Black holes, wormholes, long times and ensembles

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Motivation

Black hole horizons appear to display irreversible, dissipative behavior.

In tension with unitary quantum time evolution of the boundary theory in gauge/gravity duality.

A simple example: long time behavior of two-point functions $\langle O(t)O(0)\rangle$.
[Maldacena: Dyson-Kleban-Lindesay-Susskind, Barbon-Rabinovici]

Bulk semiclassical calculation: $\langle O(t)O(0)\rangle$ decays indefinitely – quasinormal modes. [Horowitz-Hubeny]

Boundary calculation:

$$\langle O(t)O(0)\rangle = \sum_{m,n} e^{-\beta E_m} |\langle m|O|n\rangle|^2 e^{i(E_m - E_n)t} / Z$$

We expect that black hole energy levels are discrete (finite entropy) and nondegenerate (chaos).

Then at long times $\langle O(t)O(0)\rangle$ stops decreasing. It oscillates in an erratic way and is exponentially small (in S).

What is the bulk explanation for this? An aspect of the black hole information problem...

Spectral form factor

The oscillating phases are the main actors here. Use a simpler, related diagnostic, the "spectral form factor" (SFF) [Papadodimas-Raju]:

$$\mathrm{SFF}(t) = \sum_{m,n} e^{-\beta(E_m+E_n)} e^{i(E_m-E_n)t} = Z_L(\beta+it) Z_R(\beta-it)$$

Study in SYK model [Sachdev-Ye, Kitaev]:

$$H_{SYK} = \sum_{abcd} J_{abcd} \psi_a \psi_b \psi_c \psi_d$$

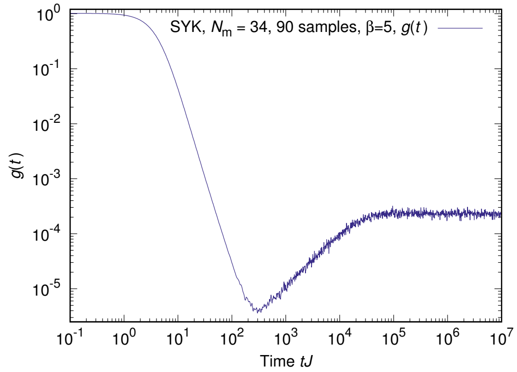
J_{abcd} are independent, random, Gaussian distributed. An **ensemble** of boundary QM systems, $\langle \cdot \rangle$

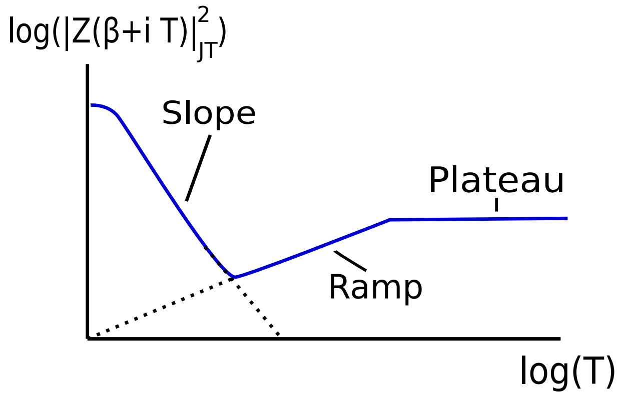
Do computer "experiments" ...

[You-Ludwig-Xu,Garcia-Garcia-Verbaaschot,

Cotler-Gur-Ari-Hanada-Polchinski-Saad-SS-Stanford-Streicher-Tezuka ...]

Random matrix statistics





SFF $= \langle Z_L(\beta + it) Z_R(\beta - it) \rangle$ – ensemble averaged result.

Slope is contained in disconnected contribution $\langle Z_L(\beta + it)\rangle \langle Z_R(\beta - it)\rangle$. This vanishes at late time.

Ramp and plateau are in the connected part. Signatures of random matrix eigenvalue statistics, quantifying long and short range level repulsion. Exponentially small effects, but leading at large time.

(Quantitatively) universal in quantum chaotic systems. Includes the boundary QM systems in standard gauge/gravity duals.

