The Weak Gravity Conjecture, Black Holes, and Cosmology

蕭文禮

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Based on work with:

Jon Brown

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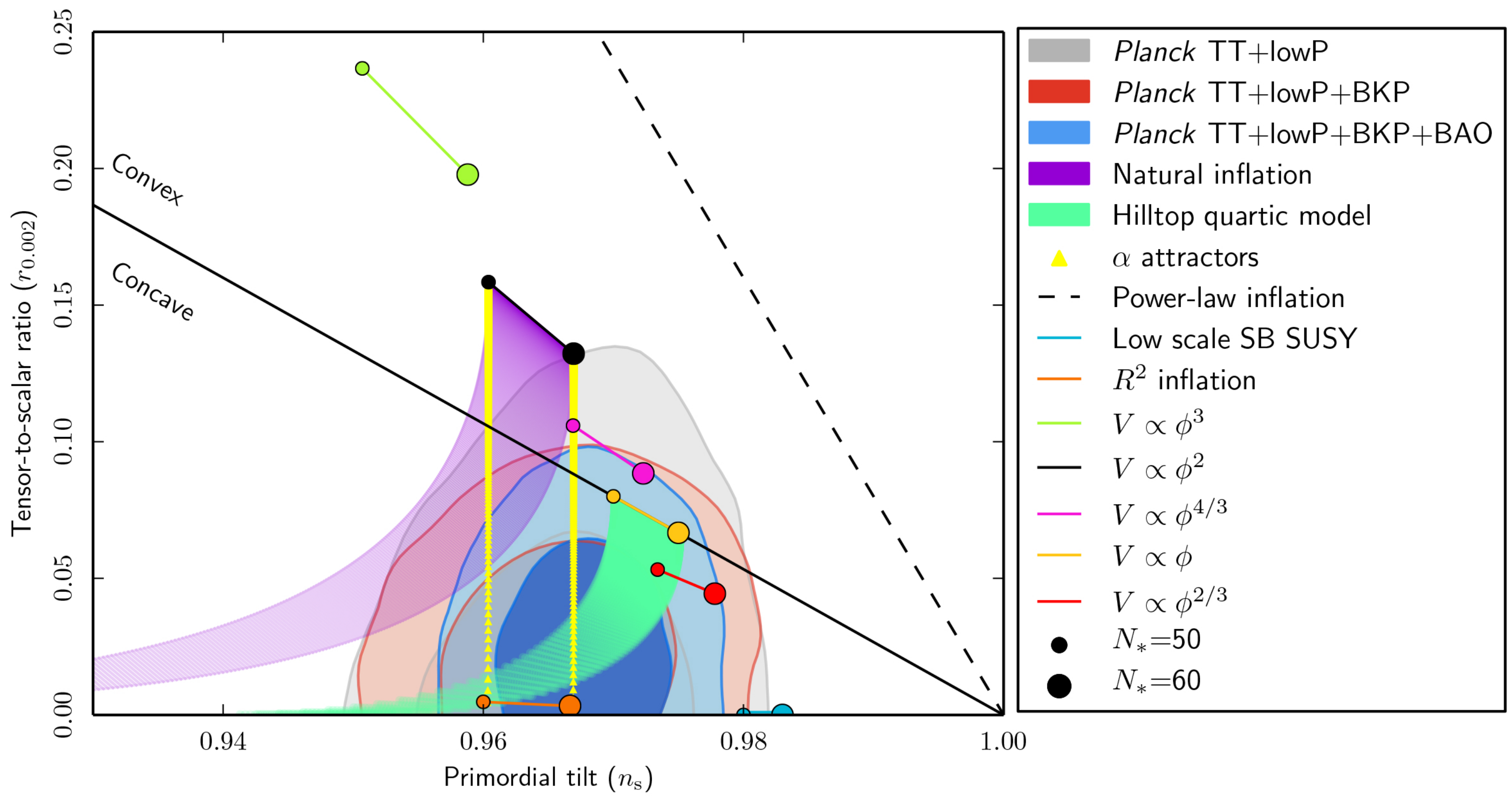
J. Brown, W. Cottrell, GS and P. Soler,
JHEP **1510**, 023 (2015),
JHEP **1604**, 017 (2016),
and arXiv:1607.00037 [hep-th]

M. Montero, GS and P. Soler, arXiv:1606.08438 [hep-th]

+ work in progress

Motivation

- An interesting observable of inflation is the tensor mode *Baumann's talk*
- Current bound (PLANCK+BICEP/KECK+BAO): r< 0.07



