Mass Renormalization and Vacuum Shift in String Theory

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Based on

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Motivation

Conventional techniques in string theory have limited use in the study of mass renormalization or vacuum shift.

We shall begin by outlining the origin of these problems.

LSZ formula for S-matrix elements in QFT

$$\lim_{k_i^2 \to -m_{i,p}^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} \{ Z_i^{-1/2} \times (k_i^2 + m_{i,p}^2) \}$$

$G^{(n)}$: n-point Green's function

$a_1, \cdots a_n$: quantum numbers, $k_1, \ldots k_n$: momenta

$m_{i,p}$: physical mass of the i-th external state

– given by the locations of the poles of two point function in the $-k^2$ plane.

Z_i: wave-function renormalization factors, given by the residues at the poles.

In contrast, string amplitudes compute ‘truncated Greens function on classical mass-shell’

$$\lim_{k_i^2 \to -m_i^2} G^{(n)}_{a_1 \cdots a_n}(k_1, \cdots k_n) \prod_{i=1}^{n} (k_i^2 + m_i^2) .$$

m_i: <u>tree level</u> mass of the i-th external state.

$k_i^2 \to -m_i^2$ condition is needed to make the vertex operators conformally invariant.

String amplitudes:

$$\lim_{k_i^2\to -m_i^2} G^{(n)}_{a_1\cdots a_n}(k_1,\cdots k_n)\prod_{i=1}^{n}(k_i^2+m_i^2)\,,$$

The S-matrix elements:

$$\lim_{k_i^2\to -m_{i,p}^2} G^{(n)}_{a_1\cdots a_n}(k_1,\cdots k_n)\prod_{i=1}^{n}\{Z_i^{-1/2}\times(k_i^2+m_{i,p}^2)\}$$

The effect of Z_i can be taken care of. **Witten**

The effect of mass renormalization is more subtle.

$\Rightarrow$ String amplitudes compute S-matrix elements directly if $m_{i,p}^2 = m_i^2$ but not otherwise.

– Includes BPS states, massless gauge particles and all amplitudes at tree level.

A common excuse

"We can find the renormalized masses by examining the poles in the S-matrix of massless and/or BPS states which do not suffer mass renormalization."

Does not always work.

Example: In SO(32) heterotic string theory there is a massive state in the spinor representation of SO(32)

– does not produce a pole in the S-matrix of massless states which belong to adjoint or singlet of SO(32).

Problem with vacuum shift

Example: In many compactifications of SO(32) heterotic string theory on Calabi-Yau 3-folds, one loop correction generates a Fayet-Ilioupoulos term.

Effect: Generate a potential of a charged scalar ϕ of the form

$$\mathbf{c}(\phi^*\phi - \mathbf{K}\,\mathbf{g}^2)^2$$

c, K: positive constants, g: string coupling

Dine, Seiberg, Witten; Atick, Dixon, A.S.; Dine, Ichinose, Seiberg
Atick, A.S.; Witten; D'Hoker, Phong; Berkovits, Witten

Conventional approach does not tell us how to carry out systematic perturbation expansion around the correct vacuum at $|\phi| = \mathbf{g}\sqrt{\mathbf{K}}$

– not described by a world-sheet CFT

How do we proceed?

Many indirect approaches to these problems have been discussed in the past.

Vacuum shift: Fischler, Susskind; · · ·

Mass renormalization: Weinberg; Seiberg; A.S.; Ooguri & Sakai; Das; Rey; · · ·

We shall follow a direct approach using off-shell Green's function.

We can think of two routes:

1. String field theory

– many attempts but not much progress beyond tree level / bosonic string theory.

Witten; Zwiebach; Berkovits; Berkovits, Okawa, Zwiebach; Erler, Konopka, Sachs; · · ·

2. Pragmatic approach: Generalize Polyakov prescription without worrying about its string field theory origin.

Vafa; Cohen, Moore, Nelson, Polchinski; Alvarez Gaumé, Gomez, Moore, Vafa; Polchinski; Nelson

We shall follow the pragmatic approach.

The main problem with this approach is that once we go off-shell, the amplitudes are not invariant under conformal transformation on the world-sheet.

They begin to depend on spurious data like the choice of

local coordinates at the punctures

where the off-shell vertex operators are inserted.

However this is not very different from the situation in a gauge theory where off-shell Green's functions of charged fields are gauge dependent.

Nevertheless the renormalized masses and S-matrix elements computed from these are gauge invariant.

Can the story be similar in string theory?

Strategy:

1. Compute the off-shell amplitudes by choosing some local coordinate system at the punctures.

2. Show that the renormalized masses and S-matrix elements do not depend on the choice of local coordinates.

Since these issues are common between bosonic string theory and superstring theories we shall discuss the results in the context of closed bosonic string theory.

Extension to super/heterotic strings involves replacing

local coordinates $\to$ local superconformal coordinates

Alvarez-Gaume, Gomez, Nelson, Sierra, Vafa; Belopolsky; Witten

(Ramond sector has some additional technical complications which have not been sorted out but do not seem unsurmountable).

It turns out that we can achieve our goal only if we impose some additional restrictions on the choice of local coordinate system at the punctures.

– gluing compatibility.

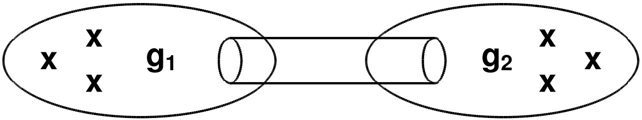
Consider a genus g_1, m-punctured Riemann surface and a genus g_2, n-punctured Riemann surface.

Take one puncture from each of them, and let w_1, w_2 be the local coordinates around the punctures at $w_1 = 0$ and $w_2 = 0$.