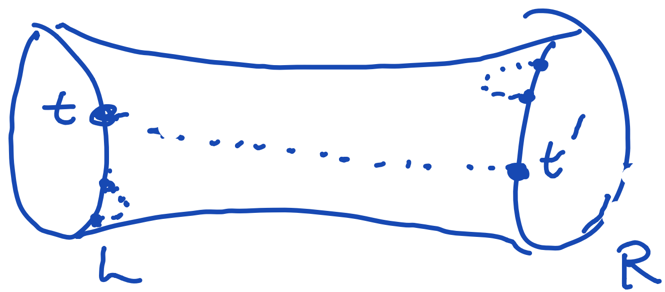
Bulk explanation?

Ramp

Look for connected SYK saddle point with nonvanishing collective field

$$G_{LR}(t,t') = \frac{1}{N}\sum_{a=1}^{N} \psi_a^L(t)\psi_a^R(t').$$

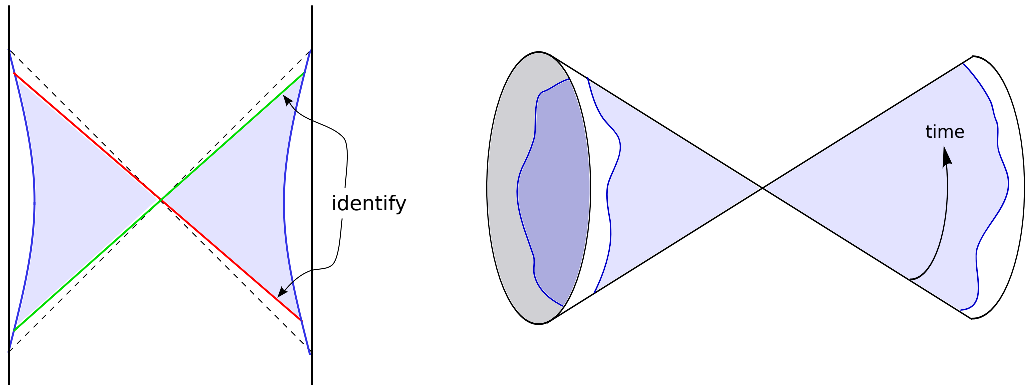
At low energies collective field saddles are given by bulk correlators on JT gravity geometries [Jensen, Maldacena-Stanford-Yang, Engelsoy-Mertens-Verlinde].



So we can think of G, Σ as proxies for bulk degrees of freedom.

Double cone

Ramp explained by the "double cone" in JT [Saad-SS-Stanford] :



- A spacetime wormhole [Maldacena-Qi].
- Identified under $\beta - it$. Complexified saddle. Regulates bifurcate point singularity.
- Topology of cylinder, $\chi = 0$. Weight $\sim 1 = e^{\chi S_0}$. Exponentially small.(Slope from disk topologies, $\sim e^{2S_0}$.)
- Zero mode from relative time rotation. Gives linear t dependence.
- Not a true saddle. Weak "pressure" to lower mass of black hole (energy of boundary system). Can stabilize by fixing energy – microcanonical partition function $Y_E(it)$.

JT gravity as an ensemble

[See Thomas Mertens' talk.]

Can work directly in JT gravity.

JT gravity is precisely dual to an ensemble of boundary Hamiltonians. Random matrix ensemble with a potential $V(H)$ tuned to give the leading JT density of states [Saad-SS-Stanford]. (Fine grained) random matrix statistics are exact.

Formally analogous to "old" matrix models of string theory. $p \to \infty$ limit of $(2, p)$ minimal strings [Seiberg-Stanford, Mertens-Turiaci].

But a different perspective:
String joining and splitting becomes JT baby universe joining and splitting. Genus expansion parameter is $e^{\chi S_0}$, nonperturbative in G_N.
Matrix is not like an $(N \times N)$ YM field. It is the full second quantized boundary H, effectively $e^S \times e^S$. "Third quantization."

Extensions:
to JT supergravity $\leftrightarrow$ Altland-Zirnbauer ensembles [Stanford-Witten].
to JT gravity with BF gauge theory $\leftrightarrow$ matrix ensembles with irrep subsectors [Iliesiu, Kapec-Mahajan-Stanford].
to general dilaton potentials and reductions of 3D gravity [Maxfield-Turiaci, Witten].

SFF as an overlap

Hilbert space interpretation of SFF.