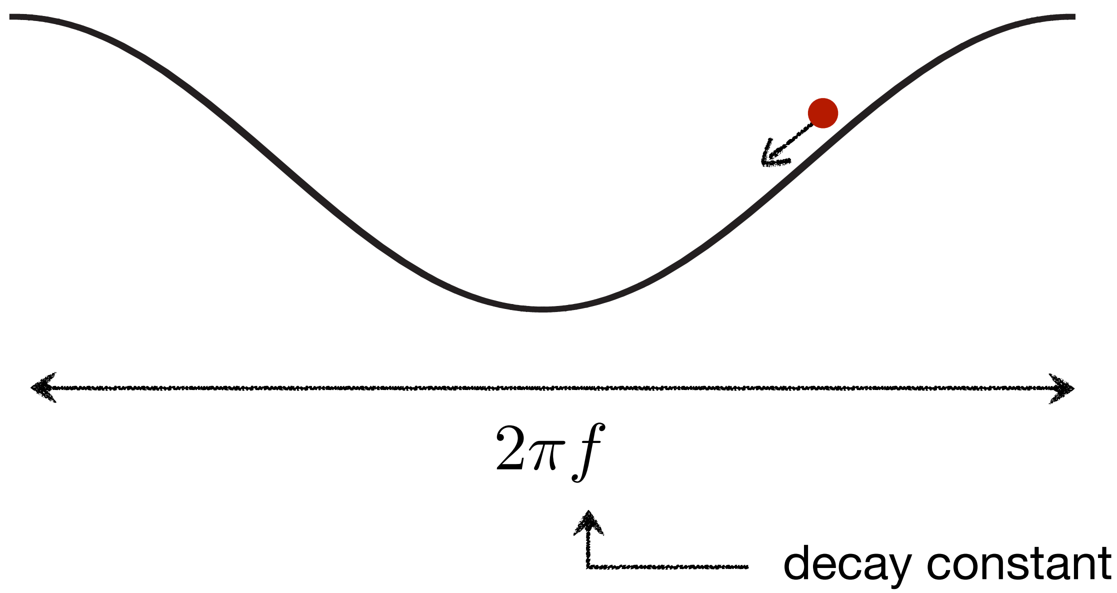
- A variety of current/near-future expts can reach r~10^{-2} (or maybe 10^{-3})
- Under some assumptions, a detection implies a strong UV sensitivity:

$$\frac{\Delta\phi}{M_{\rm pl}} \gtrsim 2 \times \left(\frac{r}{0.01}\right)^{1/2}$$

Lyth '96

Axions & Large field inflation

- Natural inflaton candidates as they enjoy a shift symmetry that is only broken by non-perturbative effects:



$$V(\phi) = \sum_k c_k \, e^{-km} \left[1 - \cos\left(\frac{k\phi}{f} \right) \right]$$

Natural Inflation
[Freese, Frieman, Olinto '90]:

$$V(\phi) = 1 - \Lambda^{(1)} \cos\left(\frac{\phi}{f} \right) + \sum_{k>1} \Lambda^{(k)} \left[1 - \cos\left(\frac{k\phi}{f} \right) \right]$$

- Controlled, slow-roll potential: $e^{-m} \ll 1 \,, \quad f > M_p$

- Axions with super-Planckian decay constants don't seem to exist in controlled limits of string theory. [Banks, Dine, Fox, Gorbatov '03]

Two Broad Classes of Models

Axion Monodromy

[Silverstein, Westphal, ‘08];
[McAllister, Silverstein, Westphal, 08];

F-term axion monodromy
(embeddable in SUGRA of string theory)
[Marchesano, GS, Uranga ’14];
[Blumenhagen, Plauschinn ’14];
[Hebecker, Kraus, Witowski, ’14];
[McAllister, Silverstein, Westphal, Wrase ’14]

Multiple Axions

Alignment
[Kim, Nilles, Peloso, ’04]

N-flation
[Dimopoulos, Kachru,McGreevy,Wacker ’05]

Kinetic and Stueckelberg mixings:
[GS, Staessens, Ye, ’15];
[Bachlechner, Long, McAllister, ’15]; …

The Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa ‘06

- The conjecture:

“Gravity is the Weakest Force”

- For every long range gauge field there exists a particle of charge q and mass m, s.t.

$$\frac{q}{m} M_P \geq \text{“1”}$$

See Harlow’s talk

$M_P \equiv 1$

Heuristic Argument

- Take a U(1) and a single family with q < m (~~WGC~~)

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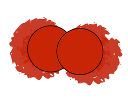
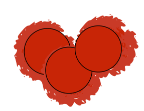
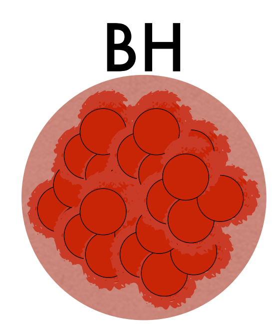
  - Infinitely many bound states







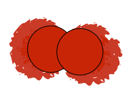
$2m > M_2 > 2q$ $\quad$ $3m > M_3 > 3q$ $\quad \cdots \quad$ $Nm > M_N > Nq$ $\quad \cdots \quad$ $M_\infty \to Q_\infty$

$M_P \equiv 1$

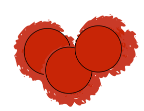
Heuristic Argument

- Take a U(1) and a single family with q < m (~~WGC~~)

- Infinitely many bound states

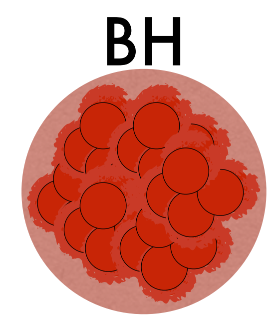


$2m > M_2 > 2q$



$3m > M_3 > 3q$

. . .