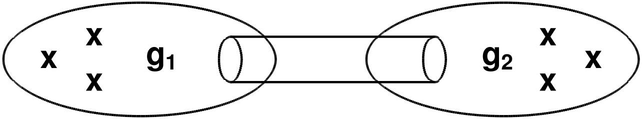
Glue them via the identification (plumbing fixture)

$$w_1 w_2 = e^{-s+i\theta}, \quad 0 \le s < \infty, \quad 0 \le \theta < 2\pi$$

– gives a family of new Riemann surfaces of genus $g_1 + g_2$ with (m+n-2) punctures.



Gluing compatibility: Choice of local coordinates at the punctures of the genus $g_1 + g_2$ Riemann surface must agree with the one induced from the local coordinates at the punctures on the original Riemann surfaces.



(Follows automatically if the choice of local coordinate system is inherited from bosonic string field theory in the Siegel gauge.)

Rastelli, Zwiebach

Gluing compatibility allows us to divide the contributions to off-shell Green's functions into 1-particle reducible (1PR) and 1-particle irreducible (1PI) contributions.

Two Riemann surfaces joined by plumbing fixture

$\Updownarrow$

Two amplitudes joined by a propagator

Riemann surfaces which cannot be obtained by plumbing fixture of other Riemann surfaces contribute to 1PI amplitudes.

1PI amplitudes do not include degenerate Riemann surfaces and hence are free from poles.

Once this division has been made we can apply the usual field theory manipulations to analyze amplitudes.

Example: Two point function

– can be used to partially resum the perturbation series and calculate mass renormalization.

Similar analysis can be used to compute S-matrix elements using LSZ procedure.

Results:

1. The renormalized masses of physical states and S-matrix elements are independent of the choice of local coordinate system.

2. Wave-function renormalization factors and the renormalized masses of unphysical states depend on the choice of local coordinates at the punctures.

3. Poles of the S-matrix of massless / BPS states occur at the renormalized masses of physical states computed from our prescription.

Everything is satisfactory!

Application 2: Shifting the vacuum

Suppose we have a scalar field ϕ with tree level potential

$$\mathbf{A}\,\phi^{\mathbf{4}} + \cdots$$

Suppose that loop correction generates a -ve mass^2 term

$$-\mathbf{C}\,\mathbf{g}^{\mathbf{2}}\phi^{\mathbf{2}} + \cdots$$

Physically we expect minima at

$$\phi^{\mathbf{2}} = \frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{C}}{\mathbf{A}}\,\mathbf{g}^{\mathbf{2}} + \cdots .$$

Question: How do we compute physical quantities in this vacuum?

We assume vanishing of all <u>other</u> massless tadpoles at this vacuum.

$\lambda \equiv$ vacuum expectation value of ϕ (unknown)

$\Gamma^{(n)}$: String amplitudes in the original vacuum (known)

$$\Gamma^{(n)} = G^{(n)} \prod_{i=1}^{n} (k_i^2 + m_i^2)$$

m_i: tree level masses

$\Gamma_\lambda^{(n)}$: String amplitudes in the shifted vacuum (unknown)

Field theory intuition tells us that

Lee; Bardakci, Halpern

$$\Gamma_\lambda^{(n)}(k_1, a_1; \cdots k_n, a_n) = \sum_{m=0}^{\infty} \frac{\lambda^m}{m!} \Gamma^{(n+m)}(k_1, a_1; \cdots k_n, a_n; \overbrace{0, \phi; \cdots ; 0, \phi}^{\longleftarrow m \longrightarrow})$$

$$
\begin{aligned}
&\Gamma^{(n)}_{\lambda}(k_1, a_1; \cdots k_n, a_n) \\
&= \sum_{m=0}^{\infty} \frac{\lambda^m}{m!} \Gamma^{(n+m)}(k_1, a_1; \cdots k_n, a_n; \overbrace{0, \phi; \cdots ; 0, \phi}^{\longleftarrow m \longrightarrow})
\end{aligned}
$$

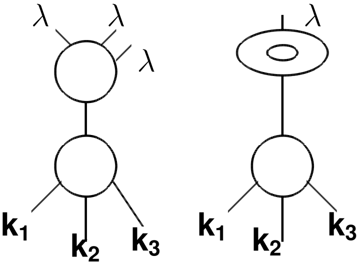
Determine λ by demanding vanishing of

$$
\begin{aligned}
\Gamma^{(1)}_{\lambda}(0, \phi) &= \sum_{m=0}^{\infty} \frac{\lambda^m}{m!} \Gamma^{(1+m)}(\overbrace{0, \phi; \cdots ; 0, \phi}^{\longleftarrow m+1 \longrightarrow}) \\
&= 4\,A\,\lambda^3 - 2\,C\,g^2 \lambda + \cdots
\end{aligned}
$$

– requires cancellation between different loop orders leading to

$$
\lambda = \sum_{n=0}^{\infty} A_n g^{2n+1}
$$

Problem: Individual contributions to $\Gamma_\lambda^{(n)}$ contain zero momentum internal massless propagators causing divergence.



Need infrared regulator at the intermediate stage of calculation.

Witten

After combining all the contributions at a given order in g we need to remove the regulator.

Results:

1. Physical quantities, as well as λ, have finite limit as the regulator is removed.

– not surprising since we are adjusting λ to cancel ϕ-tadpole at every order.

2. The shift λ depends on the choice of local coordinates.

3. However the physical quantities like S-matrix elements are independent of the choice of local coordinates.

Everything is satisfactory

Conclusion

Even if a full fledged string field theory is not available, a pragmatic definition of off-shell amplitudes in string theory serves useful purpose.

1. Mass renormalization.

2. Perturbative vacuum shift.

There could be other applications in the future.