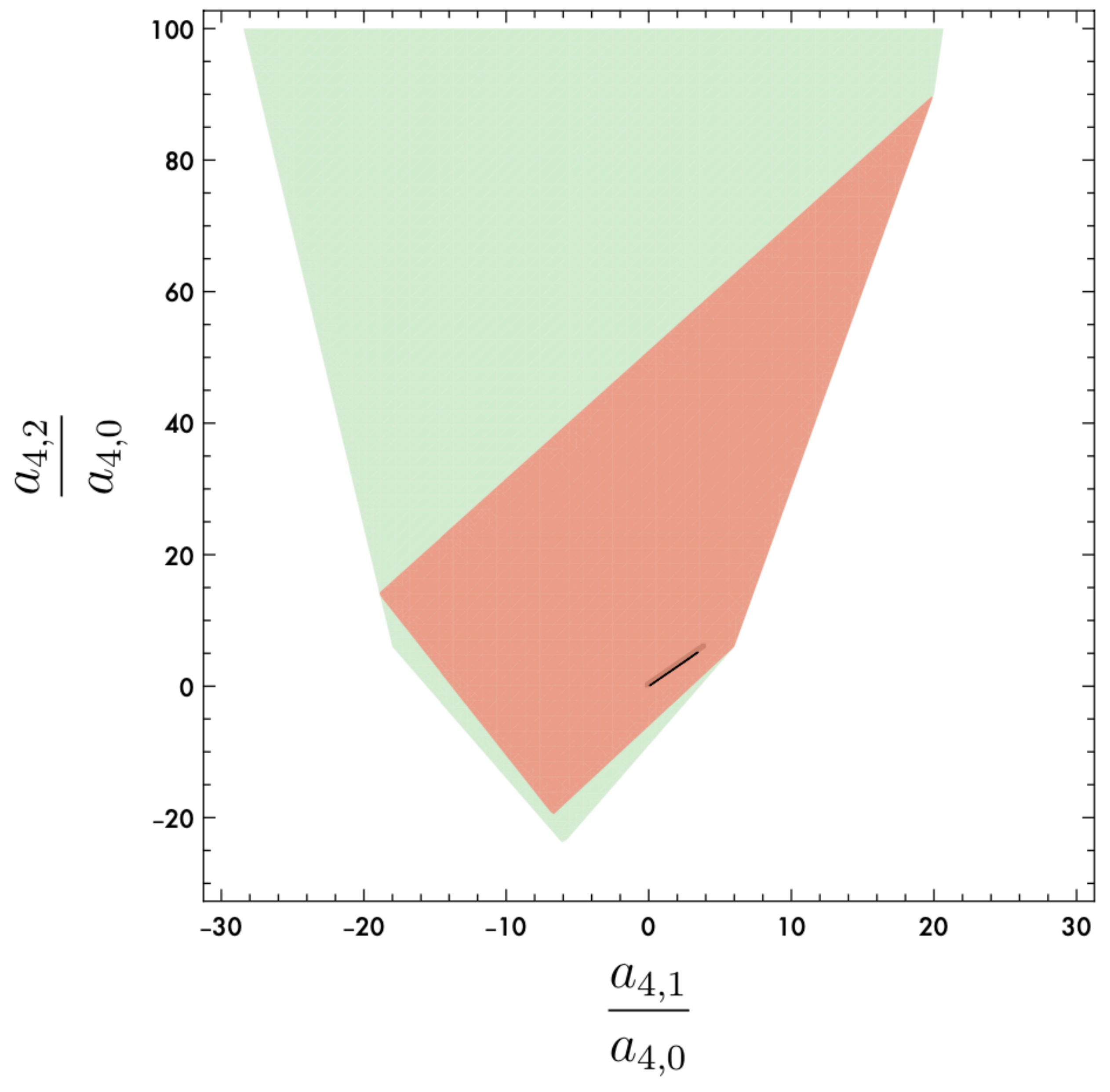
Theories with soft behavior for $\mathcal{M}$, such as with (weakly broken) Galileon symmetry, are ruled out in the sense that $m_\phi \sim$ cut-off M