Double the system and form the (unnormalized) TFD state $|TFD\rangle$. Time evolve on one side, giving $|TFD(t)\rangle = e^{-iHt}|TFD\rangle$. Then

$$Z(\beta - it) = \langle TFD|TFD(t)\rangle$$
$$Z(\beta + it)Z(\beta - it) = |\langle TFD|TFD(t)\rangle|^2$$

Time evolution begins to "randomize" state. For random vectors in Hilbert space of dimension d, $|\langle v|w\rangle|^2 \sim 1/d$. Here $d \sim e^{2S}$, consistent with beginning of ramp (after normalization). Small nonzero overlap due to finite dimensional state space.

Wormholes provide the bulk explanation for small nonzero overlaps. A recurrent theme (cf. replica wormholes).

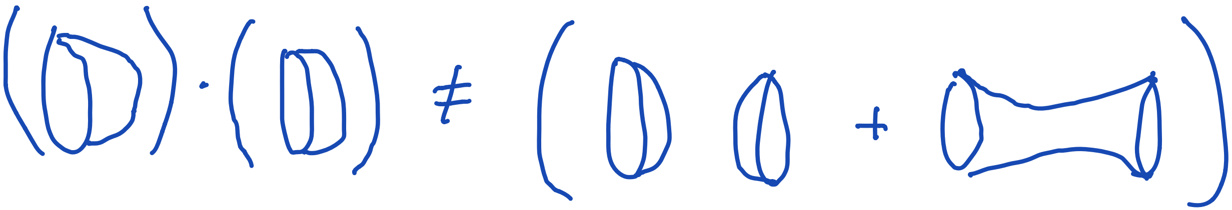
(Increases because e^{-iHt} at late times is different from a Haar random unitary U. No eigenvalue repulsion.)

Factorization puzzle

The ensemble averaged matrix element $\langle\langle TFD|TFD(t)\rangle\rangle = \langle Z(\beta - it)\rangle$ goes to zero at long time (oscillating phases, slope, disk in bulk).

But the squared matrix element $\langle|\langle TFD|TFD(t)\rangle|^2\rangle = \langle|Z(\beta - it)|^2\rangle$ does not (wormhole in bulk).

Violates factorization.



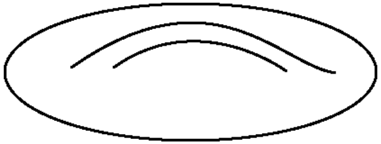
The wormhole explanation of the ramp leads to a factorization puzzle if the boundary is a single quantum system [Witten-Yau, Maldacena-Maoz].

Ensemble average of boundary systems restores consistency [Coleman, Giddings-Strominger, Maldacena-Maoz].

Correlation functions and wormholes

Long time thermal two point function $\text{Tr}[e^{-\beta H} O(t) O(0)]$:
In JT [Blommaert-Mertens-Verscheide, Saad].

One boundary. Ramp from:



$\chi = -1$. Weight $\sim e^{-S} \cdot t$.

Interpretation [Saad]:
Quantum created by $O(0)$ falls behind the horizon. Wormhole forms (e^{-S} t amplitude), allowing quantum to escape and be acted on by $O(t)$.

Bulk understanding of multi-point correlators (OTOCs) [Stanford-Yang-Yao] gives insights into aspects of the unitary black hole S-matrix.

Higher dimensions

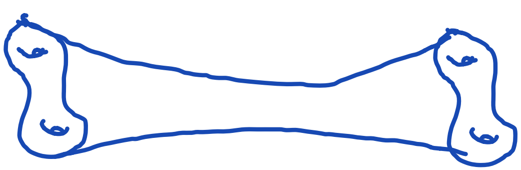
[See Kristan Jensen's talk.]

The ramp is universal, so should appear in higher dimensional AdS/CFT contexts. Perhaps there is a wormhole explanation for $|Z(\beta - it)|^2$ here as well.

[Cotler-Jensen] have found candidate higher dimensional Euclidean wormhole configurations where boundary has thermal circle.

Construction of complexified double cone from eternal AdS-Schwarzschild geometry is general. Stable against brane nucleation [Mahajan-Marolf-Santos, Cotler-Jensen].

Genus 2 boundary wormhole in 3D gravity. Making an ETH-like ansatz for OPEs, the "statistical" CFT prediction agrees with the weight of the gravitational saddle [Belin-de Boer, Pollack-Rozali-Sully-Wakeham].



