# Bulk causal features + boundary correlation in AdS/CFT

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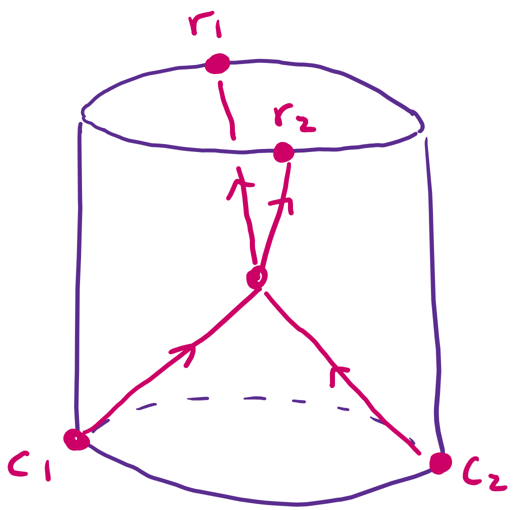
Based on: 2105.08094 AM
2101.08855 AM

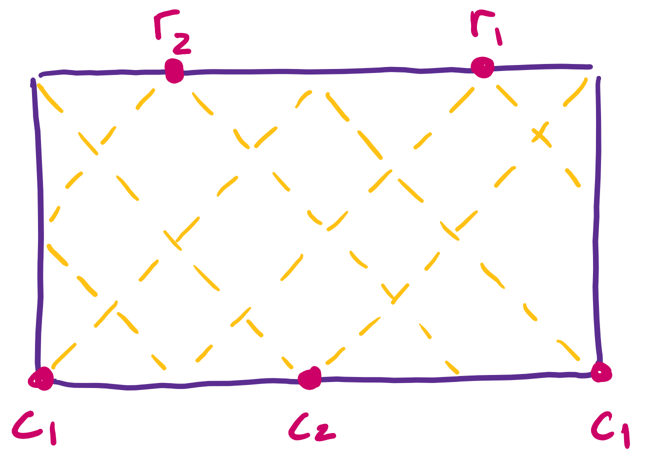
See also: 1912.05649 AM, Penington, Sorce
1902.06845 AM

# Bulk causal features

- A classic, fruitful, set-up in AdS/CFT:




can "scatter" in bulk




no place to scatter

- HPPS understood that for the boundary to reproduce bulk physics, boundary theory must be large $N$, gapped

This talk: $\boxed{\text{what do bulk causal features tell us about the boundary state?}}$

# This talk

How do bulk causal features constrain the boundary state?

1. Recall ways to measure correlation and relationship (holographically) to extremal surfaces

2. State the "Privacy-duality" theorem and give argument

3. State connected wedge theorem

4. Final comments

Follow from dynamics of quantum information in spacetime

# Measuring correlations in quantum states

# Q.I. Measures of correlation

- We will be concerned with:

① Mutual information: $I(A:B) \equiv S(A) + S(B) - S(AB)$

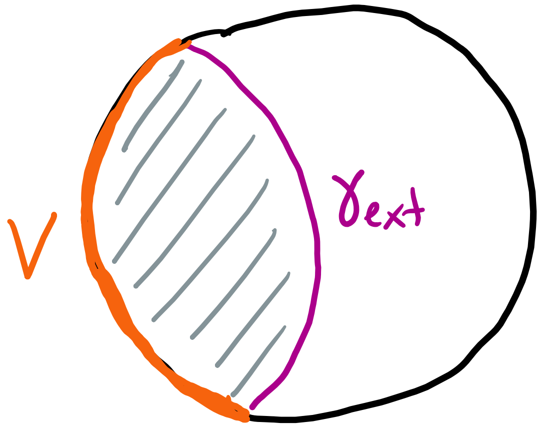
↳ Amount of correlation between A and B

② Conditional mutual info: $I(A:C|B) \equiv I(A:BC) - I(A:B)$

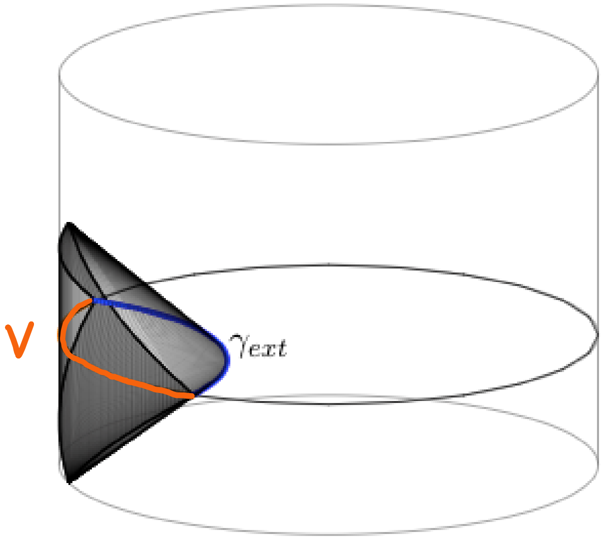
↳ Interpret as amount of correlation between between A and BC not already present between A and B

# RT formula + the entanglement wedge

- Given a boundary region $V$:




