Holographic Entanglement Entropy For $WAdS_3$

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Introduction

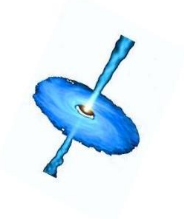
Bekenstein
Bardeen-Carter-Hawking
Strominger-Vafa
Maldacena

$$S_{BH} = \frac{A}{4G}$$

AdS/CFT

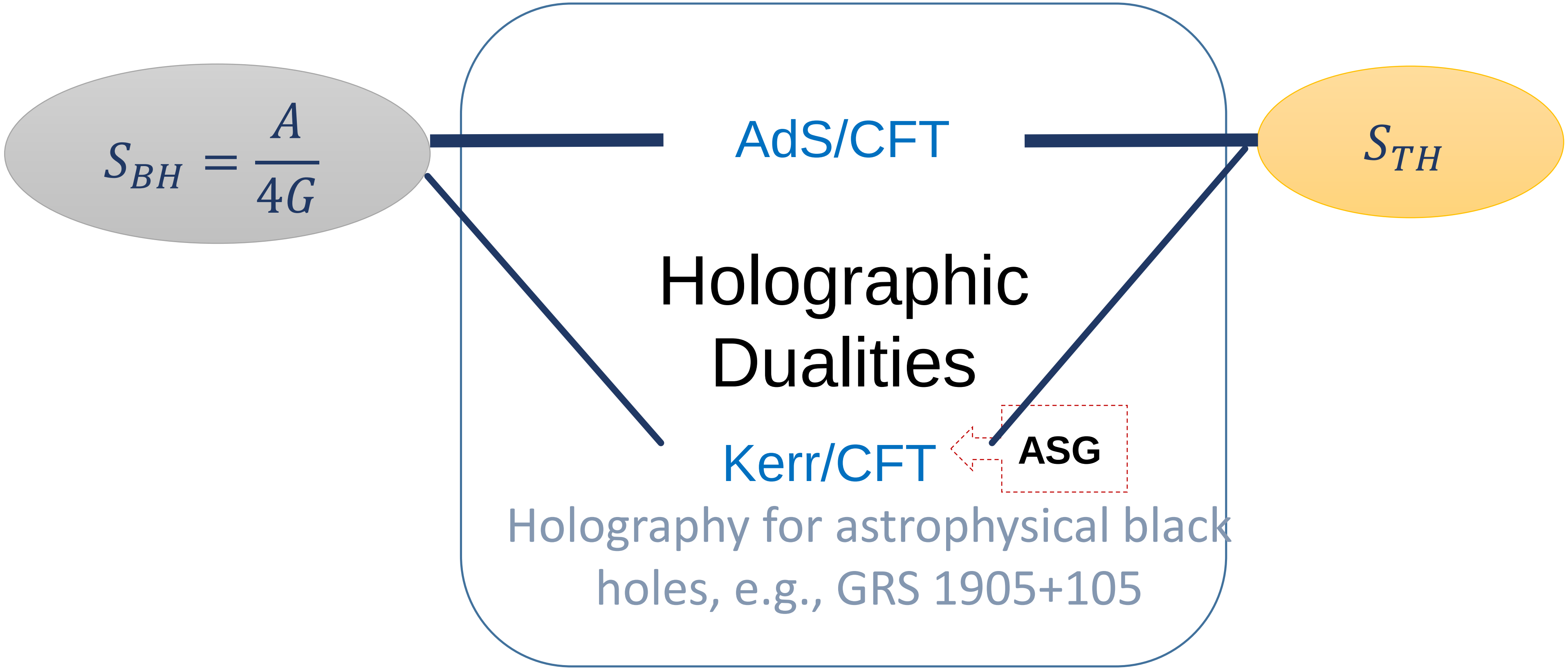
S_{TH}

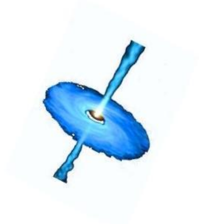
Holographic Dualities



Introduction

Guica-Hartman-WS-Strominger
Bredberg-Hartman-WS-Strominger
Castro-Maloney-Strominger





