p-adic AdS/CFT

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1. Introduction

p-adic numbers are a completion of the rationals with respect to a funny norm:

$$\left| p^v \frac{a}{b} \right|_p = p^{-v} \qquad \text{assuming} \qquad p \nmid a \quad \text{and} \quad p \nmid b \,. \tag{1}$$

The key intuition is that the prime p is small but not zero:

$$\begin{gathered} |0|_p = 0 \quad \text{(by fiat)}\,, \qquad |a|_p = 1 \quad \text{for } a = 1, 2, 3, \ldots, p-1 \\ |p|_p = \frac{1}{p}\,, \qquad |p^2| = \frac{1}{p^2}\,, \qquad |p^3| = \frac{1}{p^3} \qquad \ldots \end{gathered} \tag{2}$$

A general non-zero p-adic number can be written as a formal series

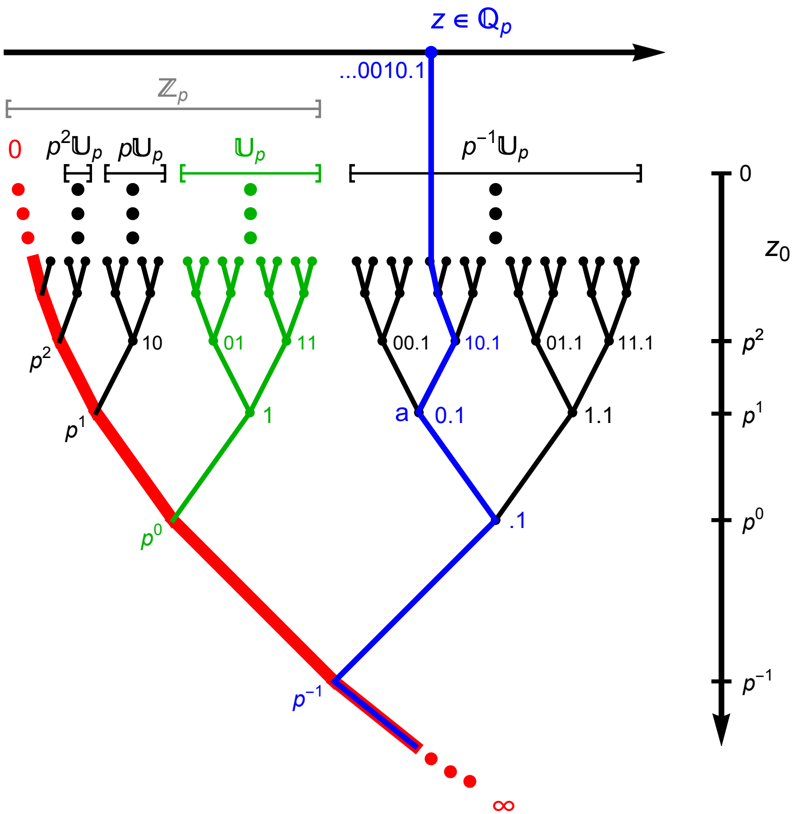
$$z = p^v \sum_{m=0}^{\infty} a_m p^m \,, \quad a_0 \neq 0 \,, \quad v \in \mathbb{Z} \,. \tag{3}$$

For example, if $p = 2$, then

$$-1 = \frac{1}{1-2} = 1 + 2 + 2^2 + 2^3 + \ldots = \ldots 1111_2 \tag{4}$$

and the sum converges in the 2-adic norm.

1.1. *p*-adic numbers are naturally holographic



Choosing a p-adic number $z \in \mathbb{Q}_p$ amounts to choosing a path up through the tree:

- Where we leave the red "trunk" determines $|z|_p$.
- The rightmost p-adic digit is non-zero, so a $(p-1)$-fold choice.
- Subsequent digits are p-fold choices that steer us up the tree.
- Each node a on the way up to z is a rational approximation to z, so that $|z-a|_p \leq |z_0|_p$.

- Note that p-adic integers $\mathbb{Z}_p$ have $|z|_p \leq 1$: a compact set!
- $z \in \mathbb{U}_p$ means $|z|_p = 1$, so $\mathbb{U}_p$ is like the unit circle.

1.2. Plan of the rest of the talk

(Not exactly in order of presentation)

- Previous work: p-adic string [Freund-Olson '87], eternal symmetree [Harlow-Shenker-Stanford-Susskind '11], p-adic field theory [Vladimirov, Zelenov, Missarov, ... c.a. '90].
- The tree is a quotient similar to $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1)$, so it's like EAdS_2.
- Field extensions of $\mathbb{Q}_p$ lead to trees analogous to EAdS_{n+1}.
- Natural classical dynamics on the tree allows to define correlators in a p-adic field theory, similar to bottom-up AdS/CFT.
- 2pt, 3pt, 4pt functions exhibit interesting comparisons to the usual formulas for real AdS_{n+1}, but "ultra-metric" properties make the 4pt function much simpler.
- A p-adic analog of Euclidean black holes is based on a discrete identification of the tree [Heydeman-Marcolli-Saberi-Stoica '16].
- Some possible future directions.