- $S(V) = \dfrac{\text{Area}[\gamma_{ext}]}{4G_N} + S[E_V]$




- $\rho_V$ records all the bulk data inside the entanglement wedge $E_V$

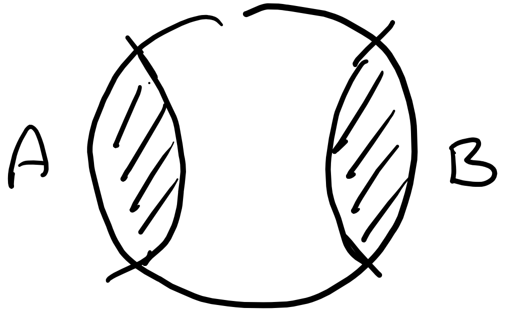
  ↪ "entanglement wedge reconstruction"

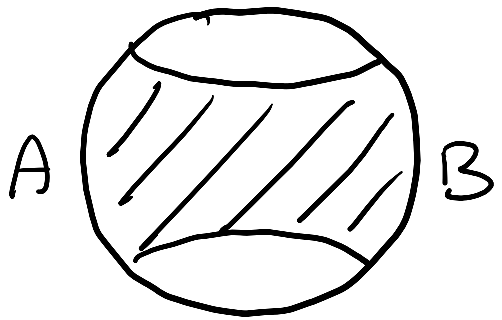
# Correlation + extremal surfaces

- The order of these correlation measures records qualitative features of extremal surfaces

E.g. $I(A:B) = O(1)$ $\qquad\qquad$ $I(A:B) = O(1/G_N)$

$\Downarrow$ $\qquad\qquad$ $\Downarrow$

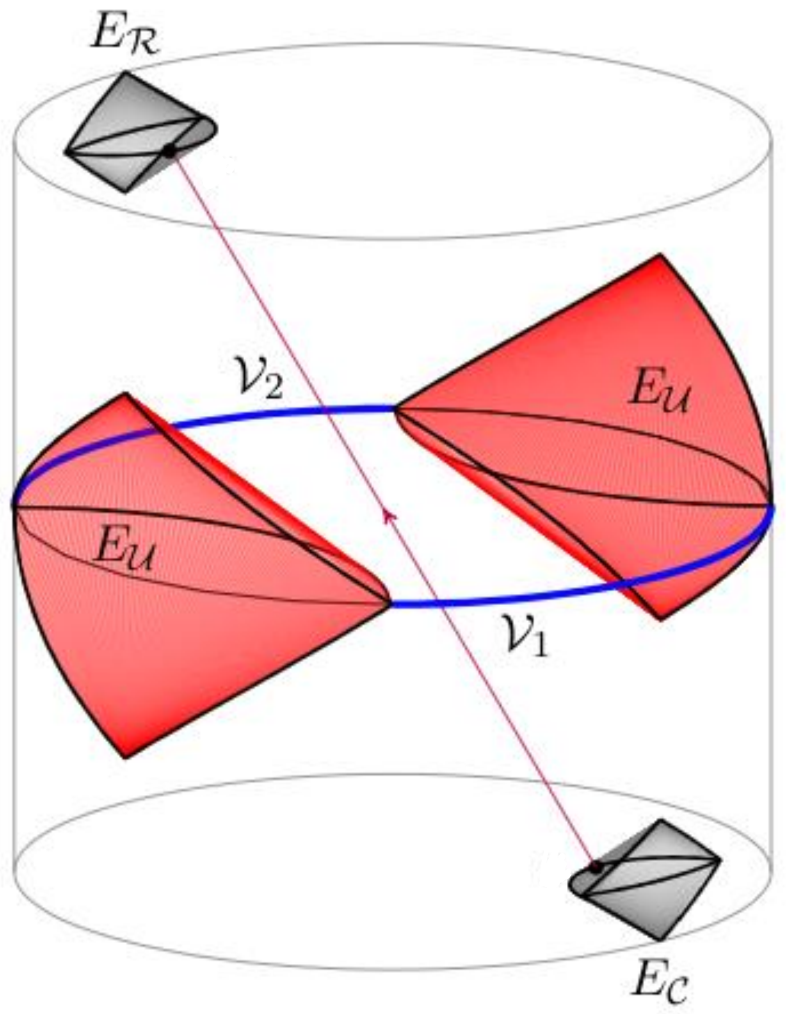



$E_{AB} = E_A \cup E_B$ $\qquad\qquad$ $E_{AB} \neq E_A \cup E_B$

# Privacy-duality theorem

Privacy - duality




"private" curve

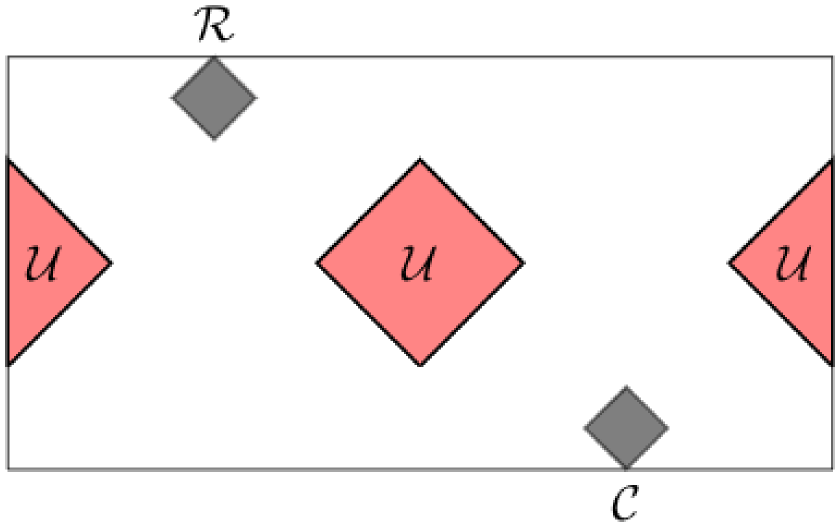
If there's a causal curve from $E_\mathcal{C}$ to $E_\mathcal{R}$ that avoids $E_\mathcal{U}$,