Introduction

Anninos-Li-Padi-WS-Strominger
Compere-Guica-Rodriguez

Detournay-Compere
Hofman-Strominger
Deournay-Hartman-Hofman
Compere-WS-Strominger

$$S_{BH} = \frac{A}{4G}$$

AdS/CFT

S_{TH}

Holographic Dualities

Kerr/CFT

ASG

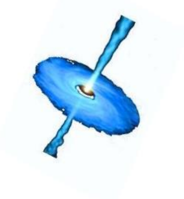
WAdS/CFT

WAdS/WCFT

SL(2,R)xU(1)

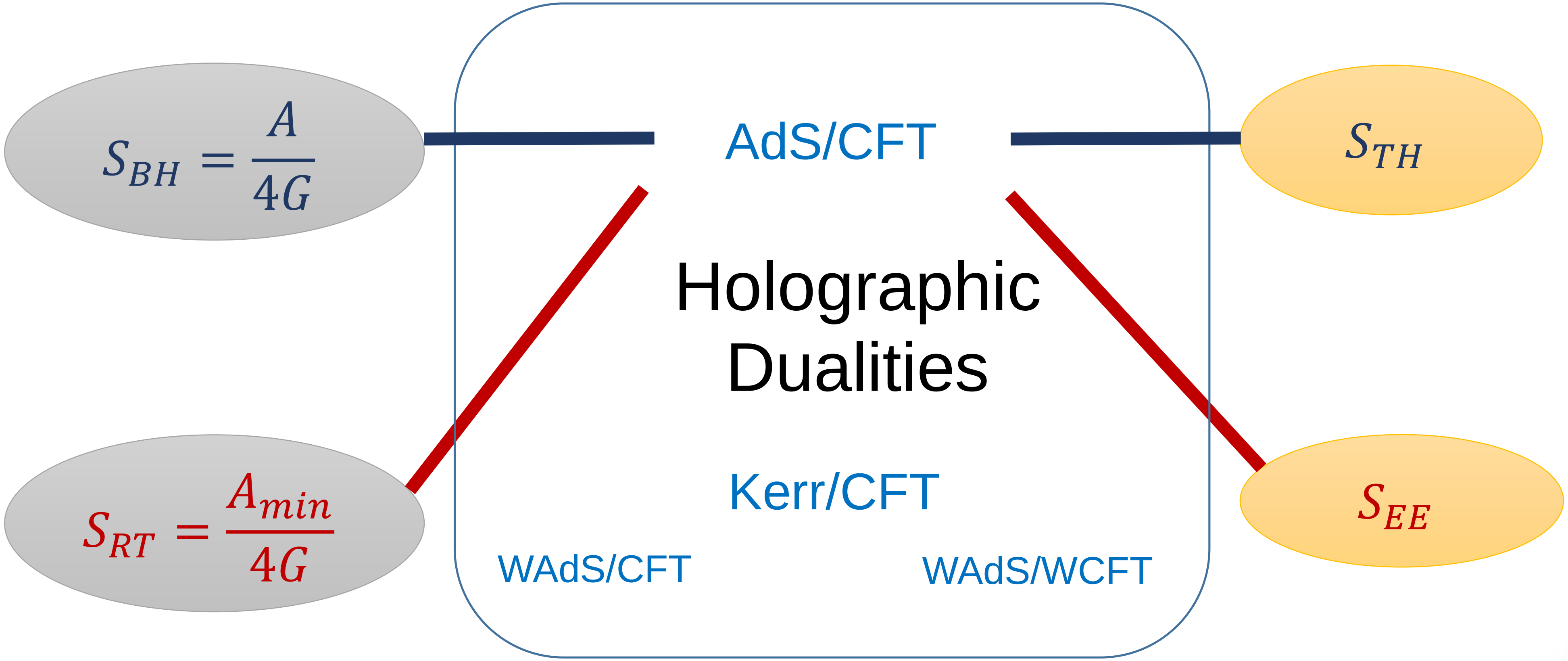
CFT:
Virasoro-Virasoro

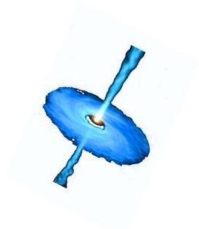
WCFT:
Virasoro-Kac-Moody



Motivation