1.3. The p-adic string

The (fully symmetrized) Veneziano amplitude has an obvious p-adic analog:

$$\begin{aligned} A_\infty^{(4)} &= \int_{\mathbb{R}} dz\, |z|^{k_1\cdot k_2} |1-z|^{k_1\cdot k_3} = \Gamma_\infty(-\alpha(s))\Gamma_\infty(-\alpha(t))\Gamma_\infty(-\alpha(u)) \\ A_p^{(4)} &= \int_{\mathbb{Q}_p} dz\, |z|_p^{k_1\cdot k_2} |1-z|_p^{k_1\cdot k_3} = \Gamma_p(-\alpha(s))\Gamma_p(-\alpha(t))\Gamma_p(-\alpha(u)) \end{aligned} \tag{5}$$

where all $k_i^2 = 2$ (tachyons), $s = -(k_1+k_2)^2$ etc., $\alpha(s) = 1 + s/2$, and we define

$$\Gamma_\infty(\sigma) \equiv 2\Gamma_{\text{Euler}}(\sigma)\cos\frac{\pi\sigma}{2} \qquad \Gamma_p(\sigma) \equiv \frac{1-p^{\sigma-1}}{1-p^{-\sigma}} \qquad \text{for } \sigma \in \mathbb{C}\,. \tag{6}$$

p-adic integration can be defined so that

$$\int_{\mathbb{Z}_p} dz = 1 \qquad \text{and} \qquad \int_{\xi S} dz = |\xi|_p \int_S dz \quad \text{for} \quad \xi \in \mathbb{Q}_p,\, S \subset \mathbb{Q}_p\,. \tag{7}$$

Amazingly, $\prod_v \Gamma_v(z) = 1$ where $\prod_v$ includes $v=\infty$ and also $v=p$ for all primes. So we get the "adelic" relation [Freund-Witten '87]

$$A_\infty^{(4)} = 1/\prod_p A_p^{(4)} \qquad \text{(cf.} \quad |z|_\infty = 1/\prod_p |z|_p \text{ for rational } z)\,. \tag{8}$$

2. p-adic AdS/CFT

A p-adic CFT should be defined by correlators, like

$$\langle \mathcal{O}(z)\mathcal{O}(0)\rangle = \frac{C_{\mathcal{O}\mathcal{O}}}{|z|_p^{2\Delta}}\,, \qquad \langle \mathcal{O}(z_1)\mathcal{O}(z_2)\mathcal{O}(z_3)\rangle = \frac{C_{\mathcal{O}\mathcal{O}\mathcal{O}}}{|z_{12}z_{23}z_{31}|_p^{\Delta}}\,, \tag{9}$$

where $z_{ij} \equiv z_i - z_j$. The z_i dependence is completely fixed by invariance under p-adic linear fractional transformations:

$$g \in \mathrm{PGL}(2,\mathbb{Q}_p) \qquad \text{acts as} \qquad g(z) = \frac{az+b}{cz+d}\,, \qquad ad-bc \neq 0\,. \tag{10}$$

The p-adic Veneziano amplitude comes from a 4pt function of vertex operators $\mathcal{V}_k(z) = :e^{ik\cdot X(z)}:$ in a free p-adic CFT: schematically,

$$A_p^{(4)} \sim \int_{\mathbb{Q}_p} dz \left\langle \mathcal{V}_{k_1}(z)\mathcal{V}_{k_2}(0)\mathcal{V}_{k_3}(1)\mathcal{V}_{k_4}(\infty)\right\rangle . \tag{11}$$

The “bulk” should be a coset space like $\mathrm{SL}(2,\mathbb{R})/\mathrm{U}(1) = \mathrm{EAdS}_2$ (cf. [Zabrodin ’89])

$\mathrm{PGL}(2,\mathbb{Z}_p) \subset \mathrm{PGL}(2,\mathbb{Q}_p)$ is the maximal compact subset, and
$\mathrm{PGL}(2,\mathbb{Q}_p)/\mathrm{PGL}(2,\mathbb{Z}_p)$ is the tree previously discussed! [Bruhat-Tits ’72]

2.1. Action in the bulk

In place of

$$S_{\text{bulk}} = \int_{\text{EAdS}_2} d^2z \sqrt{g} \left[\frac{1}{2} (\partial\phi)^2 + V(\phi) \right] \tag{12}$$

it is natural to study

$$S_{\text{tree}} = \sum_{\langle ab \rangle} \frac{1}{2} (\phi_a - \phi_b)^2 + \sum_a V(\phi_a) \tag{13}$$

where $\langle ab \rangle$ means that we are summing over adjacent vertices in the tree.