then

$$I(V_1 : V_2 \,|\, U) = O(1/G_N)$$

# Privacy-Duality theorem

- Pick $\mathcal{C}$, $\mathcal{R}$, $\mathcal{U}$ in boundary, and a Cauchy surface $\Sigma$ "through" $\mathcal{U}$

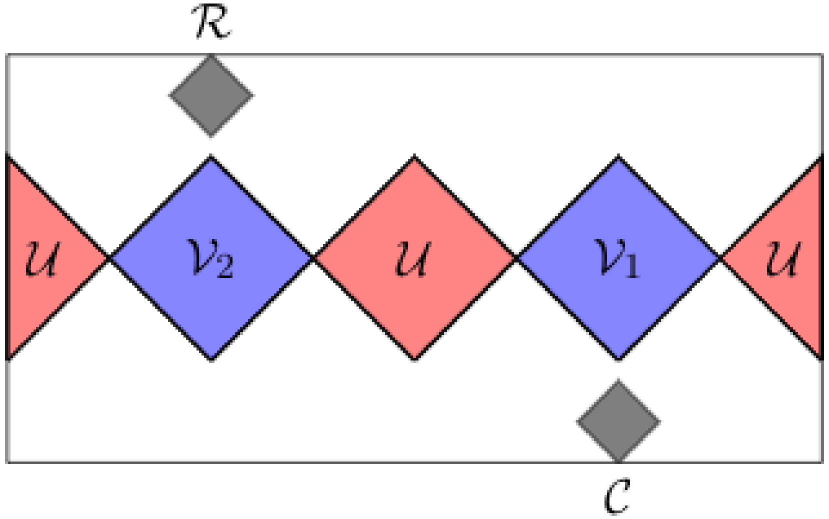


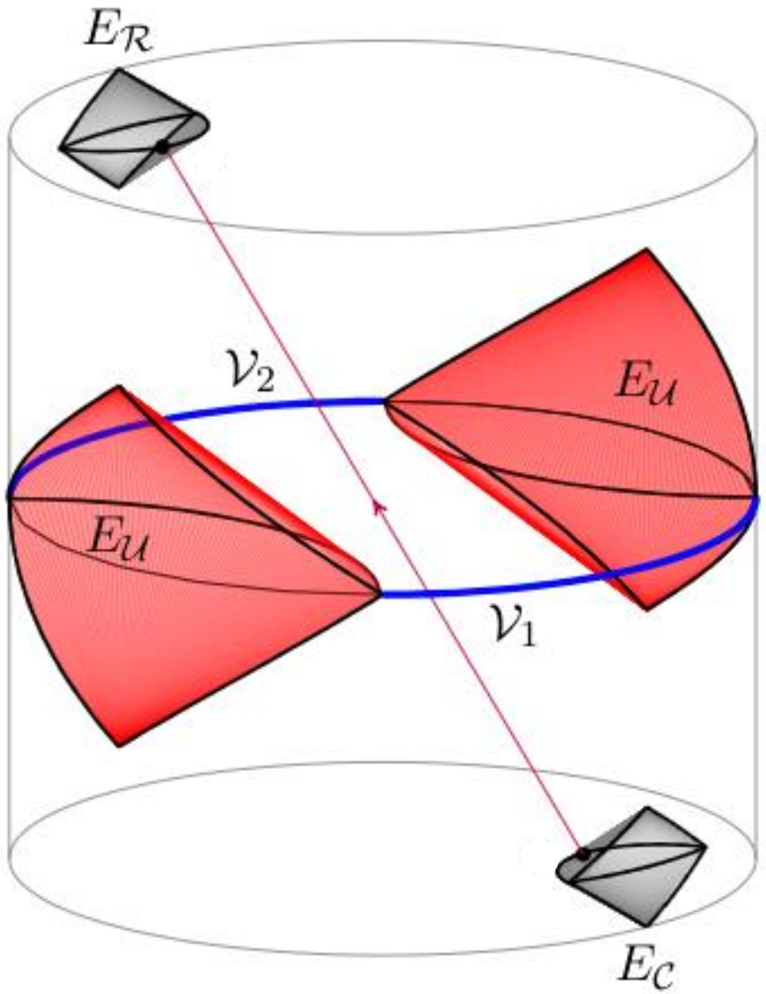

Define: $V_1 \equiv \hat{D}(J^+(\mathcal{C}) \cap \Sigma \cap \mathcal{U}')$

$$V_2 \equiv \hat{D}(J^-(\mathcal{R}) \cap \Sigma \cap \mathcal{U}')$$

# Privacy-Duality theorem

- Pick $e$, $R$, $U$ in boundary, and a Cauchy surface $\Sigma$ "through" $U$




Define: $V_1 \equiv \hat{D}(J^+(e) \cap \Sigma \cap U')$

$V_2 \equiv \hat{D}(J^-(R) \cap \Sigma \cap U')$

Then: $\boxed{\text{If } \exists \text{ a causal curve from } E_e \text{ to } E_R \text{ that avoids } E_U \text{, then } I(V_1 : V_2 | U) = O(1/G_N)}$