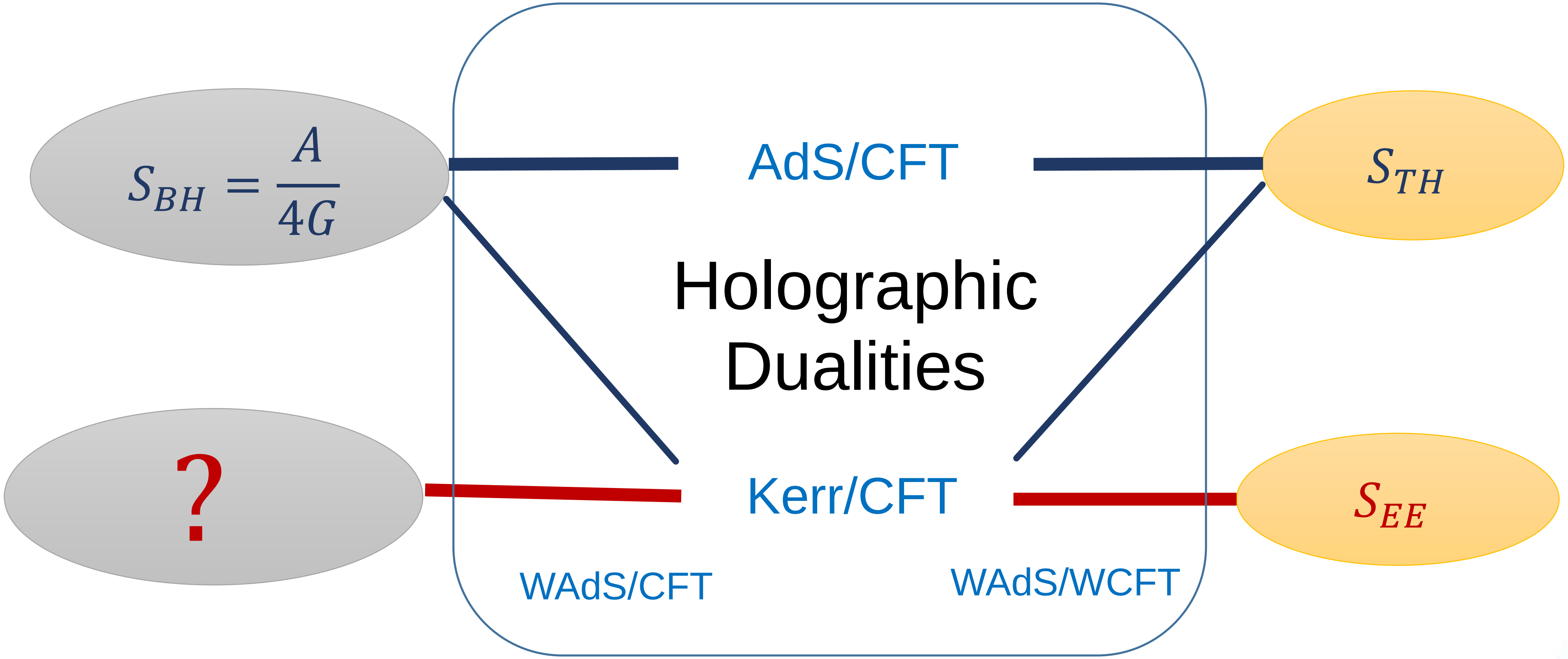
Ryu-Takayanagi
Casini-Huerta-Myers
Lewkowycz-Maldacena

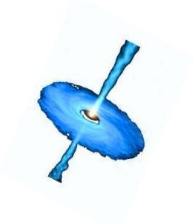




Motivation

WS-Wen-Xu





Outline

- Review of WAdS3
- EE in WAdS/CFT
- EE in WAdS/WCFT

WAdS_3 holography

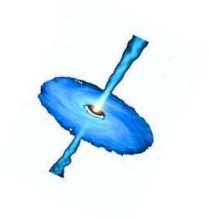
$$\underbrace{ds^2_{WAdS_3}}_{SL(2,R)_L \times U(1)_R} = \underbrace{d\hat{s}^2_{\text{AdS}_3}}_{SL(2,R)_L \times SL(2,R)_R} + \underbrace{\lambda^2 A^2}_{SL(2,R)_R \to U(1)_R}$$