The *linearized* equation of motion involves only $m_p^2 = V''(0)$:

$$(\Box + m_p^2)\phi_a = 0 \qquad \text{where} \qquad \Box\phi_a = \sum_{\substack{\langle ab \rangle \\ a \text{ fixed}}} (\phi_a - \phi_b) \,, \tag{14}$$

and the bulk-to-bulk propagator is

$$G(a,b) = \frac{\zeta_p(2\Delta)}{p^\Delta} p^{-\Delta d(a,b)} \qquad \text{where} \qquad m_p^2 = -\frac{1}{\zeta_p(\Delta-1)\zeta_p(-\Delta)} \tag{15}$$

and $d(a,b)$ is the number of steps from a to b on the tree.

2.2. Two technical detours

1. We will often encounter local zeta functions

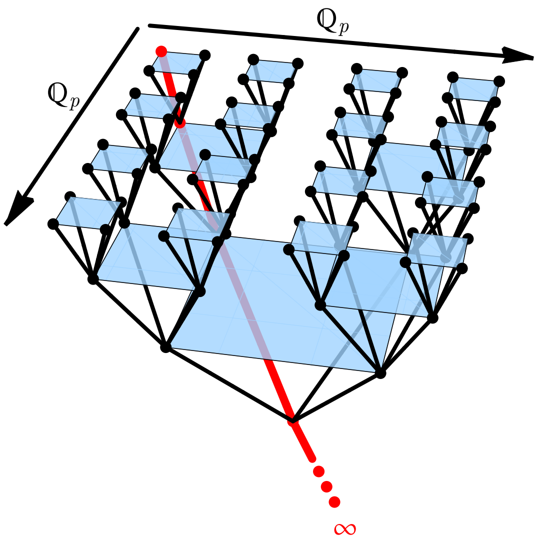
$$\zeta_\infty(\sigma) \equiv \pi^{-\sigma/2}\Gamma_{\text{Euler}}(\sigma/2) \qquad \text{and} \qquad \zeta_p(\sigma) \equiv \frac{1}{1-p^{-\sigma}}\,, \tag{16}$$

so named because

$$\zeta_{\text{Riemann}}(\sigma) = \prod_p \zeta_p(\sigma) \qquad \text{and} \qquad \Gamma_v(\sigma) = \frac{\zeta_v(\sigma)}{\zeta_v(1-\sigma)}\,, \quad v = \infty \text{ or } p\,. \tag{17}$$

2. We allow the boundary to be an n-dimensional vector space over $\mathbb{Q}_p$.

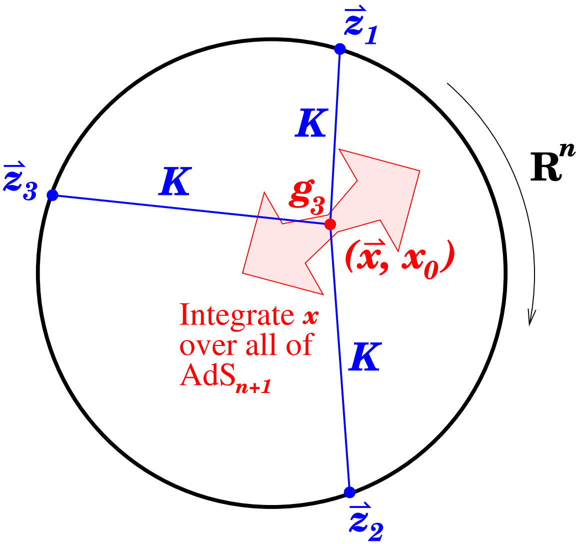
- The simplest choice is the unramified extension of $\mathbb{Q}_p$ of degree n.
- We call this extension $\mathbb{Q}_q$, with $q = p^n$.
- Pictured at right is the case $n = 2$.
- Norms $|\vec{z}|_q$ and depth direction z_0 still take values p^ω with $\omega \in \mathbb{Z}$.



2.3. Three-point function—warmup

Recall the standard story of three-point functions in Euclidean $\text{AdS}_{n+1}/\text{CFT}_n$:

$$-\log \left\langle \exp \left\{ \int_{\mathbb{R}^n} d^n z \, \phi_0(\vec{z}) \mathcal{O}(\vec{z}) \right\} \right\rangle = \underset{\phi(z_0,\vec{z}) \to \phi_0(\vec{z})}{\text{extremum}} S_{\text{bulk}}[\phi] \,, \qquad (18)$$



- Differentiate (18) with respect to $\phi_0(\vec{z}_1)$, $\phi_0(\vec{z}_2)$, $\phi_0(\vec{z}_3)$, set $g_3 = V'''(0)$.
- Bulk-to-boundary propagators are simple in terms of ζ_∞:

$$K(x; \vec{z}) = \frac{\zeta_\infty(2\Delta)}{\zeta_\infty(2\Delta - n)} \frac{z_0^\Delta}{(z_0^2 + (\vec{z} - \vec{x})^2)^\Delta}$$