# Proof strategy (general)

1. Introduce a probe system $Q$ (or systems $Q_i$) to the bulk, and exploit causal feature to have $Q$ evolve in some interesting way

2. Translate evolution of $Q$ to a boundary statement

3. Argue that for boundary to realize this evolution, while lacking same causal feature, appropriate correlations are necessary

# Q.I. argument for Privacy-duality

① Introduce a "probe" system $Q$, holding state $|\psi\rangle_Q$, and send it along $\Gamma$

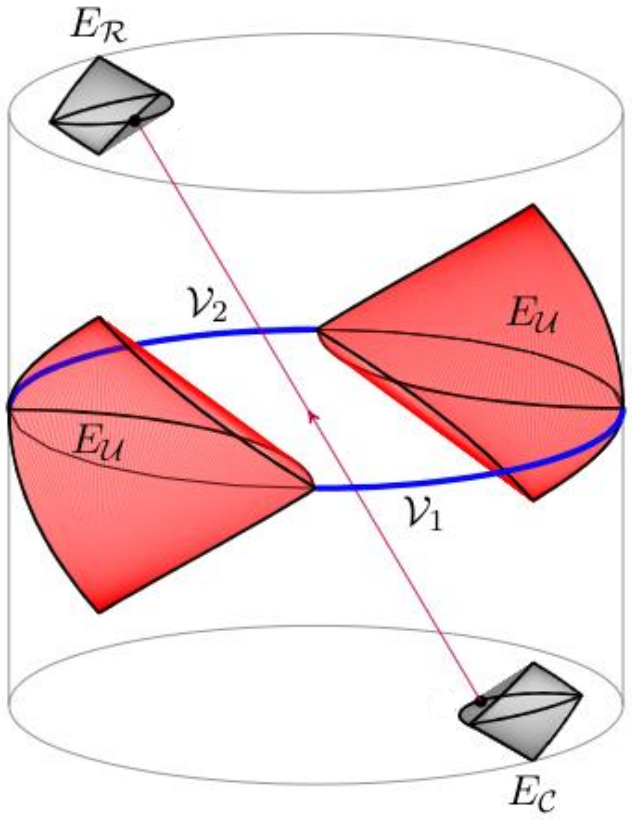
② Sends message from $E_C$ to $E_R$, while keeping message secret from $E_U$

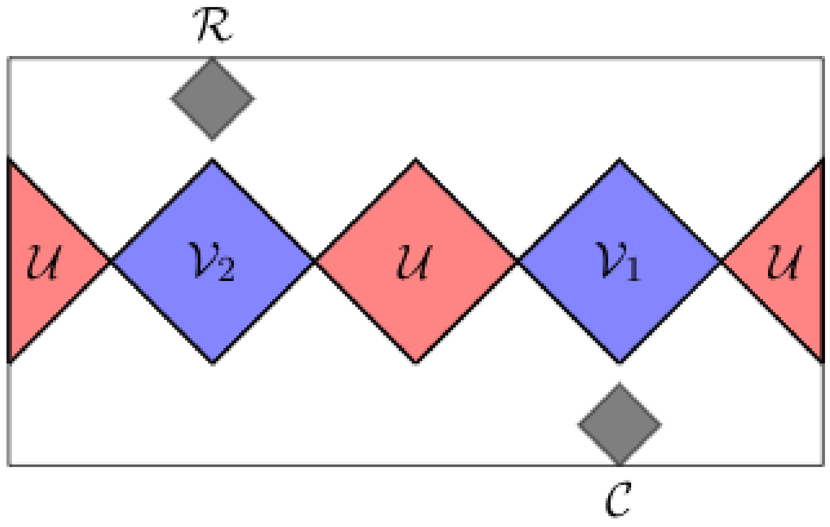
EW reconstruction

Sends message from $C$ to $R$, while keeping message secret from $U$

③ In boundary, there is no private curve.

↳ Instead, exploit correlations among subsystems to hide state on $Q$



# How to hide the state on Q

$$\rho_U \propto \sum_K P_K |\psi\rangle\langle\psi| P_K = \mathbb{I}$$

- Use the "one-time pad"