Tolley Wang Zhou

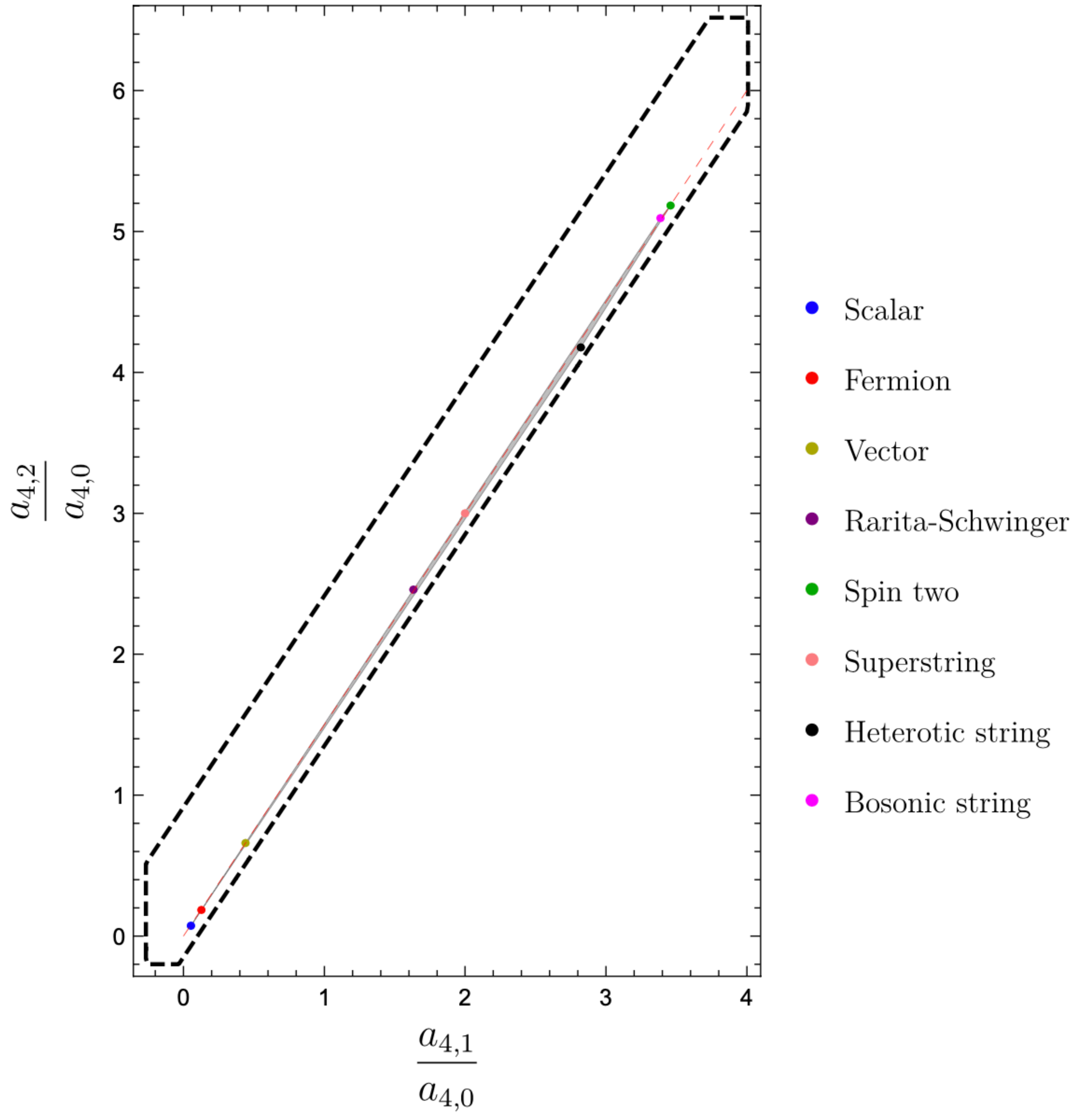
Where do actual theories sit?

Low-spin dominance

Arkani-Hamed T-C Huang Y-t Huang
Bern Kosmopoulos Zhiboedov



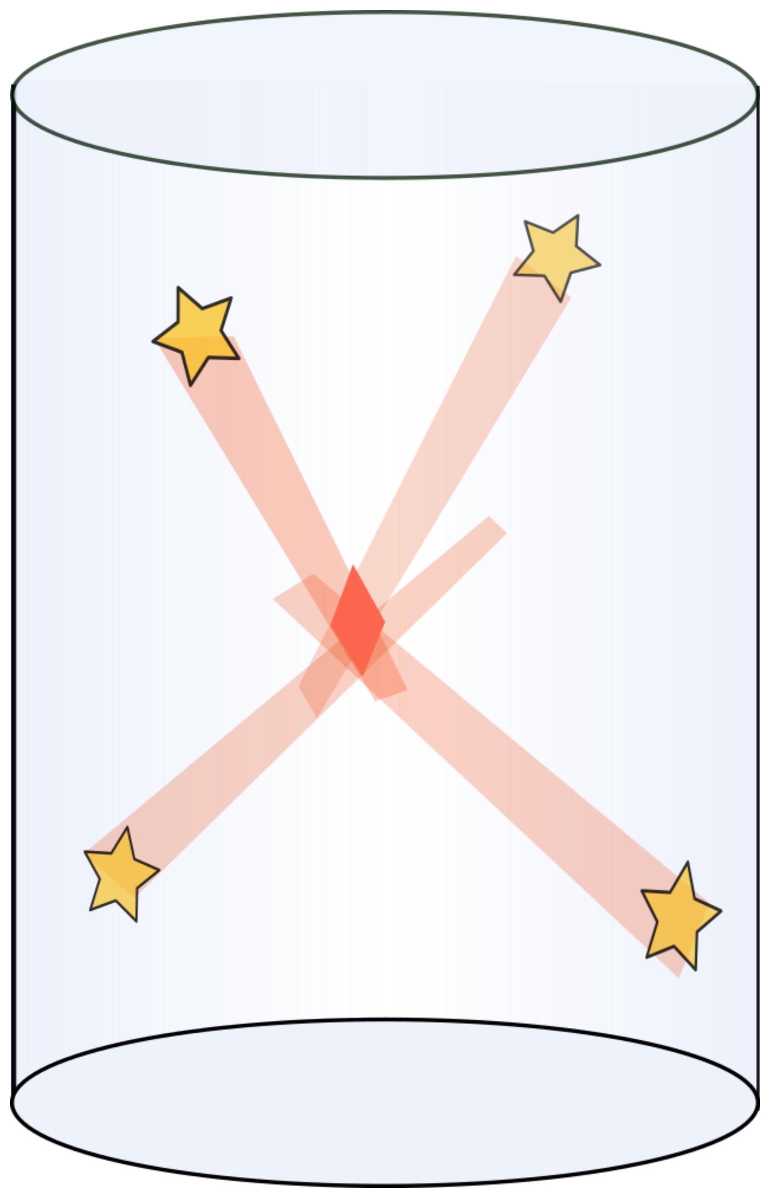
Bern Kosmopoulos Zhiboedov



AdS EFT

In purest model: graviton only state below some high scale M,

$$S_{\text{gravity}} = \frac{1}{16\pi G}\int d^D x\sqrt{-g}\,\left(-2\Lambda + \mathcal{R} + \alpha_2\mathcal{R}^2 + \alpha_3\mathcal{R}^3 + \ldots\right)$$



Assume EFT is weakly coupled at cut-off scale: $\frac{1}{R_{\text{AdS}}} \ll M \ll M_{\text{Planck}} \equiv G^{\frac{1}{2-D}}$

By power counting, expect $\alpha_n \sim 1/M^{2n-2}$.

This parametric scaling is confirmed by bulk thought experiment: large α_n lead to time *advance*

Camanho Edelstein Maldacena Zhiboedov

A corner of the conformal bootstrap, for large N CFTs with a large single-trace gap Δ_{gap}

Fully rigorous!