Maybe "3D gravity" is dual to an ensemble of 2D CFTs? (Would cure some anomalies pointed out by [Maloney-Witten]. See [Maxfield-Turiaci]).

Ensembles of CFTs

What would the ensemble of 2D CFTs be? QFTs are much more constrained than generic QM Hamiltonians.

Try averaging Narain moduli space (N free fields) [Maloney-Witten, Afkhami-Jeddi-Cohn-Hartman-Tajdini].

Consistent with a sum over partition functions of bulk $U(1)^N$ Chern-Simons theory on different 3D geometries.

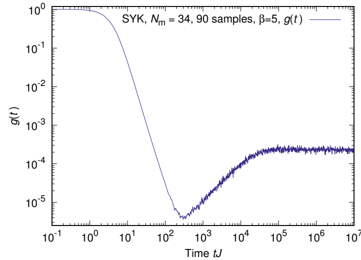
CFT is built from free fields so not chaotic. No ramp [Collier-Maloney, Cotler-Jensen].

Other (rational) examples [Benjamin, Datta, Dong, Duarie, Hartman, Jiang, Keller, Kraus, Maiti, Maloney, Meruliya, Mukhi, Ooguri, Zadeh ...].

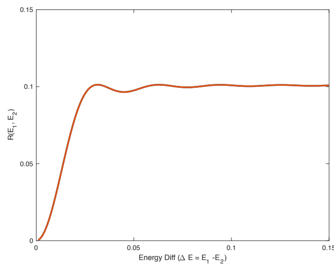
Is there a notion of an ensemble of approximate (large c, large gap) CFTs?

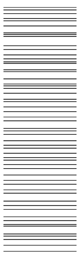
Plateau

[See Clifford Johnson's talk.]



SFF is, essentially, the Fourier transform of $\langle \rho(E)\rho(E')\rangle$:





$$\langle \rho(E)\rho(E')\rangle \sim e^{2S(\bar{E})} - \frac{1}{2(\pi(E-E'))^2}\left(1-\cos(2\pi e^{S(\bar{E})}(E-E'))\right)$$

A smoking gun for discrete eigenvalues in an averaged description.

Bulk "gravitational" interpretation? e^{ie^S} nonperturbative in genus expansion (doubly nonperturbative). D-brane effect in "JT string."

Resolvents and FZZT branes

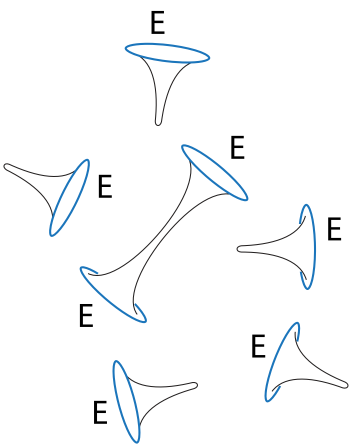
Use brane technology from topological/minimal string theory [Aganagic, Dijkgraaf, Klemm, Marino, Vafa, Fateev, Kutasov, Maldacena, Martinec, McGreevy, Moore, Seiberg, Shih, Teschner, Verlinde, Zamolodchikov, Zamolodchikov, ...].

Extract $\langle \rho(E)\rho(E')\rangle$ from double resolvent $\langle \mathrm{Tr}\frac{1}{E-H}\mathrm{Tr}\frac{1}{E'-H}\rangle$.
Then use identity

$$\mathrm{Tr}\frac{1}{E-H} = \partial_E \frac{\det(E-H)}{\det(\widetilde{E}-H)}\Big|_{\widetilde{E}\to E}.$$

Get $\langle \rho(E)\rho(E')\rangle$ from $\langle \frac{\det\det}{\det\det}\rangle$. $\det(E-H)$ inserts an FZZT brane.

- $\det(E-H) = \exp(\mathrm{Tr}\log(E-H))$
- Many disconnected "worldsheets" [Polchinski].
- Worldsheets $\leftrightarrow$ JT spacetimes.
- A "many universe" quantity.
- D-brane in "superspace."



Nonlinear sigma model

Nonlinear sigma model formalism: RMT universality from symmetry.

[Efetov, Wegner, Altshuler, Andreev, Agam, Simons, Muller, Heusler, Braun, Haake, Altland, Zirnbauer, Verbaarschot, Sonner, Bagrets, Kamenev...] [In this context, see Altland-Sonner.]

