Integrability in AdS_3/CFT_2

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based on work in collaboration with

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see in particular arXiv:1403.4543 and 1406.2971

GATIS

AdS_3/CFT_2 holography

- Many interesting properties: CFT_2, black-hole physics, higher-spin theories, rich dual gauge theory flowing to SCFT...
- In string theory, we can obtain it from RR and/or NSNS fluxes.
- For pure-NSNS, CFT techniques can be used on the worldsheet.

[Maldacena, Ooguri '00]

- RR fluxes are problematic in this approach.

Integrability and massless modes

- **Classical integrability** for maximally supersymmetric backgrounds

$$AdS_3 \times S^3 \times T^4 \qquad \text{and} \qquad AdS_3 \times S^3 \times S^3 \times S^1$$

with pure-RR and mixed flux.

[Babichenko, Stefański, Zarembo '10] [Cagnazzo, Zarembo '13]

- We would like quantum integrability, like for $AdS_5 \times S^5$.

- However, **massless modes** seemingly spoil usual approach.
Integrable massless scattering can be subtle.

[Zamolodchikov, Zamolodchikov '92] [Fendley, Saleur '93]

$\longrightarrow$ major obstacle for integrability.

Pure-RR $AdS_3 \times S^3 \times T^4$ light-cone symmetries

$$\mathfrak{psu}(1,1|2)_{\mathrm{L}} \oplus \mathfrak{psu}(1,1|2)_{\mathrm{R}}$$

$$\downarrow \quad \text{(l.c. gauge)}$$

$$\mathfrak{psu}(1|1)^4_{\text{centr.ext.}}$$

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Central charges:

H Hamiltonian,

$$\mathbf{C} = +i\,\frac{h}{2}\left(e^{+i\mathbf{P}} - 1\right),$$

M Mass,

$$\bar{\mathbf{C}} = -i\,\frac{h}{2}\left(e^{-i\mathbf{P}} - 1\right)$$

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Exact statements about massless modes

- All one-particle representations, including massless are **short**

$$\mathbf{H}^2 = \mathbf{M}^2 + 4\mathbf{C}\bar{\mathbf{C}}\,.$$

- Masslessness is protected at all-loops

$$\mathbf{M}\,|\text{massless}\rangle = 0 \qquad \text{protected by } \mathfrak{su}(2) \subset \mathfrak{so}(4).$$

- All-loop massless dispersion relation and group velocity

$$E(p) = \pm 2h \sin\frac{p}{2}, \qquad v(p) = \frac{\partial E}{\partial p} = \pm h \cos\frac{p}{2}\,.$$

The complete all-loop S matrix

- Write down irreducible representations of symmetries,

  $\bullet$ = massive, $\qquad$ $\circ$ = massless.

- Impose invariance

  $$\big[\mathbf{S}, \mathbf{Q}\big] = 0,$$

  and find

  $$\mathbf{S} = \begin{pmatrix} \mathbf{S}^{\bullet\bullet} & \mathbf{S}^{\bullet\circ} \\ \mathbf{S}^{\circ\bullet} & \mathbf{S}^{\circ\circ} \end{pmatrix}.$$

- Yang-Baxter equation holds.

Results and outlook

- For pure-RR $AdS_3 \times S^3 \times T^4$, complete exact S matrix was found.

  [Borsato, Ohlsson Sax, Stefański, AS '14]

- Validation: world-sheet perturbative calculations up to two loops.

  [Sundin, Wulff '12] [Beccaria, Levkovich-Maslyuk, Macorini, Tseytlin '12] [Abbott '13]
  [Engelund, McKeown, Roiban '13] [Babichenko, Dekel, Ohlsson Sax '14] [...]

- Mixed fluxes and $AdS_3 \times S^3 \times S^3 \times S^1$: massive sector already known, full S matrix should follow similarly.

  [Borsato, Ohlsson Sax, AS '12] [Hoare, Stepanchuk, Tseytlin '13]