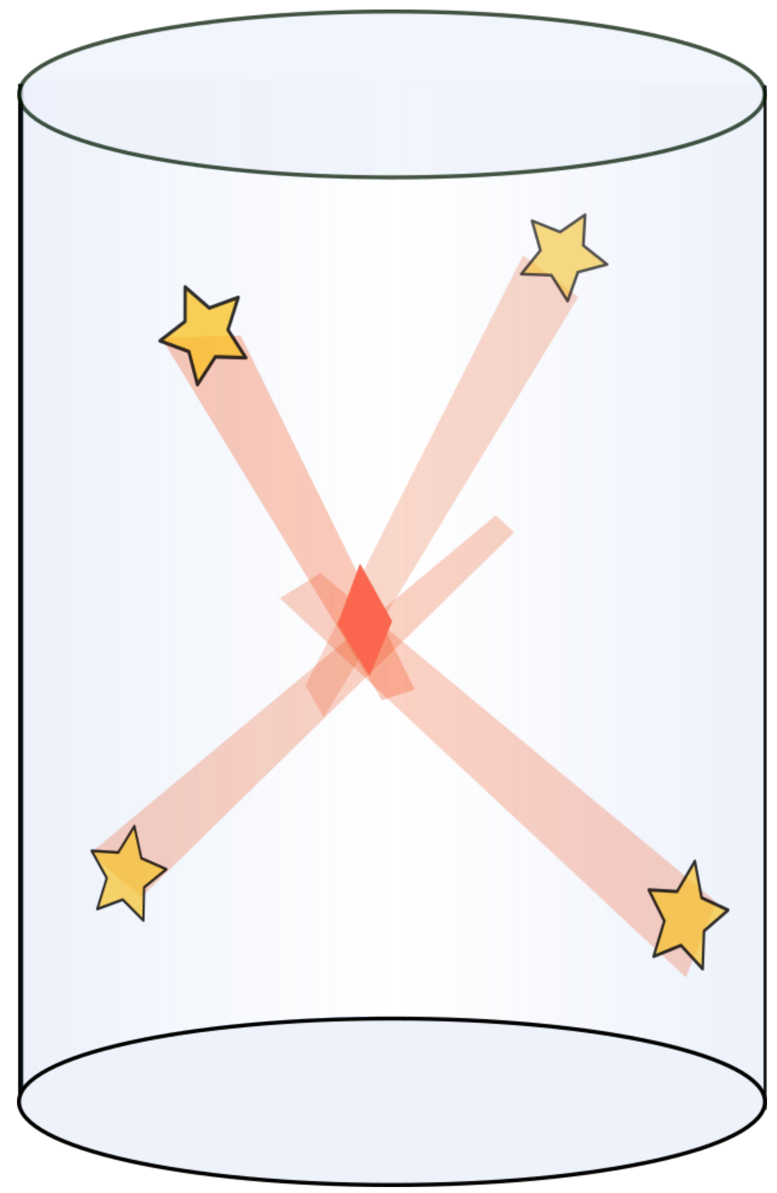
Heemskerk Penedones Polchinski Sully

Standard bootstrap methods inadequate, because OPE is polluted by double traces $\sim \mathcal{O}\,\Box^n\,\partial^J\mathcal{O}$

Right tool are *dispersive* sum rules, rooted in Lorentzian kinematics and the notion of dDisc.

For simplicity: model of a light scalar φ coupled to gravity.

$\varphi\varphi \to \varphi\varphi$ AdS ``scattering'' = CFT correlator $\langle\phi\phi\phi\phi\rangle$