so that $$\int_{\mathbb{R}^n} d^n z \, K(x; \vec{z}) = 1$$

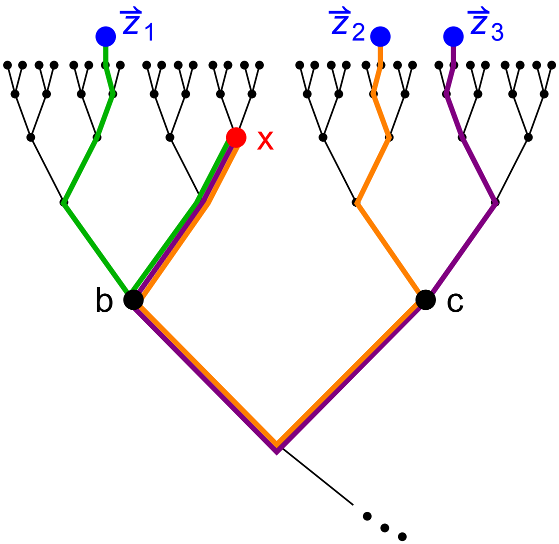
$$\langle \mathcal{O}(\vec{z}_1) \mathcal{O}(\vec{z}_2) \mathcal{O}(\vec{z}_3) \rangle = -g_3 \int_{\text{AdS}_{n+1}} d^{n+1}x \sqrt{g} \, K(x; \vec{z}_1) K(x; \vec{z}_2) K(x; \vec{z}_3)$$

$$= \frac{C^{(\infty)}_{\mathcal{OOO}}}{|\vec{z}_{12}|^\Delta |\vec{z}_{23}|^\Delta |\vec{z}_{13}|^\Delta} \,, \qquad C^{(\infty)}_{\mathcal{OOO}} = -g_3 \frac{\zeta_\infty(\Delta)^3 \zeta_\infty(3\Delta - n)}{2\zeta_\infty(2\Delta - n)^3}$$

2.4. Three-point function

The "traditional" expression for $C^{(\infty)}_{\mathcal{OOO}}$ is equivalent but involves π explicitly:

$$C^{(\infty)}_{\mathcal{OOO}} = -g_3 \pi^{-n} \frac{\Gamma_{\rm Euler}(\Delta/2)^3 \Gamma_{\rm Euler}(3\Delta/2 - n/2)}{2\Gamma_{\rm Euler}(\Delta - n/2)} \,. \tag{19}$$



The p-adic version of the three-point calculation factorizes conveniently into external legs times an internal summation:

$$\begin{aligned} \langle \mathcal{OOO} \rangle &= -g_3 \sum_x \prod_{i=1}^3 K(x; \vec{z}_i) \\ &= -g_3 \left[\prod_{i=1}^3 K(c; \vec{z}_i) \right] \times \\ &\quad \sum_x \hat{G}(c,b) \hat{G}(b,x)^3 \end{aligned} \tag{20}$$

The final result: (absence of π factors is now inevitable)

$$\langle \mathcal{O}(\vec{z}_1) \mathcal{O}(\vec{z}_2) \mathcal{O}(\vec{z}_3) \rangle = \frac{C^{(p)}_{\mathcal{OOO}}}{|\vec{z}_{12} \vec{z}_{23} \vec{z}_{13}|^\Delta} \,, \qquad C^{(p)}_{\mathcal{OOO}} = -g_3 \frac{\zeta_p(\Delta)^3 \zeta_p(3\Delta - n)}{\zeta_p(2\Delta - n)^3} \tag{21}$$

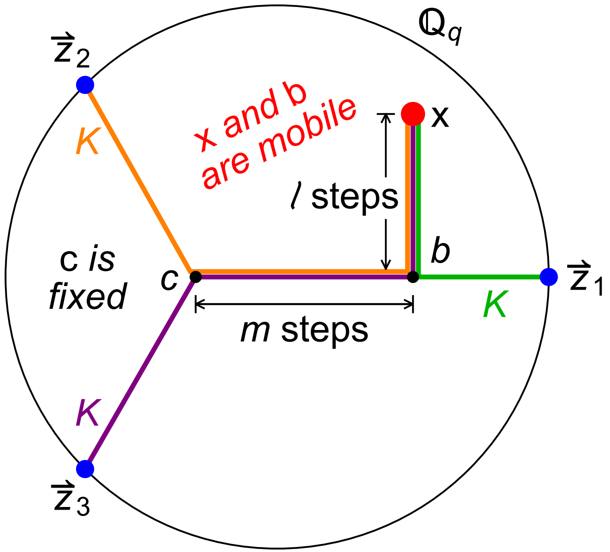
2.5. Three-point function—details

We require bulk-to-boundary and un-normalized bulk-to-bulk propagators:

$$K(x;\vec{z}) = \frac{\zeta_p(2\Delta)}{\zeta_p(2\Delta-n)} \frac{|z_0|_p^\Delta}{\left(\sup\{|z_0|_p, |\vec{z}-\vec{x}|_q\}\right)^\Delta}\,, \qquad \hat{G}(a,b) \equiv p^{-\Delta d(a,b)} \tag{22}$$