    1) Prepare a string of bits, called K, the key and make a copy K′

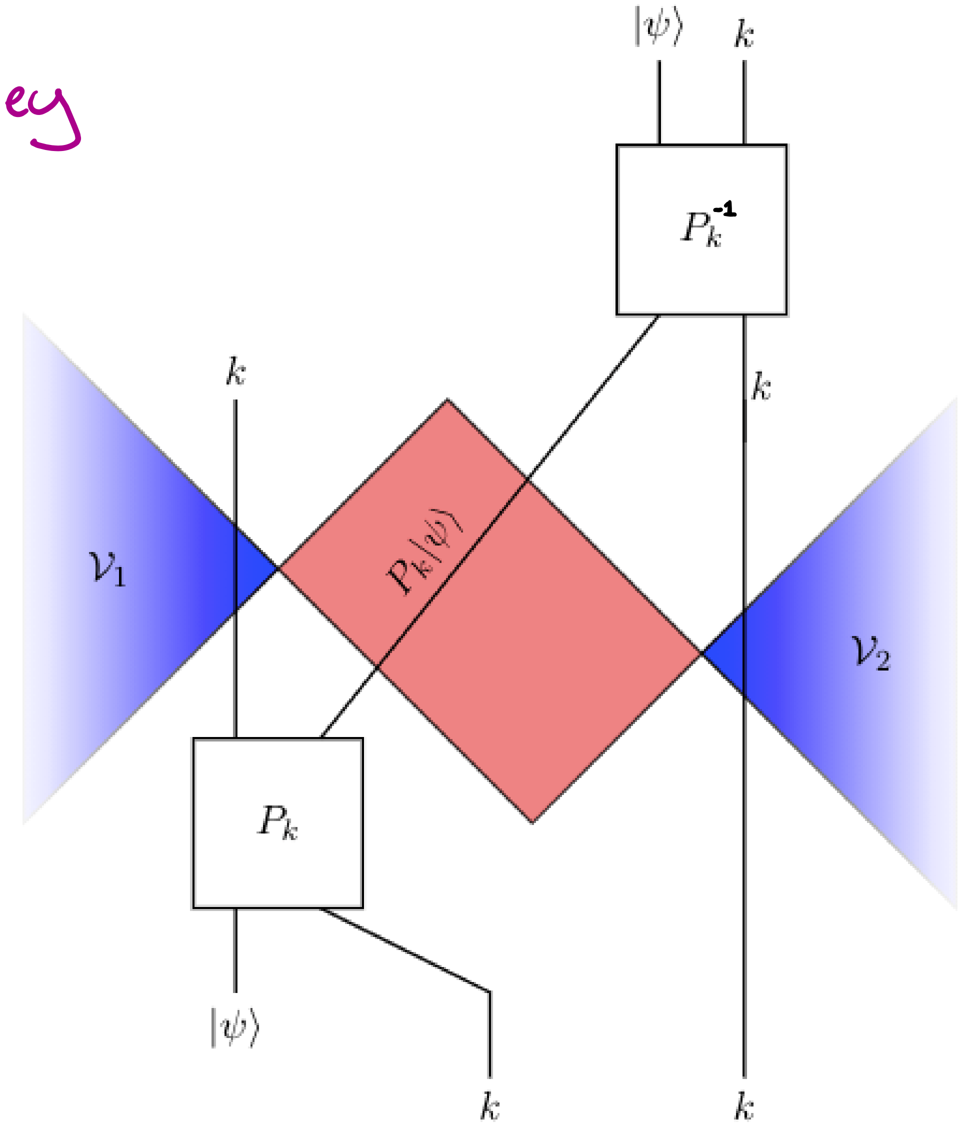
    2) Send K through $V_2$

    3) Encode $|\psi\rangle$ using K′ (do $P_{K'}$)

    4) Send "encoded" Q through U

    5) Undo $P_K$ using K, recover $|\psi\rangle$

- Can show any such procedure must have $I(V_1 : V_2 | U)$ large



(Improved)

# Connected wedge theorem

# Connected wedge theorem

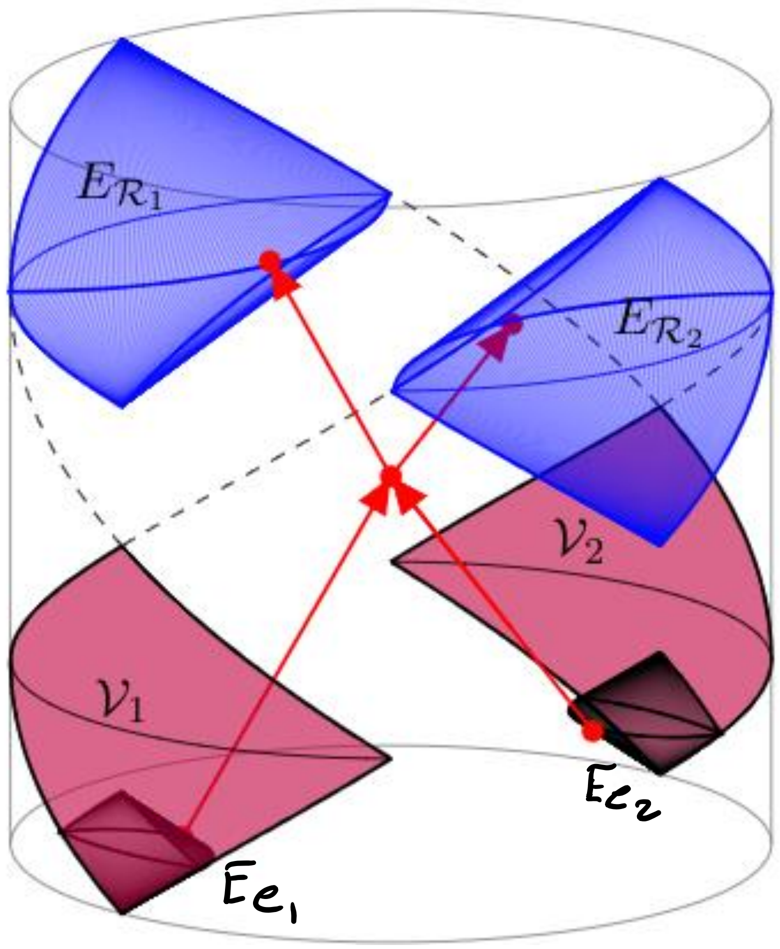
- Pick four wedges, define:

$$J^E_{12\to12} \equiv J^+(E_{e_1}) \cap J^+(E_{e_2}) \cap J^-(E_{R_1}) \cap J^-(E_{R_2})$$