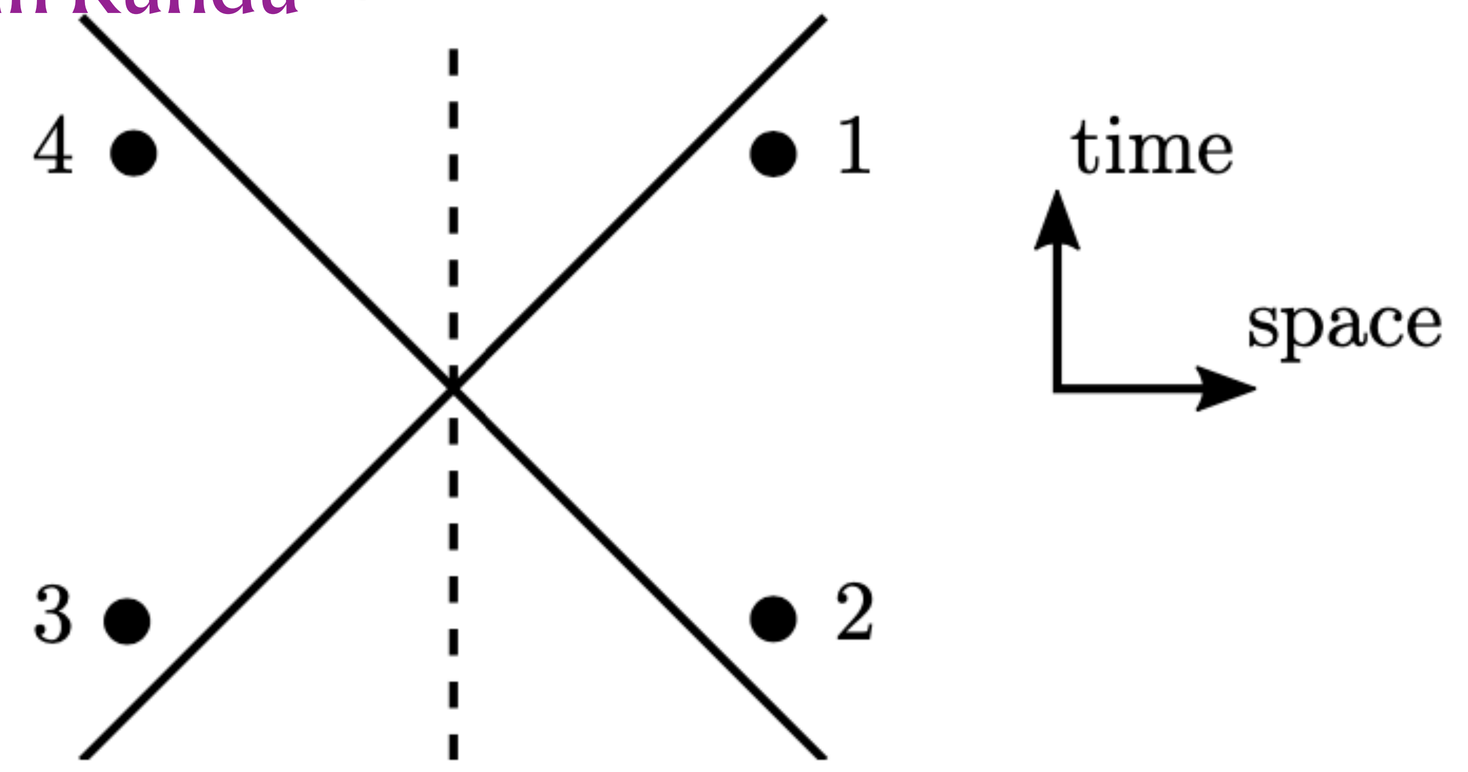


dDisc

The CFT analog of Im $\mathscr{M}$ is the double commutator (dDisc)

Caron-Huot

Hartman Jain Kundu



$$\langle \Omega | [\phi(x_1), \phi(x_2)][\phi(x_3), \phi(x_4)] | \Omega \rangle \sim \mathrm{dDisc}_s \, \mathscr{G}(z, \bar{z})$$

(Same Lorentzian kinematics as in Regge limit and in bound on chaos)

The full (subtracted) amplitude $\mathscr{M}_{\mathrm{sub}}$ is reconstructed from Im $\mathscr{M}$ on the s- and t-channel cuts.

The full (subtracted) correlator G_{sub} is reconstructed by from dDisc_s and dDisc_t . Carmi Caron-Huot

Crucially, dDisc annihilates intermediate double-traces, $\mathrm{dDisc}_s \, G^s_{2\Delta_\phi+2n+J,J} = 0$,

where $G^s_{\Delta,J}$ is the conformal block.

All CFT dispersion relations are equivalent

Caron-Huot Mazáč LR Simmons-Duffin

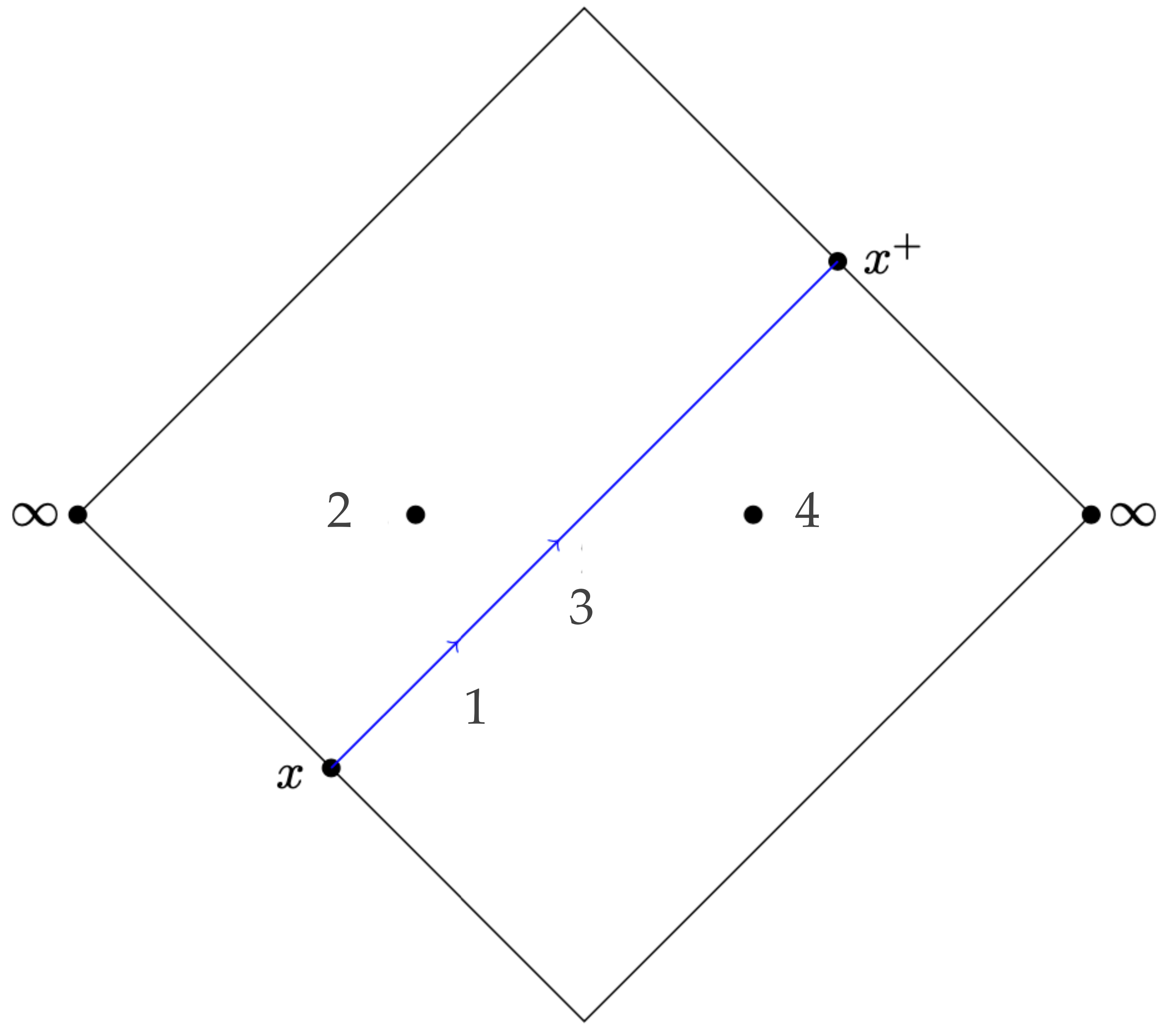
- Analytic functionals   Mazáč, Mazáč Paulos, Mazáč LR Zhou
- Mellin space dispersion   Penedones Silva Zhiboedov
- Position space dispersion   Carmi Caron-Huot
- Lightrays and superconvergence relations   Kologlu Kravchuk Simmons-Duffin Zhiboedov
- Fully crossing symmetric Polyakov-Mellin bootstrap   Gopakumar Sinha Zahed
- Momentum space   Meltzer