K solves $(\Box + m_p^2)K = 0$ with

$$\int_{\mathbb{Q}_q} d^n z\, K(x;\vec{z}) = 1\,.$$



The sum over position of x supplies an overall factor:

$$\sum_x \hat{G}(c,b)\hat{G}(b,x)^3$$
$$= 3\sum_{m=1}^\infty p^{-\Delta m}\left[1 + \sum_{\ell=1}^\infty (q-1)q^{\ell-1}\left(p^{-\Delta\ell}\right)^3\right]$$
$$+ \left[1 + \sum_{\ell=1}^\infty (q-2)q^{\ell-1}\left(p^{-\Delta l}\right)^3\right] = \frac{\zeta_p(\Delta)^3\zeta_p(3\Delta-n)}{\zeta_p(2\Delta)^3}\,,$$

3. The four-point function

The dominant contribution to the four-point function in the limit $|\vec{z}_{12}||\vec{z}_{34}| \ll |\vec{z}_{13}||\vec{z}_{24}|$ takes the form

$$\left\langle \prod_{i=1}^{4} \mathcal{O}(\vec{z}_i) \right\rangle_{\substack{\text{leading}\\ \text{log}}} = g_4 \frac{\zeta(2\Delta)^4 \zeta(4\Delta - n)}{\zeta(2\Delta - n)^4 \zeta(4\Delta)} \frac{\log(|\vec{z}_{12}||\vec{z}_{34}|/|\vec{z}_{13}||\vec{z}_{24}|)}{|\vec{z}_{13}|^{2\Delta}|\vec{z}_{24}|^{2\Delta}} \tag{23}$$

$$|\cdot| = |\cdot|_\infty \text{ or } |\cdot|_q\,, \qquad \zeta = \zeta_\infty \text{ or } \zeta_p\,, \qquad \log = \log_e \quad \text{or} \quad \log_p\,.$$

For $v = \infty$, this is a restatement of results of [D'Hoker et al '99]; full expression is available and is a complicated function of two independent cross-ratios,

$$u \equiv \frac{|\vec{z}_{12}||\vec{z}_{34}|}{|\vec{z}_{13}||\vec{z}_{24}|} \qquad \tilde{u} \equiv \frac{|\vec{z}_{14}||\vec{z}_{23}|}{|\vec{z}_{13}||\vec{z}_{24}|}\,. \tag{24}$$

For $v = p$, the *full expression* (still just the 4pt contact diagram) for $u \leq 1$ is

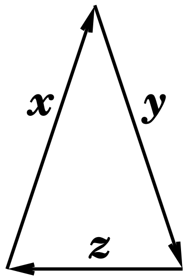
$$\left\langle \prod_{i=1}^{4} \mathcal{O}(\vec{z}_i) \right\rangle = \left[\frac{1}{\zeta_p(4\Delta)} \log_p u - \left(\frac{\zeta_p(2\Delta)}{\zeta_p(4\Delta)} + 1 \right)^2 + 3 \right] g_4 \frac{\zeta_p(2\Delta)^4 \zeta_p(4\Delta - n)/\zeta_p(2\Delta - n)^4}{|\vec{z}_{13}\vec{z}_{24}|^{2\Delta}} \tag{25}$$

$\rightarrow$ Where did $\log_p u$ come from, and why is there no $\tilde{u}$ dependence?

3.1. Ultrametricity

$|z|_p$ (and $|\vec{z}|_q$) are "ultra-metric:" $|x+y| \leq \sup\{|x|, |y|\}$.

We want this small **Theorem:** *Theorem:* $u < 1$ implies $\tilde{u} = 1$.



tall isosceles

Lemma (tall isosceles): If $x + y + z = 0$, then $|x| = |y| \geq |z|$, up to relabeling of x, y, and z.

Proof of Lemma:

Relabel so that $|z| \leq |x|$ and $|z| \leq |y|$.

Then $|x| = |y+z| \leq \sup\{|y|, |z|\} = |y|$. Likewise $|y| \leq |x|$. So $|x| = |y|$, QED.

Proof of Theorem:

Relabel so that $|z_{12}| \leq |z_{34}|$ and $|z_{13}| \leq |z_{24}|$. Then $|z_{12}| < |z_{24}|$.

$z_{12} + z_{24} - z_{14} = 0$, so $|z_{14}| = |z_{24}|$ by the Lemma.

Case I: $|z_{34}| > |z_{13}|$. Then $|z_{14}| = |z_{34}|$ by the Lemma.

$u = |z_{12}z_{34}|/|z_{13}z_{24}| = |z_{12}|/|z_{13}| < 1$, so $|z_{23}| = |z_{13}|$ by the Lemma.