And:

$$V_1 \equiv \hat{J}^+(e_1) \cap \hat{J}^-(R_1) \cap \hat{J}^-(R_2)$$

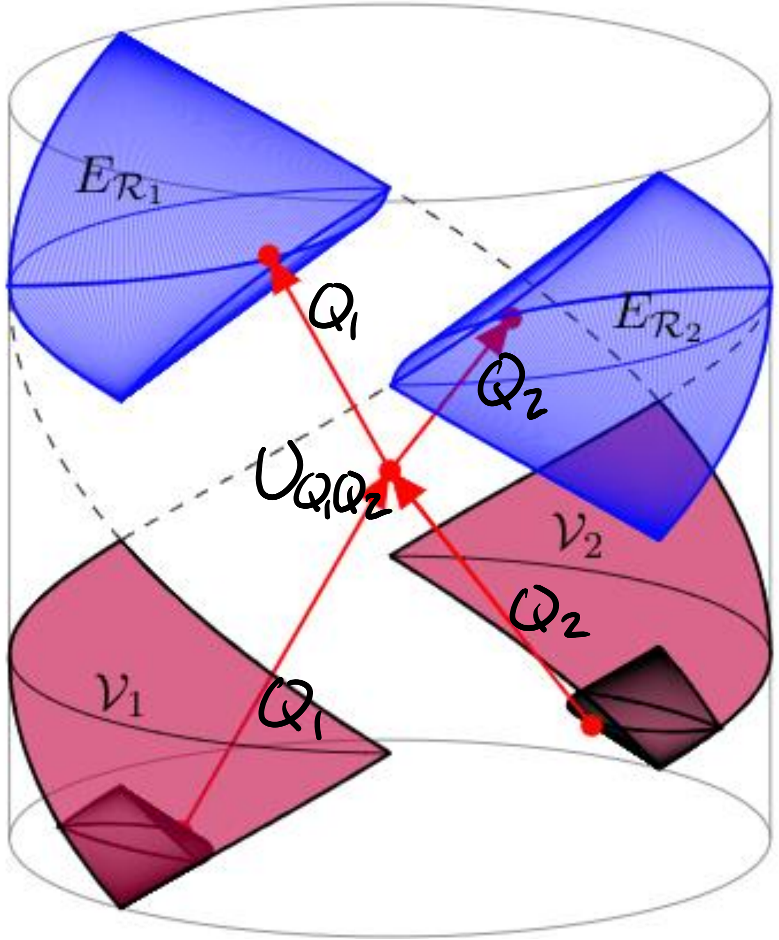
$$V_2 = \hat{J}^+(e_2) \cap \hat{J}^-(R_1) \cap \hat{J}^-(R_2)$$




If $J^E_{12\to12}$ is non-empty, then $I(V_1 : V_2) = O(1/G_N)$

# Connected wedge theorem

Use same strategy...