Dispersive sum rules from lightrays

Causality: $\langle \Omega | \phi(x_4)[\phi(x_1), \phi(x_3)]\phi(x_2) | \Omega \rangle = 0$ for $x_1 - x_3$ spacelike

Integrate x_1 and x_3 along spacelike separated null rays, with some kernel $f(x_1, x_3)$:

$$0 = \int_{-\infty}^{\infty} dx_1^+ \int_{-\infty}^{\infty} dx_3^+ \, f(x_1, x_3)\langle \Omega | \phi(x_4)\phi(x_3)\phi(x_1)\phi(x_2) | \Omega \rangle$$
$$- \int_{-\infty}^{\infty} dx_1^+ \int_{-\infty}^{\infty} dx_3^+ \, f(x_1, x_3)\langle \Omega | \phi(x_4)\phi(x_1)\phi(x_3)\phi(x_2) | \Omega \rangle$$



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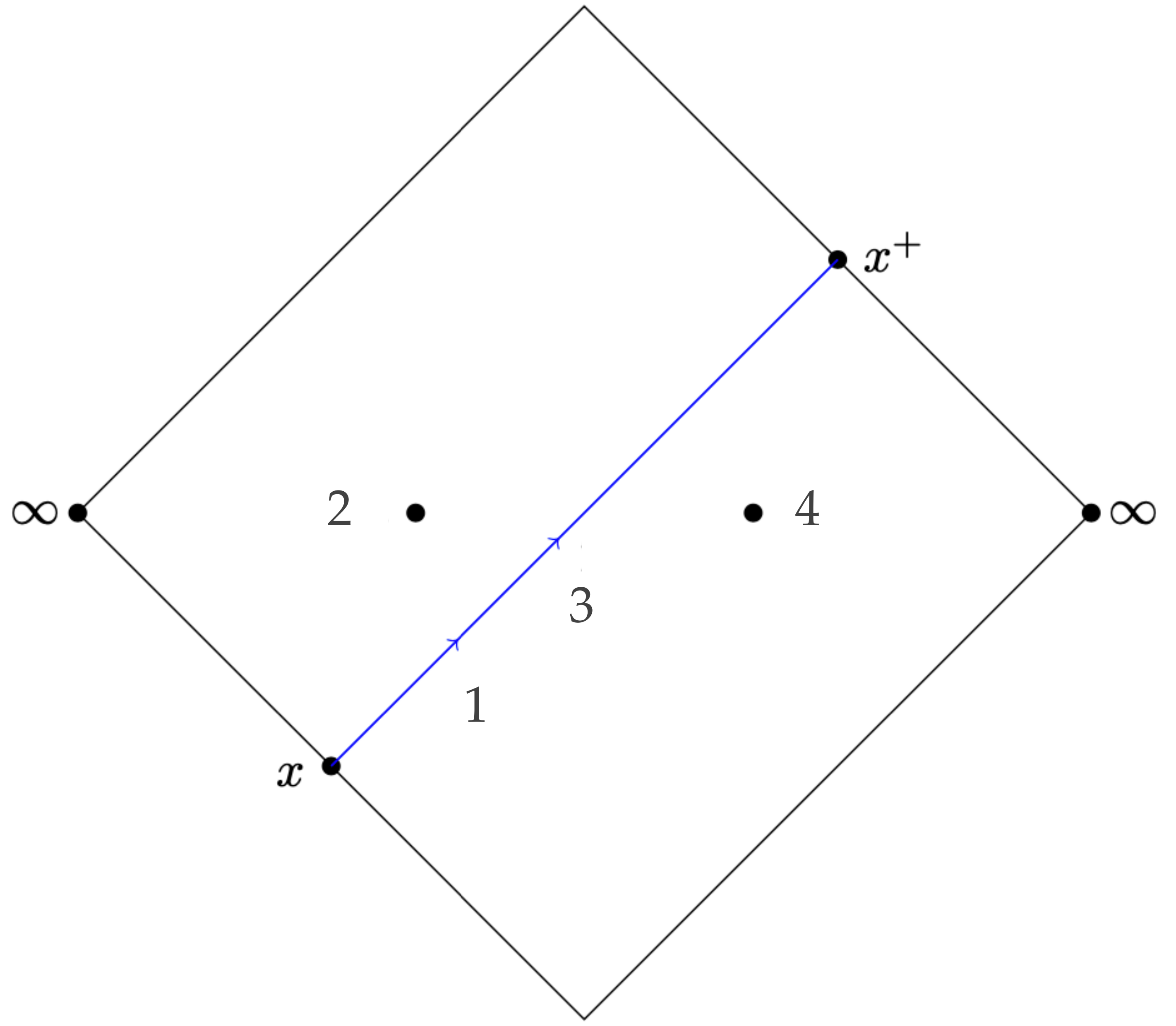
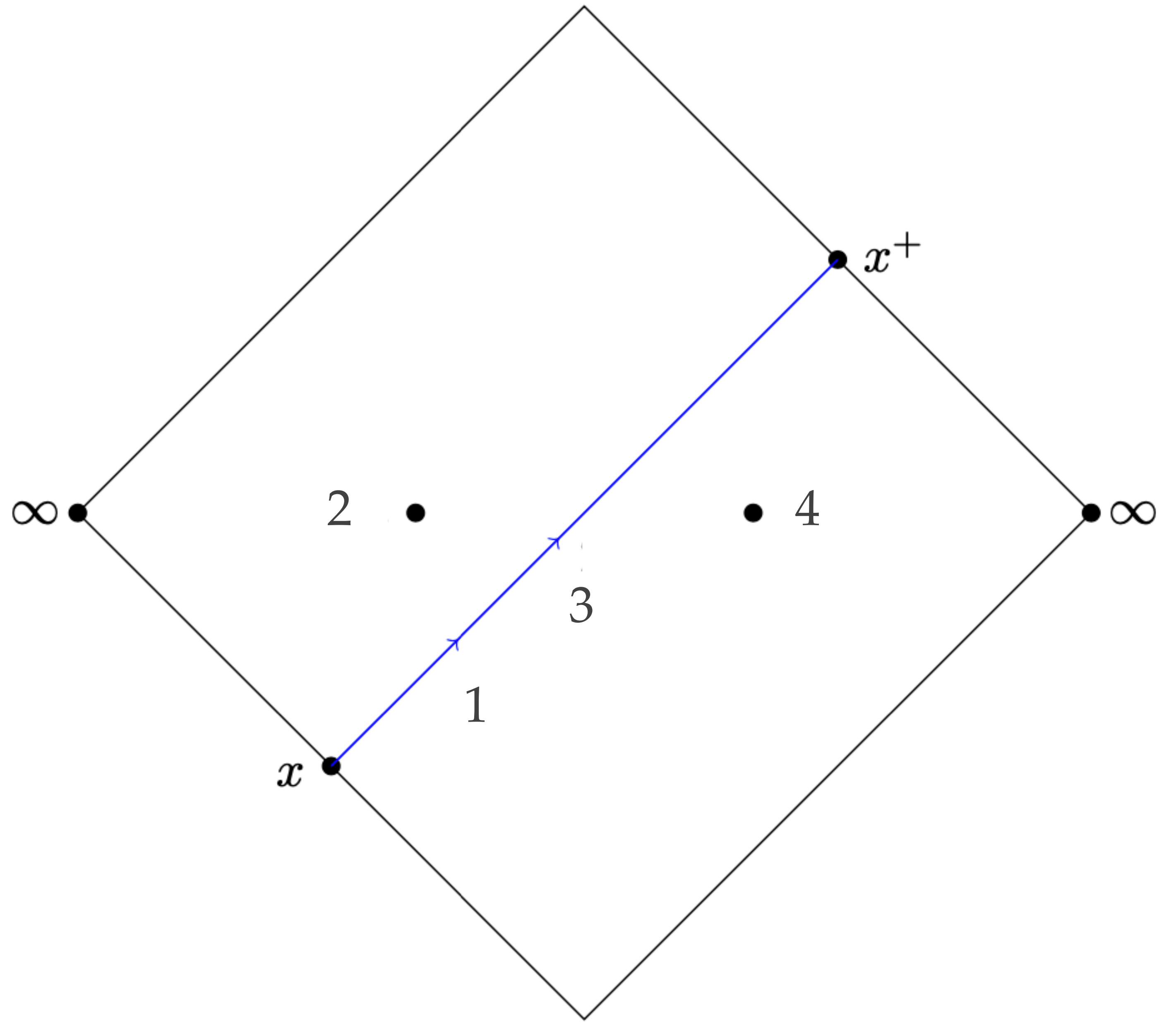
$$0 = \int_{-\infty}^{\infty} dx_1^+ \int_{-\infty}^{\infty} dx_3^+ \langle\Omega|[\phi(x_4),\phi(x_3)][\phi(x_1),\phi(x_2)]|\Omega\rangle - \int_{-\infty}^{\infty} dx_1^+ \int_{-\infty}^{\infty} dx_3^+ \langle\Omega|[\phi(x_4),\phi(x_1)][\phi(x_3),\phi(x_2)]|\Omega\rangle$$