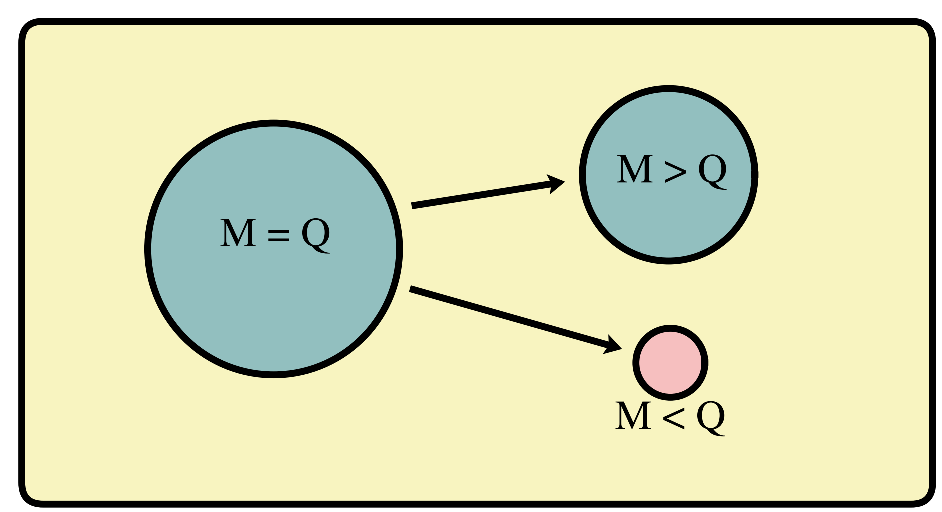
$Nm > M_N > Nq$

. . .

$M_\infty \to Q_\infty$

- *Postulate* the existence of a state with ("mild form" of WGC)

$$\frac{q}{m} \geq \text{“1”} \equiv \frac{Q_{Ext}}{M_{Ext}}$$



The Weak Gravity Conjecture

- Heuristic argument suggests ∃ a state w/ $\frac{q}{m} \geq \text{“}1\text{”} \equiv \frac{Q_{Ext}}{M_{Ext}}$
- Perfectly OK for some extremal BHs to be stable [e.g., Strominger, Vafa] as q ∊ central charge of SUSY algebra.
  - No q>m states possible (∵ BPS bound).
  - BPS BHs ***are*** the WGC states.
  - More subtle for theories with some q ∉ central charge
- One often invokes the remnants argument [Susskind] for the WGC but the situations are different (finite vs infinite mass range).
- The WGC is a conjecture on the ***finiteness of the # of stable states that are <u>not</u> protected by a symmetry principle.***
- Recent work gave more (and independent) evidences for the WGC [Montero, GS; Soler]; [Heidenreich, Reece, Rudelius]; [Harlow] (more later).

WGC and Cosmology

The Weak Gravity Conjecture

- Suggested generalization to p-dimensional objects charged under (p+1)-forms:

$$\frac{Q}{T_p} \geq \text{``}1\text{''}$$

- p=-1 applies to instantons coupled to axions:

$$e^{-S_{inst}} = e^{-m + i\phi/f} \qquad \Longrightarrow \qquad fm \leq \text{``}1\text{''}$$