Again, extract $\langle \rho(E)\rho(E')\rangle$ from $\langle \frac{\det \det}{\det \det} \rangle$.

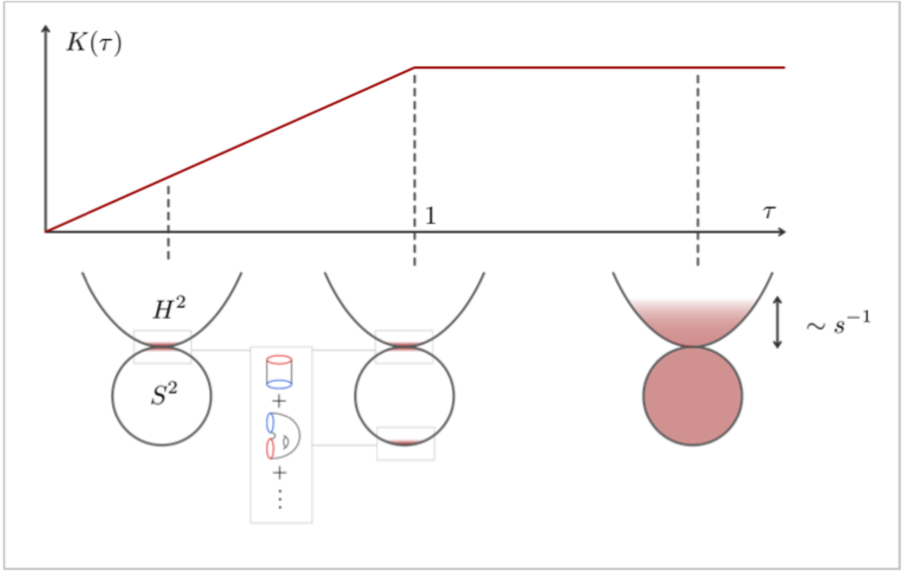
Represent $\det(E - H_{ab})$ as an integral over "matter" fields x_a^A: det from Grassmanns; $1/\det$ from scalars. Here a is a "color" index and A is a "flavor" index. System has a graded $U(2|2)$ flavor symmetry, explicitly broken by a "mass" term $(E - E')$.

Integrate out "gluons" H_{ab} (using H ensemble) to get an effective action for "meson" fields $y^{AA'} \sim x_a^A x_a^{A'}$ – open string dynamics on FZZT "flavor" branes.

Infrared dynamics governed by a nonlinear sigma model with target $U(2|2)/(U(1|1) \times U(1|1))$. The ramp is due to perturbative "pion" fluctuations. The nonperturbative effects are due to another saddle, a bit like a baryon, called the Andreev-Altshuler instanton.

Universality only depends on symmetry breaking pattern.

Causal symmetry restoration



Altland – Sonner

"Causal symmetry restoration."

Gravitational interpretation of these ideas?

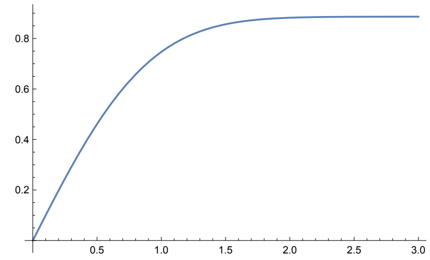
Plateau without branes

SFF is a two boundary quantity. Does it inevitably require arbitrarily many boundaries to fully describe?

Work at fixed β, not E – integrate over all different energy bands. Smooths out ripples in $\langle \rho(E)\rho(E')\rangle$.

Work in t domain. Find in simplest case, topological gravity, [Okuyama-Sakai]

$$\mathrm{SFF}(t) \sim e^{S_0}\mathrm{Erf}(e^{-S_0}t) \qquad (t \gg \beta,\ \beta \equiv 1).$$