A: $SL(2,R)_L$ Invariant 1-form

WAdS_3/CFT:	Vir $_L$-Vir $_R$	*Compere-Guica-Rodriguez*
Evidence:	$S_{BH} = S_{Cardy}$	*Anninos−Li−Padi−WS−Strominger*
EE: WS−Wen−Xu	? $= \frac{c}{3}\log(\frac{a}{\epsilon})$	*Calabrieze−Cardy*

WAdS_3/WCFT:	Vir $_L$- Kac-Moody $_L$	*Detournay-Compere*
Evidence:	$S_{BH} = S_{DHH}$	*Deournay-Hartman-Hofman*
EE: WS−Wen−Xu	? $= S_{CHI}$	*Castro−Hofman−Iqbal*

AdS_3/WCFT	*Compere-WS-Strominger*



Entanglement entropy in WAdS/CFT

- Assumptions: WAdS/CFT
- Method: Lewkowycz-Maldacena adapted
- Results: The bulk calculation agrees with the CFT expectation

The generalized gravitational entropy

Lewkowycz-Maldacena

AdS/CFT, Einstein

- Replica trick extended to the bulk

$$S_{EE} = -n\partial_n[\log \mathcal{Z}_n - n\log \mathcal{Z}_1]|_{n=1}$$

generalized gravitational entropy

$$= \partial_n(nI[g_n/Z_n] - nI[g_1])|_{n=1} = \partial_n I_{bk}[g_n/Z_n]|_{n=1}$$

- Selecting a special curve

$$ds^2_{e,n} = n^2 dr^2 + r^2 d\tau^2 + (g_{ij} + 2K^{(n)}_{aij}x^a)dy^i dy^j + \cdots$$

$n \to 1,$

$K^{(n)}_a = 0$ Extremal!

- Calculate the generalized gravitational entropy

$$S_{EE} = \int_{\gamma_{\mathcal{A}} \times S^1} \mathbf{\Theta}(\phi_i, \partial_n \phi_i)\Big|_{n\to 1, r\to 0}$$

presymplectic structure

$$S_{EE} = S_{RT}$$

Main assumption: holographic duality exist

Role of consistent asymptotic boundary conditions

AdS, Einstein

- set up a dictionary

bulk metric *boundary metric*

$$ds_0^2 = \sigma^{-2}(d\sigma^2 + \gamma_{ij}^{(0)} dx^i dx^j) + h_{\mu\nu} dx^\mu dx^\nu + \cdots$$

boundary terms only cancels out when the two metrics satisfy the same asymptotic b.c.

- Replica trick extended to the bulk

$$S_{EE} = \partial_n (nI[g_n/Z_n] - nI[g_1])|_{n=1} = \partial_n I_{bk}[g_n/Z_n]|_{n=1}$$

- Selecting a special curve

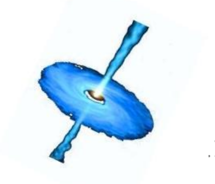
$$ds_{e,n}^2 = n^2 dr^2 + r^2 d\tau^2 + (g_{ij} + 2K_{aij}^{(n)} x^a) dy^i dy^j + \cdots$$

At n=1, the near cone expansion has to be compatible with the asymptotic expansion

- Calculate the generalized gravitational entropy

$$S_{EE} = \int_{\gamma_{\mathcal{A}} \times S^1} \mathbf{\Theta}(\phi_i, \partial_n \phi_i)\Big|_{n \to 1, r \to 0}$$

Ambiguity in the presymplectic structure



The generalized gravitational entropy in WAdS/CFT

WS-Wen-Xu

- Consistent boundary conditions

Compere-Guica-Rodriguiz

$$ds^2 = ds_0^2 + \textit{warping}\,,$$

boundary metric

$$ds_0^2 = \sigma^{-2}(d\sigma^2 + \gamma_{ij}^{(0)} dx^i dx^j) + h_{\mu\nu} dx^\mu dx^\nu \cdots$$