  - Seems to explain difficulties in finding $f > M_P$

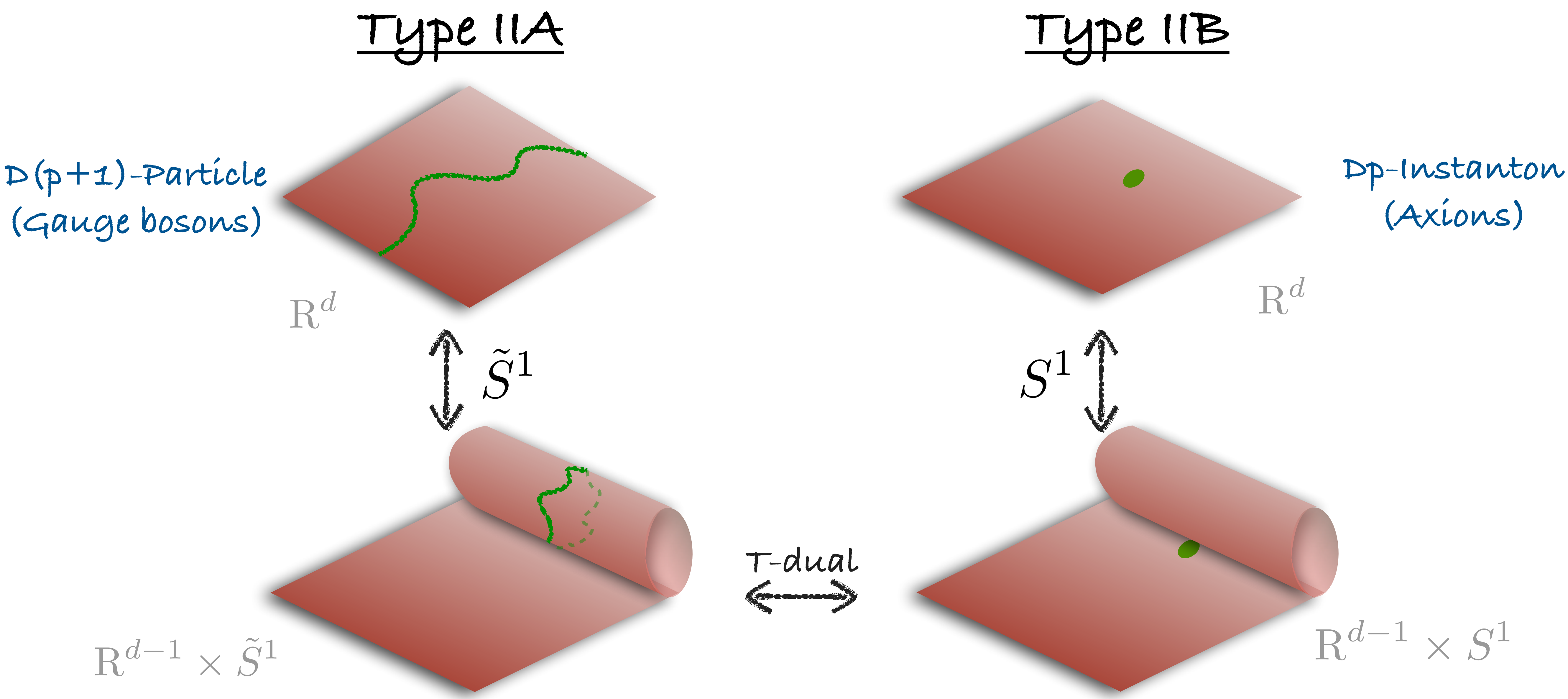
- Is there evidence for the p=-1 version of the WGC?