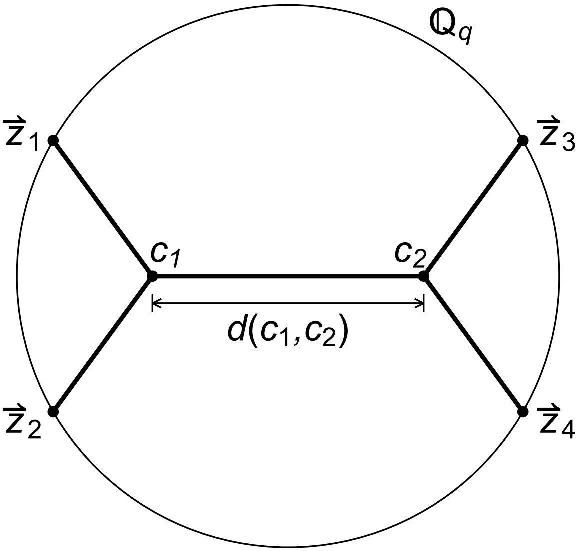
Case II: $|z_{34}| < |z_{13}|$. Then $|z_{13}| = |z_{14}| = |z_{24}| > |z_{12}|$, so again $|z_{23}| = |z_{13}|$.

Case III: $|z_{34}| = |z_{13}|$. Then $|z_{12}| = u|z_{13}| < |z_{13}|$, so again $|z_{23}| = |z_{13}|$.

Put it all together: $\tilde{u} = |z_{14}z_{23}|/|z_{13}z_{24}| = 1$, QED.

3.2. A distance formula

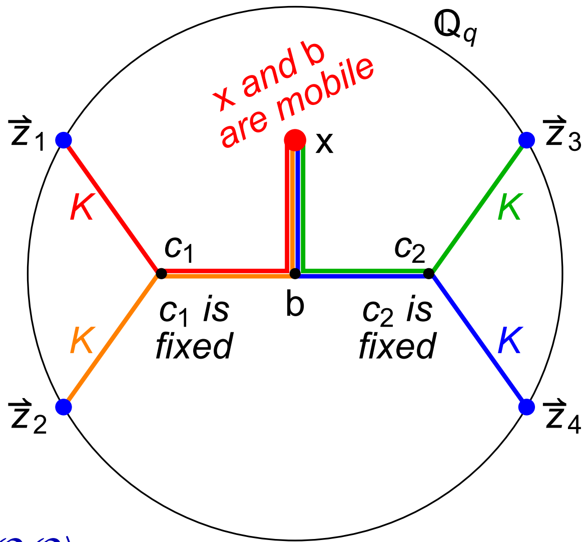
If $u < 1$, then $-\log_p u = d(c_1, c_2)$ is the number of steps between c_1 and c_2.



The situation closest to normal intuition is $|\vec{z}_{12}|, |\vec{z}_{34}| < |\vec{z}_{13}|, |\vec{z}_{14}|, |\vec{z}_{24}|, |\vec{z}_{23}|$.

Then $|\vec{z}_{13}| = |\vec{z}_{14}| = |\vec{z}_{24}| = |\vec{z}_{23}|$ (Lemma).

But the weaker result $\tilde{u} = 1$ holds even if $|\vec{z}_{34}|$ is *big*, provided only $u < 1$.



$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle = -g_4 \sum_x \prod_{i=1}^4 K(x; \vec{z}_i)$$

There are $d(c_1, c_2)$ equivalent ways to locate the b-x branch.

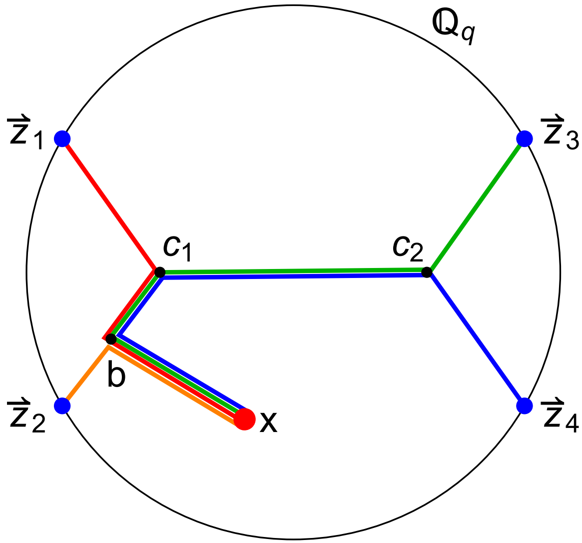
Hence we get a factor of $-\log_p u$ in $\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle$.

3.3. Form of the four-point function

Non-logarithmic terms come from "subway diagrams" of a slightly different topology:

Despite appearances, we're never including exchange diagrams!

Only the $g_4\phi^4$ interaction is included in our published calculations, but exchange diagrams don't drastically change the story: same $\log_p u$ appears.



Logarithmic behavior suggests a small anomalous dimension for $\mathcal{O}^2$:

- Suppose $\Delta_{\mathcal{O}^2} = 2\Delta_{\mathcal{O}} + \delta$ with δ small.
- $\mathcal{O}(\vec{z}_1)\mathcal{O}(\vec{z}_2) \sim |\vec{z}_{12}|^{-\delta}\mathcal{O}^2(\vec{z}_1)$ if $|\vec{z}_{12}|$ is small.
- $\left\langle \prod_{i=1}^4 \mathcal{O}(\vec{z}_i) \right\rangle \sim |\vec{z}_{12}\vec{z}_{34}|^{\delta}\langle \mathcal{O}^2(\vec{z}_1)\mathcal{O}^2(\vec{z}_3)\rangle \sim \frac{|\vec{z}_{12}\vec{z}_{34}|^{\delta}}{|\vec{z}_{13}|^{4\Delta+2\delta}} \approx \frac{1-\delta\log u}{|\vec{z}_{13}\vec{z}_{24}|^{2\Delta}}$.
- Similar mechanism is responsible for $\log u$ behavior in classic AdS/CFT.