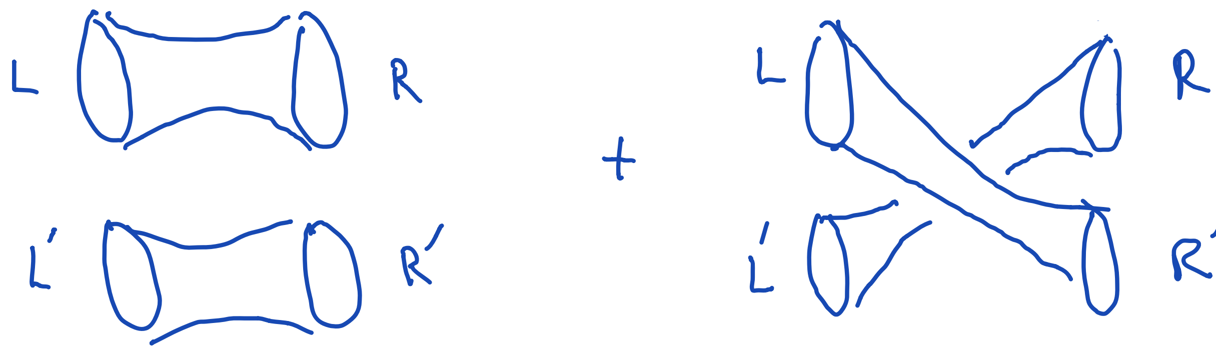
No sharp features. Has convergent power series in $e^{-S_0}t$. A genus expansion. Geometrical interpretation? (Can write down an analogous expression for JT [Stanford].)

Gauge/gravity duality without averaging

Standard gauge/gravity duals, like SYM, are not averaged. Too hard ....

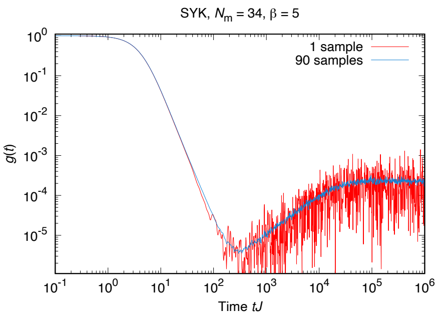
A toy model for non-averaged behavior: a single member of an ensemble.

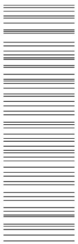
A basic result: if wormholes dominate (like on ramp), then the variance of $Z_L(it)Z_R(-it)$ in the ensemble is of order of the signal squared.



The answer depends sensitively on the element of the ensemble.
It is **noisy**.

Single sample noise

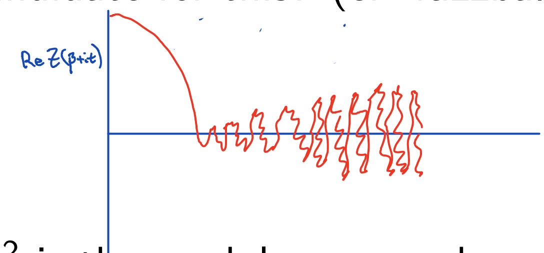




Short autocorrelation time. Different times act like different samples. RMT universality: noisy ramp is generic in gauge/gravity duality. A consequence of microstructure.

What accounts for the noise in the bulk description?

$Z(\beta - it)$ oscillates erratically around zero at late time, with variance $\sim t$. Geometric candidate for this? (cf. fuzzballs in special cases....).



SFF $= |Z(\beta - it)|^2$ is the modulus squared, producing a noisy ramp. It factorizes, and contains a wormhole signal (after time averaging). Noise and factorization parts of same puzzle. Approaches?

Universe field theory

Many baby universes, and geometries connecting them [Coleman, Giddings-Strominger, Marolf-Maxfield. See also: Blommaert-Mertens-Verscheide]. Follow [Marolf-Maxfield]:

Many boundary $\mathcal{H}$

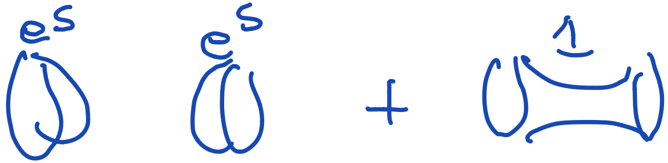
$\mathcal{H} = \{|NB\rangle, |O\rangle, |OO\rangle \cdots\}$; $\hat{Z}|NB\rangle = |O\rangle$