Brown, Cottrell, GS, Soler

WGC and Axions

Brown, Cottrell, GS, Soler

- T-duality provides a subtle connection between instantons and particles

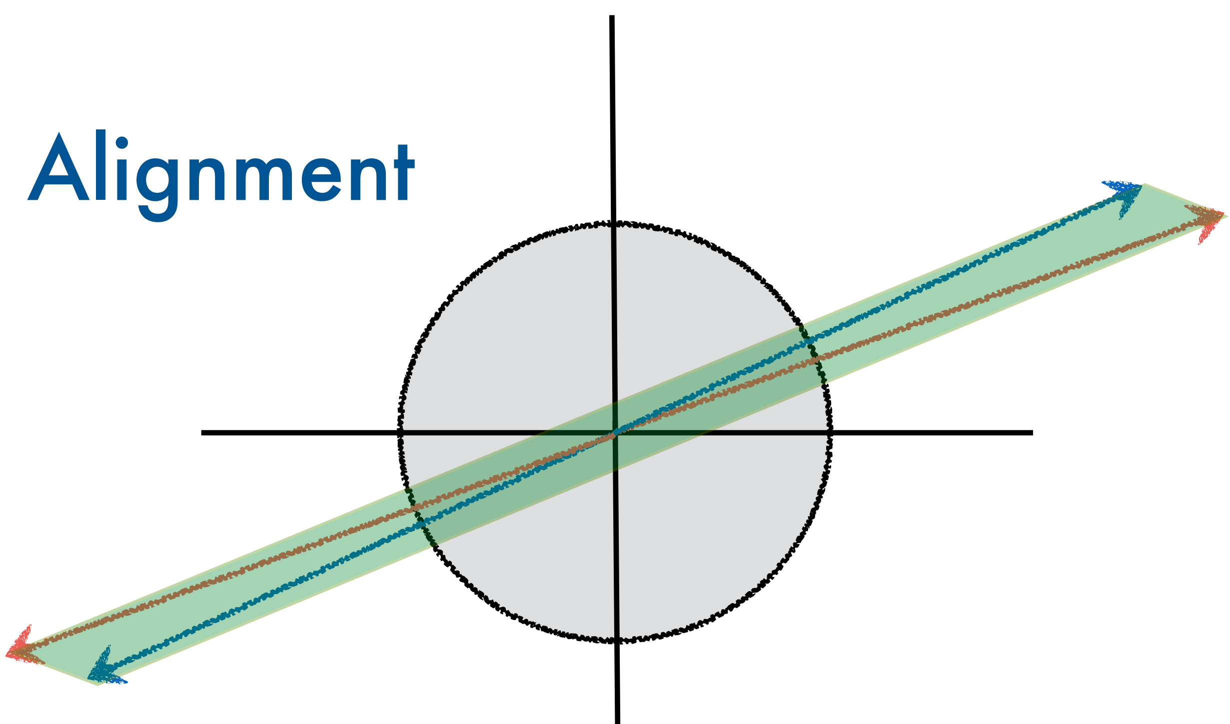


WGC and Axions

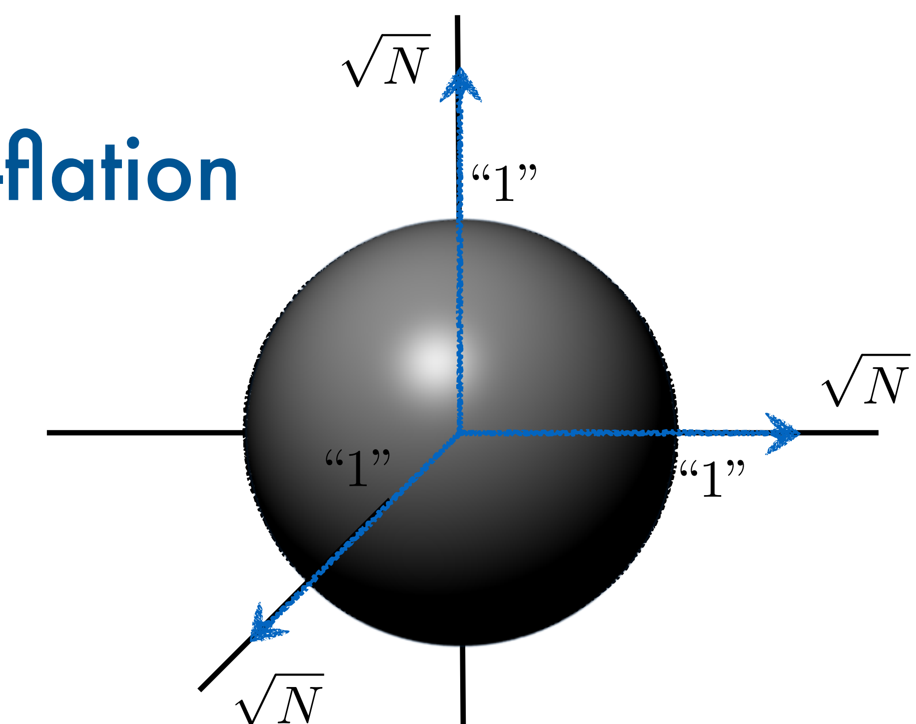
Brown, Cottrell, GS, Soler

- There is an upper bound of f·m where $e^{-S_{inst}} = e^{-m+i\phi/f}$
- For RR 2-form in IIB, we found: $$f \cdot m \leq \frac{\sqrt{3}}{2} M_P$$
- We obtained similar bounds for *other string axions*.
- Multiple axions mapped to multiple U(1)'s [where WGC was shown to imply convex hull condition [Cheung, Remmen]]

Alignment



N-flation



Axion Monodromy

- Axion is mapped to a ***massive*** gauge field.
- In F-term axion monodromy [Marchesano, GS, Uranga], axion mass is generated by fluxes or compactifcations on torsion cycles.
- Shift symmetry is *spontaneously* broken in the 4D EFT via:

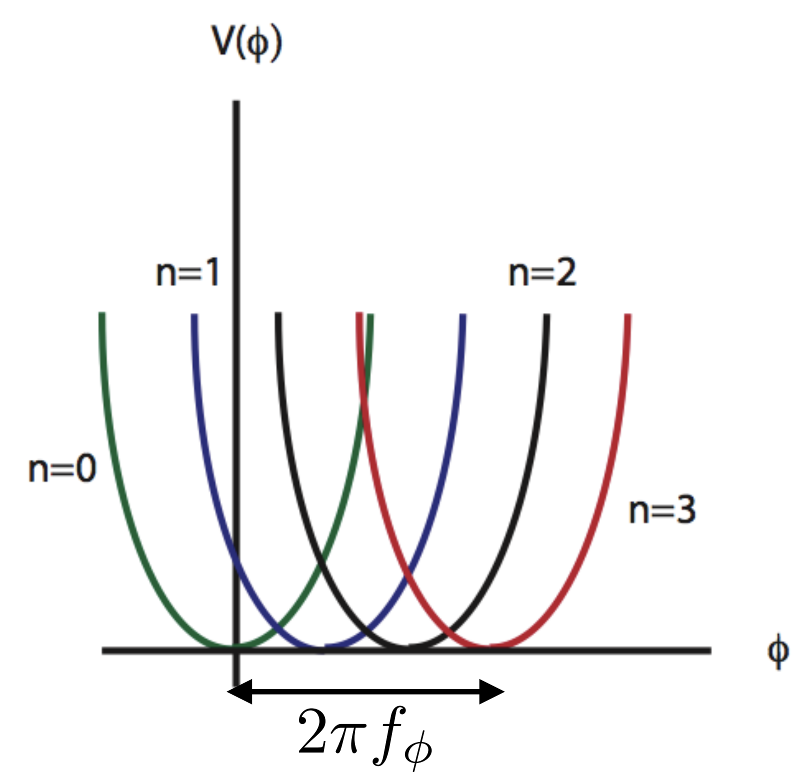
$$\int d^4x \, |F_4|^2 + |d\phi|^2 + \phi F_4$$