What about the field theory on the boundary, then?

3.4. Excursion into field theory

A Fourier transform on $\mathbb{Q}_q$ exists which exhibits most standard properties:

$$f(x) = \int_{\mathbb{Q}_q} d^n k \, \chi(kx) \hat{f}(k) \qquad\qquad \chi(\xi) = e^{2\pi i [\xi]} \,. \tag{26}$$

Details of χ are slightly tricky. Let's be satisfied with $\chi \colon \mathbb{Q}_q \to S^1 \subset \mathbb{C}$ such that $\chi(\xi_1 + \xi_2) = \chi(\xi_1)\chi(\xi_2)$: an "additive character."

Suppose $G_{\phi\phi}(k) = \langle \phi(k)\phi(-k) \rangle = \dfrac{1}{|k|^s + r}$ for real r, s but with $k \in \mathbb{Q}_q$.

Then we have UV divergences that work similarly to QFT on $\mathbb{R}^n$:

$$\int_{\mathbb{Q}_q}^{\Lambda} \frac{d^n \ell}{|\ell|^s + r} \sim \Lambda^{n-s} \qquad \text{diverges as } \Lambda \to \infty \text{ if } n > s$$

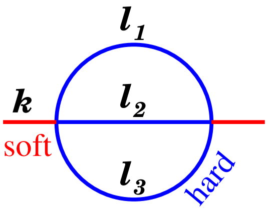
where $\int^\Lambda$ means we restrict $|\ell| < \Lambda$.

Can we get a p-adic Wilson-Fisher fixed point starting from ϕ^4 theory?

$$S = \int_{\mathbb{Q}_q} d^n k \, \frac{1}{2} \phi(-k)(|k|^s + r)\phi(k) + \int_{\mathbb{Q}_q} d^n x \, \frac{\lambda}{4!} \phi(x)^4 \,. \tag{27}$$

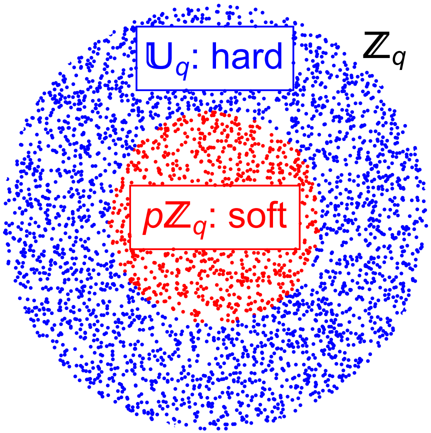
3.5. The funny thing about effective field theory

Integrating out a momentum shell generates perfectly local interactions with no derivatives [Lerner-Missarov '89]. For example: If $|k| < 1$, then



$$I_2(k) \equiv \int_{\mathbb{U}_q} \frac{d^n\ell_1\, d^n\ell_2\, d^n\ell_3}{(1+r)^3} \delta(\textstyle\sum_{i=1}^3 \ell_i - k) \doteq I_2(0)\,. \tag{28}$$

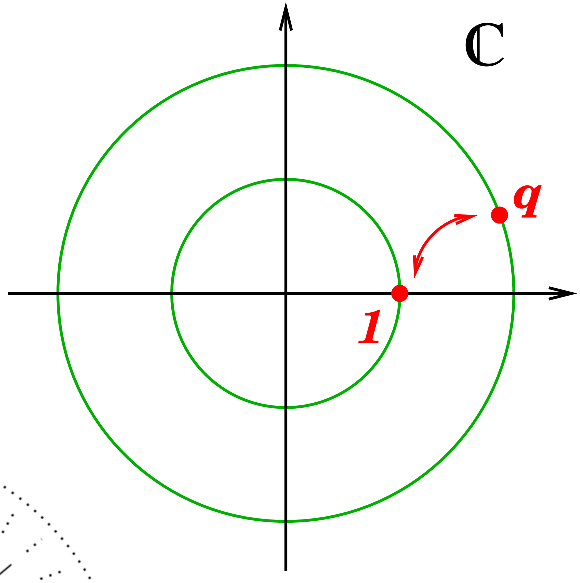
$\mathbb{U}_q = \{\ell \in \mathbb{Q}_q : |\ell| = 1\}$ is the $\Lambda = 1$ momentum shell.