Selecting a special curve

$$ds^2 = ds_e^2 + \textit{tilting}\,,$$

$$ds_e^2 = dr^2 + r^2 d\tau^2 + (\tilde{g}_{ij} + 2K_{aij}x^a) dy^i dy^j \cdots$$

$n \to 1, \quad K_a^{(n)} = 0$

NOT geodesic in WAdS,
But geodesic in AdS

Compatibility ⇒

$$ds_0^2|_{r\to 0} = ds_e^2|_{\sigma\to 0}\,,$$
$$\textit{warping}|_{r\to 0} = \textit{tilting}|_{\sigma\to 0}\,,$$

$$S_{EE} = \frac{\tilde{L}}{4G} = S_{q,m} \neq S_{RT}$$

Main Modifications to Lewkowycz-Maldacena for WAdS/CFT

- the expansion near the special surface has to be compatible with the asymptotic expansion;
- periodic conditions are imposed to coordinates in the phase space with diagonalized symplectic structure, not to all fields appearing in the action;
- evaluating the entanglement functional using the boundary term method amounts to evaluating the presymplectic structure at the special surface, where some additional exact form may contribute.

Entanglement entropy in WAdS/WCFT

- Assumptions: WAdS/WCFT, AdS/WCFT
- Method: Casini-Huerta-Myers approach adapted
- Results: The bulk calculation agrees with the WCFT calculation given by Castro-Hofman-Iqbal

CFT $z \to f(z), \quad \bar{z} \to g(\bar{z})$

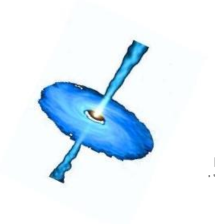
WCFT $x \to f(x), \qquad t \to t + g(x)$

Detournay-Compere
Hofman-Strominger
Deournay-Hartman-Hofman

By carefully analyzing the moduli properties and keeping track of the anomalis, a Cardy-like formula was derived

moduli transformation

$$S_{WCFT} = -\frac{4\pi i P_0 P_0^{vac}}{k} + 4\pi\sqrt{-\left(L_0^{vac} - \frac{(P_0^{vac})^2}{k}\right)\left(L_0 - \frac{P_0^2}{k}\right)}$$



EE on WCFT

Casini-Huerta-Myers
Castro-Hofman-Iqbal

- EE on an interval

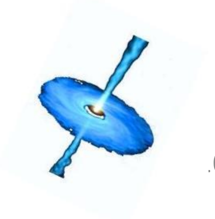
$$\mathcal{D}: \qquad (T, X) \in \left[(-\frac{l_T}{2}, -\frac{l_X}{2}), (\frac{l_T}{2}, \frac{l_X}{2}) \right] .$$

warped conformal mapping

$$\frac{\tanh \frac{\pi X}{\beta}}{\tanh \frac{l_X \pi}{2\beta}} = \tanh \frac{\pi x}{\kappa}, \qquad T + \left(\frac{\bar{\beta}}{\beta} - \frac{\alpha}{\beta} \right) X = t + \left(\frac{\bar{\kappa}}{\kappa} - \frac{\alpha}{\kappa} \right) x ,$$

- EE on $\mathcal{D}$ = Thermal entropy on $\mathcal{H}$

$$S_{EE} = i P_0^{vac} (\Delta T + \frac{\bar{\beta} - \pi}{\beta} \Delta X) + (i P_0^{vac} - 4 L_0^{vac}) \log \left(\frac{\beta}{\pi \epsilon} \sinh \frac{\Delta X \pi}{\beta} \right) ,$$



A bulk calculation

WS-Wen-Xu

- WAdS with coordinates (U,V, ρ)
- Bulk coordinate transformation preserves two explicit U(1) isometries

$$u = \frac{1}{4}\log\left(\frac{(l_U+2U)^2\rho^2-1}{(l_U-2U)^2\rho^2-1}\right)$$

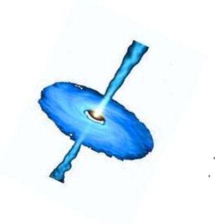
$$v = \frac{1}{4}\log\left(\frac{(1+2\rho U)^2-l_U^2\rho^2}{(1-2\rho U)^2-l_U^2\rho^2}\right)+V\,,$$