[see also in Kaloper, Sorbo]

- Gauge symmetry ⇒ UV corrections only depend on F_4

$$\sum_n c_n \frac{F^{2n}}{\Lambda^{4n}} \longrightarrow \mu^2\phi^2 \sum_n c_n \left(\frac{\mu^2\phi^2}{\Lambda^4}\right)^n$$

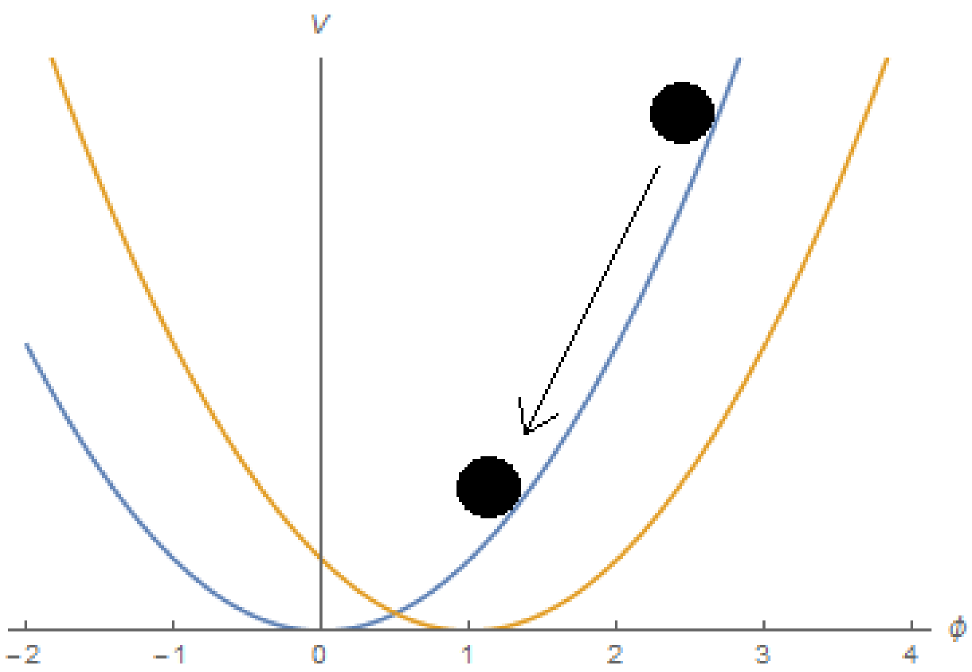
- Multi-branched potential:



Axion Monodromy

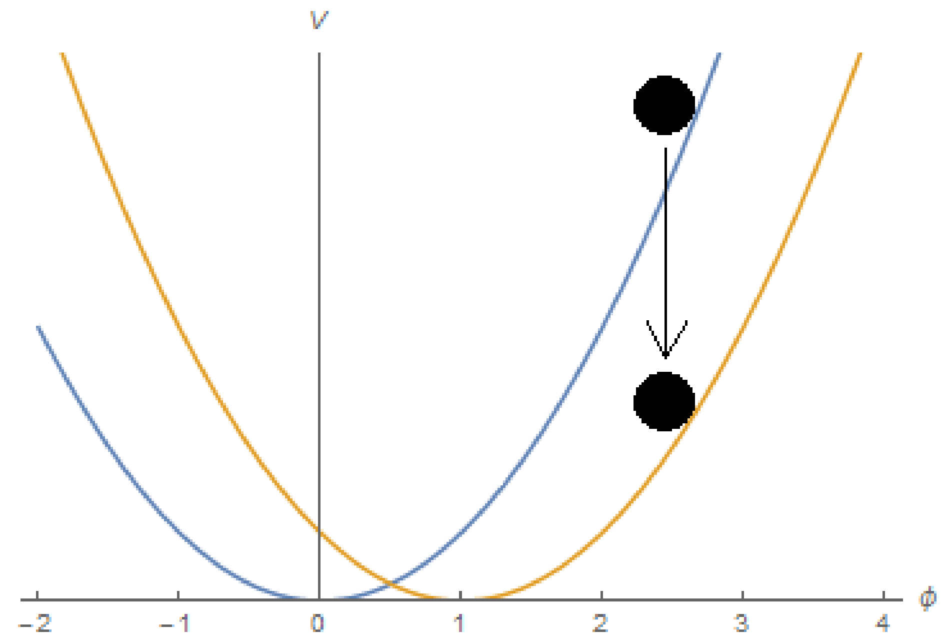
- Possible tunneling to different branches of the potential:

$$V(\phi) = \frac{1}{2}(ne + \mu\phi)^2$$



Slow Roll Inflation

$$V(\phi) = \frac{1}{2}(ne + \mu\phi)^2$$



Tunneling Event: $n \to n-1$

- Suppressing this tunneling can lead to a bound on field range (hence r).