- To demonstrate $\doteq$ we simply set $\tilde{\ell}_3 = \ell_3 - k$. Given $|k| < 1$, we have $|\ell_3| = 1$ iff $|\tilde{\ell}_3| = 1$, by the Lemma.
- $\ell_3 \to \tilde{\ell}_3$ is a bijection on $\mathbb{U}_q$, and $d^n\ell_3 = d^n\tilde{\ell}_3$. Similar arguments apply for any graph.
- $S_{\rm eff} = \int d^n k\, \frac{1}{2}\phi(-k)|k|^s\phi(k) + \int d^n x\, V_{\rm eff}(\phi)\,.$
- Operators $\phi(x)^m$ mix among themselves but not with $\phi\partial^\ell\phi$ like in real case.
- I suspect this is closely related to simplicity of 4pt function: OPE of $\mathcal{O}(x)\mathcal{O}(0)$ is probably very sparse, with no descendants.

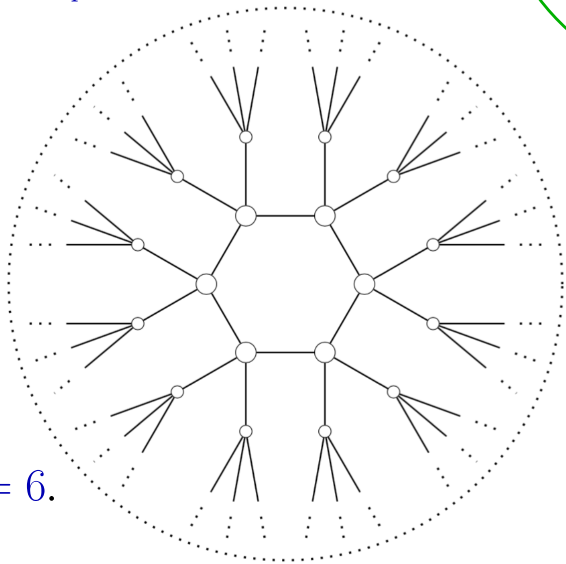
4. Loose ends and future directions

4.1. A BTZ-like construction



In [Heydeman-Marcolli-Saberi-Stoica '16] a p-adic version of Euclidean BTZ was suggested.

- Inspiration is the representation of a torus as $\mathbb{C}^\times$ modulo the map $z \to qz$ where $q = e^{2\pi i\tau}$.
- Similarly identify $\mathbb{Q}_p^\times$ modulo $z \to qz$ where $|q| > 1$.



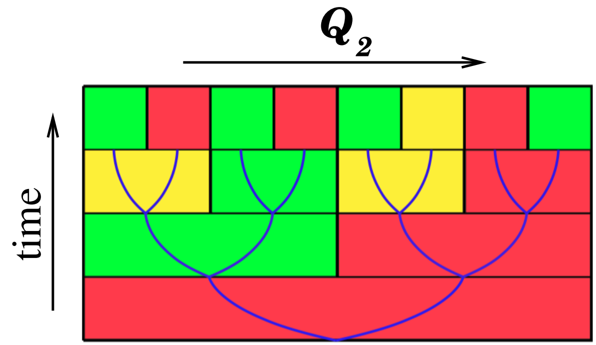
Here $p = 3$ and $\log_p |q| = 6$.

- Identifying the bulk by $\begin{pmatrix} q & 0 \\ 0 & 1 \end{pmatrix} \in \text{PGL}(2, \mathbb{Q}_p)$ gives a cycle of length $\log_p |q|$.

4.2. A connection to inflation

Eternal symmetree [Harlow-Shenker-Stanford-Susskind '11]:

- An idealization of eternal inflation.
- A given causal region can nucleate new de Sitter vacua which fall out of causal contact and split again.
- A statistical model is defined on the tree for $\mathbb{Q}_2$ where each node is a causal patch.
- The forward-time boundary is $\mathbb{Q}_2$, which is more descriptive of the late-time multiverse than the forward-time boundary of de Sitter.



- 2-adic symmetry is realized on statistical correlators of observables at this forward-time boundary.

Single-cell probabilities evolve by $P_m(\text{child}) = G_{mn} P_n(\text{parent})$ where m, n run over colors representing different dS vacua.

N-point correlators comes from joint probabilities $P_{n_1 \ldots n_N}$ where n_i specifies color of cell a_i, eventually all taken to the far future.

4.3. Conclusions and future directions

- A holographic direction is natural in describing the p-adic numbers: z_0 is the p-adic accuracy of a truncated p-nary expansion of z.
- A simple classical action on the tree whose boundary is $\mathbb{Q}_q$ gives rise to N-point functions closely analogous to standard results in AdS/CFT.
  - Normalizations are naturally expressed in terms of local zeta functions $\zeta_v(\Delta)$.
- Why are the p-adic results so close to classic AdS/CFT for $N = 3$ and $N = 4$, but not quite the same? ($N = 2$ suffers some IR regulator issues.)
- What about fluctuating geometry in the tree graph?
- What about Lorentzian signature?
- Ultra-metricity leads to some interesting simplifications in QFT.
  - Momentum shell integration, counter-terms, and OPE all seem simpler.
  - Form of holographic 4pt function hints at similar simplifications.
- Can we give a more symmetry-based presentation of p-adic CFT (cf. [Melzer '89]), and/or find explicit AdS/CFT dual pairs?