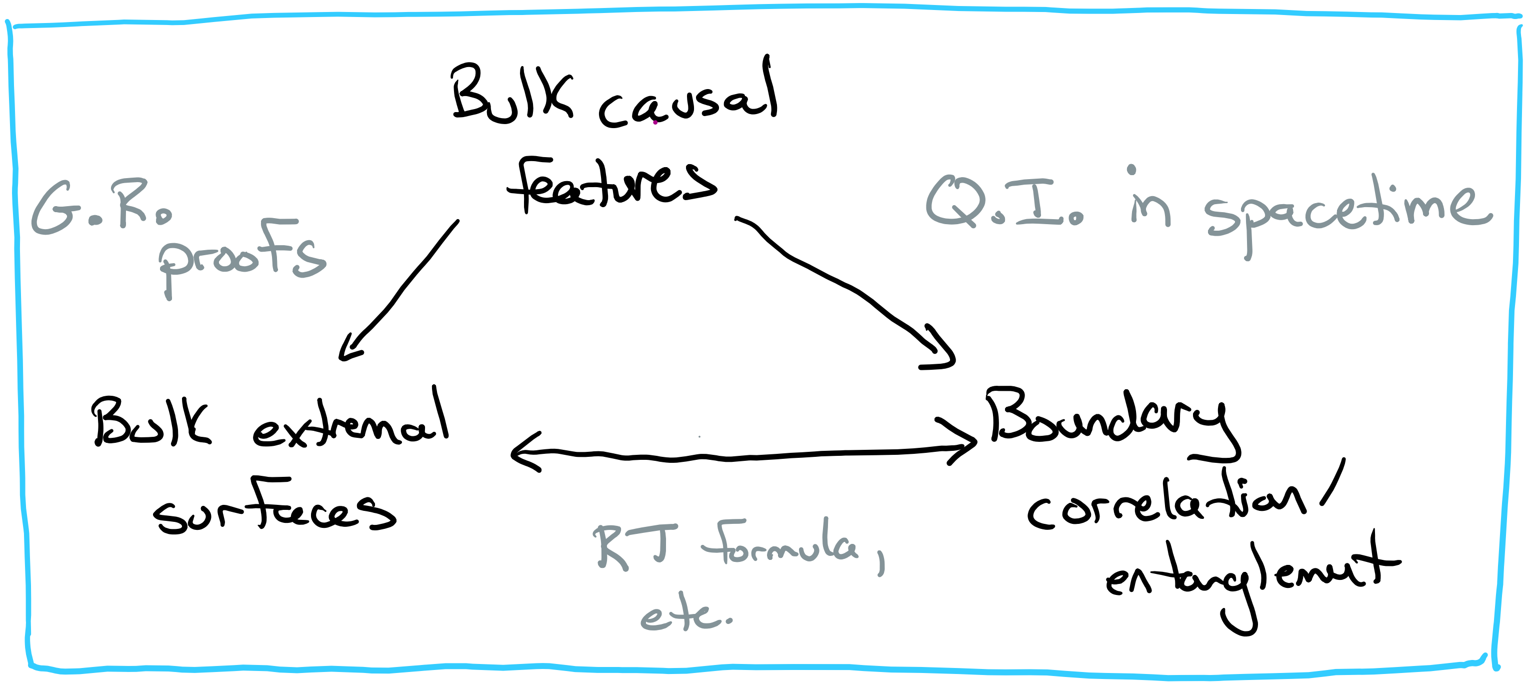



For the boundary to reproduce the bulk unitary $U_{Q_1Q_2}$

$V_1$ and $V_2$ must share lots of entanglement $\longrightarrow$ $I(V_1 : V_2) = \mathcal{O}(1/G_N)$

# Final Remarks

# Comments




- Two theorems together suggest a general causal feature ~ entanglement connection

- Complements usual extremal surface ~ entanglement connection given by RT

- Interesting to revisit problems studied using extremal surfaces, and look for a causal perspective:

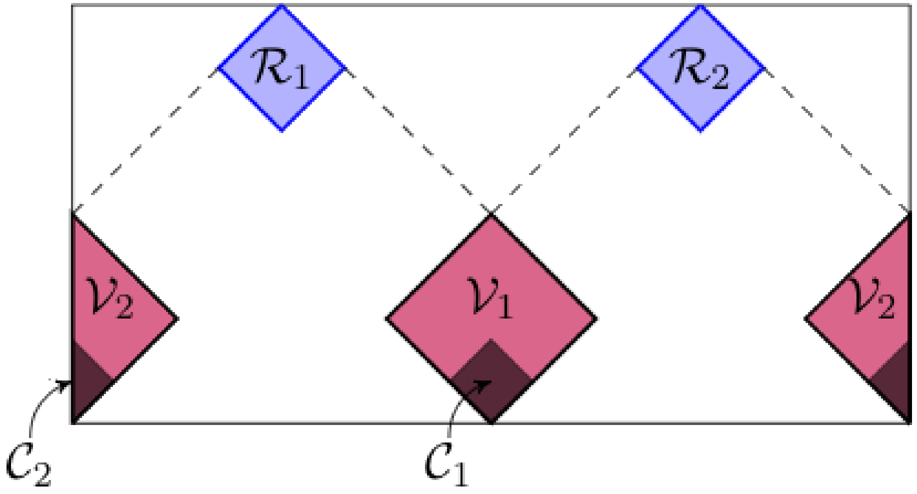
  ↳ e.g. In the BH + islands context, found causal condition for island formation (at least in toy models) (2102.01810 AM, David Wakeham)

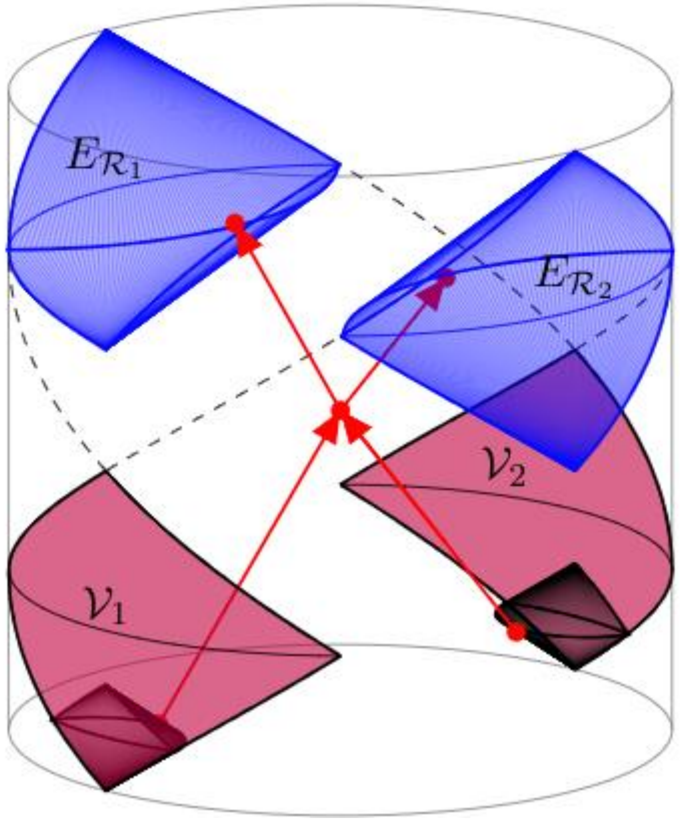
Thanks! Questions?