- Subtleties vs Coleman's vacuum decay (e.g, tunneling between non-metastable states) Brown, Cottrell, GS, and Soler, 1607.00037 [hep-th]

Evidences for the Weak Gravity Conjecture

Evidences for the Weak Gravity Conjecture

- Lots of work in using the WGC to constrain axion inflation [De la Fuente, Saraswat, Sundrum '14];[Rudelius '14,'15];[Montero, Uranga, Valenzuela '15]; [Brown, Cottrell, GS, Soler '15] (x2); [Bachlechner, Long, McAllister '15]; [Hebecker, Mangat, Rompineve, Witkowski '15];[Junghans '15]; [Heidenreich, Reece, Rudelius '15] (x2), [Palti '15]; [Kooner, Parameswaran, Zavala '15]; ….
- Loopholes were suggested, e.g., by exploiting the "mild form".
- But string theory seems to satisfy stronger versions of the WGC [Brown, Cottrell, GS, Soler '15]; [Heidenreich, Reece, Rudelius, '15]
- The WGC is suggestive based on analyticity of amplitudes [Cheung, Remmen] and holography [Nakayama, Nomura];[Harlow];[Benjamin, Dyer, Fitzpatrick, Kachru] but no formal proof is given.
- [Montero, GS, Soler '16], took a modest step in this direction. We found modular invariance + charge quantization imply a version of this conjecture (see [Heidenreich, Reece, Rudelius '16] for similar conclusion).

The Weak Gravity Conjecture & Holography

- We will explore the WGC in AdS spacetimes, in particular in 3D.

- Advantages:
    - Behavior of gravity and gauge fields much simpler
    - Greatly enhanced CFT symmetry algebra
    - Extra constraints on CFT, in particular modular invariance

- Main disadvantage:
    - d=3 so different than d>3 that any relation with higher d WGC is uncertain at best

Gravity and gauge theories in three dimensions

U(1) gauge theories in 3d

- U(1) gauge theories are special in 3d: electrostatic energy of charged particles is IR divergent

- Gauge coupling runs and becomes strongly coupled in IR. Electric charge confines. [Polyakov]

- Alternatively, in the presence of a Chern-Simons term, the gauge field becomes massive:

$$\frac{\mu}{2}\int F \wedge A$$

  - At low energy, gauge boson behaves as scalar with mass μ
  - This term is also required by holography for the dual CFT to have non-trivial unitary representations.

U(1) gauge theories in 3d

- CS-term modifies the e.o.m: $d * F = *j_e + \mu F$

…and hence Gauss' law: $\int_{S^1} *F = Q_e + \mu \int_{S^1} A$

- Electric charge can be measured at infinity:

$$Q_e = -\mu \int_{S^1_\infty} A$$

- Compactness of U(1) gauge group implies
  - Charge quantization: $\mu = \frac{Ng^2}{2\pi}$, quantized CS level $N \in \mathbb{Z}$
  - Aharanov-Bohm exp. measures charge mod N. Full U(1) charge is nevertheless conserved.

U(1) gauge theories in 3d

- Gravity is also special (topological) in 3d: metric has no propagating degrees of freedom

- Nevertheless, black hole solutions exist, albeit only in AdS spacetime [Bañados, Teitelboim, Zanelli]

$$ds^2 = -\left(-8GM + \frac{r^2}{\ell^2} + 16\frac{(GJ)^2}{r^2}\right)^2 dt^2 + \frac{dr^2}{\left(-8GM + \frac{r^2}{\ell^2} + 16\frac{(GJ)^2}{r^2}\right)^2} + r^2\left(d\phi - 4\frac{GJ}{r^2}dt\right)^2$$

$(\ell \equiv \ell_{AdS})$

- Finite horizon at $r_+ = \ell\left[4GM\left(1 + \sqrt{1 - \left(\frac{J}{M\ell}\right)^2}\right)\right]^{\frac{1}{2}}$

U(1) gauge theories in 3d

- 3d no-hair theorem implies BHs cannot source electric field

- BTZ metric has a non-contractible one-cycle on which a flat connection can be turned:

$$Q_e = -\mu \int_{S^1} A$$

- Although charged BHs exist:
    - No backreaction on the metric (even after including higher derivative corrections)
    - No apparent notion of extremality
    - No straightforward connection to WGC in d>3

The CFT perspective

- Weakly coupled AdS_3 is dual to a CFT_2 at large central charge

$$c = \frac{3\ell}{2G}$$

- Bulk U(1) is dual to (holomorphic) CFT current j(z) at level N>0:

$$[j_m, j_n] = N\delta_{m+n,0} \qquad [L_m, j_p] = -p\, j_{m+p}$$

  - j_0 is proportional to Q (bulk electric charge)
  - $[L_0, j_0] = 0 \quad \Rightarrow$ electric charge is exactly conserved
  - N>0 required for non-trivial unitary representation

The CFT perspective

- In the presence of U(1) currents, the CFT stress energy tensor admits a Sugawara decomposition

$$T(z) = T'(z) + T^S(z), \quad T^S(z) = \frac{1}{2} : jj(z) :$$

- The Virasoro generators also split $L_m = L'_m + L^S_m$
  The unitarity bound arises

$$L_0 = L'_0 + L^S_0 \qquad \Longrightarrow \qquad L_0 \geq L^S_0 \geq \frac{Q^2}{2N}$$