$\hat{Z}|O\rangle = |OO\rangle$

$\vdots$

Inner product from geometry

$\langle O|O\rangle = \langle NB|\hat{Z}^2|NB\rangle =$



doesn't factorize

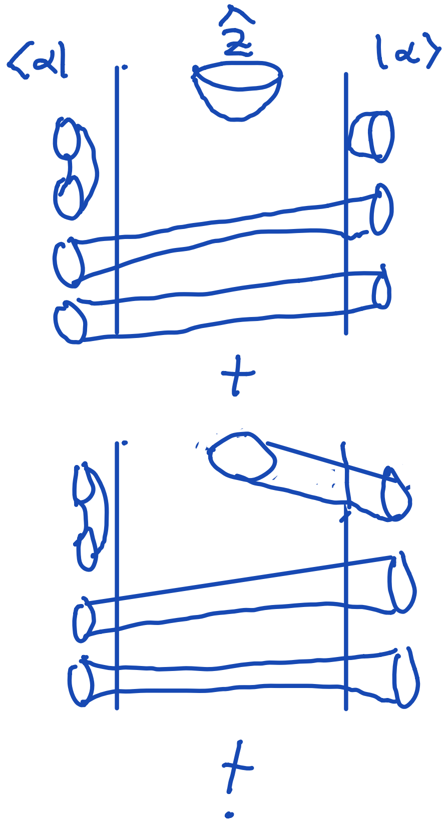
Factorization in α state

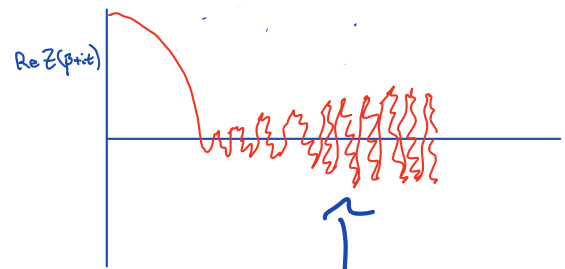
$\hat{Z}|\alpha\rangle = \alpha|\alpha\rangle$; $|\alpha\rangle$ one element of ensemble

Universe field theory, contd.

$$|\alpha\rangle = c_0|NB\rangle + c_1|O\rangle + c_2|OO\rangle + \cdots$$

$$\langle\alpha|\hat{Z}|\alpha\rangle \quad C$$

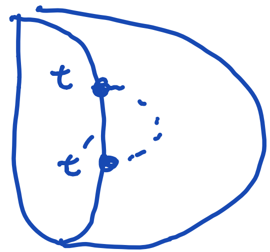




noisy contribution

SYK with fixed couplings

Can formulate a collective field (G, Σ) description of SYK with **fixed** couplings [Saad-SS-Stanford-Yao].



Only a toy model of gravity, But we have an explicit representation for the entire "gravitational" (G, Σ) path integral. Must contain noise somewhere.

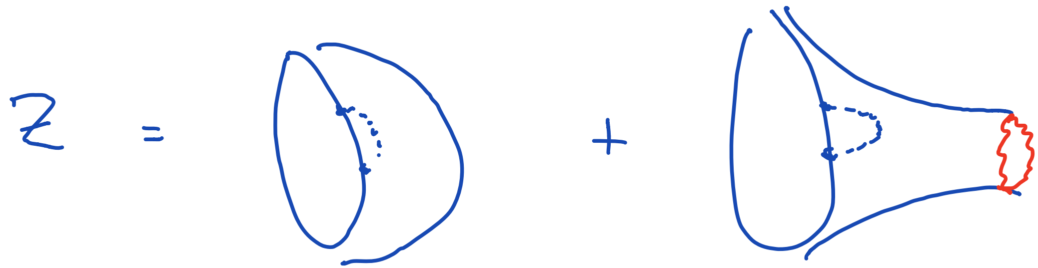
$$Z = \int dG(t, t')d\Sigma(t, t') \exp(I_{\text{fixed}}(N, G, \Sigma))$$

In principle we can explore its nooks and crannies. (Detailed analysis only in a toy^2 model with one time point.)