Without $f(x_1, x_3)$, each term would become a dDisc, because null-integrated operators kill the vacuum



Dispersive sum rules from lightrays

Causality: $\langle\Omega|\phi(x_4)[\phi(x_1),\phi(x_3)]\phi(x_2)|\Omega\rangle = 0$ for $x_1 - x_3$ spacelike



Integrate x_1 and x_3 along spacelike separated null rays, with some kernel $f(x_1, x_3)$.

The kernel is needed for convergence at the endpoints of null integrals. Poles of $f(x_1, x_3)$ introduce additional contributions.

All in all, sum rule $\sum_{\Delta,J} p_{\Delta,J}\, \omega[G^s_{\Delta,J}] = 0$

ω is a *dispersive functional*: it has double zeros on all double traces with twist $\tau > \tau_{\min}$

Sum rules for AdS EFT

$$\langle\phi\phi\phi\phi\rangle = \underbrace{G_{\mathbb{1}} + \sum G_{[\phi\phi]_{n,\ell}} + G_{T_{\mu\nu}} + G_{[\text{composites}]}}_{\tau<\Delta_{\text{gap}}} + \underbrace{\sum G_{\text{heavy}}}_{\tau>\Delta_{\text{gap}}}$$

Apply to this equation a dispersive functional ω. Splitting light and heavy contributions,

$$\omega|_{\text{light}} = \sum_{\tau\leq\Delta_{\text{gap}}} p_{\Delta,J}\,\omega[G^s_{\Delta,J}]\,, \qquad \omega|_{\text{heavy}} = \sum_{\tau>\Delta_{\text{gap}}} p_{\Delta,J}\,\omega[G^s_{\Delta,J}]$$

$$-\,\omega|_{\text{light}} = \omega|_{\text{heavy}}$$

Crucially, $\omega_{\text{light}} = O(1/N^2)$ and can be computed from low-energy EFT.

If we construct a heavy non-negative ω, we have a constraint on EFT couplings:

$$-\,\omega|_{\text{light}} \geq 0\,.$$

Completely analogous to flat space sum rule!