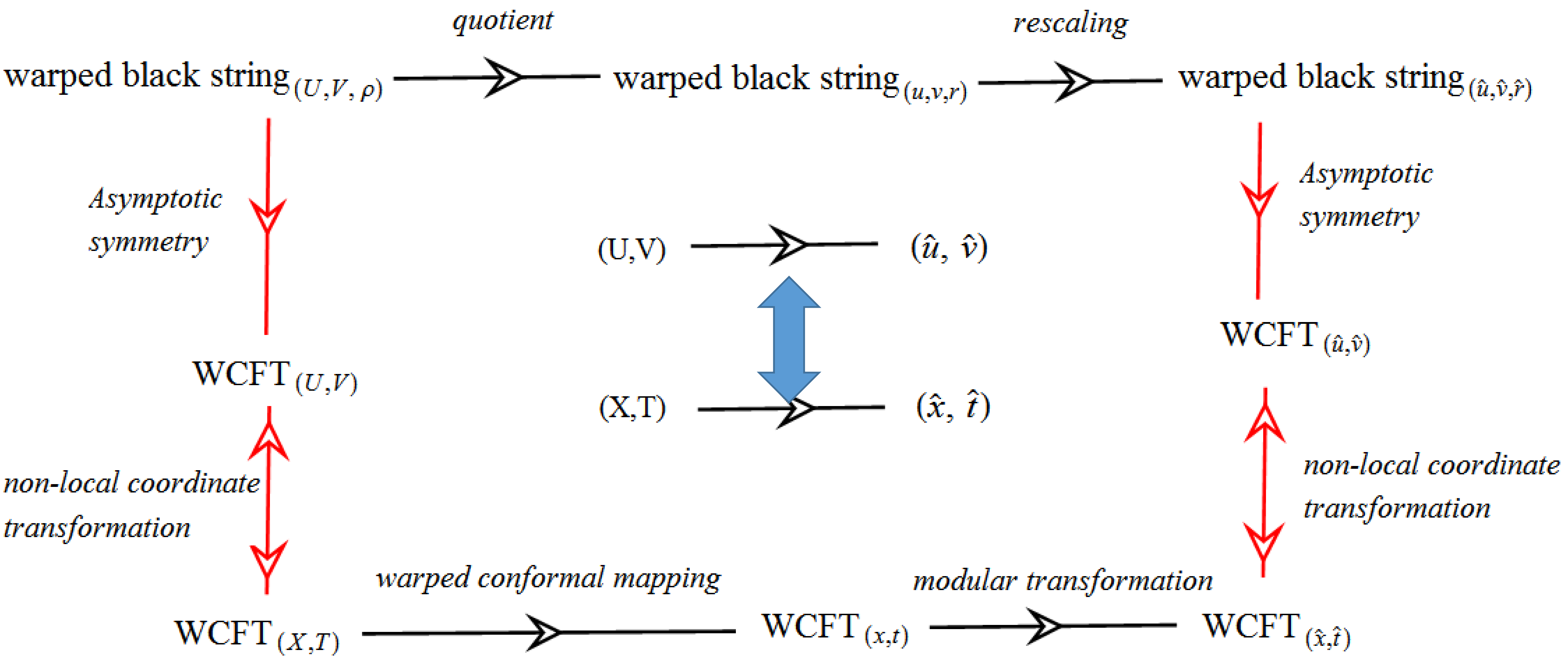
$$r = \frac{1+\rho^2\left(l_U^2-4U^2\right)}{2l_U\rho}\,,$$

SL(2,R)x U(1) quotient

- Black string with coordinates (u,v, r) and thermal entopy:

$$S_{thermal} = \frac{\ell}{4G}\Delta V + \frac{\ell}{4G}\log\frac{\Delta U}{\eta}\,.$$





$$S_{thermal} = S_{EE}$$

Summary of our results

- Assuming WAdS/CFT, we take the LM approach, and derive a holographic calculation for the entanglement entropy. The bulk result agrees with the CFT results.

- Assuming WAdS/WCFT, we take the CHM approach, and derive a holographic calculation for entanglement entropy. The bulk result agrees with the WCFT results.

Thank you!