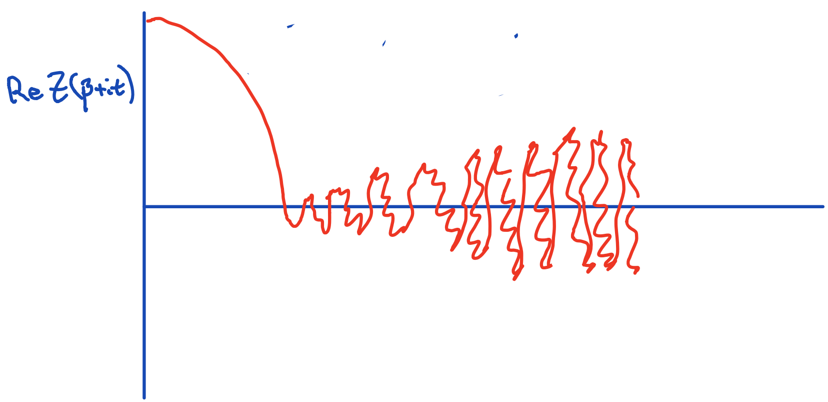
Where does the noise come from?

Half-wormholes

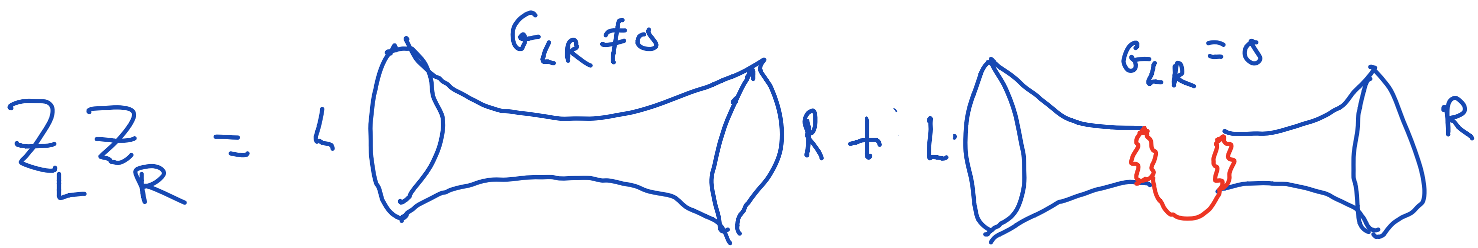
Scenario motivated by simple model. Z given by disk and another saddle point – “half-wormhole.”



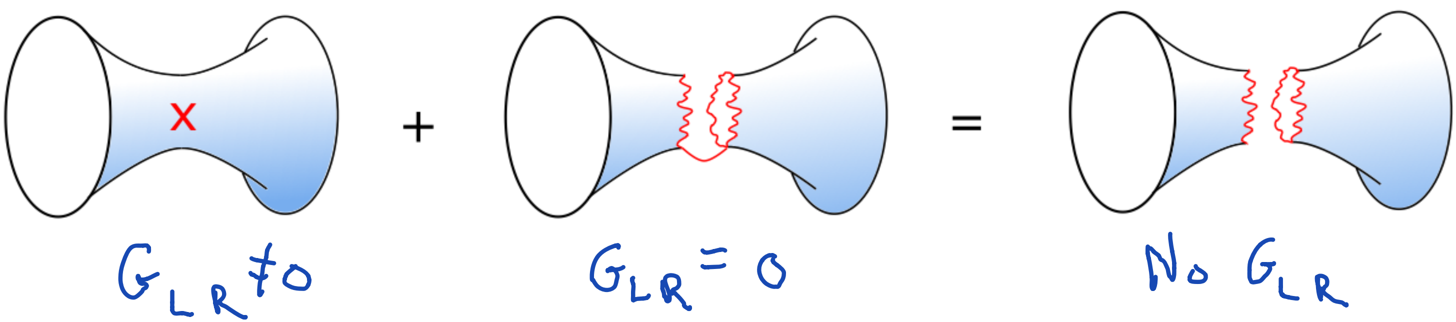
Half-wormhole is the source of the noisy late-time behavior. Vanishes on averaging.



The noisy ramp



The wormhole plus the half-wormholes combine to restore factorization.



A multiplicity of bulk descriptions...

[cf. Marolf-Maxfield, Jafferis-Schneider, Eberhardt, Mukhametzhanov...].

Questions

Lots...

Are these ideas relevant for standard holographic systems?

Discussion: Friday June 25, 12:20 - 13:10 (BRT)
[8:20 - 9:10 (PDT)]

Also:

Structure of black hole microstates: Discussion June 22

Black hole information problem: Discussion June 25

Talks:
Mertens (6/23); Jensen, Johnson (6/25); Eberhardt (6/30)

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