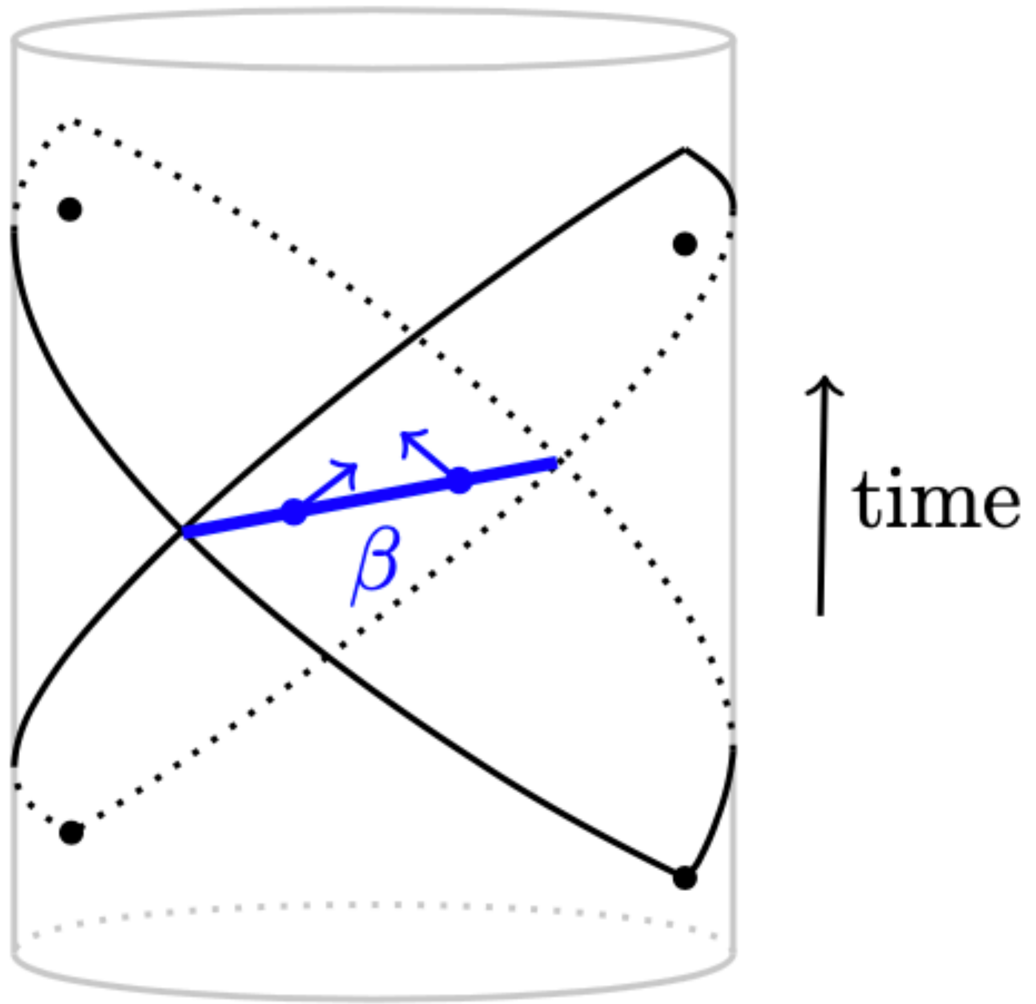
Construct AdS analogs of the flat space sum rules $\mathscr{C}_{k,u}$

Family of CFT sum rules $C_{k,\nu}$ that achieve bulk focussing:

couplings are measured at small AdS impact parameter $\beta \sim 2J/\Delta \ll 1$.

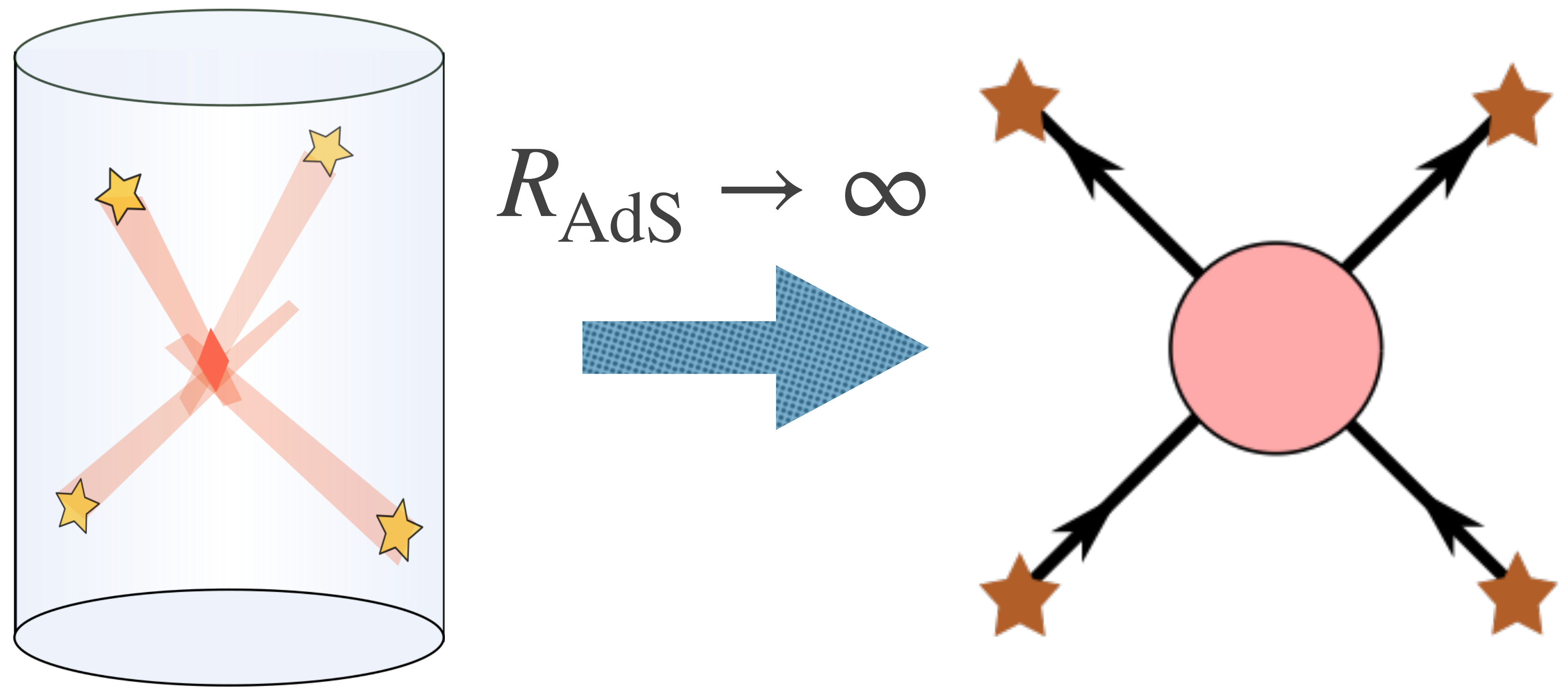
Uplift to AdS of the flat space bounds!