(also, available on Slack
or may@phas.ubc.ca)

# Improved connected wedge theorem

- Pick $\mathcal{C}_1, \mathcal{C}_2, \mathcal{R}_1, \mathcal{R}_2$:



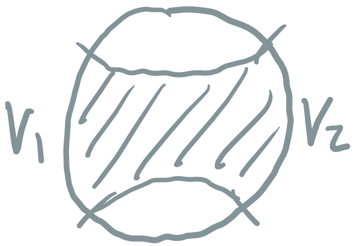

"entanglement scattering region"

Define: $\mathcal{V}_1 \equiv \hat{J}^+(\mathcal{C}_1) \cap \hat{J}^-(\mathcal{R}_1) \cap \hat{J}^-(\mathcal{R}_2)$

$\mathcal{V}_2 \equiv \hat{J}^+(\mathcal{C}_2) \cap \hat{J}^-(\mathcal{R}_1) \cap \hat{J}^-(\mathcal{R}_2)$, $\quad J^E_{12\to 12} \equiv J^+(E_{\mathcal{C}_1}) \cap J^+(E_{\mathcal{C}_2}) \cap J^-(E_{\mathcal{R}_1}) \cap J^-(E_{\mathcal{R}_2})$

Then:
$$\boxed{\begin{array}{c} J^E_{12\to 12} \neq \emptyset \\ \Downarrow \\ I(\mathcal{V}_1 : \mathcal{V}_2) = O(1/G_N) \end{array}}$$

ie $\quad E_{\mathcal{V}_1\mathcal{V}_2} \neq E_{\mathcal{V}_1} \cup E_{\mathcal{V}_2}$,



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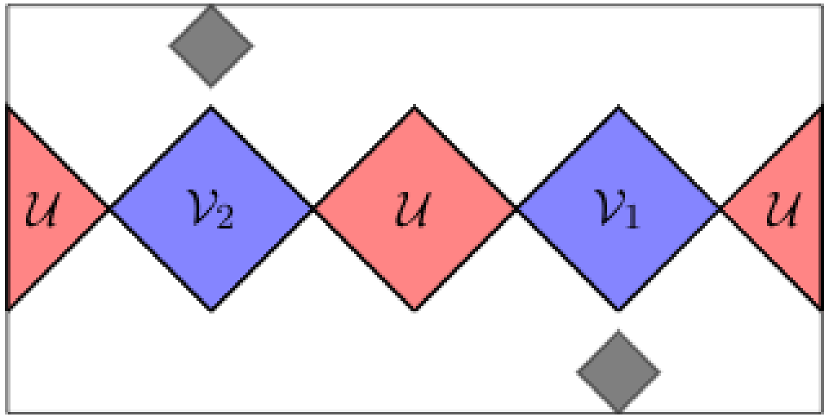


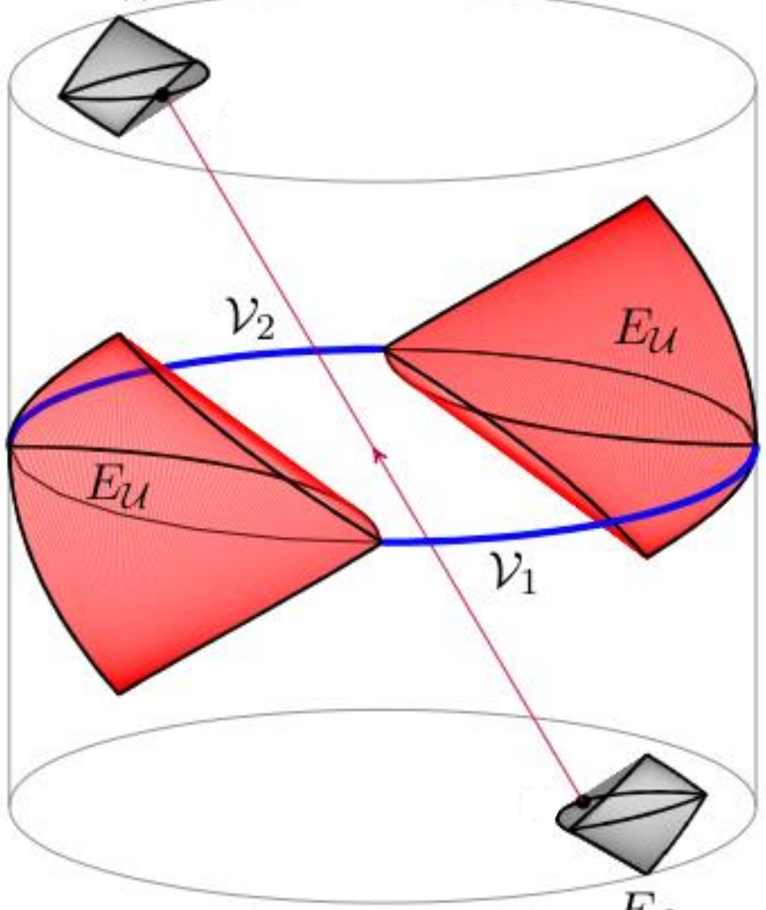


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Then: If $\exists$ a causal curve from $E_\mathcal{C}$ to $E_\mathcal{R}$ that avoids $E_\mathcal{U}$, then $I(V_1 : V_2 | \mathcal{U}) = \mathcal{O}(1/G_N)$