  - Eigenvalues h of L_0 measure the total energy of the bulk
  - Same story holds for anti-holomorphic part $\tilde{T}$ when N < 0

The CFT perspective

- Both L_0' and L_0^S can be directly obtained from the bulk for BTZ charged BH, given the explicit solution:

$$h'_{M,J} = \frac{c}{24} + \frac{1}{2}(M\ell + J)\,, \qquad h^S = \frac{Q^2}{2N}$$

- Hence, BHs satisfy from the CFT perspective the bound

$$h_{BH} > \frac{c}{24} + \frac{Q^2}{2N}$$

- Can regard this as 3d ``extremality bound". A WGC could postulate the existence of charged states

$$h_{BH} > h_{WGC} \geq h_{\text{Unit}} \qquad \Longleftrightarrow \qquad \frac{c}{24} + \frac{Q^2}{2N} > h_{WGC} \geq \frac{Q^2}{2N}$$

  - Our goal is to find such "super-extremal" states

Modular invariance and “super-extremal” states

Modular invariance & super-extremal states

- Take CFT partition function with chem. potential

$$Z(\tau, z) = \mathrm{Tr}\left(q^{L_0 - \frac{c}{24}}\bar{q}^{\tilde{L}_0 - \frac{\tilde{c}}{24}} e^{2\pi i z Q}\right)$$

  - **Charge quantization** implies $Z(\tau, z) = Z(\tau, z+1)$

- On the other hand, **modular invariance** implies:

$$Z(\tau', z') = \exp\left(i\pi N \frac{z^2}{c\tau + d}\right) Z(\tau, z)\,, \qquad \tau \to \frac{a\tau + b}{c\tau + d},\ z \to \frac{z}{c\tau + d}$$

- Together, these mean

$$Z(\tau, 0) = \exp\left(-i\pi N \tau\right) Z(\tau, \tau) = \mathrm{Tr}\left(q^{L_0 - \frac{c}{24} + Q + \frac{N}{2}}\bar{q}^{\tilde{L}_0 - \frac{\tilde{c}}{24}}\right)$$

Modular invariance & super-extremal states

- Conclusion: Modular invariance and charge quantization imply invariance under spectral flow

$$L_0 \to L_0 + Q + \frac{N}{2}, \qquad Q \to Q + N, \qquad \tilde{L}_0 \to \tilde{L}_0$$

- Acting k times on the vacuum $(L_0 = \tilde{L}_0 = Q = 0)$ we infer the existence of states with

$$Q = kN \qquad \text{and} \qquad L_0 = k^2 \frac{N}{2} = \frac{Q^2}{2N} = h_{\text{Unit}} < h_{BH}$$

- These states saturate the unitarity bound and lie below the BH threshold.
    - 3d WGC satisfied in the sector of charges $Q = N \cdot \mathbb{Z}$

Modular invariance & super-extremal states

- Remarks: Usual WGC heuristics do not apply in AdS in three dimensions:
  - Gauge field is massive due to CS term. There is no tunable gauge coupling and no obvious $g \to 0$ limit.
  - Large BHs (larger than ℓ_{AdS}) do not evaporate, no trouble with remnants
  - Small BHs are subject to large quantum corrections
- However, modular invariance + charge quantization imply a certain version of WGC for $Q = N \cdot \mathbb{Z}$
  - Sub-lattice WGC

The $\mathbb{Z}_n$ charge

- Can modular invariance test WGC for 0<Q<N?
  - Partition function splits into $\mathbb{Z}_N$ sectors $$Z(\tau) = \sum_{Q=0}^{N-1} Z_Q(\tau)$$
  - In the low T limit ($\tau_2 \to \infty$), $Z_Q(\tau)$ gives the conformal weight of the lightest state with charge Q mod N
  - $\mathbb{Z}_N$ - WGC: $Z_Q > e^{-\tau_2 \frac{Q^2}{N}} \,, \qquad \forall Q \neq 0 \mod N$
- Modular invariance and spectral flow can be used to constrain the spectrum of $\mathbb{Z}_N$ - charged states
  - Modular bootstrap [Benjamin, Dyer, Fitzpartrick, Kachru]
- These are however not sufficient to prove $\mathbb{Z}_N$-WGC

Conclusions

Conclusions

- Motivated by gravity waves & large field inflation, we have revisited the WGC and the “Swampland” proposal.
- We have formulated the WGC for (a large class of) axions which can be dualized to U(1) gauge fields.
- Constraints on multiple axions in terms of convex hull (bound on the “diameter” of axion space):
  - KNP, N-flation, kinetic mixing,…
- String theory examples suggest stronger versions of the WGC.

Conclusions

- Evidence of the WGC in AdS_3/CFT_2. Key ingredients are modular invariance & compactness of Abelian group.

- Exciting interface between Black holes, Inflation & String Theory.

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谢谢！

THANKS