Proof of bulk locality with sharp inequalities, e.g. $\dfrac{g_2}{8\pi G} \geq \dfrac{\alpha(D)}{{\Delta_{\text{gap}}}^2}\left[1 + O(\Delta_{\text{gap}}^{-2})\right]$



Caron-Huot Mazáč LR Simmons-Duffin

From AdS to flat space



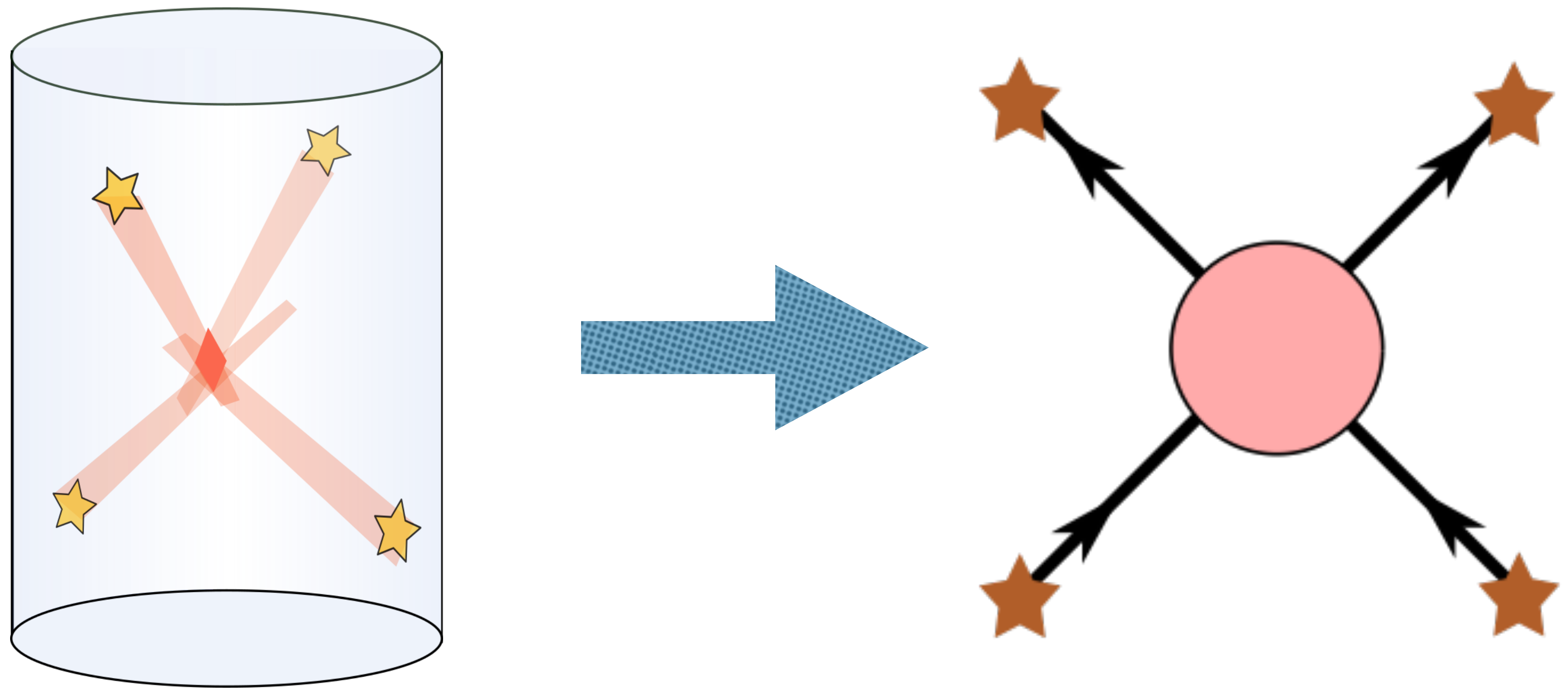
CFT bootstrap $\xrightarrow{?}$ S-matrix bootstrap

Justify assumptions about flat space $\mathscr{M}$ from this limit?

In AdS, causality and analyticity directly follow from bootstrap axioms
Regge boundedness with intercept ≤ 1 at non-perturbative level Caron-Huot

Any S-matrix that arises from AdS obeys a twice-subtracted dispersion relation.
This has implications for classical Regge growth conjecture [Chowdhury et al.]

Summary



- In (asymptotically) flat space, first steps of S-matrix bootstrap for weakly coupled EFTs, both with and without gravity. Must make plausible physical assumptions.
  Bounds with correct EFT scaling.
- In asymptotically AdS, a corner of the CFT bootstrap. Fully rigorous.
  Proof that large N CFTs with large gap have a local AdS dual, with sharp bounds.
- Causality is really powerful!

Much more to do…

- Generalizations: spin; multiple correlators/amplitudes; EFT loops; n-point functions
- Many potential physical applications (large N gauge theories, BSM, …)
- Interesting theories at boundaries/kinks/islands?
- Direct constraints on the spectrum?
- AdS bounds stronger than flat space bounds?
- Deep swampland questions (e.g. existence of “pure” AdS gravity)?
- $\Lambda > 0$?
- Deeper reformulation where positivity is the primitive notion? [Arkani-Hamed]

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